**Centralizers on Prime and Semiprime Γ-rings**

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**Abstract**

 In this paper, we will generalized some results related to centralizer concept on prime and semiprime Γ-rings of characteristic different from 2 .These results relating to some results concerning left centralizer on Γ-rings.

**Keywords:** Semiprime Γ-ring , Centralizers .

**تمركزات على الحلقات الاولية وشبه اولية من النمط كاما**

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**الخلاصة**

 في هذه البحث ، سوف نعمم بعض النتائج المتعلقة بمفهوم التمركز على الحلقات الاولية وشبه الاولية من النمط كاما التي ممثلها لا يساوي 2 هذه النتائج متعلقة مع بعض النتائج للتمركز الايسر على الحلقات من النط كاما.

1. **Introduction**

 Nobusawa in [1] presented the idea of a Γ-ring , the concept of Γ-ring is more general of the Ring Barnes in [2] the definition of the Γ-ring with less conditions . On the basis of these two definitions many researchers in pure mathematics have made working on Γ-ring sense Barnes and Nobusawa see [3-6] , which parallel results in the Ring theory, Barnes in [2] defined it as following : suppose N and Γ be an additive abelaine groups , if there exists a map from N$×$Γ$×$N to N, for all a, b, c $\in $N and γ, δ $\in $Γ satisfying the following conditions :

1. aγb $\in $ N .
2. (a+b)γc=aγc+bγc , a(γ+δ)b=aγb+aδb and aγ(b+c)=aγb+aγc
3. (aγb)δc=aγ(bδc) .

 Then N is called Γ-ring.

 Some preliminaries of Γ-rings was given by S.Kyuno [7] as following : "Let I be a non-zero subset of a Γ-ring N, then I is called a left (right) ideal , if I be an additive subgroup of N and NΓI$⊆$I (IΓN$⊆$I), if I be a left and right ideal then I is called an ideal of N . N is called 2-torsion free if 2a=0 obtain a=0 , a$\in $N . A Γ-ring N is said to be prime if aΓNΓb=(0) with a,b $\in $N , obtain a=0 or b=0 and it simeprime if aΓNΓa=(0) with a$\in $N, obtain a=0 . A Γ-ring N is called commutative if aγb=bγa, for all a,b $\in $Γ and γ$\in $Γ . The subset Z(N)={a$\in $N| aγb=bγa, for ـ

all a$\in $N and γ$\in $Γ } of a Γ-ring N is called center of N ". An additive mapping T:N$\rightarrow $N is called left (right) centralizer if T(aγb)=T(a)γb (T(aγb)=aγT(b)) for all a,b $\in $N and γ$\in $Γ, and T is called Jordan left (right) centralizer if T(aγa)=T(a)γa (T(aγa)=aγT(a)) for all a $\in $N and γ$\in $Γ. If T are both left and right centralizer then T is called centralizer . Also the element (aγb-bγa) $\in $ N is called the commutater of a and b with respect to γ which is denoted by [a,b]γ . In [8] S. Chakraboty and A.C. Paul show that if N is a Γ-ring for all a ,b, c $\in $N and γ,δ$\in $Γ , then

1. [a+b,c]γ=[a,c]γ+[b,c]γ
2. [a,b+c]γ=[a,b]γ+[a,c]γ
3. [aδb,c]γ =aδ[b,c]γ +[a,c]γδ b+aδcγb-aγcδb

 In this paper we assume that aδcγb=aγcδb which represent by ($\*$) then from equation (iii) , we get [aδb,c]γ =aδ[b,c]γ +[a,c]γδ b .In [9] M.F. Hoque and A.C.Paul proved that if N be a semiprime Γ-ring of characteristic different from 2 with condition($\*$) then the Jordan left centralizer is left centralizer on N and they proved if N be a semi-prime Γ-ring of characteristic different from 2 with condition($\*$) then the Jordan centralizer is a centralizer on N In this paper we show that if N be a 2-torsion free semi-prime Γ-ring with condition ($\*$), I be an ideal of N and T:N$⟶$N be a Jordan left centralizer on I, then N contains a central ideal ideal. and if is a prime Γ-ring of characteristic different from 2 with the same above hypotheses then N is commutative Γ-ring.

**2. The Results**

 To prove the main result , we begin with some lemmas:

**Lemma 2.1. [9]** Suppose N be a semi-prime Γ-ring, if a,b, $\in $N and γ,δ$\in $Γ, such that aγcδb=0 for all c$\in $N , then aγb=bγa=0.

**Lemma 2.2. [9]** Suppose N be a semi-prime Γ-ring and F:N$×$N$\rightarrow $N , bi-additive mapping. If F(a,b)γcδF(a,b)=0 for all a,b,c $\in $N and γ,δ$\in $Γ , then F(a,b)γcδF(u,v)=0 , a,b,c,u,v $\in $N.

**Lemma 2.3.[9]** Suppose N be a semi-prime Γ-ring with condition ($\*$) and x be a fixed element in N. If xδ[a,b]γ=0, for all a, b $\in $N and δ , γ$ \in $Γ, then N have central ideal I , such that x$\in $I$⊂$N.

**Theorem 2.4.** Suppose N be a 2-torsion free semi-prime Γ-ring with condition ($\*$),I be an ideal of N and T:N$⟶$N be a Jordan left centralizer on I , then N contains a central ideal.

**Proof:**

for all a$\in $I and γ$\in $Γ, then T(aγa)=T(a)γa (1)

 ifwe replace a by (a+b) in (1), we get for for all γ$\in $Γ T(aγb+bγa)=T(a)γb+T(b)γa (2)

in (2) replace b by aγb+bγa and γ by δ , for all b$\in $I and δ$\in $Γ , we obtain

T(aδ(aγb+bγa)+(aγb+bγa)δa) =T(a)δaγb+T(a)δbγa+T(a)δaγb+T(b)γaδa= T(a)δaγb+2T(a)δbγa+ T(b)γaδa (3)

Calculate (3) By deferent way then

T(aδ(aγb+bγa)+(aγb+bγa)δa) =T(aδaγb+bγaδa)+2T(aδbγa) = T(a)δaγb+ T(b)γaδa+2T(aδbγa) (4)

By subtracting Eq.3 from Eq. 4 resulting in

T(aδbγa)=T(a)δbγa (5)

In Eq. 5 replace a by a+c for all c$\in $I , we obtain

T((a+c)δbγ(a+c))=T((a+c))δbγ(a+c) =(T(a)+T(c)) δbγ(a+c)

=T(a) δbγ(a+c)+T(c)δbγ(a+c)= T(a) δbγa+ T(a) δbγc+ T(c)δbγa+ T(c)δbγc (i)

And we can show that

T((a+c)δbγ(a+c))=T(aδbγa+aδbγc+cδbγa+cδbγc)

T((a+c)δbγ(a+c))=T(a)δbγa+T(aδbγc+cδbγa)+T(c)δbγc (ii)

From (i) and (ii) , we get

 T(aδbγc+cδbγa)= T(a)δbγa+ T(c)δbγa (6)

Suppose that J=T(aγbδcαbβa+bγaδcαaβb) , for all a,b,c$\in $I and γ,δ,α,β$\in Γ$ , and calculate J by two deferent way as follows:

By using Eq. 5 resulting in

J=T(a)γbδcαbβa+T(b)γaδcαaβb (7)

And by Eq. 6 resulting in

 J=T(aγb)δcαbβa+T(bγa)δcαaβb (8)

By subtracting Eq.8 from Eq. 7 resulting in

0=(T(aγb)$-$ T(a)γb)δcαbβa + (T(bγa)$-$ T(b)γa)δcαaβb (9)

Suppose the following bi-additive map F(a,b)= T(aγb)$-$ T(a)γb, and we can show that F(a,b)=$ -$F(b,a). So Eq. 9 become 0= F(a,b) δcαbβa$+$ F(b,a) δcαaβb and

F(a,b) δcα[a,b]β=0 , using Lemma 2.2**.** we have F(a,b) δcα[u,v]β=0 in this equation fix some a,b $\in $I and let F= F(a,b) , then Fδcα[u,v]β=0 , for all u,v $\in $I that mean by lemma 2.1.Fδ [u,v]β=0 and by lemma 2.3. we get N have central ideal.

 From Theorem 2.4. and using some lemmas in Γ-rings corresponding to lemmas in the Rings Theory we can prove some results.

**Lemma 2.5.** Suppose N be a semi-prime Γ-ringwith condition ($\*$)and I be a left ideal of N then Z(I)$⊆$Z(N) .

**Proof :** if a$\in $Z(I) , since I is left ideal then xγa$\in $I , for all x$\in $N and γ$\in $Γ also 0=[a,xγa]γ , that lead to

0=[a,x]γγa (1)

By Eq. 1 for all y$\in $N , then

0=[a,x]γγaδy , for all δ$\in $Γ (2)

In Eq. 1 replace x by xδy we obtain

0=[a,x]γδyγa(3)

From Eq. 2 and Eq. 3 we obtain

 0=[a,x]γδ[a,y]γ (4)

In Eq.4 replace y by yαx , for all α$\in $Γ , we get

0=[a,x]γδyα[a,x]γ. By the sime-primeness of N ,[a,x]γ**=**0.

**Lemma 2.6.** Suppose N be a semi-prime Γ-ring with condition ($\*$)and let I be a non-zero left ideal of N . if I be a commutative as a Γ-ring, then I$⊆$Z(N), if in addition N is a prime Γ-ring, then N must be commutative.

**Proof:**

By Lemma 2.5. , we get our first desired.

I$⊆$Z(I)$⊆$Z(N) (1)

For all x$\in $N and a$\in $I , then xΓa$⊆$I and by Eq. 1, xΓa$⊆$Z(N) , also for all α$\in $Γ and y$\in $N

Then (0)=[y, xΓa ]α=[y,x]αΓa , in general

[y,x]αΓI=(0) (2)

Since I is left idea and by Eq. 2 , then [y,x]αΓNΓI=(0), but N is prime Γ-ring and I non-zero ideal that means [y,x]α=0 , for all x,y$\in $N and α$\in $Γ.

**Corollary 2.7.** Suppose N be a prime Γ-ring of characteristic different from 2, with condition ($\*$)**,** I be an ideal of N and T:N$⟶$N be a Jordan left centralizer on I , then N is commutative.

**Proof:** by Theorem 2.4. , then N contains a central ideal and by Lemma 2.6. then N is commutative Γ-ring.

**Corollary 2.8.** Suppose N be a prime Γ-ring of characteristic different from 2, with condition ($\*$)**,** if T:N$⟶$N be a left centralizer on N, then T is centralizer on N .

**Proof:** for all a$\in $N and γ$\in $Γ , T(aγa)=T(a)γa , by corollary 2.7. then N is commutative, for all a,b$\in $N and γ$\in $Γ ,T(aγb)=T(bγa)=T(b)γa=aγT(b) , that is T also right centralizer.

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