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On Two Sided α -*n*-Derivations in Prime near – Rings

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Abstract

In this paper, we investigate prime near – rings with two sided *α*-*n*-derivations satisfying certain differential identities. Consequently, some well-known results have been generalized**.** Moreover, an example proving the necessity of the primness hypothesis is given.

Keywords: prime near-ring, semi group ideal, derivation, n-derivation, two sided α-derivation, two sided α-n-derivation.

ثنائية الجانب على الحلقات المقترية الأولية $\alpha{-}$n الاشتقاقات

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الخلاصة

في هذه الورقة بحثنا في الاشتقاقات $\alpha{-}$n ثنائية الجانب على الحلقات المقتربة الأولية . وقمنا بوضع بعض الفرضيات على هذه الاشتقاقات، وتبعا لذلك قمنا بتعميم بعض النتائج المعروفة . كذلك قمنا بإعطاء مثال يبين ضرورة فرض أن الحلقة المقتربة المعطاة تكون أولية.

Introduction

A right near – ring (resp. left near ring) is a set N together with two binary operations (+) and (.) such that (i) $(N,+)$ is a group (not necessarily abelian). (ii) $(N,.)$ is a semi group. (iii) For all $a,b,c \in N$; we have $(a + b).c = a.c + b.c$ (resp. $a.(b + c) = a.b + b.c$. Trough this paper , N will be a zero symmetric left near – ring (i.e., a left near-ring N satisfying the property $0.x = 0$ for all $x\in N$). we will denote the product of any two elements x and y in N ,i.e.; $x.y$ by xy . The symbol Z will denote the multiplicative centre of N, that is $Z = \{x \in N \mid xy = yx \text{ for all } y \in N\}$. For any $x, y \in N$ the symbol $[x, y] = xy - yx$ stands for multiplicative commutator of x and y, while the symbol $x\circ y$ will denote $xy + yx$. N is called a prime near-ring if $xNy = \{0\}$ implies either $x = 0$ or $y = 0$. A nonempty subset U of N is called semigroup left ideal (resp. semigroup right ideal) if $NU \subseteq U$ (resp. $UN \subseteq U$) and if U is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. For terminologies concerning near-rings ,we refer to Pilz [1].

An additive mapping $\delta : N \longrightarrow N$ is said to be a derivation if $\delta(xy) = \delta(x)y + x\,\delta(y)$, (or equivalently $\delta(xy) = x\,\delta(y) + \delta(x)y$ for all $x, y \in N$, as noted in proposition 1 of [2]). The concept of derivation has been generalized in several ways by various authors. Two sided α-derivation has been introduced already in near-rings by N. Argac [3]. An additive mapping $d : N \longrightarrow N$ is called two sided α-derivation if there exist a function $\alpha : N \longrightarrow N$ such that $d(xy) = d(x)y + \alpha(x)d(y)$ and $d(xy) = d(x)\,\alpha(y) + xd(y)$ for all $x, y \in N$.

Also the notion of permuting *n*-derivations in near-rings has been introduced already by M. Ashraf, M.A. Siddeeque [4, 5].A map $d: \underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ is said to be permuting if the equation

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$d(x_1, x_2, \dots, x_n)$=$d(x_{\pi(1)}, x_{\pi(2)}, .., x_{\pi(n)})$ holds for all $x_1, x_2, \dots, x_n \in N$ and for every permutation $\pi \in S_n$ where S_n is the permutation group on $\{1,2,\dots,n\}$.

Let n be a fixed positive integer. An additive (i.e.; additive in each argument) mapping $d:\underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ is said to be *n*-derivation if the relations

$$d(x_1\ x_1', x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)x_1' + x_1\ d(x_1', x_2, \dots, x_n)$$

$$d(x_1\ , x_2x_2', \dots, x_n) = d(x_1\ , x_2, \dots, x_n)x_2' + x_2\ d(x_1\ , x_2', \dots, x_n)$$

$$\vdots$$

$$d(x_1\ , x_2, \dots, x_nx_n') = d(x_1\ , x_2, \dots, x_n)x_n' + x_n\ d(x_1\ , x_2, \dots, x_n')$$

Hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in N$. If in addition d is a permuting map then d is called a permuting *n*-derivation of *N*.

In the present paper, inspired by these concepts, we define a two sided α -*n*-derivation of near-ring *N*, which gives a generalization of *n*-derivation of near-ring.

Let *n* be a fixed positive integer. An additive (i.e.; additive in each argument) mapping $d:\underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ is said to be two sided α -*n*-derivation if the relations

$$d(x_1\ x_1', x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)x_1' + \alpha(x_1\ )d(x_1', x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + x_1\ d(x_1', x_2, \dots, x_n)$$

$$d(x_1\ , x_2x_2', \dots, x_n) = d(x_1\ , x_2, \dots, x_n)x_2' + \alpha(x_2\ )d(x_1\ , x_2', \dots, x_n) = d(x_1\ , x_2, \dots, x_n)\alpha(x_2') + x_2\ d(x_1\ , x_2', \dots, x_n)$$

$$\vdots$$

$$d(x_1\ , x_2, \dots, x_nx_n') = d(x_1\ , x_2, \dots, x_n)x_n' + \alpha(x_n\ )d(x_1\ , x_2, \dots, x_n') = d(x_1\ , x_2, \dots, x_n)\alpha(x_n') + x_n\ d(x_1\ , x_2, \dots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in N$. If in addition d is a permuting map then d is called a permuting two sided α-n-derivation of N .

For $\alpha = I_N$, a two sided α-*n*-derivation is of course the usual *n*-derivation.

2. Preliminary results

Through the present paper, d is two sided α-*n*-derivation associated with an homomorphism α of *N*. We begin with the following lemmas which are essential in developing the proof of our main results. Proof of the first three lemmas can be seen in [6].

Lemma 2.1. Let *N* be a prime near-ring and U a nonzero semigroup ideal of *N* . If $x, y \in N$ and $xUy = \{0\}$ then either $x = 0$ or $y = 0$.

Lemma 2.2. Let *N* be a prime near-ring and *U* a nonzero semigroup right ideal (resp, semigroup left ideal) and *x* is an element of *N* such that $Ux = \{0\}$ (resp, $xU = \{0\}$), then $x = 0$.

Lemma 2.3. Let *N* be a prime near-ring and Z contains a nonzero semigroup left ideal or nonzero semigroup left ideal, then *N* is a commutative ring.

Lemma 2.4 . Let *N* be a near-ring . Then *d* is a two sided α-*n*-derivation of *N* if and only if

$$d(x_1\ x_1', x_2, \dots, x_n) = x_1\ d(x_1', x_2, \dots, x_n) + d(x_1\ , x_2, \dots, x_n)\alpha(x_1')$$

$$d(x_1\ , x_2x_2', \dots, x_n) = x_2\ d(x_1\ , x_2', \dots, x_n) + d(x_1\ , x_2, \dots, x_n)\alpha(x_2')$$

$$\vdots$$

$$d(x_1\ , x_2, \dots, x_nx_n') = x_n\ d(x_1\ , x_2, \dots, x_n') + d(x_1\ , x_2, \dots, x_n)\alpha(x_n')$$

Proof . By hypothesis , we get

$$d(x_1\ (x_1' + x_1'), x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)\alpha(x_1' + x_1') + x_1\ d((x_1' + x_1'), x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + x_1\ d(x_1', x_2, \dots, x_n) + x_1\ d(x_1', x_2, \dots, x_n)\ . \quad (1)$$

And

$$d(x_1\ (x_1' + x_1'), x_2, \dots, x_n) = d(x_1\ x_1' + x_1\ x_1', x_2, \dots, x_n) = d(x_1\ x_1', x_2, \dots, x_n) + d(x_1\ x_1', x_2, \dots, x_n) = d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + x_1\ d(x_1', x_2, \dots, x_n) + d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + x_1\ d(x_1', x_2, \dots, x_n) \quad (2)$$

Comparing the two equations (1) and (2) , then we conclude that

$$d(x_1\ , x_2, \dots, x_n)\alpha(x_1') + x_1\ d(x_1', x_2, \dots, x_n) =$$

$$x_1\, d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\alpha(x_1')$$

Similarly we can prove the remaining (n−1) relations . Converse can be proved in a similar manner.

Lemma 2.5. Let *N* be a near-ring admitting a two sided α-*n*-derivation d. Then

$$(d(x_1, x_2, \dots, x_n)x_1' + \alpha(x_1)d(x_1', x_2, \dots, x_n))y = d(x_1, x_2, \dots, x_n)x_1'y + \alpha(x_1)d(x_1', x_2, \dots, x_n)y$$

$$(d(x_1, x_2, \dots, x_n)x_2' + \alpha(x_2)d(x_1, x_2', \dots, x_n))y = d(x_1, x_2, \dots, x_n)x_2'y + \alpha(x_2)d(x_1, x_2', \dots, x_n)y$$

$$\vdots$$

$$(d(x_1, x_2, \dots, x_n)x_n' + \alpha(x_n)d(x_1, x_2, \dots, x_n'))y = d(x_1, x_2, \dots, x_n)x_n'y + \alpha(x_n)d(x_1, x_2, \dots, x_n')y$$

Hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n'$,y ∈ N.

Proof. for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in$ N ,

$$d((x_1\, x_1')x_1'', x_2, \dots, x_n) = d(x_1\, x_1', x_2, \dots, x_n)x_1'' + \alpha(x_1\, x_1')d(x_1'', x_2, \dots, x_n)$$
$$= (d(x_1, x_2, \dots, x_n)x_1' + \alpha(x_1)d(x_1', x_2, \dots, x_n))x_1'' + \alpha(x_1)\alpha(x_1')d(x_1'', x_2, \dots, x_n) . \quad (3)$$

Also

$$d((x_1\, (x_1'x_1''), x_2, \dots, x_n) = d(x_1, x_2, \dots, x_n)x_1'x_1'' + \alpha(x_1)\, d(x_1'x_1'', x_2, \dots, x_n)$$
$$= d(x_1, x_2, \dots, x_n)x_1'x_1'' + \alpha(x_1)d(x_1', x_2, \dots, x_n)x_1'' + \alpha(x_1)\alpha(x_1')\, d(x_1'', x_2, \dots, x_n) . \quad (4)$$

Combining relations (3) and (4) , we get

$$(d(x_1, x_2, \dots, x_n)x_1' + \alpha(x_1)d(x_1', x_2, \dots, x_n))x_1'' = d(x_1, x_2, \dots, x_n)x_1'x_1'' + \alpha(x_1)d(x_1', x_2, \dots, x_n)x_1''$$

Putting y in place of x_1'',we find that

$$(d(x_1, x_2, \dots, x_n)x_1' + \alpha(x_1)d(x_1', x_2, \dots, x_n))y = d(x_1, x_2, \dots, x_n)x_1'y + \alpha(x_1)d(x_1', x_2, \dots, x_n)y$$

Similarly other (n−1) relations can be proved .

Using Lemma 2.4 and similar techniques as used to prove the above lemma, one can easily get the following:

Lemma 2.6. Let *N* be a near-ring admitting a two sided α-*n*-derivation *d* of *N* . Then

(i) $(\alpha(x_1)d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)x_1')y = \alpha(x_1)d(x_1', x_2, \dots, x_n)\, y + d(x_1, x_2, \dots, x_n)x_1'y$

$(\alpha(x_2)d(x_1, x_2', \dots, x_n) + d(x_1, x_2, \dots, x_n)x_2')y = \alpha(x_2)d(x_1, x_2', \dots, x_n)y + d(x_1, x_2, \dots, x_n)x_2'y$

$\vdots$

$(\alpha(x_n)d(x_1, x_2, \dots, x_n') + d(x_1, x_2, \dots, x_n)x_n')y = \alpha(x_n)d(x_1, x_2, \dots, x_n')\, y + d(x_1, x_2, \dots, x_n)x_n'y$

Hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n'$,y ∈ N.

(ii) $(d(x_1, x_2, \dots, x_n)\alpha(x_1') + x_1\, d(x_1', x_2, \dots, x_n))y = d(x_1, x_2, \dots, x_n)\alpha(x_1')\, y + x_1\, d(x_1', x_2, \dots, x_n)y$

$(d(x_1, x_2, \dots, x_n)\alpha(x_2') + x_2\, d(x_1, x_2', \dots, x_n))y = d(x_1, x_2, \dots, x_n)\alpha(x_2')\, y + x_2\, d(x_1, x_2', \dots, x_n)y$

$\vdots$

$(d(x_1, x_2, \dots, x_n)\alpha(x_n') + x_n\, d(x_1, x_2, \dots, x_n'))y = d(x_1, x_2, \dots, x_n)\alpha(x_n')y + x_n\, d(x_1, x_2, \dots, x_n')y$

(iii) $(x_1\, d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\alpha(x_1'))y = x_1\, d(x_1', x_2, \dots, x_n)y + d(x_1, x_2, \dots, x_n)\alpha(x_1')y$.

$(x_2\, d(x_1, x_2', \dots, x_n) + d(x_1, x_2, \dots, x_n)\alpha(x_2'))\, y = x_2\, d(x_1, x_2', \dots, x_n)y + d(x_1, x_2, \dots, x_n)\alpha(x_2')\, y$.

$\vdots$

$(x_n\, d(x_1, x_2, \dots, x_n') + d(x_1, x_2, \dots, x_n)\alpha(x_n'))y = x_n\, d(x_1, x_2, \dots, x_n')y + d(x_1, x_2, \dots, x_n)\alpha(x_n')y$

Lemma 2.7. Let *N* be a prime near ring , *d* a nonzero two sided α- *n*-derivation of *N* , and $U_1, U_2, \dots, U_n$ be a nonzero semigroup ideals of N . If $d(U_1, U_2, \dots, U_n) = \{0\}$, then

$d(N, N, \ldots, N) = \{0\}$.

Proof. By our hypothesis, we have $d(U_1,U_2,\ldots,U_n)x = \{0\}$, i.e.;

$d(u_1,u_2,\ldots,u_n) = 0$. (5)

for all $u_1 \in U_1$, $u_2 \in U_2, \ldots, u_n \in U_n$. putting u_1r_1 for u_1 in(5) ,where $r_1 \in N$, we get then

$0 = d(u_1 r_1,u_2,\ldots,u_n) = u_1d(r_1,u_2,\ldots,u_n) + d(u_1,u_2,\ldots,u_n)\,\alpha(r_1) = u_1d(r_1,u_2,\ldots,u_n)$, hence $u_1td(r_1,u_2,\ldots,u_n) = 0$, where $t \in N$, i.e. ; $U_1Nd(r_1,u_2,\ldots,u_n) = \{0\}$. But $U_1 \neq \{0\}$ and N is prime near ring , we conclude that

$d(r_1,u_2,\ldots,u_n) = 0$ (6)

Now putting $r_2\, u_2 \in U_2$ in place of u_2, where $r_2 \in N$, in (6) and proceeding as above we get $d(r_1, r_2, \ldots, u_n) = 0$. Proceeding inductively as before we conclude that $d(r_1, r_2, \ldots, r_n) = 0$ for all $r_1, r_2, \ldots, r_n \in N$, this shows that $d(N, N, \ldots, N) = \{0\}$.

Lemma 2.8. Let N be a prime near ring , d a nonzero two sided α- n-derivation of N , and $U_1,U_2,\ldots,U_n$ be a nonzero semigroup ideals of N .

(i) If $x \in N$ and $d(U_1,U_2,\ldots,U_n)x = \{0\}$, then $x = 0$.

(ii) If $x \in N$ and $xd(U_1,U_2,\ldots,U_n) = \{0\}$, then $x = 0$.

Proof. (i) By our hypothesis, we have $d(U_1,U_2,\ldots,U_n)x = \{0\}$, i.e.;

$d(u_1,u_2,\ldots,u_n)x = 0$ (7)

for all $u_1 \in U_1$, $u_2 \in U_2, \ldots, u_n \in U_n$. Putting r_1u_1 for u_1 in(7), where $r_1 \in N$, we get

$0 = d(r_1u_1,u_2,\ldots,u_n)x = \alpha(r_1)d(u_1,u_2,\ldots,u_n)x + d(r_1,u_2,\ldots,u_n)\,u_1x$. Using the hypothesis again we get $d(r_1,u_2,\ldots,u_n)u_1x = 0$. Replacing u_1 by u_1s where $s \in N$ in preceding relation we obtain $d(r_1,u_2,\ldots,u_n)\,u_1sx = 0$, i.e.; $d(r_1,u_2,\ldots,u_n)\,u_1Nx = \{0\}$.Since N is a prime near-ring, either $d(r_1,u_2,\ldots,u_n)\,u_1 = 0$ or $x = 0$. Our claim is that $d(r_1,u_2,\ldots,u_n)\,u_1 \neq 0$, for some $r_1 \in N$, $u_1 \in U_1$, $u_2 \in U_2, \ldots, u_n \in U_n$. For otherwise if $d(r_1,u_2,\ldots,u_n)\,u_1 = 0$ for all $r_1 \in N$, $u_1 \in U_1$, $u_2 \in U_2, \ldots, u_n \in U_n$, then $d(r_1,u_2,\ldots,u_n)\,tu_1 = 0$ where $t \in N$, i.e.; $d(r_1,u_2,\ldots,u_n)\,Nu_1 = \{0\}$. As $U_1 \neq \{0\}$, primeness of N yields $d(r_1,u_2,\ldots,u_n) = 0$ for all $r_1 \in N$, $u_2 \in U_2, \ldots, u_n \in U_n$. Now proceeding as in the proof of lemma 2.7, we can show that $d(N, N, \ldots, N) = \{0\}$ leading to a contradiction . Therefore, our claim is correct and now we conclude that $x = 0$.

(ii) It can be proved in a similar way.

3. Main results

In the year 1987 H. E. Bell ([7],Theorem 2) proved that if a 2-torsion free zero symmetric prime near-ring N admits a nonzero derivation d for which $d(N) \subseteq Z$, then N is a commutative ring. Further, this result was generalized by K. H. Park ([8],Theorem 3.1) in the year 2010 for permuting tri-derivation, who showed that if 3!-torsion free zero symmetric prime near-ring N admits a nonzero permuting tri-derivation d for which $d(N,N,N) \subseteq Z$, then N is a commutative ring. M. Ashraf, M.A. Siddeeque in the year 2013 showed that 2-torsion free and 3!-torsion free restrictions in the above results used by Bell and Park are superfluous. In fact, Ashraf ([4], theorem 3.2) have obtained that if d is a nonzero permuting n-derivation of prime near-ring N such that $d(N,N,\ldots,N) \subseteq Z$, then N is a commutative ring. In the year 2014 Ashraf ([5],Theorem 3.3) proved that if N is a prime near-ring and d is a nonzero n-derivation of N such that $d(U_1, U_2, \ldots, U_n) \subseteq Z$, where $U_1, U_2, \ldots, U_n$ are nonzero semigroup right ideals of N, then N is a commutative ring. L. Oukhtite , A. Raji ([9] theorem 1) in 2015 proved that if N is a prime near-ring and I is a nonzero semigroup ideal of N and d is a nonzero two sided α-derivation such that $d(I) \subseteq Z(N)$, then N is a commutative ring Motivated by these results we have proved the following theorem in the setting of two sided α- n-derivation associated with an homomorphism α :

Theorem 3.1. Let N be a prime near ring , d a nonzero two sided α- n-derivation of N , and $U_1,U_2,\ldots,U_n$ be a nonzero semigroup ideals of N . If $d(U_1,U_2,\ldots,U_n) \subseteq Z$, then N is a commutative ring.

Proof . We are given that $d(u_1,u_2,\ldots,u_n) \in Z$ for all $u_1 \in U_1$, $u_2 \in U_2,\ldots,u_n \in U_n$. (8)

Hence $t\, d(u_1 u_1',u_2,\ldots,u_n) = d(u_1 u_1',u_2,\ldots,u_n)\, t$ for all u_1 , $u_1' \in U_1$, $u_2 \in U_2,\ldots,u_n \in U_n$, $t \in N$. By lemma 2.6 (iii) we get

$tu_1d(u_1',u_2,\ldots,u_n) + td(u_1,u_2,\ldots,u_n)\alpha(u_1') = u_1d(u_1',u_2,\ldots,u_n)\, t + d(u_1,u_2,\ldots,u_n)\,\alpha(u_1')\, t$.

Using (8) again ,we obtain

$$d(u_1',u_2,\ldots,u_n)\, t\, u_1 + d(u_1,u_2,\ldots,u_n)t\,\alpha(u_1') = d(u_1',u_2,\ldots,u_n)\, u_1t + d(u_1,u_2,\ldots,u_n)\,\alpha(u_1')t . \quad (9)$$

Replacing t by $\alpha(u_1')$ in (9) ,we get

$d(u_1',u_2,...,u_n)\,\alpha(u_1')\,u_1 = d(u_1',u_2,...,u_n)\,u_1\,\alpha(u_1')$ for all $u_1, u_1' \in U_1, u_2 \in U_2,...,u_n \in U_n$, hence $d(u_1',u_2,...,u_n)N[\alpha(u_1'), u_1] = 0$, primeness of N yields either $d(u_1',u_2,...,u_n) = 0$ or $[\alpha(u_1'), u_1] = 0$ .If $d(u_1',u_2,...,u_n) = 0$ then by lemma 2.7 we conclude that $d(N,N,...,N) = \{0\}$, leading to a contradiction as *d* is a nonzero *d* two sided α- n-derivation of N. Therefore there exist $x_1 \in U_1, x_2 \in U_2,...,x_n \in U_n$ all being nonzero such that $d(x_1,x_2,\cdots,x_n) \neq 0$ such that $\alpha(x_1)u = u\alpha(x_1)$ for all $u \in U_1$. Replacing u by ut where $t \in N$, we get $U_1[\alpha(x_1), t] =\{0\}$, for all $t \in N$. By lemma 2.2 we get $\alpha(x_1) \in Z$. Taking x_1 instead of u_1', x_2 instead of u_2,..., x_n instead of u_n in (9) ,we obtain $d(x_1,x_2,...,x_n)\,t\,u_1 = d(x_1,x_2,...,x_n)\,u_1 t$ for all $u_1 \in U_1$, $t \in N$, i.e.;

$d(x_1,x_2,...,x_n)[t, u_1] = 0$, accordingly $d(x_1,x_2,...,x_n)N[t, u_1] = 0$ for all $u_1 \in U_1$, $t \in N$. Primeness of N and $d(x_1,x_2,\cdots,x_n) \neq 0$ yield that $U_1 \subseteq Z$, by lemma 2.3 we conclude that N is a commutative ring .

Corollary 3.1 ([5] Theorem 3.3). Let *N* be a prime near ring, *d* a nonzero *n*-derivation of N, and $U_1, U_2, ..., U_n$ be a nonzero semigroup ideals of N. If $d(U_1,U_2,...,U_n) \subseteq Z$, then *N* is a commutative ring .

Corollary 3.2.([9], Theorem 1). Let N be a prime near ring, d is a nonzero two sided α-n- derivation of *N*, and *U* be a nonzero semigroup ideal of *N* . If $d(U) \subseteq Z$, then *N* is a commutative ring .

As an application of theorem 3.1, we get the following theorems.

Theorem 3.2. Let N be a prime near-ring admitting a nonzero two sided *α*- *n*-derivation . Let $U_1, U_2, ...,U_n$ be nonzero semigroup ideals of N. If $d([u_1, u'_1], u_2, ..., u_n) = 0$, for all $u_1, u'_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, then *N* is a commutative ring**.**

Proof . Since $d([u_1, u'_1], u_2, ..., u_n) = 0$, for all $u_1, u'_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$**.** Replacing u'_1 by $u_1u'_1$ in preceding relation and using it again we get $d(u_1, u_2, ..., u_n)[u_1, u'_1] = 0$,i.e.;

$d(u_1, u_2, ..., u_n)u_1u'_1=d(u_1, u_2, ..., u_n)u'_1 u_1$ (10)

Replacing u'_1 by u'_1r, where $r \in N$, in relation (10) and using it again we get $d(u_1, u_2, ..., u_n)u'_1[u_1, r]= 0$, i.e.; $d(u_1, u_2, ..., u_n)U_1[u_1, r] = \{0\}$, By using lemma 2.1, we conclude that for each $u_1 \in U_1$ either $u_1 \in Z$ or $d(u_1,u_2,...,u_n) = 0$. (11)

Let $x_1 \in U_1 \cap Z$, by lemma 2.4 and defining property of d, we have for all $y \in N$,

$d(x_1y, u_2, ..., u_n) = x_1 d(y, u_2, ..., u_n) + d(x_1, u_2, ..., u_n)\alpha(y) = d(yx_1, u_2, ..., u_n) =$

$d(y, u_2, ..., u_n)x_1 + \alpha(y)d(x_1, u_2, ..., u_n)$, this implies $d(x_1, u_2, ..., u_n)\alpha(y) = \alpha(y)d(x_1, u_2, ..., u_n)$.

Hence , for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, $y \in N$ we get

$d(u_1, u_2, ..., u_n)\alpha(y) = \alpha(y)d(u_1, u_2, ..., u_n)$. (12)

On the other hand , from

$d(x_1t, u_2, ..., u_n) = d(x_1, u_2, ..., u_n)t + \alpha(x_1)d(t, u_2, ..., u_n) = d(tx_1, u_2, ..., u_n) = td(x_1, u_2, ..., u_n) + d(t, u_2, ..., u_n)\alpha(x_1)$ for all $t \in N, u_2 \in U_2,...,u_n \in U_n$. It follows that for all $t \in N, u_2 \in U_2,...,u_n \in U_n$ we get

$$d(x_1, u_2, ..., u_n)t + \alpha(x_1)d(t, u_2, ..., u_n) = td(x_1, u_2, ..., u_n) + d(t, u_2, ..., u_n)\alpha(x_1) \quad (13)$$

In particular , taking $t \in U_1$ in (13) and using (12), we get

$d(x_1, u_2, ..., u_n)t = t\,d(x_1, u_2, ..., u_n)$ for all $t \in U_1, u_2 \in U_2,...,u_n \in U_n$.

Replacing t by ty in the preceding equation , where $y \in N$, we get

$t\,y\,d(x_1, u_2, ..., u_n) = d(x_1, u_2, ..., u_n)\,t\,y = td(x_1, u_2, ..., u_n)\,y$ for all $t \in U_1, u_2 \in U_2,...,u_n \in U_n$, $y \in N$, that is:

$t[d(x_1, u_2, ..., u_n), y] = 0$ for all $t \in U_1, u_2 \in U_2,...,u_n \in U_n$, $y \in N$.

So that

$U_1[d(x_1, u_2, ..., u_n), y] = 0$, by lemma 2.2 we get $d(x_1, u_2, ..., u_n) \in Z$. According to (11) we conclude that $d(u_1, u_2, ..., u_n) \in Z$ for all $u_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, and hence N is commutative ring by application of theorem 3.1.

Corollary 3.3 ([5], Theorem 3.6). Let N be a prime near-ring admitting a nonzero n-derivation d of N. Let $U_1, U_2, ...,U_n$ be nonzero semigroup ideals of N. If $d([u_1, u'_1], u_2, ..., u_n) = 0$, for all $u_1, u'_1 \in U_1, u_2 \in U_2,...,u_n \in U_n$, then N is commutative ring**.**

Corollary 3.4. ([9], Corollary 2). Let N be a prime near-ring admitting a nonzero two sided α-derivation d. Let U be nonzero semigroup ideal of N. If $d([x, y])= 0$, for all $x, y \in U$, then N is commutative ring**.**

If *N* is 2-torsion, the following theorem shows that the conclusion of theorem 3.2 is not true if we replace [x , y] by xoy .

Theorem 3.3. Let N be a 2-torsion free prime near-ring and $U_1, U_2, ..., U_n$ be nonzero semigroup ideals of N , then then there exist no nonzero two sided α- n-derivation d of N such that $d(u_1 o u'_1, u_2, ..., u_n) = 0$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ **.**

Proof . Assume that

$d(u_1 o u'_1, u_2, ..., u_n) = 0$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ **.** (14)

Substituting $u_1 u'_1$ for u'_1 in (14) we obtain $d(u_1(u_1 \circ u'_1), u_2, ..., u_n) = 0$, i.e.;

$d(u_1, u_2, ..., u_n) (u_1 \circ u'_1) + \alpha (u_1)d((u_1 \circ u'_1), u_2, ..., u_n) = 0$. By hypothesis we get $d(u_1, u_2, ..., u_n) (u_1 \circ u'_1) = 0$, i.e.;

$d(u_1, u_2, ..., u_n) u_1 u'_1 = - d(u_1, u_2, ..., u_n) u'_1 u_1$ (15)

Putting $u'_1 z$ for u'_1, where $z \epsilon$ N, in (15) we get $d(u_1, u_2, ..., u_n) u_1 u'_1 z = - d(u_1, u_2, ..., u_n) u'_1 z u_1$ and using (15) again we get($-d(u_1, u_2, ..., u_n) u'_1 u_1)z = - d(u_1, u_2, ..., u_n) u'_1 z u_1$ that is $d(u_1, u_2, ..., u_n) u'_1(-u_1)z + d(u_1, u_2, ..., u_n) u'_1 z u_1 = 0$. Now replacing u_1 by $-u_1$ in preceding relation we have $d(-u_1, u_2, ..., u_n) u'_1 u_1 z + d(-u_1, u_2, ..., u_n) u'_1 z(-u_1) = 0$,i.e.; $d(-u_1, u_2, ..., u_n) u'_1[u_1 z , z u_1] = 0$, that is $d(-u_1, u_2, ..., u_n) U_1[u_1 z , z u_1] = 0$. For each fixed $u_1 \epsilon U_1$ lemma 2.1 yields either $u_1 \epsilon Z$ or $d(-u_1, u_2, ..., u_n) = 0$. Since $d(u_1, u_2, ..., u_n) = - d(-u_1, u_2, ..., u_n)$, so we get

either $u_1 \epsilon Z$ or $d(u_1, u_2, ..., u_n) = 0$ (16)

which is identical with the relation (11) in theorem 3.2. Now arguing in the same way in the theorem 3.2 we conclude that N is a commutative ring**.** In this case , returning to hypothesis ,we find that $d(u_1 u'_1, u_2, ..., u_n) = 0$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ **.** In particular $0 = d((zu_1) u'_1), u_2, ..., u_n) = d(z(u_1 u'_1), u_2, ..., u_n) = d(z, u_2, ..., u_n) u_1 u'_1 + \alpha (z)d(u_1 u'_1, u_2, ..., u_n) = d(z, u_2, ..., u_n) u_1 u'_1$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, $z \epsilon N$. we conclude that $d(z, u_2, ..., u_n) U_1 u'_1 = 0$, since $U_1 \neq 0$, then by lemma 2.1 we get $d(z, u_2, ..., u_n) = 0$ for all $u_2 \epsilon U_2,...,u_n \epsilon U_n$, $z \epsilon N$ which is identical with the relation (6). Now arguing in the same way in the lemma 2.7 we conclude d = 0 ,which contradicts our original assumption that $d \neq 0$.

Corollary 3.5 ([5], Corollary 3.9). Let N be a prime near-ring , then N admits no n-derivation d such that $d(x_1 o x'_1, x_2, ..., x_n) = 0$, for all $x_1, x'_1, x_2 \epsilon,...,x_n \epsilon N$ **.**

In the following two theorems, we assume that the α is an automorphism.

Theorem 3.4 Let N be a prime near-ring admitting a nonzero two sided α- n-derivation d. Let $U_1, U_2, ..., U_n$ be nonzero semigroup ideals of N. If $d([u_1, u'_1], u_2, ..., u_n) = \pm [u_1, u'_1]$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, then N is commutative ring**.**

Proof . Since $d([u_1, u'_1], u_2, ..., u_n) = \pm[u_1, u'_1]$, for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$**.** Replacing u'_1 by $u_1 u'_1$ in preceding relation and using it again we get $d(u_1, u_2, ..., u_n) \alpha ([u_1, u'_1]) = 0$, i.e.;

$d(u_1, u_2, ..., u_n) \alpha (u_1) \alpha (u'_1)=d(u_1, u_2, ..., u_n) \alpha (u'_1) \alpha (u_1)$, let $\alpha (U_1) = V_1$ since α is surjective , then V_1 is a semigroup ideal of N . Now let $\alpha (u'_1) = v_1$, where $v_1 \epsilon V_1$, so we have

$d(u_1, u_2, ..., u_n) \alpha (u_1) v_1 = d(u_1, u_2, ..., u_n) v_1 \alpha (u_1)$. (17)

Replacing v_1 by $v_1 r$,where $r \epsilon N$, in relation (17) and using it again we get

$d(u_1, u_2, ..., u_n) v_1 [\alpha (u_1) , r] = 0$, then we obtain $d(u_1, u_2, ..., u_n) V_1 [\alpha (u_1) , r] =\{0\}$, by lemma 2.1 we get for all $u_1 \epsilon U_1$

either $\alpha (u_1) \epsilon Z$ or $d(u_1, u_2, ..., u_n) = 0$ for all $u_2 \epsilon U_2,...,u_n \epsilon U_n$. (18)

Let $u \epsilon U_1$ such that $d(u, u_2, ..., u_n) = 0$ for all $u_2 \epsilon U_2,...,u_n \epsilon U_n$, then

$d(vu, u_2, ..., u_n) = d(v, u_2, ..., u_n) u + \alpha (v) d(u, u_2, ..., u_n) = d(v, u_2, ..., u_n)u$

and

$d(vu, u_2, ..., u_n) = d(v, u_2, ..., u_n) \alpha (u) + vd(u, u_2, ..., u_n) = d(v, u_2, ..., u_n) \alpha (u)$

for all $v \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$.

Combining both expressions of $d(vu, u_2, ..., u_n)$, we obtain

$d(v, u_2, ..., u_n) (\alpha (u) – u) = 0$ for all $v \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ (19)

Replacing v by vw , where $w \epsilon U_1$, in (19) we get $d(v, u_2, ..., u_n) w(\alpha (u) – u) = 0$ for all $v,w \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, i.e.; $d(v, u_2, ..., u_n) U_1(\alpha (u) – u) = 0$ for all $v \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, by lemma 2.1 we conclude that either $d(v, u_2, ..., u_n) = 0$ for all $v \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ or $\alpha (u) = u$.

If $d(v, u_2, ..., u_n) = 0$, then by lemma 2.7 we conclude d = 0, which contradicts our original assumption that $d \neq 0$.

Hence we conclude that $\alpha (u) = u$, so we get $d(\alpha (u), u_2, ..., u_n) = 0$. According to (18) we arrive at a conclusion

$\alpha(u_1) \in Z$ or $d(\alpha(u_1), u_2, \ldots, u_n) = 0$ for all $u_1 \in U_1$. It follows that for all $v_1 \in V_1$, we get either $v_1 \in Z$ or $d(v_1, u_2, \ldots, u_n) = 0$ which is identical with the relation (11) in theorem 3.2. Now arguing in the same way in the theorem 3.2 we conclude that N is a commutative ring.

Corollary 3.6 ([5], Theorem 3.7) Let N be a prime near-ring admitting a nonzero n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If $d([u_1, u'_1], u_2, \ldots, u_n) = \pm [u_1, u'_1]$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, then N is commutative ring.

Theorem 3.5. Let N be a 2-torsion free prime near-ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N , then N admits no two sided α- n-derivation d associated with a nonzero two sided α- n-derivation d such that $d(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm(u_1 \circ u'_1)$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$.

Proof . We are assuming that for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, we have

$$d(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm(u_1 \circ u'_1) \quad (20)$$

Substituting $u_1 u'_1$ for u'_1 in (20) we obtain $d(u_1(u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$, i.e.; $d(u_1, u_2, \ldots, u_n)\,\alpha(u_1 \circ u'_1) + u_1 d((u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$. By hypothesis we get $d(u_1, u_2, \ldots, u_n)\,\alpha(u_1 \circ u'_1) = 0$, i.e.;

$$d(u_1, u_2, \ldots, u_n)\,\alpha(u'_1)\,\alpha(u_1) = -\,d(u_1, u_2, \ldots, u_n)\,\alpha(u_1)\,\alpha(u'_1) \quad (21)$$

, let $\alpha(U_1) = V_1$ since α is surjective, then V_1 is a semigroup ideal of N. Now let $\alpha(u'_1) = v_1$, where $v_1 \in V_1$, so we have

$$d(u_1, u_2, \ldots, u_n)\,v_1\,\alpha(u_1) = -\,d(u_1, u_2, \ldots, u_n)\,\alpha(u_1)\,v_1 \quad (22)$$

Replacing v_1 by $v_1 r$, where $r \in N$, in relation (22) and using it again we get

$d(u_1, u_2, \ldots, u_n) v_1\, r\, \alpha(u_1) = d(u_1, u_2, \ldots, u_n) v_1\, \alpha(u_1)\, r$, which can be written as for all $u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $r \in N$

$d(u_1, u_2, \ldots, u_n)\, v_1 [\alpha(u_1), r] = 0$, then we obtain $d(u_1, u_2, \ldots, u_n)\, V_1 [\alpha(u_1), r] = \{0\}$, by lemma 2.1 we get for all $u_1 \in U_1$

$$\text{either } \alpha(u_1) \in Z \text{ or } d(u_1, u_2, \ldots, u_n) = 0 \text{ for all } u_2 \in U_2, \ldots, u_n \in U_n. \quad (23)$$

which is identical with the relation (18) in theorem 3.4. An argument similar to that used in the proof of theorem 3.4 shows N is a commutative ring. By 2-torsion freeness of N , we have

$$d(u_1 u'_1, u_2, \ldots, u_n) = u_1 u'_1, \text{ for all } u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n. \quad (24)$$

$$z u_1 u'_1 = d((z u_1) u'_1), u_2, \ldots, u_n) = d(z(u_1 u'_1), u_2, \ldots, u_n) = d(z, u_2, \ldots, u_n)\,\alpha(u_1)\,\alpha(u'_1) + z d(u_1 u'_1, u_2, \ldots, u_n) = d(z, u_2, \ldots, u_n)\,\alpha(u_1)\,\alpha(u'_1) + z u_1 u'_1,$$

so we get $d(z, u_2, \ldots, u_n)\,\alpha(u_1)\,\alpha(u'_1) = 0$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in N$. we conclude that $d(z, u_2, \ldots, u_n)\, v_1 v'_1 = 0$ for all $v_1, v'_1 \in V_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in N$,consequently by lemma 2.1 we obtain that $d = 0$, which contradicts our original assumption that $d \neq 0$.

Corollary3.7. Let N be a 2-torsion free prime near-ring and $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N , then thre is no n-derivation d such that $d(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm(u_1 \circ u'_1)$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$.

Corollary3.13([9], Corollary 6) Let N be a 2-torsion free prime near-ring . N admits no a nonzero two sided α -derivation d such that $d(x \circ y) = x \circ y$ for all $x, y \in N$.

The following example proves that the hypothesis of primness in various theorems is not superfluous. Let S be a 2-torsion free zero-symmetric left near-ring. Let us define :

$$N = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix}, x, y, 0 \in S \right\}$$ is zero symmetric near-ring with regard to matrix addition and matrix multiplication .

$$U_1 = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, x, 0 \in S \right\}$$

Define $d: \underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ such that

$$d\left(\begin{pmatrix} 0 & 0 & 0 \\ x_1 & 0 & y_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ x_2 & 0 & y_2 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & 0 & 0 \\ x_n & 0 & y_n \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 & 0 \\ x_1 x_2 \ldots x_n & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now we define $\alpha : N \rightarrow N$ by

$$\alpha\begin{pmatrix}0 & 0 & 0\\ x & 0 & y\\ 0 & 0 & 0\end{pmatrix} = \begin{pmatrix}0 & 0 & 0\\ y & 0 & x\\ 0 & 0 & 0\end{pmatrix}$$

It is easy to verify that N is not prime near-ring , U_1 is a nonzero semigroup ideal of N and d is a nonzero two sided α- n-derivation of N satisfying

(i) $d(U_1, U_1,..., U_1) \subseteq Z$ (iv) $d([A,B],A_2,...,A_n) = [A,B]$.

(ii) $d([A,B],A_2,...,A_n) = 0$ (v) $d(A \text{ o } B,A_{2,...,}A_n) = A \text{ o } B$

(iii) $d(A \text{ o } B,A_{2,...,}A_n) = 0$ for all $A,B,A_2,...,A_n \in U_1$, but N is not commutative ring.

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