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The Trivial Extension of SAG-Algebra

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Abstract

The string almost gentle algebras (SAG-algebras) are studied in this paper. Generalizing the properties of string almost gentle algebras has also given. Let $A = \frac{KQ}{I}$ be the string almost gentle algebras, with quiver Q and admissible ideal I of algebra. We show that the radical of string almost gentle algebras A can be written as a direct sum of uniserial modules. After that the quiver Q^* is constructed from Q and an extension of the quiver Q is showed. Also, the quiver Q^* which is a quiver of trivial extension of string almost gentle algebras A has been studied. Consequently, the relations I^* of algebra $A^* = \frac{KQ^*}{I^*}$ have been described. As well as, we will show that the algebra $A^* = \frac{KQ^*}{I^*}$ is an extension to string almost gentle algebras. Furthermore, we describe the trivial extension of string almost gentle algebras A, we prove and that the trivial extension of A is isomorphic to the algebra A^*.

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Keywords: Admissible ideal, Almost gentle algebra, Maximal path, String almost gentle algebra, Trivial extension.

الامتداد البسيط للجبر من نوع SAG

رؤى يوسف جواد

معهد اعداد المدربين التقنين,الجامعه التقنيه الوسطى, بغداد, العراق

الخلاصة:

في هذا البحث تمت دراسة الجبر من نوع SAG. حيث تم إعطاء تعميم لخواص الجبر. ليكن الجبر $A = \frac{KQ}{I}$ مع جعبة Q والمثالي I المحتوى في KQ . وقد بيناّ ن أنه يمكن كتابة جذر السلسلة الجبرية A كمجموع مباشر للوحدات غير المتسلسلة. بعد ذلك ، تم إنشاء الجعبة Q^* من Q وقد برهننا انه امتداد للجعبة Q. تبين أن الجعبة Q^*هي جعبة للامتداد البسيط للجبر A . ان العلاقات الجديدة I^* في الجبر المبني $A^* = KQ^*/I^*$ تم وصفها في هذه الورقة البحثية. وقد برهننا وعرضنا ان الجبر $A^* = KQ^*/I^*$ الذي تم إنشاؤه في هذه الورقة هو امتداد لسلسلة الجبر A. واكثر من ذلك قمنا بوصف هذا الامتداد وبينا ان الامتداد البسيط للجبر A هو مشابه للجبر المبني A^*.

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1. Introduction

The classes of string almost gentle algebras named (SAG-algebra) are studied in this paper. An algebra $A = \frac{KQ}{I}$ is string almost algebra if it is special multiseiral, I is given by paths of length 2 and for all $v \in Q$, v is the source of at most two arrows, and v is the target of at most two arrows. The almost gentle algebras are generalized to classes of SAG-algebras.

The SAG-algebra introduced by Franco, Giraldo, and Rizzo in [1]. SAG-algebra arises from the intersection of two classes which are string algebra and almost gentle algebra. Green and Schroll defined the almost gentle algebra in [2]. The almost gentle algebras are monomial and special multiserial algebras. Also, they show that the trivial extension of an almost gentle algebra is a symmetric special multiserial algebra. The string algebra is special biserial algebra and I is generated by zero relations. The string algebras are generalized to gentle algebras [3] and moreover are generalized to SAG-algebra.

The global dimension of almost gentle algebra could be finite or infinite. While the string almost gentle algebras have an infinite global dimension which is proved in [1]. Algebra is backbone for all science like [4], [5], and [6].

We begin this paper by recalling the definition of string almost gentle algebras (SAG-algebras) and giving properties for SAG-algebras. Also, the $rad(A)$ can be written as a direct sum of uniserial modules which is proved in section two.

In section three, the new quiver Q^* is built from the quiver Q and also the relations I^* are defined. Where we show that the quiver Q^* is a quiver of trivial extension of SAG-algebra A. This construction is applied to example, where we describe in this example the new quiver Q^*. In addition, the dimension of $A^* = \frac{KQ^*}{I^*}$ has been calculated and given through section three.

In section four, the trivial extension of SAG-algebras is given, and we proved the main theorem

Theorem:
Let $A = \frac{KQ}{I}$ be a SAG-algebra and $T(A)$ be the trivial extension of A arising from $D(A)$. Then $A^* \cong T(A)$.

We fix some notation through this paper. Let $A = \frac{KQ}{I}$ be a finite dimensional algebra over an algebraically closed field. Our quiver Q is a finite and I is an admissible ideal, and we call Q_0 the set of vertices and Q_1 the set of arrows. All modules are finitely generated right module. We denoted the trivial path by e_v which corresponds to a vertex v. We set P to be the set of all paths in A that are not in I with lengths greater than and equal to 1.

2. String Almost Gentle Algebras

The string almost gentle algebra (SAG-algebra) is defined in this section. The class of almost gentle algebra is a special case to class of almost gentle algebra. Franco, Giraldo, and Rizzo established a new class named String almost gentle algebra, this class come from the intersection of two classes: string algebra and almost gentle algebra [1]. We recall a definition of gentle algebra from Scroll [7].

Definition 2.1: The algebra A is gentle if it is Morita equivalent to an algebra of the form KQ where

(S0) The relations I are generated by paths of length two.

(S1) For every $\alpha \in Q_1$, there is at most one element (arrow) $\beta \in Q_1$, such that $\alpha\beta \notin I$ and at most one element (arrow) $\gamma \in Q_1$, such that $\gamma\alpha \notin I$.

(S2) For every $\alpha \in Q_1$there is at most one element (arrow) $\beta \in Q_1$, such that $\alpha\beta \in I$, and at most one element (arrow) $\gamma \in Q_1$, such that $\gamma\alpha \in I$.

(S3) For every $v \in Q_0$,the vertex v is the source of at most two arrows, and is the target of at most two arrows.

The algebra $A = \frac{KQ}{I}$ is a special multiserial algebra [7] if it is Morita equivalent and satisfying the condition (S1).

Definition 2.2[2]: Algebra is almost gentle if it is Morita equivalent and satisfying (S0) and (S1).

The definition of string almost gentle algebra is used from Franco, Giraldo, and Rizzo [1] or simply we call it SAG-algebra.

Definition 2.3: We say that the algebra $A = \frac{KQ}{I}$is string almost gentle if the following conditions hold:

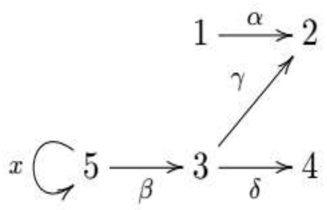
1. For all $v \in Q_0$, v is the source of at most two arrows. And for all $v \in Q_0$, v is the target of at most two arrows.
2. Given an $\alpha \in Q_1$, there is at most one element (arrow) $\beta \in Q_1$, where $s(\beta) = t(\alpha)$ and $\alpha\beta \notin I$.
3. Given an $\alpha \in Q_1$, there is at most one element (arrow) $\gamma \in Q_1$, where $s(\alpha) = t(\gamma)$ and $\gamma\alpha \notin I$.
4. The admissible ideal I is generated by paths of length 2.

It is clear that the almost gentle algebra is generalized for two classes which are gentle algebra and SAG-algebra.

Let A be a SAG-algebra, A path $m = v_1\alpha_1 \cdots v_n\alpha_n v_{n+1}$ or, simply $m = \alpha_1 \cdots \alpha_n$ in P, is maximum if for all a and b in Q_1, we have am and mb in I. We denoted to $\mathcal{M}$to be the set of maximum paths. It is clear that $\mathcal{M} \subseteq P$.

The next example shows that the set $\mathcal{M}$.

Example 2.4: Let A be a SAG-algebra over K which given by a quiver



and relations $I = \langle \beta\gamma, \beta\delta, x^2, x\beta \rangle$.

We have the set of vertices $Q_0 = \{v_1, v_2, \dots, v_5\}$ and the set P containing the following paths

- paths of length 1 are $\alpha, \beta, \gamma, \delta, x$.
- paths of length 2 are $x^2, x\beta, \beta\gamma, \beta\delta$.
- paths of length 3 are $x^3, x^2\beta, x\beta\delta, x\beta\gamma$.
- paths of length 4 are $x^4, x^3\beta, x^2\beta\delta, x^2\beta\gamma$.
- continuing in the same way we get paths of length n which are $x^n, x^{n-1}\beta, x^{n-2}\beta\delta, x^{n-2}\beta\gamma$.

So $P = \{\alpha, \beta, \gamma, \delta, x, x^2, x\beta, \beta\gamma, \beta\delta, x^3, x^2\beta, x\beta\delta, x\beta\gamma, \ldots, x^n, x^{n-1}\beta, x^{n-2}\beta\delta, x^{n-2}\beta\gamma, \ldots\}$. It is clear that the maximum path set $\mathcal{M}$ is $\{x, \beta, \gamma, \delta\}$.

Defining the function ϕ from KQ to A via $\alpha \mapsto \bar{\alpha}$, for all α in Q_1. So ϕ is a canonical surjection. For all $\alpha \in Q_1$, we let $\bar{\alpha}A$ to be A-module which is generated by $\bar{\alpha}$. In case that A is SAG- algebra, the A-modules, $\bar{\alpha}A$ are uniserial [8] and [9].
The following lemma is a generalization of Lemma 12, [1].

Lemma 2.5: Let A be a SAG-algebra, then
1. There are no repeated arrows in maximal path m, for all $m \in \mathcal{M}$.
2. v lies in two maximal paths m_1 and m_2 if and only if $m_1 \cap m_2 = \{0\}$.
3. v is in a unique maximal path $m \in \mathcal{M}$ if and only if $s(m) = v$.
4. v is in a unique maximal path $m \in \mathcal{M}$ if and only if $t(m) = v$.

Lemma 2.6: Let A be a SAG-algebra, then the maximal paths are elements in the basis of $soc_{A^e}A$

Proof: Let $m = v_i m v_j$ be a maximal path. We look at $soc_{A^e}A$ as vector space over K, then we have

$$soc_{A^e}A = \bigoplus_{v_i, v_j \in Q_0} vi(soc_{A^e}A)v_j.$$

We want to show that each maximal path m belongs to the basis of $soc_{A^e}A$, in particular, $v_i\, m v_j$ from a basis $v_i soc_{A^e}Av_j$. Since A is a SAG-algebra, then there is at most a non-zero path p and q such that $s(p) = s(q) = v_i$ and $t(p) = t(q) = v_j$. Set $p = \alpha_1 \cdots \alpha_m$ and $q = \beta_1 \cdots \beta_n$. So, we have four different cases. First, by assuming that the only arrows start from v_i are α_1 and β_1, and the only arrows end at v_j are α_m, and β_m. Hence, p and q are in $\mathcal{M}$, particularly $\mathcal{M} = \{p, q\}$. Thus, $\{p, q\}$is a basis of $v_i soc_{A^e}Av_j$.

Second, by letting $\alpha \in Q_1$, such that $t(\alpha) = s(p) = s(q)$. So, if $\alpha\, p$ lies in I, then p maximum (using definition of maximum). If not, we have either $\alpha p \notin I$ or, $\alpha q \notin I$. It is enough to discuss one case, assume $\alpha p \notin I$, then $p \notin \mathcal{M}$ and hence $q \in \mathcal{M}$. Therefore, $v_i soc_{A^e}Av_j = \{q\}$.

Third, let $\alpha, \beta \in Q_1$ such that $t(\alpha) = t(\beta) = s(p) = s(q)$. Then we have either (i) $\alpha\alpha_1 \in I$ and $\beta\beta_1 \in I$ or (ii) $\alpha\beta_1 \in$ I and $\beta\alpha_1 \in I$. Suppose (i) holds, then $p, q \notin \mathcal{M}$ and $\{0\}$ is a basis of $v_i soc_{A^e}Av_j$.

Finally, assume that $\alpha, \beta, \gamma \in Q_1$ such that $t(\alpha) = t(\beta) = s(p) = s(q)$ and $s(\gamma) = t(p) = t(q)$. Without loss of generality, we suppose that $\beta_n\gamma \in I$ and so $q \in \mathcal{M}$. However, $p \notin \mathcal{M}$. Then $v_i soc_{A^e}Av_j = \{q\}$. By a similar argument, we get other cases.

Remark 2.7: From the definition of SAG-algebra we can see that SAG-algebra is a special multiserial algebra.

We denoted the length of path p by $\ell(p)$. Since A is a SAG- algebra and so monomial then the K basis of A is the set $\{p + I | p \in Q \; and \; p \notin I\}$ where p is a path in Q and p is a subpath of $m, m \in \mathcal{M}$.

Proposition 2.8: Let A be a SAG-algebra. Let m be a maximum path such that a is a beginning arrow in m. Then $rad\ (A) = \oplus_{m\in\mathcal{M}}\, mA$, where mA is the uniserial module generated by the a.

Proof: Let $m \in \mathcal{M}$ and let p be a path in Q and p is a subpath of m. Then we have two cases: if $\ell(p) = 0$, then $p = e_v$ which is a trivial path for some $v \in Q$ and so $e_v \in Soc_k(A)$. If $\ell(p) \geqslant 1$, then we set a path $p = \alpha_1 \cdots \alpha_n \in Q$. Hence $p + I$ in $\overline{\alpha_1}\, A$. As $rad(A)$ is generated by the image of arrows, we get $\sum_{\alpha\in Q} \overline{\alpha_1}\, A = rad(A)$. Since A is a monomial algebra, then $rad\ (A) = \bigoplus_{\alpha\in Q_1} \overline{\alpha_1}\, A$. Since A is a SAG-algebra and each maximum path has no repeated arrow or common arrow with a different maximum path.
So $p + I$ in mA, for all $m \in \mathcal{M}$. Thus $rad\ (A) = \oplus_{m\in\mathcal{M}}\, mA$.

Consequently, we get the following corollary.

Corollary 2.9: Let A be a SAG-algebra. Then $\dim_K A = |Q_0| + \sum_{m\in\mathcal{M}}(\ell(mA)\ell(mA) + 1)/2$.

Proof: We have $|Q_0|$vertices and so there are $|Q_0|$trivial paths. Using the above proposition we get $dim\left(rad\ (A)\right) = \oplus_{m\in\mathcal{M}} \dim\ (mA)$. Let $m \in \mathcal{M}$, then mA <u>unisearal</u> module generated by the first arrow and moreover has length $\ell(mA)$. So, dim(rad(A))= $\sum_{m\in\mathcal{M}}(\ell(mA)\ell(mA) + 1)/2$.

3. The extension of quiver Q and relations I

In this section the quiver Q^* is created from the quiver Q, and also the relations I^* are defined. We let $= \frac{KQ}{I}$ be a SAG-algebra which is given by quiver Q and relations I.

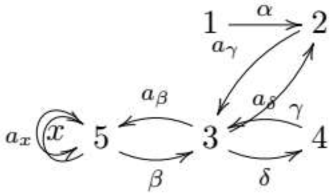
We start defining a quiver Q such that:

1. $Q^* = Q$,
2. $Q_1^* = Q_1 \cup \{\alpha_m | m \in \mathcal{M} \text{ and } s(\alpha_m) = t(m), t(\alpha_m) = s(m)\}$.

Proposition 3.1: Let A be a SAG-algebra and let Q^* built as above. Then Q^* is a quiver of $T(A)$.

Proof: By using [7] and Lemma 2.6, we have the set of arrows which obtain from Q in A and the set $\{\alpha_m | m \in \mathcal{M}\}$, where $s(\alpha_m) = t(m), t(\alpha_m) = s(m)$. Thus, the quiver of the trivial extension of A is Q^*.

Recall Example 2.4, to show that our construction of quiver Q^*. So we have $Q_0^* = \{v_1, v_2, v_3, v_4, v_5\}$ and $Q_1^* = \{\alpha, \beta, \gamma, \delta, x, a_\beta, a_\gamma, a_\delta, a_x\}$ and so we have the following quiver:



Following the approach of Green and Schroll [2], we have for all $m \in \mathcal{M}$, a cycle element $c_m = ma_m$ in Q. Defining a set $\mathcal{S}_m$ to be a set $\{c^* | c^*\ a\ cyclic\ permution\ of\ c,\ for\ some\ m \in \mathcal{M}\}$. The function $\mu: \mathcal{S}_m \longrightarrow \mathbb{Z}$ is defined as follow $\mu(c^*) = 1$, for all $c^* \in \mathcal{S}_m$.

Definition 3.2 [10]: Let $\mathcal{T}$ be a set of a cycle in Q such that there is no common arrows in cycle (simple), and let $\mathcal{V}: \mathcal{T} \longrightarrow \mathbb{Z}_{\geq 0}$ We call $(\mathcal{T}, \mathcal{V})$ defining pair in if satisfying:

1. If there is a loop $c \in \mathcal{T}$ at $v \in Q$, then $\mathcal{V}(c) > 1$.
2. Whenever a simple cycle c contain in $\mathcal{T}$, then c^* (cycle permutation) of c contain in $\mathcal{T}$.

3. Let $c \in \mathcal{T}$ and let $c^* \in \mathcal{T}$ (cycle permutation) of c. Then $\mathcal{V}(c) = \mathcal{V}(c^*)$.
4. If an arrow appears in two simple cycles which are in $\mathcal{T}$. Then the cycle of each one is a cycle permutation of each other.

Then $(\mathcal{T}, \mathcal{V})$ is defining pair.

Hence $(\mathcal{S}_m, \mu)$ gives the algebra over K with Q^* and ideal I^* generated by all relations of the following form:

1. $c_1 - c_2$, where $c_1, c_2 \in \mathcal{S}_m$ and cycles at vertex $v \in Q_0$.
2. ca, where $c = aa_1 \cdots a_n$ and $a, a_i \in Q_0$, where $i = 1, \ldots, n$.
3. ab, where a, b are arrows in Q and ab is not subpath of c, for all $c \in \mathcal{S}_m$.

We get the following theorem using results in [11].

Theorem 3.3: Let A be a SAG-algebra and let A^* defined by $(\mathcal{S}_m, \mu)$. Then A^* is a symmetric special multiserial algebra.

The dimension of A^* is characterized in the next result.

Proposition 3.4: Let A be a SAG-algebra and let A^* is constructed as mention above. Then $dim_k(A^*) = 2|Q_0| + \ell(m)^{\cdot}\ell(m) + 1$.

Proof: The algebra A^* has Kbasis which is all paths of length more than and equal to 1 together with the trivial path e_v. From the construction of quiver Q^*, we have $|Q_0|$and so there are $|Q_0|$trivial paths. Since A^* is symmetric, then $dim\,(soc(A^*)) = |Q_0|$. Let $m \in \mathcal{M}$, then $c_m = ma_m$ in $\mathcal{S}_m$with length $\ell(m) + 1$. Since A is SAG-algebra and the maximum path m is unique for each arrow, then we have the starting arrow a in m and moreover a in c_m. It follows aA^* is a uniserial module of length $\ell(m) + 1$. So $dim\,(\frac{aA^*}{aA^*} \cap soc(A^*)) = \ell(m)\ell(m) + 1$.Therefore, $dim\,rad(A^*)/soc(A^*) = \sum_{m \in \mathcal{M}} \ell(m)\ell(m) + 1$ and so $dim_k(A^*) = 2|Q_0| + \sum_{m \in \mathcal{M}} \ell(m)\ell(m) + 1$.

4. Trivial extension of SAG-algebra

We define the trivial extension of a finite-dimension algebra A as in literature, where we let $D(A) = Hom_K(A, K)$ be the dual of A. The trivial extension of A is given by $T(A) = A \rtimes D(A)$, where $T(A)$ is symmetric algebra and viewed as vector space $T(A) = A \oplus D(A)$ with multiplication which is given by $(a_1, f_1)(a_2, f_2) = (a_1a_2, a_1f_2 + f_1a_2)$ for all $a_1, a_2 \in A$ and $f_1, f_2 \in D(A)$.

The basis of $D(A)$ is $P^\vee = \{p^\vee | p \in P\}$. We remind the reader of an element $p \in P$ in particular in A, then $p^\vee \in D(A)$ [2].

Proposition 4.1 [2]: Let A be finite dimension algebra. Then the set $\{(\alpha, 0) | \alpha \in Q_1 \cup \{(0, m^\vee) | m \in \mathcal{M}\}$ generate $T(A)$.

The next result show that $A^* \cong T(A)$.

Theorem 4.2: Let $A = KQ/I$ be a SAG-algebra and $T(A)$ be the trivial extension of A arise from $D(A)$. Then $A^* \cong T(A)$.

Proof: Starting by defining an algebra homomorphism $\phi: KQ^* \longrightarrow T(A)$ via

1. Let $v \in Q_0^*$, then $\phi(v) = (v, 0)$.

2. Let $\alpha \in Q_1^*$. then

$$\phi(\alpha) = \begin{cases} (\alpha, 0) & \alpha \in Q_1 \\ (0, m^\vee) & \alpha \in \{\alpha_m | m \in \mathcal{M} \text{ and } s(\alpha_m) = t(m), t(\alpha_m) = s(m)\}. \end{cases}$$

It is clear that from the Proposition 2, the ring homomorphism is a surjection. We want to show that $I^* \subseteq Ker\,\phi$. We start with the first type. (i) Let c_1, c_2 in $\mathcal{S}_m$ at vertex $v \in Q_0^*$. Then $\phi(c_1 - c_2) = \phi(c_1) - \phi(c_2)$. Let $\mathrm{m} \in \mathcal{M}$ and $m = pq$, where $p, q \in P$. Since c_1, c_2 in $\mathcal{S}_m$, then we write $c_1 = p\alpha_m q$ *and*

$c_2 = p'\alpha_{m'} q'$. From [2, Lemma 4.1], we have $\phi(c_1) = \phi(p\alpha_m\, q) = (q, 0)(0, m^\vee)(p, 0) = (0, pm^\vee\, p) = (0, r^\vee)$, this gives $m = prq$. By hypothesis $m = pq$, so $r = e_v$. Hence $\phi(c_1) = (0, r^\wedge \vee) = (0, e_v)$.[3]
Similarly we get $\phi(c_2) = (0, e_v)$ and so $\phi(c_1 - c_2) = 0$.

(ii) Let c in $\mathcal{S}_m$ at v and write $c = q\alpha_m p$ where α is a first arrow in c. So $\phi(c\alpha) = \phi(q\alpha_m\, p)\phi(\alpha) = (p, 0)(0, m^\vee)(q, 0)\phi(\alpha) = (0, pm^\vee\, q)\phi(\alpha) = (0, r^\vee)\phi(\alpha) = (0, e_v)\phi(\alpha)$. In this case either $\alpha \in Q_1$ or $\alpha = \alpha_m$. If $\alpha \in Q_1$, then $(0, e_v)\phi(\alpha) = (0, e_v)(\alpha, 0) = (0, \alpha\, e_v)$. Here two cases are appeared: if $(0, \alpha\, e_v) = 0$, then $\phi(c\alpha) = 0$. Suppose that $(0, \alpha\, e_v) \neq 0$, then $(0, \alpha\, e_v) = (0, r^\vee)$ and so $e_v = r\alpha$ this is a contradiction with assumption $\alpha \in Q_1$. If $\alpha = \alpha_m$, then $\phi(c\alpha) = (0, e_v)(0, m^\vee) = 0$ by using [Lemma 4.1, 2].

(iii) Let α and $\backslash beta$ in Q_1, since $\alpha\beta \in I^*$, then $\alpha\beta = 0$ and hence $\phi(\alpha\beta) = 0$.
Let $\alpha \in Q_1$ and $\beta = \alpha_m$ for some m in $\mathcal{M}$. Then $\phi(\alpha\alpha_m) = (\alpha, 0)(0, m^\vee) = (0, \alpha\, m^\vee)$. If $(0, \alpha\, m^\vee) \neq 0$, we have $(0, \alpha\, m^\vee)(o, r^\vee)$ and hence $m = r\alpha$ this is a contradiction with hypothesis. Let $\alpha = \alpha_m$ *and* $\beta = \alpha_{m'}$, for some m, m' in $\mathcal{M}$. By [Lemma 4.1(4), 2] we have $\phi(\alpha_m\alpha_{m'}) = (0, m^\vee)(0, m'^\vee) = 0$. So $Ker\ \phi = I^*$. Using the First isomorphism theorem we get $\frac{KQ^*}{I} \cong T(A)$.

Corollary 4.3: Let $A = \frac{KQ}{I}$ be a SAG-algebra and $T(A)$ be the trivial extension of A arise from $D(A)$. Then $dim_K(A^*) = dim_K\,(T(A))$.

5. Conclusions

In this article the string almost algebras $A = KQ/I$ are investigated. Many properties of SAG-algebra are given. The quiver $Q^*$was constructed from the quiver Q and the admissible ideal was defined. The algebra A^* which was given by quiver Q^* and relations $I^*$was defined. We proved that the trivial extension of algebra A was isomorphic to the algebra A^*.

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