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On A Class of W-Valent Functions With Two Fixed Points Involving Hypergeomatric Function with Generalization Integral Operator

Rafid Habib Buti[1*], Kassim A. Jassim[2]
[1]Department of Mathematics and Computer Applications, College of Sciences, University of Al-Muthanna, Al-Muthanna, Iraq
[2]Department of Mathematics, College of Sciences, University of Baghdad, Baghdad, Iraq

Abstract
In this paper we have studied a generalization of a class of (w-valent) functions with two fixed points involving hypergeometric function with generalization integral operator . We obtain some results like, coefficient estimates and some theorems of this class.
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Keywords: w-valent functions, generalization integral operator, hypergeometric function.

حول عائلة من الدوال المتعددة القيم من الصنف w مع نقطتين ثابتتين والمتضمنة الدالة الفوق هندسية مع المؤثر التكاملي الموسع

رافد حبيب بطي*[1]، قاسم عبد الحميد جاسم[2]
[1]قسم الرياضيات وتطبيقات الحاسوب ، كلية العلوم ، جامعة المثنى ، المثنى، العراق
[2]قسم الرياضيات ، كلية العلوم ، جامعة بغداد ، بغداد، العراق

الخلاصة
في هذا البحث نحن درسنا تعميم لعائلة من الدوال المتعددة القيم من الصنف w مع نقطتين ثابتتين متضمنة دالة الفوق هندسية مع المؤثر التكاملي الموسع وقد حصلنا غلى بعض النتائج كتخمين المعاملات وبعض النظريات لتلك العائلة .

1. Introduction:

Let RZ denoted by the class of w- valent functions defined by the following:

$$f(v) = mv^w + \sum_{q=w-1}^{\infty} t_{q-w+1} v^{q+w-1} - {}_2F_1(a,b;c;v), \quad |v| < 1$$

$$= mv^w - \sum_{q=w+1}^{\infty} g_q v^q, \quad w \in N \quad , \tag{1}$$

where ${}_2F_1(a,b;c;v)$ is defined by Gaussian hypergeometric function defined by

$${}_2F_1(a,b;c;v) = \sum_{q=0}^{\infty} \frac{(a)_q (b)_q}{(c)_q\, q!} v^q, \quad |v| < 1$$

*Email: 1rafidhbuti@yahoo.com

where $(a)_q = \dfrac{\Gamma(a+q)}{\Gamma(a)}$ $c > b > 0$ and $c > a+b$ if $w = p$, see [1]such that Γ is gamma function and $m > 0$, $t_{q-w+1} = \dfrac{(a,q-w+1)(b,q-w+1)_q}{(c,q-w+1)_q (q-w+1)!}$, $q > w+1$ (2)

and

$$g_q = \frac{\Gamma(a+q)\Gamma(b+q)\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c+q)\Gamma(q+1)} \quad , \quad q > w+1 \ .$$

Definition 1: A function $f(v)$ is given by (1) be in the class $RZ_*^{w,m}(u, \varphi, \vartheta, \upsilon.\lambda, \wp, \psi)$ if and only if it the condition:

$$\left| \frac{u\varphi\left[v^3 (HR^w(f(v)))''' - mw(w-1)(w-2)v^w\right]}{m\vartheta(1-\zeta M) + \lambda\left[v(w-1)(HR^w(f(v)))' - v^2 (HR^w(f(v)))''\right]} \right| < (\wp - \psi), \tag{3}$$

where $\vartheta, m > 0, \lambda > 0, 0 \le u < 1, 0 \le \varphi < 1, 0 < \psi < \wp, \wp > 0, 0 \le M\varsigma < 1$, and $HR^w(f(v))$ is generalization integral operator defined as following :

$$HR^w(f(v)) = \frac{(1-\mu)^{\theta[r+\eta(1+w)+1]}}{r^w \xi^{r+\eta(w+1)+1}\Gamma(r+\eta(w+1)+2)} \int_0^\infty \tau^{r+\eta} e^{-(\frac{1-\mu}{\xi})^\theta \tau} f(rv\tau^\beta)dt, \tag{4}$$

where $r, \xi > 0$,$0 < \mu \le 1, \eta \ge 0, \theta > 0$.

if $w = 1$, we have integral operator is introduced by H. J.Abdul Hussein and R.H.Buti[2].

So from (4) we get

$$HR^w(f(v)) = mv^w - \sum_{q=w+1}^\infty g_q \Delta(q, \xi, \eta, r, \mu, \theta) v^q, \tag{5}$$

where $\Delta(q, \xi, \eta, r, \mu, \theta) = \dfrac{r^{n-w}(1-\mu)^{\theta\eta[n++w-2]}\xi^{\eta(n-w)}\Gamma(r+\eta(n-1)+2)}{\Gamma(r+\eta(w-1)+2)} = \mathrm{E}$.

We note that, if $\Delta(q, \xi, \eta, r, \mu, \theta) = \Delta(q, \xi, 0, 1, \mu, 0)$, $\left[HR^w(f(v))\right]^{(n)} = f^{(n-1)}(z)$ and $w = (w-1), (w-2) = 1$, we have a class studied by [3].

For a given real $v_0 (0 < v_0 < 1)$ .Let $RZ^{wi}(i = 0,1)$ be a subclass of RZ satisfies the following **conditions**

$v_0^{-w} f(v_0) \ge 1$, and $\dfrac{1}{w} z^{1-w} f'(v_0) \le 1$ respectively

Let $RZ_{*,m}^{wi}(u, \varphi, \vartheta, \upsilon.\lambda, \wp, \psi, v_0) = RZ_*^{w,m}(u, \varphi, \vartheta, \upsilon.\lambda, \wp, \psi) \cap RZ^{wi}(i = 0,1)$.

Some another classes are by W.G.Atshan and S.R.Kulkarni [4,5], S.R.Kulkarni and Mrs. S.Joshi[6] , consisting of p-valent meromorphic and univalent meromorphic functions. So the hypergeometric function studied by another researcher R.Ezhilarasi, et.al.[7] and Rabha et.al.[8,9]

2. Coefficient Estimation

In the next theorem, we obtain a coefficient estimation for the function to be in the class $RZ_*^{w,m}(u, \varphi, \vartheta, \upsilon.\lambda, \wp, \psi)$.

Theorem 1: A function $f(z)$ defined by (1) be in the class $RZ_*^{w,m}(u, \varphi, \vartheta, \upsilon.\lambda, \wp, \psi)$ if and only if

$$\sum_{q=w+1}^\infty q\left[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)\right]\mathrm{E} g_q \le m\vartheta(\wp-\psi)(1-\varsigma M), \tag{6}$$

where $\vartheta, m > 0, \lambda > 0, 0 \le u < 1, 0 \le \varphi < 1, 0 < \psi < \wp, \wp > 0, 0 \le M\varsigma < 1$.

Proof: Let the inequality (6) holds true and $|v| < 1$, we get

$$\left|u\varphi\left[z^{3}(HR^{w}(f(v)))''' - mw(w-1)(w-2)v^{w}\right] - \right.$$
$$\left. -(\wp-\psi)\right|m\vartheta(1-\zeta M)+\lambda\left[v(w-1)(HR^{w}(f(v)))' - v^{2}(HR^{w}(f(v)))''\right]$$
$$=\left|-\sum_{q=w+1}^{\infty} u\varphi q(q-1)(q-2)\mathrm{E}g_{q}v^{q}\right| - (\wp-\psi)\left|m\vartheta(1-\zeta M)-\sum_{q=w+1}^{\infty} q\lambda(w+q-2)\mathrm{E}g_{q}v^{q}\right|$$
$$\leq \sum_{q=w+1}^{\infty} u\varphi q(q-1)(q-2)\mathrm{E}g_{q}|v|^{q} - \left[(\wp-\psi)m\vartheta(1-\zeta M)\right] +$$
$$+\sum_{q=w+1}^{\infty} q\lambda(\wp-\psi)(w+q-2)\mathrm{E}g_{q}|v|^{q}$$
$$\sum_{q=w+1}^{\infty} q\left[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)\right]\mathrm{E}g_{q} - m\vartheta(\wp-\psi)(1-\varsigma M)\leq 0..$$

Hence by maximum modulus, $f(z)$ belongs to the class $RZ_{*}^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$.

Conversely, let $f(v)\in RZ_{*}^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$, then

$$\left|\frac{u\varphi\left[v(HR^{w}(f(v)))''' - mw(w-1)(w-2)v^{w}\right]}{m\vartheta(1-\zeta M)+\lambda\left[v(w-1)(HR^{w}(f(v)))' - v^{2}(HR^{w}(f(v)))''\right]}\right| < (\wp-\psi).$$

That is

$$\left|\frac{-\sum_{q=w+1}^{\infty} u\varphi q(q-1)(q-2)\mathrm{E}g_{q}v^{q}}{m\vartheta(1-\zeta M)-\sum_{q=w+1}^{\infty} q\lambda(w+q-2)\mathrm{E}g_{q}v^{q}}\right| < (\wp-\psi).$$

We have $|\mathrm{Re}(v)|\leq|v|$ for any v, so

$$\mathrm{Re}\left\{\frac{\sum_{q=w+1}^{\infty} u\varphi q(q-1)(q-2)\mathrm{E}g_{q}v^{q}}{m\vartheta(1-\zeta M)-\sum_{q=w+1}^{\infty} q\lambda(w+q-2)\mathrm{E})g_{q}v^{q}}\right\} < (\wp-\psi).$$

We choosing values of z on the real axis and letting $v\to 1^{-}$, we get

$$\sum_{q=w+1}^{\infty} u\varphi q(q-1)(q-2)\mathrm{E}g_{q} + \sum_{q=w+1}^{\infty} q\lambda(\wp-\psi)(w+q-2)\mathrm{E}g_{q}$$
$$\leq(\wp-\psi)m\vartheta(1-\zeta M).$$

Then

$$\sum_{q=w+1}^{\infty} q\left[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)\right]\mathrm{E}g_{q} \leq m\vartheta(\wp-\psi)(1-\varsigma M).$$

The proof is completes.

Corollary 1: If $f(v)\in RZ_{*}^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$.Then

$$g_{q}\leq\frac{m\vartheta(\wp-\psi)(1-\varsigma M)}{q\left[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)\right]\mathrm{E}}. \tag{7}$$

Theorem 2: Let $f(z)\in RZ_{*}^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$. Then $f(z)\in RZ_{*,m}^{w0}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_{0})$ if and only if $\sum_{q=w+1}^{\infty}\left[\frac{q\left[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)\right]\mathrm{E}}{\vartheta(\wp-\psi)(1-\varsigma M)}+v_{0}^{q-w}\right]g_{q}\leq 1.$ (8)

Proof: Since $f(z) \in RZ^{w0}_{*,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,z_0)$, then we have $z_0^{-w} f(z_0) = m - \sum_{q=w+1}^{\infty} g_q z_0^{q-w}$, since $v_0^{-w} f(v_0) \le 1$, so we get $m - \sum_{q=w+1}^{\infty} g_q v_0^{q-w} \le 1$, or equivalent to

$$m \le 1 + \sum_{q=w+1}^{\infty} g_q v_0^{q-w}. \tag{9}$$

Put m in (6) we get

$$\sum_{q=w+1}^{\infty} q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E} g_q \le m\vartheta(\wp-\psi)(1-\varsigma M)$$

$$\le (1 + \sum_{q=w+1}^{\infty} g_q v_0^{q-w})\vartheta(\wp-\psi)(1-\varsigma M),$$

which is equivalently to

$$\sum_{q=w+1}^{\infty} \left[q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E} - \vartheta(\wp-\psi)(1-\varsigma M)v_0^{q-w}\right] g_n \le \vartheta(\wp-\psi)(1-\varsigma M).$$

Or

$$\sum_{q=w+1}^{\infty} \left[\frac{q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{\vartheta(\wp-\psi)(1-\varsigma M)} + v_0^{q-w}\right] g_q \le 1.$$

Conversely, let (8) is holds true, then

$$m\sum_{q=w+1}^{\infty} \left[\frac{q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{m\vartheta(\wp-\psi)(1-\varsigma M)}\right] g_q - \sum_{q=w+1}^{\infty} z_0^{q-w} g_q \le 1$$

$$\le m - \sum_{q=w+1}^{\infty} g_q v_0^{q-w} \le 1,$$

So $v_0^{-w} f(v_0) \le 1$, then $f(v) \in RZ^{w0}_{*,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_0)$.

Corollary2: Let $f(v) \in RZ^{w0}_{*,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_0)$. Then

$$g_q \le \frac{\vartheta(\wp-\psi)(1-\varsigma M)}{\left[q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E} - \vartheta(\wp-\psi)(1-\varsigma M)v_0^{q-w}\right]}.$$

Theorem 3: Let $f(v) \in RZ^{w,m}_{*}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$. Then $f(z) \in RZ^{w1}_{*,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_0)$ if and only if $\sum_{q=w+1}^{\infty} \left[\frac{q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{\vartheta(\wp-\psi)(1-\varsigma M)} + (\frac{q}{w})v_0^{q-w}\right] g_q \le 1$ (10)

Proof : Since $f(z) \in RZ^{w1}_{*,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_0)$. Then we have

$$\frac{1}{w} v^{1-w} f'(z_0) = m - \sum_{q=w+1}^{\infty} (\frac{q}{w}) g_q v_0^{q-w}.$$

Which given

$$m \le 1 + \sum_{q=w+1}^{\infty} (\frac{q}{w}) g_q v_0^{q-w} \tag{11}$$

Put in (6), we get

$$\sum_{q=w+1}^{\infty} q[u\varphi(q-1)(q-2) + \lambda(\wp-\psi)(w+q-2)]\mathrm{E} g_q \le \vartheta(\wp-\psi)(1-\varsigma M)$$

$$\le (1 + \sum_{q=w+1}^{\infty} (\frac{q}{w}) g_q z_0^{q-w})\vartheta(\wp-\psi)(1-\varsigma M).$$

Or equivalent

$$\sum_{q=w+1}^{\infty}\left[q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}-(\frac{q}{w})\vartheta(\wp-\psi)(1-\varsigma M)z_0^{q-w}\right]g_q \le \vartheta(\wp-\psi)(1-\varsigma M)$$

Or

$$\sum_{q=w+1}^{\infty}\left[\frac{q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{\vartheta(\wp-\psi)(1-\varsigma M)}+(\frac{q}{w})v_0^{q-w}\right]g_q \le 1.$$

Conversely , let (10) is holds true , then

$$\sum_{q=w+1}^{\infty}\left[\frac{q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{\vartheta(\wp-\psi)(1-\varsigma M)}+(\frac{q}{w})v_0^{n-p}\right]g_q \le 1$$

Or equivalent to

$$m\sum_{q=w+1}^{\infty}\left[\frac{q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}}{m\vartheta(\wp-\psi)(1-\varsigma M)}\right]g_q-\sum_{q=w+1}^{\infty}(\frac{n}{p})v_0^{n-p}g_q \le 1$$

$$\le m-\sum_{q=w+1}^{\infty}(\frac{n}{p})v_0^{n-p}g_q \le 1,$$

So $\frac{1}{w}v^{1-w}f'(v_0)\le 1$, then $f(v)\in RZ_{*,m}^{p1}(A,B,k,\lambda.h,\ell,\alpha,v_0)$.

The proof is complete .

Corollary 3: Let $f(z)\in RZ_{*,m}^{w1}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi,v_0)$. Then

$$g_n \le \frac{\vartheta(\wp-\psi)(1-\varsigma M)}{\left[q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}-(\frac{q}{w})\vartheta(\wp-\psi)(1-\varsigma M)z_0^{q-w}\right]}.$$

3. Application Multiplier transformation and integral operator

The multiplier transformation is defined as following

$$I_w(d,y)(f(v))=mv^w-\sum_{q=w+1}^{\infty}\left(\frac{q+y}{w+y}\right)^d g_q v^q,\ d>0,\quad y\ge 0 \quad . \tag{12}$$

The operator $I_w(d,y)$ was studied by W. G. Atshan and S.R.Kulkarni [10] and Tehrnchi [8] subclass of meromorphic univalent functions and so Cho , Kwon and Srivastava [1].

Now, the integral operator $G(z)$ is defined by the following form:

$$G(v)=\frac{y+w}{z^y}\int_0^v h^{y-1}r(h)dh \quad , \quad y>-w. \tag{13}$$

And $r(v)=mv^w-\sum_{q=w+1}^{\infty}j_q v^q$, $w\in N$, so from (13), we have

$$G(v)=mv^w-\sum_{q=w+1}^{\infty}\Lambda(q,y,w)j_q v^q. \tag{14}$$

And $\Lambda(q,y,w)=\frac{y+w}{y+q}$.

So, from (12) and (14), we defined Hadamard product or (convolution) as the following

$$T(v)=I_w(d,y)(f(v))*G(v)=mv^w-\sum_{q=w+1}^{\infty}\left(\frac{q+y}{w+y}\right)^d\Lambda(q,y,w)g_q j_q v^q. \tag{15}$$

In the next theorem we show that the function $T(v)$ is defined by (15) be in the class $RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$.

Theorem 4: Let $f(v)$ and $r(v)$ be in the class $RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$. Then the function $T(v)$ defined by (15) be in the class $RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$ if $d=1$, $j_w\le 1$.

Proof: We have from (15)

$$T(z)=I_w(d,y)(f(v))*G(v)=mv^w-\sum_{q=w+1}^{\infty}\left(\frac{q+y}{w+y}\right)^d\Lambda(q,y,w)g_q j_q v^q$$

$$=mv^w-\sum_{q=w+1}^{\infty}\left(\frac{q+y}{w+y}\right)^d\left(\frac{y+w}{y+q}\right)g_q j_q v^q .$$

Now to show that $T(v)\in RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$, we must to show that

$$\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}\left(\frac{q+y}{w+y}\right)^{d-1}g_q j_q\le m\vartheta(\wp-\psi)(1-\varsigma M).$$

Now,

$$\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}\left(\frac{q+y}{w+y}\right)^{d-1}g_q j_q$$

$$\le\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}g_q\le m\vartheta(\wp-\psi)(1-\varsigma M).$$

Hence $T(v)\in RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$

The proof is complete.

In the, next theorem, we show that the integral operator $F_{\zeta}(v)$ defined by the form

$$F_{\Im}(v)=m(1-\Im)v^p-\Im p\int_0^v\left(\frac{f(u)}{u}\right)du\quad,\quad \Im\ge 0,\tag{16}$$

be in the class $RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$.

Theorem 5: Let $f(v)\in RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$. Then $F_{\Im}(v)\in RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$ if $0\le\Im\le\frac{1+w}{w}, m>0.$

Proof: By virtue of (16) it follows from (1) that

$$F_{\Im}(v)=mv^w-\sum_{q=w+1}^{\infty}\Xi(q,\Im,w)g_q v^q,\quad \text{Where } \Xi(q,\Im,w)=\frac{\Im w}{q}\quad .\tag{17}$$

Now,

$$\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\Xi(q,\Im,w)\mathrm{E}g_q$$

$$=\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\frac{\Im w}{q}\mathrm{E}g_q$$

$$\le\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\frac{\Im w}{w+1}\mathrm{E}g_q .$$

Since $\left(\frac{\Im w}{w+1}\right)\le 1$, then

$$\le\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\frac{\Im w}{w+1}\mathrm{E}g_q$$

$$\le\sum_{q=w+1}^{\infty}q[u\varphi(q-1)(q-2)+\lambda(\wp-\psi)(w+q-2)]\mathrm{E}g_q\le m\vartheta(\wp-\psi)(1-\varsigma M)$$

So $F_{\Im}(v)\in RZ_*^{w,m}(u,\varphi,\vartheta,\upsilon.\lambda,\wp,\psi)$.

The proof is complete.

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