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Characterizing Jordan Higher Centralizers on Triangular Rings through Zero Product

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Abstract

In this paper , we prove that if T is a 2-torsion free triangular ring and $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$ then φ satisfying $X\varphi_i(Y) + \varphi_i(Y)X = 0\ \forall\ i \in N$ whenever $X, Y \in T, XY = YX = 0$ ifand only if φ is a higher centralizer which is means that φ is Jordan higher centralizer on 2-torsion free triangular ring if and only if φ is a higher centralizer and also we prove that if $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$ satisfying the relation $\varphi_n(XYX) = \sum_{i=1}^{n} X\varphi_i(Y)X \quad \forall\ X, Y \in T$, Then φ is a higher centralizer.

Keywords: higher centralizer, Jordan higher centralizer

تمييز تطبيقات جوردان المركزية من الرتب العليا على حلقات المصفوفات المثلثية العليا من خلال الضرب الصفري

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الخلاصة

برهنا في هذا البحث ، اذا كانت T حلقة مصفوفات مثلثية عليا طليقة الالتواء من النمط الثاني و $\varphi = (\varphi_i)_{i\in N}$عائلة من التطبيقات الجمعية $\varphi_i: T \to T$ اذن φ تحقق $X\varphi_i(Y) + \varphi_i(Y)X = 0$ لكل$i \in N$, $X, Y \in T$ بحيث $XY = YX = 0$ اذا وفقط اذا كان φ تطبيق مركزي من الرتب العليا اي انφ يكون تطبيق جوردان المركزي من الرتب العليا على حلقة المصفوفات المثلثية العليا طليقة الالتواء من النمط الثاني اذا وفقط اذا كان φ تطبيق مركزي من الرتب العليا وكذلك برهنا اذا كانت $\varphi = (\varphi_i)_{i\in N}$ عائلة من التطبيقات الجمعية $\varphi_i: T \to T$ التي تحقق $\varphi_n(XYX) = \sum_{i=1}^{n} X\varphi_i(Y)X \quad \forall\ X, Y \in T$ تكون تطبيق مركزي من الرتب العليا.

1. Introduction

Let R be a ring with center Z(R).Recall that an additive map $\varphi: R \to R$ is said to be a right (resp., left)) centralizer if $\varphi(XY) = X\varphi(Y)$ (resp., $\varphi(XY) = \varphi(X)Y$) $\forall\ X, Y \in R$ and is called a centralizer if it is both left and right centralizer .In case R has a unity 1 , φ is a centralizer iff $\varphi(X) = \varphi(1)X\ \forall X \in R$where $\varphi(1) \in Z(R)$. We say that φ is a Jordan centralizer if $\varphi(XY + YX) = X\varphi(Y) + \varphi(Y)X\ \forall\ X, Y \in R$ .Clearly each centralizer is a Jordan centralizer but the converse in general ,not true see [1, Example 2.6] , the question under what conditions that a map becomes a centralizer attracted much attention of

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mathematicians. Vukman [2] has showed that an additive map $\varphi: R \to R$ where R is a 2-torsion free semi-prime ring with the property that $2\varphi(X^2) = X\varphi(X) + \varphi(X)X \quad \forall X \in R$ is a centralizer .Hence any Jordan centralizer on a 2-torsion free semi-prime ring is a centralizer .Vukman [3] has showed the following result if $\varphi: R \to R$ is an additive mapping, where R is a 2-torsion free semi-prime ring satisfying the relation $\varphi(XYX) = X\varphi(Y)X, \ \forall X \in R$ Then φ is a centralizer . In [4] authors present and study the concept of higher (σ, τ) -centralizer ,Jordan higher (σ, τ) –centralizer and Jordan Triple higher (σ, τ) –centralizer and their generalization on the ring . In [5] characterized Jordan derivations of matrix rings through zero product . In this paper , we characterized Jordan higher centralizer on triangular ring through zero product by proving that if T is a 2-torsion free triangular ring and if $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$ then φ satisfying $X\varphi_i(Y) + \varphi_i(Y)X = 0 \ \forall i \in N$ whenever $X, Y \in T, \ XY = YX = 0$ iff φ is a higher centralizer which is means that φ is a Jordan higher centralizer on 2-torsion free triangular ring iff φ is a higher centralizer and also we prove that if $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$satisfying the relation $\varphi_n(XYX) = \sum_{i=1}^{n} X\,\varphi_i(Y)X \quad \forall X, Y \in T$ Then φ is a higher centralizer.

2. Preliminaries

Recall that triangular ring Tri(R,M,S) is a ring of the form

Tri(R,M,S):=$\left\{\begin{bmatrix} r & m \\ 0 & s \end{bmatrix} : r \in R, s \in S, m \in M\right\}$

Under the usual matrix operations ,where R and S are unital rings and M is a unital (R,S)-bimodule which is faithful as a left R-modules as well as a right S-module, the most important example of triangular rings are upper triangular matrices over a ring R Recently ,there has been a growing interest in the study of linear maps that preserve zero products .Throughout this paper ,R and S are unital 2-torsion free rings ,M is a unital 2-torsion free (R,S)-bimodule which is faithful as a left R-module and also as a right S-module .Also T denotes the triangular ring Tri(R,M,S) which is 2-torsion free ring .Let 1_R and 1_S be identities of the rings R and S ,Respectively .We denote the identity of the triangular ring T ,i.e the identity matrix $\begin{bmatrix} 1_R & 0 \\ 0 & 1_S \end{bmatrix}$ by1, also ,throughout this paper we shall use the notation P=$\begin{bmatrix} 1_R & 0 \\ 0 & 0 \end{bmatrix}$ and Q =$\begin{bmatrix} 0 & 0 \\ 0 & 1_S \end{bmatrix}$

We immediately notice that P and Q are the standard idempotents (i.e $P^2 = P$ and $Q^2 = Q$) in T such that P+Q=1 and PQ=QP=0. We should mentioned the reader that the following definitions equivalent to the definitions found in [4, Definition 2.1, 2.3]here we suppose that $\sigma = \tau = I$ and the ring is a triangular ring

Definition 2.1:- Let T be a triangular ring and $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$. then φ is called a Higher Centralizer on T if the following condition satisfies

$$\varphi_n(XY) = \sum_{i=1}^{n} X\,\varphi_i(Y) = \sum_{i=1}^{n} \varphi_i(X)Y \qquad \forall X, Y \in T.$$

Definition 2.2:-Let T be a triangular ring and $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$ then φ is called a Jordan Higher Centralizer on T if the following condition satisfies

$$\varphi_n(XY + YX) = \sum_{i=1}^{n} X\,\varphi_i(Y) + \sum_{i=1}^{n} \varphi_i(Y)X \qquad \forall X, Y \in T.$$

Also $2\varphi_n(X^2) = \sum_{i=1}^{n} X\,\varphi_i(X) + \sum_{i=1}^{n} \varphi_i(X)X \qquad \forall X \in T.$

It is easy to see that every higher centralizer be a Jordan higher centralizer but the converse is not true in general, so we give the following example

Example 2.3:- let A=B= R be a ring such that $x_1 x_2 \neq x_2 x_1$ but $x_1 x_3 = x_3 x_1$ for some $x_1, x_2, x_3 \in R$, $M = \{0\}$ and let $t=(t_i)_{i\in N}$ is a higher centralizer on R. let U= $\left\{\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} : x \in R\right\}$and let $T=(T_i)_{i\in N}$ is a family of additive mapping satisfying

$$T_n\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\right) = \begin{bmatrix} t_n(x) & 0 \\ 0 & t_n(x) \end{bmatrix}$$

It is easy to see that

$$2T_n\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\right) = 2T_n\left(\begin{bmatrix} x^2 & 0 \\ 0 & x^2 \end{bmatrix}\right) = \begin{bmatrix} 2t_n(x^2) & 0 \\ 0 & 2t_n(x^2) \end{bmatrix}$$

Since t is a higher centralizer on R then it is Jordan higher centralizer on R.

So $2T_n\left(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\right) = \begin{bmatrix} \sum_{i=1}^{n}(xt_i(x)+t_i(x)x) & 0 \\ 0 & \sum_{i=1}^{n}(xt_i(x)+t_i(x)x)\end{bmatrix}$

Also

$\sum_{i=1}^{n}(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix} T_i\left(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\right) + T_i\left(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\right)\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix})=$

$$= \sum_{i=1}^{n}\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\begin{bmatrix} t_i(x) & 0 \\ 0 & t_i(x)\end{bmatrix} + \begin{bmatrix} t_i(x) & 0 \\ 0 & t_i(x)\end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}$$

$= \begin{bmatrix} \sum_{i=1}^{n}(xt_i(x)+t_i(x)x) & 0 \\ 0 & \sum_{i=1}^{n}(xt_i(x)+t_i(x)x)\end{bmatrix}$

Then T_n is a Jordan higher centralizer on U. But

$T_n\left(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\begin{bmatrix} y & 0 \\ 0 & y\end{bmatrix}\right) = T_n\left(\begin{bmatrix} xy & 0 \\ 0 & xy\end{bmatrix}\right)$

$=\begin{bmatrix} t_n(xy) & 0 \\ 0 & t_n(xy)\end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} xt_i(y) & 0 \\ 0 & \sum_{i=1}^{n} xt_i(y)\end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} t_i(x)y & 0 \\ 0 & \sum_{i=1}^{n} t_i(x)y\end{bmatrix}$

Since $x_1 x_2 \neq x_2x_1$ but $x_1 x_3 = x_3x_1$ for some $x_1, x_2, x_3 \in$ R.It is easy to see that

$$\sum_{i=1}^{n}\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix} T_i\left(\begin{bmatrix} y & 0 \\ 0 & y\end{bmatrix}\right) = \begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\sum_{i=1}^{n} T_i\left(\begin{bmatrix} y & 0 \\ 0 & y\end{bmatrix}\right) = \begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\begin{bmatrix} \sum_{i=1}^{n} t_i(y) & 0 \\ 0 & \sum_{i=1}^{n} t_i(y)\end{bmatrix}$$

$=\begin{bmatrix} \sum_{i=1}^{n} xt_i(y) & 0 \\ 0 & \sum_{i=1}^{n} xt_i(y)\end{bmatrix} \neq \begin{bmatrix} \sum_{i=1}^{n} xt_i(y) & 0 \\ 0 & \sum_{i=1}^{n} t_i(x)y\end{bmatrix}$

And

$$\sum_{i=1}^{n} T_i\left(\begin{bmatrix} x & 0 \\ 0 & x\end{bmatrix}\right)\begin{bmatrix} y & 0 \\ 0 & y\end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} t_i(x) & 0 \\ 0 & \sum_{i=1}^{n} t_i(x)\end{bmatrix}\begin{bmatrix} y & 0 \\ 0 & y\end{bmatrix}$$

$=\begin{bmatrix} \sum_{i=1}^{n} t_i(x)y & 0 \\ 0 & \sum_{i=1}^{n} t_i(x)y\end{bmatrix} \neq \begin{bmatrix} \sum_{i=1}^{n} xt_i(y) & 0 \\ 0 & \sum_{i=1}^{n} t_i(x)y\end{bmatrix}$

Then T_n is not higher centralizer on U.

3-Result

Theorem 3.1:-Let T be a triangular ring and $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i: T \to T$ then φ satisfying $\forall X, Y \in T$, $X\varphi_i(Y) + \varphi_i(Y)X = 0 \quad \forall i \in N$ whenever XY=YX=0 if and only if φ is a higher centralizer.

Proof:-Let X and Y be arbitrary elements in T.

Since P(QXQ)=(QXQ)P=0.Then

$$P\,\varphi_i\,(QXQ)+\varphi_i\,(QXQ)P=0 \qquad \forall i \in N \tag{3.1}$$

And so

$$\sum_{i=1}^{n}(P\,\varphi_i\,(QXQ) + \varphi_i\,(QXQ)P) = 0$$

Then multiplying this identity by P both on the left and on the right, we find

$2\quad P\,\varphi_i\,(QXQ)P=0 \;\; \forall i = 1, \dots, n$,and so

$$P\,\varphi_i\,(QXQ)\,P=0\;\forall i \in N \tag{3.2}$$

Now, multiplying (3.1) from the left by P and from the right by Q

$$P\,\varphi_i\,(QXQ)\,Q=0 \qquad \forall i \in N \tag{3.3}$$

From Q(PXP)=(PXP)Q=0, we have

$Q\varphi_i$ (PXP) + φ_i (PXP) Q=0 $\forall$ i $\in$ N

$$\sum_{i=1}^{n}(\, Q\, \varphi_i\, (PXP) + \varphi_i\, (PXP)Q) = 0$$

By this identity and using similar methods as above, we obtain

Q φ_i (PXP)Q=0 and P φ_i (PXP)Q=0 $\forall$ i $\in$ N (3.4)

Since (P-PXQ) (Q+PXQ)= (Q+PXQ) (P-PXQ)=0,it follows that

(P-PXQ) $\varphi_i(Q + PXQ) + \varphi_i$(Q+PXQ) (P-PXQ)=0 $\forall$ i $\in$ N .

Multiplying this identity by P both on the left and on the right and by the fact that

P φ_i (Q) P= 0 $\forall$ i $\in$ N , we see that

$$P\ \varphi_i\ (PXQ)P = 0\ \ \forall\, i \in N \tag{3.5}$$

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXQ)P = 0$$

From (PXP-PXPYQ)(Q+PYQ) = (Q+PYQ)(PXP-PXPYQ)=0

(Q+PYQ) φ_i (PXP-PXPYQ)+ φ_i (PXP-PXPYQ) (Q+PYQ)=0 $\forall$ i $\in$ N (3.6)

And so

$$\sum_{i=1}^{n}(Q + PYQ)\ \varphi_i\ (PXP - PXPYQ) + \varphi_i\ (PXP - PXPYQ)\ (Q + PYQ) = 0$$

Let X=P and multiplying above identity by Q both on the left and on the right and the fact that

Q φ_i (P) Q=0 $\forall$ i $\in$ N

And so

$$\sum_{i=1}^{n} Q\ \varphi_i\ (P)Q = 0 \tag{3.7}$$

We obtain

$$\sum_{i=1}^{n} Q\ \varphi_i\ (PYQ)Q = 0$$

Multiplying (3.6) by P on the left and by Q on the right,from (3.4),(3.5)and (3.7)

We arrive P φ_i (PXPYQ) Q=P φ_i (PXP) PYQ $\forall$ i $\in$ N (3.8)

And so

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXPYQ)Q = \sum_{i=1}^{n} P\, \varphi_i\ (PXP)\ PYQ$$

Replacing X by P in above equation ,we get

$$P\ \varphi_i\ (PYQ)\ Q = P\ \varphi_i\ (P)\ \ PYQ\ \ \forall\, i \in N \tag{3.9}$$

$$\sum_{i=1}^{n} P\ \varphi_i\ (PYQ)Q = \sum_{i=1}^{n} P\, \varphi_i\ (P)\ PYQ$$

So from (3.8) and (3.9), it follows that

P φ_i (PXP)PY Q=P φ_i (PXPYQ) Q= P φ_i (P) PXPY Q $\forall$ i $\in$ N

And so

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXP)PY\, Q = \sum_{i=1}^{n} P\, \varphi_i\ (PXPYQ)Q = \sum_{i=1}^{n} P\ \varphi_i\ (P)\ PXPY\, Q$$

And hence (P φ_i (PXP)P-P φ_i (P) PXP) PYQ=0 $\forall$ i $\in$ N (3.10)

$$\sum_{i=1}^{n}(\, P\ \varphi_i\ (PXP)P - P\, \varphi_i\ (P)\ PXP\,)\ PYQ = 0$$

Since $Y \in T$ is arbitrary and M is faithful left R-module ,we find from (3.10)

$$P\ \varphi_i\ (PXP)P = P\ \varphi_i\ (P)\ PXP \qquad \forall\ i \in N \tag{3.11}$$

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXP)P = \sum_{i=1}^{n} P\ \varphi_i\ (P)PXP$$

From (P-PXQ)(PXQYQ+QYQ)=(PXQYQ+QYQ)(P-PXQ)=0

We have

$(P-PXQ)\ \varphi_i(PXQYQ + QYQ) + \varphi_i(PXQYQ + QYQ)(P - PXQ) = 0 \quad \forall\ i \in N$.

And so

$$\sum_{i=1}^{n}(P - PXQ)\varphi_i(PXQYQ + QYQ) + \varphi_i(PXQYQ + QYQ)(P - PXQ) = 0$$

Multiplying this identity by P on the left and by Q on the right ,from (3.2),(3.3),(3.5)and (3.7),we see that $P\ \varphi_i\ (PXQYQ)\ Q=PXQ\ \ \varphi_i\ (QYQ)Q \qquad \forall\ i \in N$ (3.12)

And so

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXQYQ)Q = \sum_{i=1}^{n} PXQ\ \ \varphi_i\ (QYQ)Q$$

Replacing Y by Q in above equation ,we get

$$P\ \varphi_i\ (PXQ)\ Q= PXQ\ \ \varphi_i\ (Q)Q \qquad \forall\ i \in N \tag{3.13}$$

$$\sum_{i=1}^{n} P\ \varphi_i\ (PXQ)Q = \sum_{i=1}^{n} PXQ\ \ \varphi_i\ (Q)Q$$

By (3.12) and (3.13),using similar methods as above and the fact M is a faithful right S-module ,we obtain

$$Q\varphi_i\ (QYQ)Q= QYQ\varphi_i\ (Q)Q \qquad \forall\ i \in N \tag{3.14}$$

And so

$$\sum_{i=1}^{n} Q\ \varphi_i\ (QYQ)Q = \sum_{i=1}^{n} QYQ\varphi_i\ (Q)Q$$

By (3.9) and (3.13),we have

$$P\varphi_i\ (P)PXQ = PXQ\ \varphi_i\ (Q)Q \qquad \forall\ i \in N \tag{3.15}$$

So that

$$\sum_{i=1}^{n} P\varphi_i\ (P)PXQ\ = \sum_{i=1}^{n} PXQ\ \varphi_i\ (Q)Q$$

$$P\varphi_i(P)PXPYQ =PXPYQ\varphi_i(Q)Q=PXP\varphi_i(P)\ PYQ \qquad \forall\ i \in N$$

$$\sum_{i=1}^{n} P\varphi_i(P)PXPYQ = \sum_{i=1}^{n} PXPYQ\varphi_i(Q)Q = \sum_{i=1}^{n} PXP\varphi_i(P)\ PYQ$$

And hence

$$P\varphi_i(P)PXP=PXP\varphi_i(P)\ P \qquad \forall\ i \in N \tag{3.16}$$

$$\sum_{i=1}^{n} P\varphi_i(P)PXP = \sum_{i=1}^{n} PXP\varphi_i(P)P$$

Since M is faithful left R –Module ,similarly ,from (3.15),we get

$$Q\varphi_i(Q)QXQ = QXQ\varphi_i(Q)Q \quad \forall\ i \in N \tag{3.17}$$

Now by (3.2),(3.3),(3.4),(3.15),(3.16) and (3.17)

We have

$$X\varphi_i(1) = PXP\varphi_i(P)P + PXQ\varphi_i(Q)Q + QXQ\varphi_i(Q)Q$$
$$=P\varphi_i(P)PXP + P\varphi_i(P)PXQ + Q\varphi_i(Q)QXQ$$
$$=\varphi_i(1)X \tag{3.18}$$

Then $X\varphi_i\ (1) = \varphi_i(1)X \quad \forall\ i \in N$

And from (3.2),(3.3),(3.4),(3.5),(3.7),(3.9),(3.11),(3.14) and (3.18), we arrive that

$$\varphi_i(X) = P\varphi_i(PXP)P + P\varphi_i(PXQ)Q + Q\varphi_i(QXQ)Q$$
$$=P\varphi_i(P)PXP + P\varphi_i(P)PXQ + QXQ\varphi_i(Q)Q$$

$= \varphi_i(1)X$

Then

$$\varphi_n(X) = \sum_{i=1}^{n} \varphi_i(1)X = \sum_{i=1}^{n} X\,\varphi_i(1)$$

These results show that φ is a higher centralizer.

Since every higher centralizer is a Jordan higher centralizer and every Jordan higher centralizer satisfies the requirements in theorem because

$\varphi_n(XY + YX) = \sum_{i=1}^{n} X\,\varphi_i(Y) + \sum_{i=1}^{n} \varphi_i(Y)X$

If n=1 and XY=YX=0 then $X\varphi_1(Y) + \varphi_1(Y)X = 0$ and if n=2 and XY=YX=0 then $X\varphi_2(Y) + \varphi_2(Y)X = 0$ And so for every n, then the proof is complete.

also the following corollary is clear .

Corollary 3.2:- Let T be a triangular ring and suppose that $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i : T \to T$ then φ is a Jordan higher centralizer if and only if φ is a higher centralizer.

Also ,from this result we can obtain the following corollary

Corollary 3.3:-Let T be a triangular ring and $\varphi = (\varphi_i)_{i\in N}$ be a family of additive mapping $\varphi_i : T \to T$ satisfying the relation

$$\varphi_n(XYX) = \sum_{i=1}^{n} X\,\varphi_i(Y)X \quad \forall\, X, Y \in T.$$

Then φ is a higher centralizer.

Proof:-Let X and Y be arbitrary elements in T , replacing X by X+1 in the above relation, we obtain

$$\varphi_n\big((X+1)Y(X+1)\big) = \sum_{i=1}^{n} (X+1)\,\varphi_i(Y)(X+1)$$

$$\varphi_n(XY + XYX + YX + Y) = \sum_{i=1}^{n} X\varphi_i(Y)X + X\varphi_i(Y) + \varphi_i(Y)X + \varphi_i(Y)$$

Then

$$\varphi_n(XY + YX) = \sum_{i=1}^{n} (X\varphi_i(Y) + \varphi_i(Y)X)$$

So φ is a Jordan higher centralizer and by [Corollary 3.2], it is a higher centralizer.

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