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On (m,n) (U,R) – Centralizers

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Abstract
Let $m \geq 1, n \geq 1$ be fixed integers and let R be a prime ring with $char(R) \neq 2$ and $(m+n)$. Let T be a $(m,n)(U,R)$-Centralizer where U is a Jordan ideal of R and $T(R) \subseteq Z(R)$ where $Z(R)$ is the center of R ,then T is (U,R)- Centralizer.

Keywords: Prime ring, Semiprime ring, Left (right) Centralizer, Left(right) Jordan Centalizer, (m,n)-Jordan Centralizer, Left(righ)(m,n)(U,R)-Centralizer, $(m,n)(U,R)$-Centalizer.

حول التمركزات– (U،R) من النوع (m،n)

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الخلاصة
ليكن m ≥ 1 و n ≥ 1 عددان صحيحان و R هي حلقة اوليه حيث ان $(m+n)$,$char(R) \neq 2$.
ليكن T تمركز– (U,R) من النوع (m,n) عندما U مثالي جوردان في R و $T(R) \subseteq Z(R)$ حيث ان $Z(R)$ هو مركز الحلقة R ، فان T تمركز– (U,R) .

Introduction

Throughout, R will represent an associative ring with center $Z(R)$. The characteristic of R is the smallest positive integer n such that $na=0$ for all $a \epsilon R$ .As usual the commutator as follows $xy-yx$ will be denoted by $[x,y]$.We shall use the commutator identities $[xy,z]=[x,z]y+x[y,z]$ and $[x,yz]=[x,y]z+y[x,z]$, for all $x,y,z \epsilon R$ .Recall that a ring R is prim if for all $a,b \epsilon R$, $aRb=\{0\}$ implies that either $a=0$ or $b=0$.Aring R is called semiprime if $aRa=\{0\}$ implies $a=0$ .An additive subgroup U of R is said to be a Jordan ideal of R if $ur+ru \epsilon U$,for all $u \epsilon U$ and $r \epsilon R$ .An additive mapping $D:R \rightarrow R$ is called a derivation if $D(xy)=D(x)y+xD(y)$holds for all $x,y \epsilon R$ and is called a Jordan derivation if $D(x^2) =D(x)x+xD(x)$ is fulfilled for all $x \epsilon R$. One can easily prove that every derivation is a Jordan derivation, but converse is in general not true. A classical result due to Herstein [1, Theorem 3.3] asserts that a Jordan derivation on prime ring of characteristic different from two is a derivation. A brief proof of Herstienś result can be found in [2],this result was extended to a characteristic different from two semi prime rings by Cusack [3] (see[2] for an alternative proof).An additive mapping $T:R \rightarrow R$ is called a left (right) Centralizer if $T(xy)= T(x)y$ ($T(xy)=xT(y)$) holds for all pairs $x,y \epsilon R$ see[4].If R has the identity element,then $T:R \rightarrow R$ is left Centralizer if and only if T is of the form $T(x)=ax$ for all $x \epsilon R$ where $a \epsilon R$ is a fixed element .An additive mapping $T:R \rightarrow R$ is called a left (right) Jordan Centralizer if $T(x^2)=T(x)x$ ($T(x^2)=xT(x)$) holds for all $x \epsilon R$ see[5] .We call an additive mapping $T:R \rightarrow R$ *a* two sided Centralizer (a two-sided Jordan Centralizer) if T is both a left and right Centralizer (a left and right Jordan Centralizer).In [6] Zalar has proved that any left (right) Jordan Centralizer on a characteristic different from two semi prime ring is a left (right) Centralizer. In[7]Vukman defined an (m,n)-Jordan Centralizer as follows,an additive mapping $T:R \rightarrow R$ is called (m,n)-Jordan Centralizer if $(m+n)T(x^2) =$

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$mT(x)x + nxT(x)$ holds for all $x\epsilon R$. In the case when $m = n = 1$ we have the relation $2T(x^2) = T(x)x + xT(x)$ for all $x\epsilon R$, Vukman [5] has proved that every additive mapping $T:R\to R$, where R is a 2-torsion free semiprime ring ,satisfying the relation above is a two-sided Centralizer.Fošner [8]defined an generalized (m,n)-Jordan Centralizers as follows,an additive mapping $T:R\to R$ is called generalized (m,n)-Jordan Centralizer if there exists an (m,n)-Jordan centralizer $T_o:R\to R$ such that $(m+n)T(x^2) = mT(x)x + nxT_o(x)$ holds for all $x\epsilon R$.

In the following we define (m,n)(U,R)-centralizer where U is a Jordan ideal of R. Thes definition has no related with (m,n)-jordan Centralizer.
The following example illustrates the above definition

DEFINITION

Let $m\geq 1$, $n\geq 1$ be fixed integers and let R an arbitrary ring .An additive mapping $(m+n)T(ur+ru) = 2(m+n)T(r)u$ **(**$(m+n)T(ur+ru) = 2(m+n)uT(r)$**)** (*)
holds for all $r\epsilon R, u\epsilon U$.
We call an additive mapping $T:R\to R$ is $(m,n)(U,R)$-Centralizer if T is both a left and right $(m,n)(U,R)$-Centralizer for all $r\epsilon R$, $u\epsilon U$.
The following example illustrates the above definition.

Example 1:

Let R={$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$; a,b ϵ S }be the ring of 2×2 matrices over a commutative ring S of characteristic two.
Let U ={$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$; a,b ϵ S }. It is clear that U is a Jordan ideal of R.
Define d:R→R by T ($\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$) = $\begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$, it is clear that Tis left (m,n)(U,R)-Centralizer.

Example 2:

Let R={$\begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$; a,b ϵ S }be the ring of 2×2 matrices over a commutative ring S of characteristic two.
Let U ={$\begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$; a,b ϵ S }. It is clear that U is a Jordan ideal of R.
Define d:R→R by T ($\begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$) = $\begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$, it is clear that Tis Right (m,n)(U,R)-Centralizer.

Example 3:

Let R={$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$; a,b ϵ S }be the ring of 2×2 matrices over a commutative ring S of characteristic two.
Let U ={$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$; a,b ϵ S }. It is clear that U is a Jordan ideal of R.
Define d:R→R by T ($\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$) = $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, it is clear that Tis (m,n)(U,R)-Centralizer.
In this paper R will denote an associative prime ring of characteristic defferent from 2 and $(m+n)$,U will denote a Jordan ideal of R and T is $(m,n)(U,R)$-Centralizer.
Put $(U)^r = T(ur) - uT(r)$ for all $r\epsilon R$ *and* $u\epsilon U$.
And $(r)^u = T(ru) - uT(r)$ for all $r\epsilon R$ *and* $u\epsilon U$.
So we can show by using equation (*) that :$(u)^r = -(r)^u$.
In this paper we went to proved that an additive mapping $T:R\to R$, where R is a characteristic different from two and $(m+n)$ prim ring, satisfying the relation (*) and $T(R) \subseteq Z(R)$, where $Z(R)$ is center of R, then T is (U,R)-Centralizer.

Theorem 1 .Let R be prime ring of characteristic different from 2 and $(m+n)$,U be a Jordan ideal of R and T be $(m,n)(U,R)$-Centralizer and $T(R) \subseteq Z(R)$ where $Z(R)$ is a center of R ,then T is (U,R)-Centralizer.
In the proof of the above theorem we will need the next lemmas.

Lemma 1. For all $r\epsilon R$ and $u\epsilon U$,
$T(uru)=uT(r)u$.

Proof. In(*),replace r by $u2r+2ru$, then we get
$(m+n)T(u(u2r+2ru)+(u2r+2ru)u)=2(m+n)uT(u2r+2ru)$
Since the characteristic of R is different from $(m+n)$ we get

$T(u(u2r+2ru)+(u2r+2ru)u)=2uT(u2r+2ru)$
$T((u^2 2r+2uru)+(2uru+2ru^2))=4uT(ur)+4uT(ru)$
$2T(u^2r)+4T(uru)+2T(ru^2)=8u^2T(r)$
$4u^2T(r)+4T(uru)=8u^2T(r)$
$4T(uru)=4u^2T(r)$
Since the chcaracteristic of R is different from 2 and $T(R) \subseteq Z(R)$ we get
$T(uru)=uT(r)u$
For all $r \in R$ and $u \in U$.This completes the proof.
Lemma 2. For all $v \in U$ and $r \in R$,
$[v^2,r](v^2)^r=0$ and $(v^2)^r[v^2,r]=0$.
Proof.Since $(m+n)T(ur+ru)=2(m+n)uT(r)$,for all $u \in U$,and $r \in R$, and characteristic of R is different from $(m+n)$ we get
$T(ur+ru)=2uT(r)$
$2T(u^2)=2uT(u)$, for all $u \in U$ then by[9, Theorem 1.1.13],T is Centralizer on U .
i.e $(T(uv)-T(u)v)=0$, for all $u,v \in U$.
i.e $(uv-vu)(T(uv)-T(u)v)=0$.
Replace u by $2vr+2rv$, for all $r \in R$, we get
$((2vr+2rv)v-v(2vr+2rv))(T((2vr+2rv)v)-T(2vr+2rv)v)=0$.
$(2vrv+2rv^2-2v^2r-2vrv)(2T(vrv)+2T(rv^2)-4vT(r)v)=0$.
By using lemma 1,we have
$(2rv^2-2v^2r)(2T(rv^2)-2vT(r)v)=0$.
By using the relation $(u)^r = -(r)^u$ for all $u \in U$ and $r \in R$, we get
$(2v^2r-2rv^2)(2T(v^2r)-2v^2T(r))=0$
i.e $[v^2,r](v^2)^r=0$ for all $v \in U$ and $r \in R$.
similarly,we can prove $(v^2)^r[v^2,r]=0$.
Corollary.
(i\) $[u^2,r](u^2)^s + [u^2,s](u^2)^r=0$, for all $u \in U$ and $r,s \in R$.
(ii) $(u^2)^r[u^2,r] + (u^2)^r[u^2,s]=0$, for all $u \in U$ and $r,s \in R$.
Proof of theorem 1.
Replace r by ur in equation (*) , then we get
$(m+n)T(uur+uru)=2(m+n)uT(ur)$.
Since the characteristic of R is different from $(m+n)$ we get
$T(uur+uru)=2uT(ur)$. (1)
So,
$T(uur+uru)=T(u^2r+uru)$
$=T(u^2r)+uT(r)u$. (2)
But, by [9,Lemma (2.2.6)]
$(u^2)^r=0=T(u^2r)-u^2T(r)$,for all $u \in U$ and $r \in R$,
$T(u^2r)=u^2T(r)$
so,equation (2) becomes
$T(u^2r+uru)=u^2T(r)+uT(r)u$. (3)
By comparing equation (1) and (3), we get
$u^2T(r)+uT(r)u=uT(ur)+T(ur)u$.
$u(uT(r)-T(ur))=(T(ur)-uT(r))u$.
$u(T(ur)-uT(r))=-(T(ur)-uT(r))u$.
$u(u)^r+(u)^ru=0$, for all $u \in U$ and $r \in R$
lineaizing the a bove equation on u,we get
$u(v)^r+v(u)^r+(u)^rv+(v)^ru=0$.
Replace v by $2v^2$ and use[9, Lemma (2.2.6)],we get
$2v^2(u)^r+2(u)^rv^2=0$.
i.e
$v^2(u)^r+(u)^rv^2=0$, for all $u,v \in U, r \in R$.
And so by [9, Lemma (2.2.3)] we get
$(u)^r=0$, for all $u \in U$ and $r \in R$.
i.e

$T(ur)=uT(r)$, for all $u\epsilon U$ and $r\epsilon R$.

CONJECTURE

Let $m \geq 1$, $n \geq 1$ be some fixed integer s, let R be a prime ring with suitable torsion restrictions, and $T : R \rightarrow R$ be a $(m,n)(U,R)$-Centralizer. Then T is (U,R)-Centralizer.

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