Taher and Abdulla *Iraqi Journal of Science, 2024, Vol. 65, No. 10, pp: 5849-5863*
DOI: 10.24996/ijs.2024.65.10.41

ISSN: 0067-

Determining the Relationship between the Crescent Visibility Factors and the Coordinates of the Sun and Moon

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Received: 7/3/2023 Accepted: 1/9/2023 Published: 30/10/2024

Abstract

The study of the relationship between the coordinates of the sun and the moon with the crescent visibility factors has not been previously treated in a detailed and accurate way in research and previous studies, despite its religious importance. Accordingly, this paper aims to study the relationship between the crescent visibility factors (age, lag time, elongation (ARCL), arc of vision or relative altitude (ARCV), relative azimuth (DAZ), and crescent width (W), with coordinates of the sun and the moon), and how it varies during the day of the crescent's observation. In this paper, Matlab programs were designed to calculate the ecliptic sun and moon coordinates (λ, β) and in the presence of all perturbation impacts (planets), then convert these coordinates to the equatorial (α, δ) and horizontal coordinates (A, a). The results were compared with other programs in this field and references such as the ephemeris, accurate times, and astronomical programs. The variation of the sun and moon coordinates (ecliptic, equatorial, and horizontal) was studied with time, and the relationship of those coordinates was explained with visibility crescent factors during the day of the crescent's observation. Finally, the relationship of those factors with each other was studied. The results indicated that there was a relationship between the sun and the moon coordinates with some factors of crescent visibility in critical and standard observations, where the critical observations of the moon (age, lag time, elongation) were increased when the sun and the moon coordinates approach from two equinox points (spring and autumn), while the other factors, such as the relative azimuth and relative altitude, were independent on the change of the coordinates through the year. Also, a relationship was found between the visibility factors with each other, this led to a direct relationship between increasing the elongation with the crescent width. The values of the relative altitude were also increased with the increase in the lag time. Lastly, there was a direct relationship between the increase in the relative azimuth and the relative altitude.

Keywords: Visibility factors, coordinates of the sun and the moon, sun-moon elongation, orbit mechanics

تحديد العلاقة بين عوامل رؤية الهلال مع احداثيات الشمس والقمر

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الخلاصة:

ان دراسة العلاقة بين احداثيات الشمس والقمر وعوامل رؤية الهلال لم تعالج من قبل الدراسات والبحوث السابقة بشكل مفصل ودقيق على الرغم من اهمية مسألة رؤية الهلال الدينية لذلك يهدف هذا البحث الى التركيز على دراسة العلاقة بين عوامل رؤية الهلال(العمر(Age)، مدة المكث (Lag)، الاستطالة (ARCL)، فرق السمت (DAZ)، فرق الارتفاع(ARCV)، سمك الهلال(W)) مع احداثيات الشمس والقمر وكيفية تغيرها في يوم مراقبة الهلال . تم إعداد برنامج(ماتلاب) لحساب الاحداثيات البروجية (λ ,β) للشمس والقمر بدقة عالية مع وجود جميع الاضطرابات المؤثرة(مثل الكواكب), ثم تحويل هذه الاحداثيات الى الاحداثيات الاستوائية (α،δ) وألافقية (A,a) . التأكد من صحة النتائج بمقارنتها بالبرامج والمراجع المعتمدة مثل التقويم الفلكي وبرنامج المواقيت الدقيقة وبرامج فلكية اخرى. التغيرفي احداثيات الشمس والقمر (البروجية، الاستوائية، الافقية) تمت دراستها مع الزمن ودراسة علاقة هذه الاحداثيات مع عوامل رؤية الهلال في يوم المراقبة . واخيراً علاقة هذه العوامل مع بعضها البعض .بينت النتائج الى ان هنالك علاقة بين احداثيات الشمس والقمر مع بعض عوامل رؤية الهلال في الارصاد الحرجة والقياسية حيث تزداد الارصاد الحرجة لعمر الهلال ومدة المكث والاستطالة مع اقتراب احداثيات الشمس والقمر من نقطتي الاعتدالين (الربيعي والخريفي) بينما لا تعتمد بقية العوامل مثل فرق السمت وفرق الارتفاع كثيراً على تغير الاحداثيات خلال السنة وايضاً تم ايجاد علاقة بين عوامل الرؤية مع بعضها البعض وهذا يقودنا الى ان هنالك علاقة طردية حيث بزيادة الاستطالة يزداد سمك الهلال وتزداد ايضا قيم فرق الارتفاع بزيادة مدة مكث الهلال واخيرا هنالك علاقة مباشرة وطردية بين زيادة فرق السمت وفرق الارتفاع .

1. Introduction

The relationship between the coordinates of the sun and the moon and crescent visibility factors was very important in determining the form and validity of the criteria for crescent sighting[1]. Unfortunately, the Arab and international libraries are devoid of research and studies sufficient to study the subject in detail and accuracy, despite its religious importance. The importance of the subject lies in the different criteria for visibility of the crescent at different ages, as no criterion has been agreed upon to date because the criteria depend on the coordinates of the sun and the moon in special cases during observation time.

The coordinates of the moon are related to its rotation around the Earth, and both the moon and the Earth’s rotation around the sun [2- 4].
The moon appears to be the most moving celestial star and changes in its illumination in a striking way, as it does not follow a path that will return to it in the following month or the following year[2][5][6][7]. Compared with the movement of the stars, its movement is complex and subject to the laws of mechanics of multiple bodies due to the effects of the Earth’s attraction to it, and the effects of the sun and the rest of the multiple planets, and the change in distance between the Earth and the sun, etc. Some influences cause the moon’s orbit to be disturbed, which requires making several corrections to obtain the exact location of the moon. [8-12].

In this subject, the relationship between the sun and the moon coordinates was discussed, as their relationship with the observer's position on the surface of the Earth, through calculating high-precision programs. This paper, tried to find the relationship between the coordinates of the moon and the sun with visibility factors of a lunar crescent.

2. The System (Earth-Moon) Orbits around the Sun

From a considerable distance, a celestial observer would not be able to see the moon spiraling around the Earth in space. It is viewed in orbit around the sun, just like the Earth, but due to the Earth's influence, the moon’s orbit wiggles a little bit as the Earth and moon’s relative positions vary, as shown in Figure (1). This is due to the moon's gravitational pull from the sun being significantly weaker than the Earth's. [13-16].

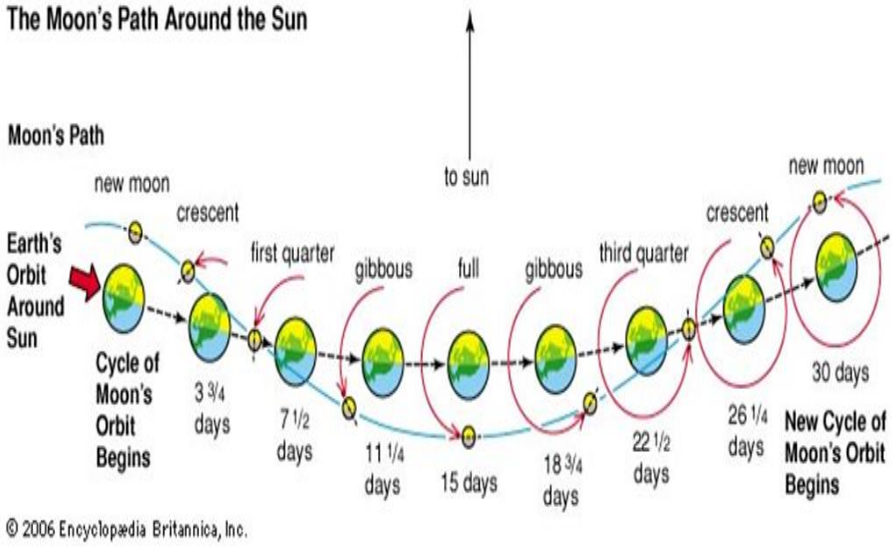

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Figure 1: The System (Moon-Earth) Orbit around the Sun [9]

3. Methodology

3.1. Celestial Coordinate Systems:

Spherical astronomy coordinates of the moon and the sun systems can be classified as the following[9][17]:

3.1.1 The Ecliptic System

The ecliptic longitude (λ) was calculated positively eastward in the main plane (the ecliptic) from 0° to 360°. And also measured the angular distance of an object along the ecliptic from the primary direction (vernal equinoxes)[13].

The ecliptic latitude (β) is a measurement of an object's angular distance from the ecliptic about either the north (+90) or south (90) ecliptic pole[18].

The longitude of the sun on the epoch J2000.0 is 280°.46, and the rate at which the Earth is going around the sun is 0°.985647359 per day (from equinox to equinox), the mean longitude of the sun is given by:[19][20][21]:

$$L = 280^\circ.46 + 0^\circ.985647359 * n \tag{1}$$

Where J2000 was the Julian Date for the Noon at Greenwich on 1st January 2000

And n: the mean angular velocity of the planet

The following formula can be used to determine the sun's true longitude:

$$\lambda s = L + 1^\circ.915262268 * \sin(M) + 0^\circ.020008486 * \sin(2M) + 0^\circ.000289389 * \sin(3M) \tag{2}$$

Where: M is the mean anomaly, which is represented by M= 357°.52911+ 0°.985600281*n

The sun's latitude (β_s) remains on the ecliptic and can be roughly regarded as zero [20].

Geocentric ecliptic coordinates representing the moon location are (λ, β).

The formula for the moon longitude (λ_m) is [19] [20]

$$\lambda m = 218.316 + 481267.881T2 + 6.29\,sin\,(134.9 + 477198.85\,T2) - 1.27\,sin\,(259.2 - 413335.38\,T2) + 0.66\,sin\,(235.7 + 890534.23\,T2) + 0.21\,sin\,(269.9 + 954397.7\,T2) - 0.19\,sin\,(357.5 + 35999.05\,T2) - 0.11\,sin\,(186.6 + 966404.05\,T2) \qquad (3)$$

Periodic terms and perpetration have been added to the moon's longitude and latitude [20][13]. The formula for the moon's ecliptic latitude (β_m) is [20]:

$$\beta m = 5.13 \sin(93.3 + 483202.03\,\mathrm{T2}) + 0.28 \sin(228.2\ 960400.87\,\mathrm{T2}) - 0.28 \sin(318.3 + 6003.18\,\mathrm{T2}) - 0.17 \sin(217.6 - 407332.2\,\mathrm{T2}) \qquad (4)$$

Where: T2 is the Julian number centuries from the epoch J2000.0 which can be calculated by equation T2= (JD – 2451545) / 36525 (5)

3.1.2. The Equatorial system

To convert ecliptic coordinates into equatorial coordinates (α, δ),
The following equations were used:

$$\tan\alpha = (sin\,\lambda cos\varepsilon - \tan\beta \sin\varepsilon) / \cos\lambda \qquad (6)$$

And

$$\sin\delta = \sin\beta\cos\varepsilon + cos\beta\sin\varepsilon\sin\lambda \qquad (7)$$

Where: (ε) is the obliquity angle[22].

3.1.3. The Horizontal (alt– azimuth) System

The definition of altitude (a), which is measured in degrees, is the angle measured along the vertical circle passing through the celestial object from the horizon[17][23].

Azimuth (A) is the angle between the vertical via the south point and the vertical through the object, measured in degrees westwards along the horizon from 0° to 360°, or in degrees eastwards or westwards from 0° to 180° along the horizon[17]. This simplest system is most closely associated with the sense of the observer being on a flat plane and in the middle of a large globe across which the heavenly bodies travel. The boundary between the Earth and space is known as the horizon. [24]. The horizontal system is divided into three parts:

1- Local horizon (geographic horizon): The line that may be seen separating the Earth from the sky is called the Earth-sky horizon and there could be mountains, buildings, and trees on the immediate horizon.
2- Celestial horizon: Astronomers use celestial horizons and calculations of where the Earth is about the rest of the sky. The imaginary horizontal plane that is constantly at a 90° angle from the observer's zenith is known as the astronomical horizon (the point directly above the observer) [25][26].
3- Sea-level horizons: The geographical horizon at sea level is known as the sea-level horizon.

It is possible to translate the equatorial coordinates (α, δ) into local horizontal coordinates (A, a) by using the following formula [26]:

$$\tan A = \sin H / (\cos H \sin\phi - \tan\delta\cos\phi) \qquad (8)$$

$$\sin a = \sin\phi\sin\delta + \cos\phi\ \cos\delta\cos H \qquad (9)$$

where (ϕ) is the observer's latitude.

3.2. Calculating the Parameters of Crescent Visibility

The local coordinates (altitude and azimuth) of the moon and the sun serve as the basis for the criterion for the visibility of the new crescent at the time of observation locally (after sunset on the day following conjunction)[27][28][29][30]. Below are some astronomical terms used in observing the crescent, some of which are shown in Figure (2).

1. Age of the Moon (**Age**): Interval time between conjunction and the time of observation, measured in hours[20][31] [32]

$$Age = T_s - T_C \quad (10)$$

Where T_s: Time of sunset or any relevant time observation
And T_C: Time of conjunction with the sun

2. Lag time of the moon (Lag) or (Makth time): The time interval between sunset ($T_{s.s}$) and moonset ($T_{m.s}$) is obtained by[20][31][33].

$$Lag = T_{m.\,s} - T_{s.s} \quad (11)$$

3. Altitude of the moon (Alt): The moon's angular separation above the local horizon, calculated by equation (9)[18].

4. Moon arc of light (ARCL): The sun's and the moon's angular separation (elongation) after sunset, measured in degrees and obtained by[20][31].

$$\cos(ARCL) = \cos(ARCV)\cos(DAZ) \quad (12)$$

5. Arc of vision (ARCV): The angular difference in altitude between the sun (a_s) and the moon (a_m) is obtained by[20][31].

$$ARCV = a_m - a_s \quad (13)$$

6. The moon relative azimuth (DAZ): between the sun (A_s) and the moon (A_m)[20][31].

$$DAZ = / A_s - A_m / \quad (14)$$

Where: A_s: Sun azimuth
A_m: Moon azimuth

7. Crescent width (W): The width of the moon's light region as measured along its diameter in arc minutes. The width of the moon crescent is directly proportional to the semi-diameter of the moon (SD) and the ARCL, then the crescent width (W) is given by the formula[34][35]:

$$W = SD\,(1 - \mathrm{Cos}\,(ARCL)) \quad (15)$$

Where: SD: moon's semi-diameter as determined by the relationship [34],[20][36]:

$$SD = 0.2725\,\text{Л}$$

(16)

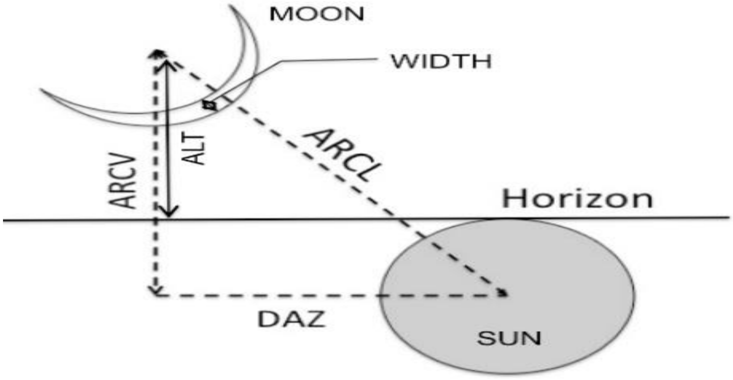

Figure 2: Basic geometric variables for crescent visibility prediction[37][29]

4. Results and Discussion

Matlab programs were designed to plot the result. Three types of results were obtained, the first was the sun and the moon coordinates, the second was crescent visibility factors with the coordinates system of the moon, and finally, determining the relationship between crescent visibility factors of each other, where an accurate program was used to compute the following:

A. Coordinates for the Moon and the Sun:

1. Ecliptic Coordinates (λ, β) of the Moon and the Sun with a Period:

A period of two years was taken to apply the results, as shown in Figure (3). This figure explained the relationship between the latitude of the moon as a function of time, this relationship was regular from (-5°) to (+5°) while the same longitude of the sun and the moon has a period from (0°-360°) as shown in figure (4):

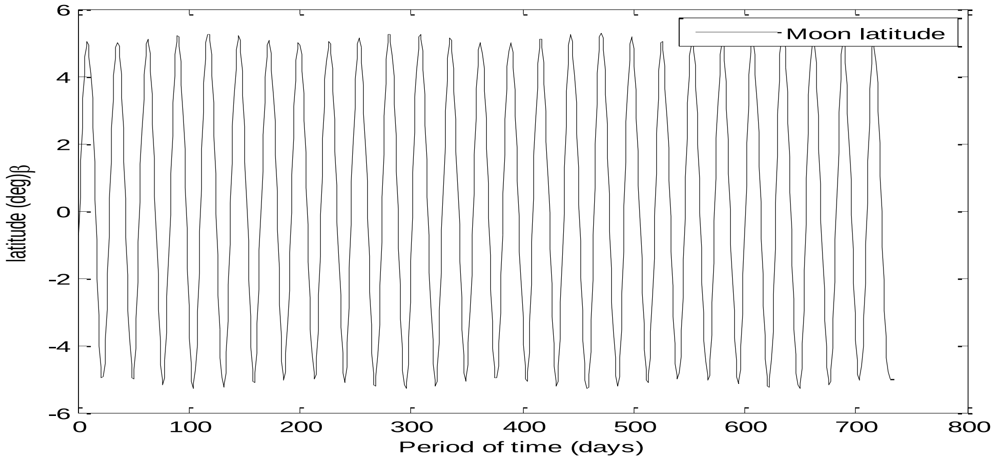

Figure 3: Represents the variation of ecliptic latitude(β) of the moon

Notice that the maximum longitude of the sun value is (359.8°) and the minimum value is (0.1°), as shown in Figure (4). The longitude of the sun was not raised at a constant rate of 360° for every year, but the ecliptic latitude was nearly to be 0° for all times because the sun's orbit was originally located on the ecliptic circle.

The maximum longitude value for the moon was 359.8° and the minimum value was 0.5°, they were repeated for every lunar month.

The lunar orbit has an inclination value of 5.9° to the ecliptic. Even the nodes of the orbit, which mark the point where the lunar orbit intersects the ecliptic, are not fixed and revolve around it in 18.6 years while oscillating by much as 1.67◦. The longitudes were measured positively to the west and this was the reason for the longitude of their central meridian being visible from the Earth. It was getting bigger as time passes.

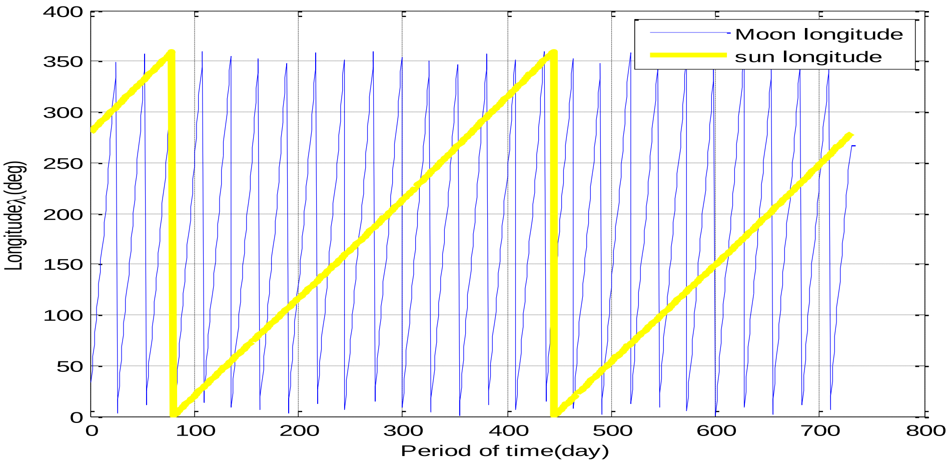

Figure 4: Represents the variation of ecliptic longitude (λ) of the moon and the sun with a period of time (two years)

2. The Sun's and the Moon's Equatorial Coordinates (α, δ).

In this part, also two years were taken to apply the results of the variation for the equatorial coordinates of the moon's and the sun's right ascension (α), then the relationship between the right ascension (α) with time was studied, as shown in Figure (5). It has a regular relationship with an increase and decrease from (0°-359°) with time.

The maximum right ascension value of the sun was 359.8° and the minimum value was 0.1°, they were repeated every 360^d and they were calculated from the vernal equinox.
Where: d: represented number of days.

The maximum right ascension value of the moon was 359.1° and the minimum value was 0.3° and they were repeated about every (27.5^d). This event occurred twice each draconic month.

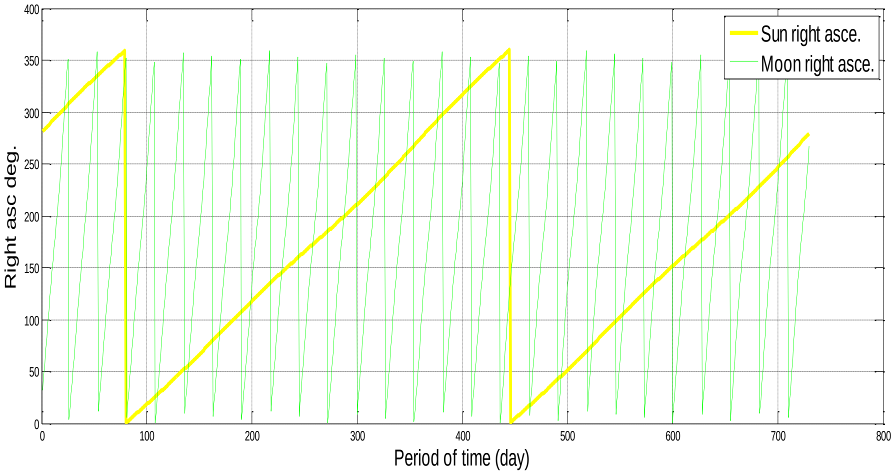

Figure 5: Represents the variation of equatorial coordinates (α) of the moon and the sun over a period of time (two years)

The variation of the equatorial coordinates of the moon and the sun declination (δ) and the study of the relationship between the declination (δ) with time, as shown in Figure (6).

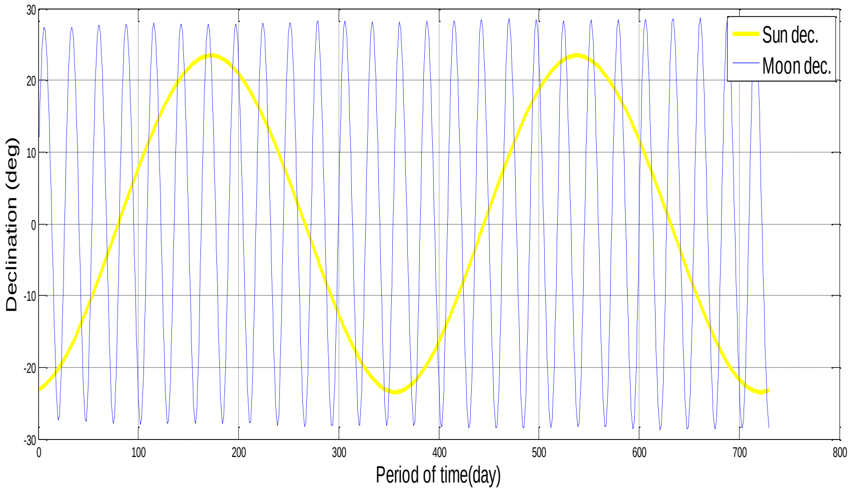

Figure 6: Represents the variation of equatorial coordinates (δ) of the moon and the sun over a period of time (two years)

Notice that the maximum declination value for the sun is (23.4◦) and the minimum value is (-23.4◦) and they were repeated every 365^d, while the moon's southern (or negative) declination ranges from roughly -18.2° to -28.7° at its extremes. Similar to the moon, the northern (or positive) declination extremes vary from approximately +18.2° to +28.7°, the scientific explanation for the above values is that the moon is inclined from the ecliptic (the path of the sun) about (5.9°) and the path of the sun is inclined at an angle (23.5°), that is, when adding or subtracting the value of the moon inclination with the angle of the path of the sun, we get the high and low values. The months are low and the values of the moon inclination are unstable due to the perpetrations in its orbit as a result of the influence of each of the sun, the Earth, and the planets on it. During the course of the draconic month, the moon's declination changes quickly along with it. There are two lunar standstills per the draconic month (27.3^d), which occur when the moon reaches its minimum or maximum declination. The moon steadily rotates 19° westward on its axis per year due to the gravitational pull of the sun and the nodes of the lunar orbit undergo one full rotation during a period of 18.6 years. The lunar standstill points and declination extremes of the moon gradually change from month to month as a result. The lunar nodes' orbital precession, which lasts 18.6 years, further modulates the monthly declination extremes (and lunar standstills).

3. The Sun's and the Moon's Horizontal Coordinates (A, a).

From the astronomical calendar, azimuth, or the angle of the sun at sunrise and sunset, it can be described as a divergence from the north in degrees (with east at 90◦). From 57◦ at the summer solstice to 122◦ at the winter solstice, it oscillated by around 66◦ over the year or east at ± 33◦)

There are seasonal variations in the sun's altitude at noon, which ranges from 71◦ on the summer solstice to 24◦ on the winter solstice.

Additionally, daylight hours range from 15.3^h in the summer to 9.1^h in the winter.

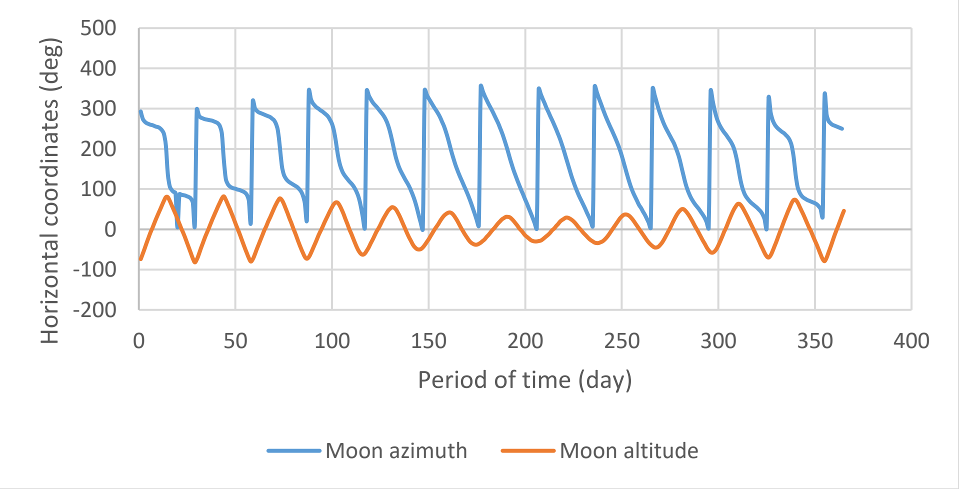

Figure 7: Represents the moon altitude and azimuth variation through one year

Every day, the moon location shifts in azimuth and altitude. Each month, the moon will transit the sky higher than the summer sun and lower than the winter sun, rising and setting in directions that are more northerly and more southerly than the solar extremes. The maximum moon altitude value is 82◦ and the minimum value is -82◦ and they were repeated every lunar month 29.5^d and azimuth maximum value is 356◦ and the minimum value is nearly 0◦ and they were repeated every day, as shown in Figure (7).

B. The Relationship between the Crescent Visibility Factors and the Coordinates System of the Moon:

We took 25 critical observations from a statistical study of astronomical tables for 100 years and previous studies such as from Schaefer [38] and Yallop[34] and which were updated from the Islamic Project website until up to date[39]. We obtained an inductive table(1), and by drawing this table with the relationships, the following figures with these critical observations were studied:

Table 1: Critical observations of crescent visibility[34][38][39]

No.	Moon age (h)	Moon Lag Time(min)	Elongation(ARCL)(deg)	Relative Azimuth(DAZ)(deg)	Relative Altitude(ARCV)(deg)	Crescent Width(deg)
1	13.06	39	7.7	2	7.5	0.0025
2	40.3	29	19.4	18.4	5.9	0.0141
3	15.016	39	7.6	0.6	7.6	0.0025
4	12.38	24	6	3.4	5	0.0014
5	25.1833	20	12.1	11.4	4.1	0.0056
6	14.1833	26	6.8	-2.7	6.3	0.0019
7	23	34	10.4	8.3	6.3	0.0041
8	14	34	7.6	3.1	7	0.0025
9	16.01	35	7.8	3.3	7	0.0022
10	13.066	34	7.5	2.8	7	0.0025
11	15.05	33	8.3	4.7	6.9	0.003
12	17.083	34	9.4	6.4	6.9	0.0036
13	17.083	35	8.3	3.8	7.4	0.0027
14	18.15	36	10.6	8.2	6.7	0.0044
15	16.13	36	8.2	3.6	7.4	0.0025
16	17.11	35	9.6	6.5	7	0.0038
17	17.066	36	10.3	7.6	6.9	0.0044
18	19.016	40	7.8	3.3	7.5	0.0025
19	13.11	41	7.9	1.6	7.8	0.0027
20	14.083	47	8	0.4	8	0.0027
21	14.01	43	8.1	0.3	8.1	0.0027
22	16.13	36	8.6	1.8	8.4	0.003
23	20.066	51	10.1	0.1	10.1	0.0038
24	22.066	46	10.7	0.3	10.7	0.0044
25	15.083	39	7.6	0.6	7.6	0.0025

1- The Coordinates' System of the Moon with Moon Age:

Figure (8) shows the earliest crescent visible with the optical aid of topocentric Moon Age was (12^h23^m) when the Moon's coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) were close to the vernal equinox point, and with the naked eye, the earliest crescent found was at (15^h 33^m) when the moon's coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) were close to the vernal autumnal point.

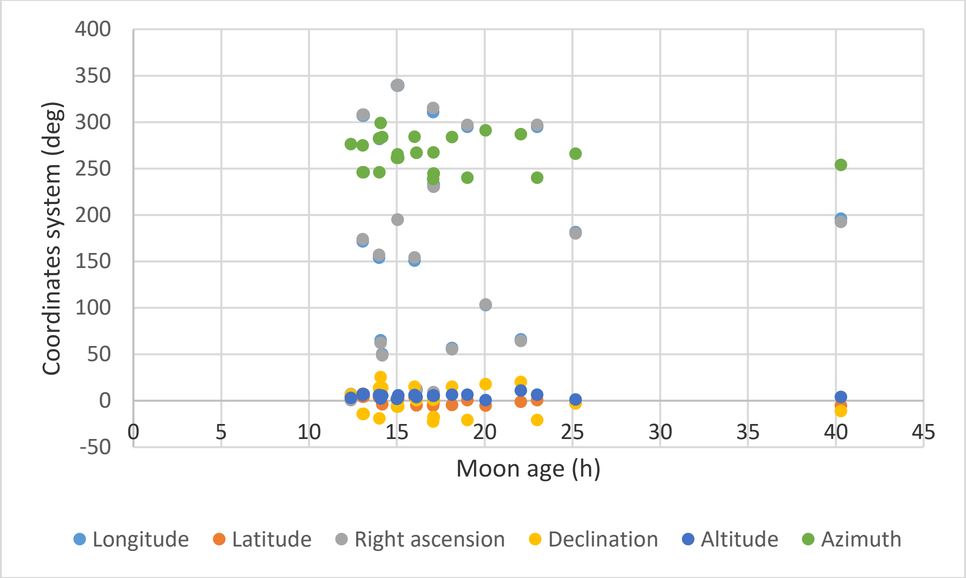

Figure 8: Represents the relationship between the moon's coordinates system with Moon Age

2- The Coordinates' System of the Moon with Moon Lag Time:

Figure (9) shows the Lag time, whether by telescope or with the naked eye, that reached the critical time when the moon coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) were almost close to the vernal and autumnal equinox points, with approximate values ranging between (±3°) from vernal points.

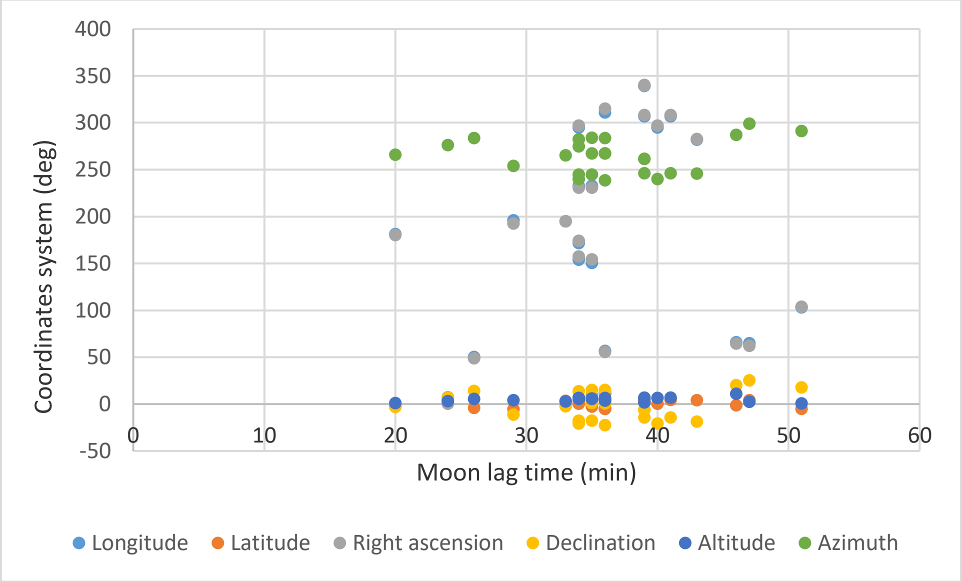

Figure 9: Represents the relationship between the moon's coordinates system with Moon

3- The Coordinates' System of the Moon with Elongation(ARCL):

The size and brightness of the lunar crescent depend on only one astronomical quantity (elongation (ARCL)) angular separation from the moon to the sun, in addition, elongation is one basic factor for the possibility of crescent visibility, where if its value exceeds 5° or more, then it is a good factor in visibility, noting that it is the least critical observation that has been observed, and it is about 6◦ using the telescope, as shown in Figure (10), knowing that the geometric condition for vision with the naked eye, which was called danjon limit is 7◦.

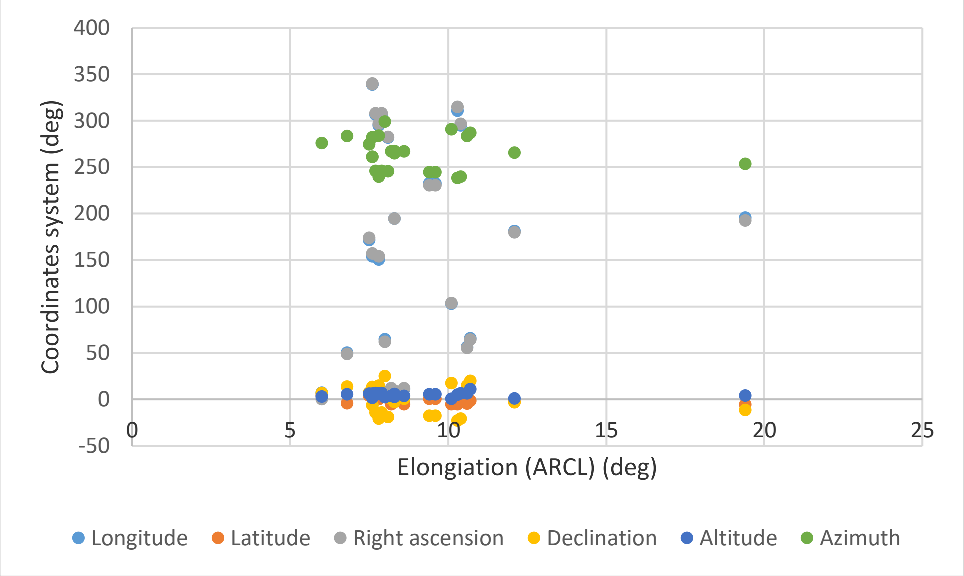

Figure 10: Represents the relationship between the moon's coordinates system with Elongation(ARCL)

4- The Coordinates' System of the Moon with Relative Azimuth(DAZ):

Figure (11) explains the most critical observations of relative azimuth(DAZ) ranged between two values (0◦-5◦) and when the moon's coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) were almost close to the vernal and autumnal equinox points.

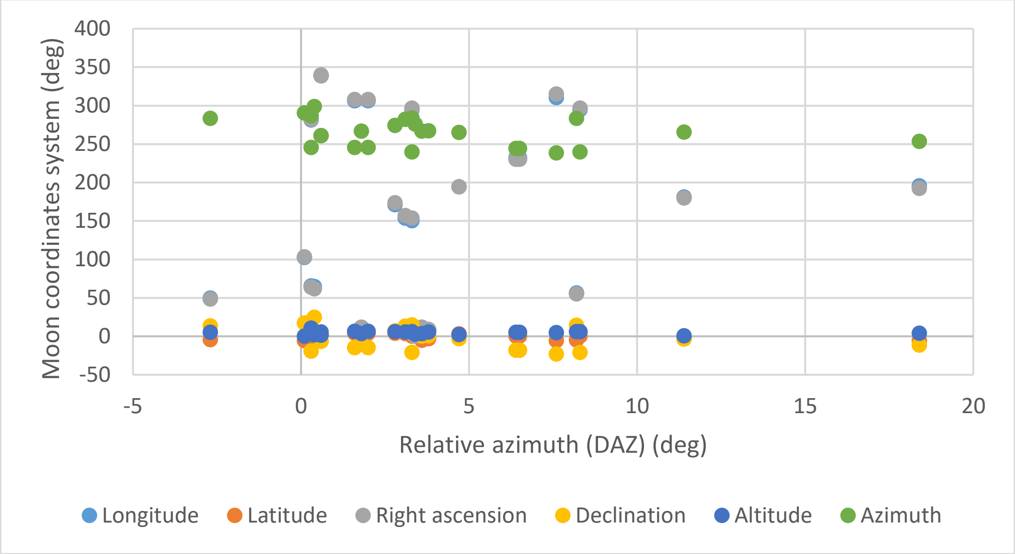

Figure 11: Represents the relationship between the moon's coordinates system with relative azimuth(DAZ)

5- The Coordinates' System of the Moon with Relative Altitude (ARCV):

Figure (12) explains the most critical observations of Relative Altitude (ARCV) ranged between two values (4◦-11◦) and when the moon's coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) were almost close to the vernal and autumnal equinox points.

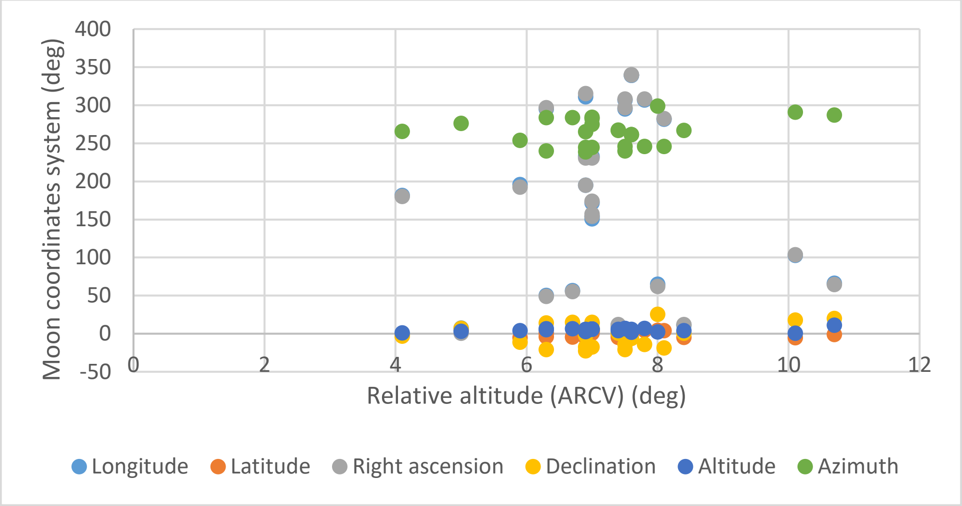

Figure 12: Represents the relationship between the moon's coordinates system with Relative altitude (ARCV)

6- The Coordinates' System of the Moon with Crescent Width (W):

The crescent width is associated with elongation, as we noticed that most of the increase in the crescent width occurred with an increase in elongation, and the moon's coordinates (ecliptic (λ, β), equatorial (α, δ), and horizontal (A, a) had no effect on this factor, as shown in Figure (13).

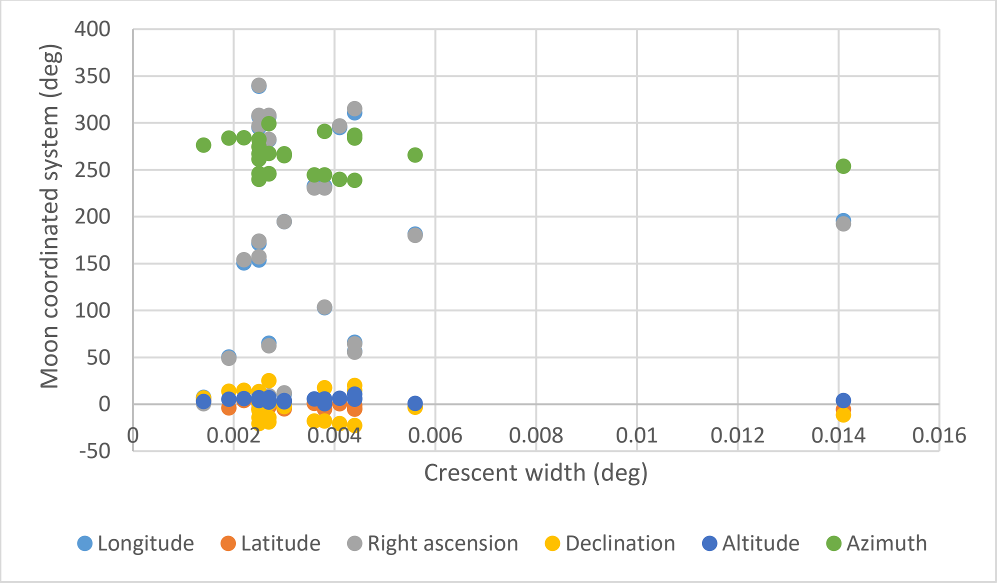

Figure 13: Represents the relationship between the moon's coordinates system with crescent width (W)

C. The Relationship between Each of the Crescent Visibility Factors:

1- Elongation (ARCL) with crescent width:

Figure (14) shows that the increase in elongation means the increase in the width of the crescent in a direct relationship, according to equation 15, and the surface brightness of the crescent was also rapidly increasing.

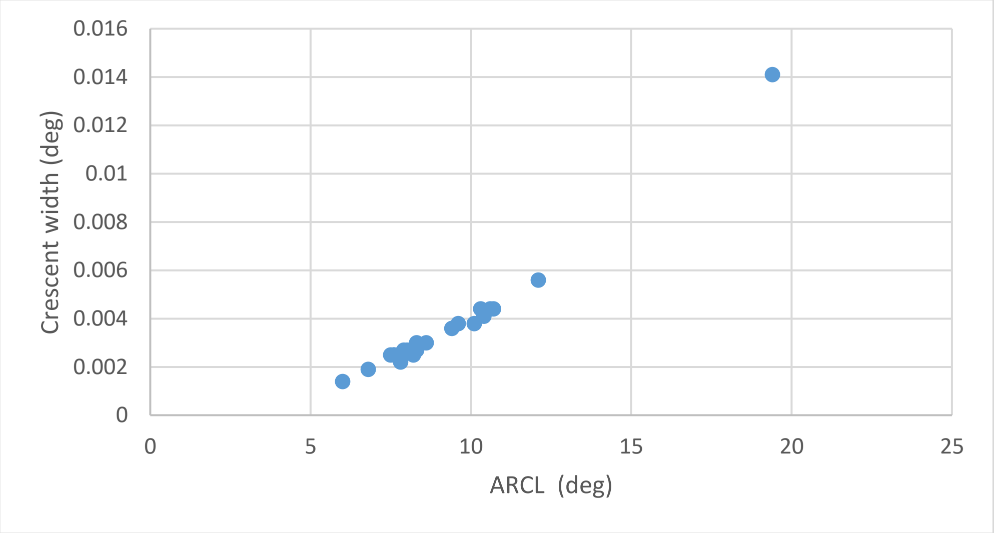

Figure 14: Represents the relationship between Elongation (ARCL) with crescent width

2- Lag Time with Relative Altitude (ARCV):

There is a direct relationship between the increase in the lag time with the relative altitude (ARCV). As we see in Figure (15), the good values of lag time are between (30^m-40^m) with a perfect value of relative altitude between (6°-8°). Also, the Figure shows that a large number of observations have a linear relationship. There is some dispersion around the best-fit line, which indicates that a lunar crescent with a particular lag time may or may not be visible depending on the ARCV value. We note that the longer the lag time, the higher the relative altitude (ARCV) of the lunar crescent and the darker the sky.

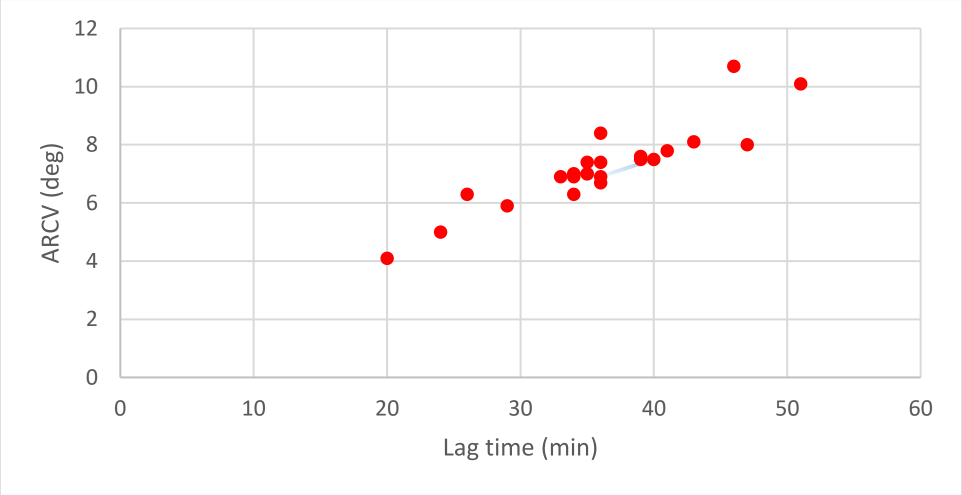

Figure 15: Represents the relationship between Lag time with Relative altitude (ARCV) crescent width

3- Relative Azimuth (DAZ) with Relative Altitude (ARCV):

Figure (16) shows that there is a direct relationship between the increase in the relative azimuth (DAZ) with the relative altitude (ARCV).

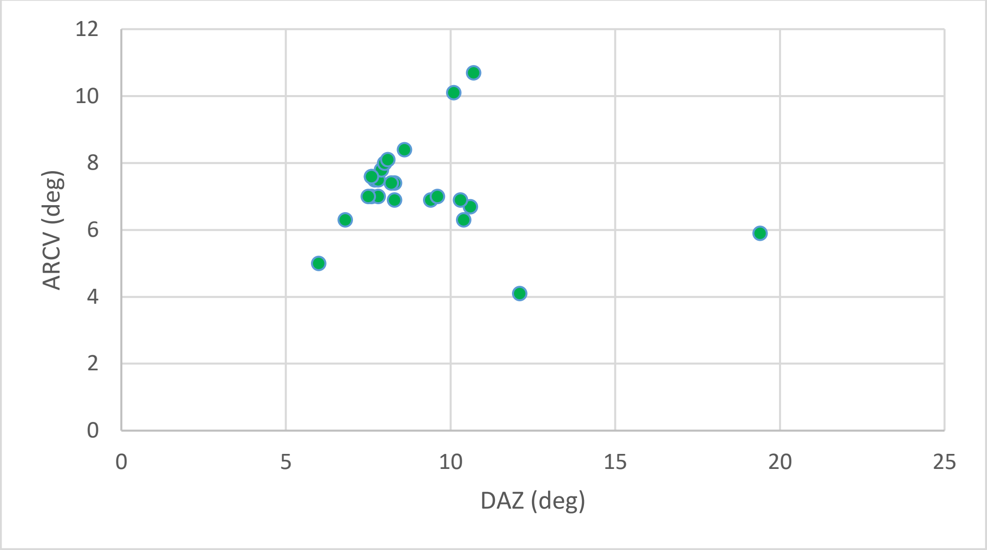

Figure 16: Represents the relationship between relative azimuth (DAZ) with relative altitude (ARCV)

5. Conclusion

The results were compared with[20][40] and showed a high convergence of about 95%, and calculations were made after taking (25) standard and critical observations, that there was a relationship between the coordinates of the sun and the moon, especially when approaching the months of spring and autumn equinoxes, where the critical values of the crescent visibility factors increased because the ecliptic sun made a relatively steep angle to the western horizon during these months. The extreme angle means that the moon's altitude would be greatest just after sunset. In addition, we concluded that the elongation factor is a more reliable parameter to use as a starting point in evaluating the visibility of the lunar crescent at any given date and time.

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