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$\mathcal{J}$-ω*-Continuous and $\mathcal{J}$-ω*- Open Functions

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Abstract:
In ideal topological space, we define and study a new class of functions known as $\mathcal{J}$-ω^*-continuous functions by using $\mathcal{J}$-ω^*-open sets. This idea is stronger than each of the classes (ω-continuous, ωb-continous, α-$\mathcal{J}_\omega$-continuous, pre-$\mathcal{J}_\omega$-continuous, β-$\mathcal{J}_\omega$-continuous, $faintly$- ω-continuous, $slightly$- ω-continuous, $\mathcal{J}_\omega$-c-continuous). Furthermore, relationships between this class and other relevant classes of functions are investigated, and some characterizations of this new class of functions are studied. Finally, we introduce the $\mathcal{J}$-ω^*- open function as an extension of the ω- open function through $\mathcal{J}$-ω^*-open sets, and some results are provided.

Keywords: Ideal topological space, $\mathcal{J}$-ω^*-open set, ω-continuous function, $\mathcal{J}$-ω^*-continuous function, $\mathcal{J}$-ω^*-open function.

الدوال المستمرة−$\mathcal{J}-\omega^*$ و المفتوحة −$\mathcal{J}-\omega^*$

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الخلاصة
في الفضاء التبولوجي المثالي، نقوم بتعريف ودراسة صنف جديد من الدوال تسمى بالدوال المستمرة $\mathcal{J}$− ω^*− باستخدام المجموعات المفتوحة $\mathcal{J}-\omega^*$−. هذه الفكرة أقوى من كل صنف من اصناف الدوال (مستمرة− ω، مستمرة− ω b ، مستمرة− α $\mathcal{J}_\omega$−، مستمرة−قبليا−$\mathcal{J}_\omega$−، مستمرةβ−$\mathcal{J}_\omega$−، مستمرة− ω −ضعيفا، مستمرة− ω −قليلاً، مستمرة$\mathcal{J}_\omega$−c−). علاوة على ذلك، يتم ايجاد العلاقات بين هذا الصنف واصناف الدوال الأخرى ذات الصلة، ودراسة بعض المكافئات لهذا الصنف الجديد من الدوال. أخيرًا، نقدم دالة المفتوحة $\mathcal{J}-\omega^*$− كامتداد للدالة مفتوحة −ω من خلال المجموعات المفتوحة $\mathcal{J}-\omega^*$−، مع تقديم بعض النتائج.

1. Introduction:
The fundamental and central concept in the theory of classical point set topology and various disciplines of mathematics, which has been extensively researched by numerous authors, is continuity of functions. This idea has been developed to include $\mathcal{J}$-continuity of

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functional settings. Jankovice and Hamlett [1] presented the concept of $\boldsymbol{\mathcal{J}}$-open sets in topological spaces. Abd El- Monsef et al. [2] further investigated $\boldsymbol{\mathcal{J}}$-open sets and $\boldsymbol{\mathcal{J}}$-continuity functions. The concept of $\boldsymbol{\alpha}$-$\boldsymbol{\mathcal{J}_{\omega}}$-open ($\boldsymbol{resp.}$ $\boldsymbol{pre}$-$\boldsymbol{\mathcal{J}_{\omega}}$-open, $\boldsymbol{b}$-$\boldsymbol{\mathcal{J}_{\omega}}$-open) sets to obtain decomposition of continuity was presented by O.Ravi et al [3]. Moreover, [4] has established the notions of locally $\boldsymbol{\mathcal{J}_{\omega^*}}$-closed sets and $\boldsymbol{\mathcal{J}\omega}$-$\boldsymbol{c}$-continuous functions. The notions of $\boldsymbol{\mathcal{J}}$-open [**2**] ($\boldsymbol{resp.\,\omega\beta}$-open [**5**]) functions introduced via $\boldsymbol{\mathcal{J}}$-open ($\boldsymbol{resp.\,\omega\beta}$-open) and discussed some properties of them.

Wadei AL-Omeri et al. [6] **e**-$\boldsymbol{\mathcal{J}}$-open, which is a generalization of semi $*$ -$\boldsymbol{\mathcal{J}}$-open sets and pre $*$ -$\boldsymbol{\mathcal{J}}$-open sets, as well as a strong $\mathbf{B}^*{}_{\boldsymbol{\mathcal{J}}}$- set that obtains a decomposition of continuing by idealization. Furthermore, the **e**-$\boldsymbol{\mathcal{J}}$-continuous function was introduced by using **e**-$\boldsymbol{\mathcal{J}}$-open sets in ideal space to characterize and analyze the relationship of **e**-$\boldsymbol{\mathcal{J}}$-continuous functions with other types of functions. N.R. Pachon Rubiano [7] recently introduced the concept of C-continuous and D-continuous functions in an ideal topological space. New forms of functions called semi totally $\boldsymbol{B_c}$-continuous functions and totally $\boldsymbol{B_c}$-continuous functions were introduced in topological space by Jasem and Tawfeeq [8].

The aim of this work is to present a new kind of function. $\mathcal{J}$-ω^*-open sets are used to define and study $\mathcal{J}$-ω^*- continuous functions in ideal topological space. Each of the classes (ω-continuous, ωb-continous, α-$\mathcal{J}_{\omega}$-continuous, pre-$\mathcal{J}_{\omega}$-continuous, β-$\mathcal{J}_{\omega}$-continuous, $faintly$- ω-$c$ontinuous, $slightly$- ω-continuous, $\mathcal{J}_{\omega}$-c-continuous) is weaker than this one. A few characterizations of this new class of functions are also analyzed, in addition to the connections between these classes and other relevant classes. The $\mathcal{J}$-ω*- open function is finally introduced as an extension of the ω- open function through $\mathcal{J}$-ω*-open sets, and some results are shown.

This study is organized as follows: In Section 2, we recall the necessary background on some sets and some functions. In Section 3, by utility $\mathcal{J}$-ω^*-open sets we define a new kind of functions called $\mathcal{J}$-ω^*-continuous function in ideal topological space. Moreover, we try to discuss some fundamental properties. In Section 4, we establish a new category of function by using $\mathcal{J}$-ω^*-open in ideal topological space called $\mathcal{J}$-ω^*-open function which is stronger than $\omega\beta$-open. Moreover, some characterizations of this function are presented. Finally, we conclude the research and suggest future work.

2. Preliminaries:

Throughout this paper, the set of all real ($resp.$ rational, irrational, natural) numbers by R ($resp.\ Q, Irr, N$). An ideal $\mathcal{J}$ on a non-empty set $\mathcal{X}$ is a non-empty collection of subsets of $\mathcal{X}$, which satisfies the following conditions:1- $\mathcal{M} \in \mathcal{J}, \mathcal{H} \subseteq \mathcal{M}$ implies $\mathcal{H} \in \mathcal{J}$. 2- If $\mathcal{M} \in \mathcal{J}, \mathcal{H} \in \mathcal{J}$ implies $\mathcal{M} \cup \mathcal{H} \in \mathcal{J}$ [3]. Let $(\mathcal{X}, \mathcal{T}, \mathcal{J})$ be an ideal topological space and if $P(\mathcal{X})$ is the set of all subsets of $\mathcal{X}$, a set operator [3] $(^*): P(\mathcal{X}) \longmapsto P(\mathcal{X})$, called the local function of $\mathcal{M}$ with respect to $\mathcal{J}$ and ζ is defined as for $\mathcal{M} \subseteq \mathcal{X}$: $\mathcal{M}^* = \{x \in \mathcal{X}: L \cap \mathcal{M} \notin \mathcal{J}$ for each $x \in L \subseteq \zeta\}$. Besides, in [1,9] introduced $cl^*(.)$ defined by $cl^*(\mathcal{M}) = \mathcal{M} \cup \mathcal{M}^*$ which constructs a new topology on $\mathcal{X}$ finer than ζ, it denoted by ζ^* called $*$-topology on $\mathcal{X}$, the members of ζ^* are called ζ^* -open ($*$ - open) sets. We will write the interior of $\mathcal{M}$ by $int^*(\mathcal{M})$ in $(\mathcal{X}, \zeta^*)$ for each subset $\mathcal{M}$ of an ideal space $(\mathcal{X}, \zeta, \mathcal{J})$. By a space, we always signify a topological space $(\mathcal{X}, \mathcal{T})$ with no separation properties assumed. If $\mathcal{M} \subseteq \mathcal{X}$, the closure ($resp.$ interior) of $\mathcal{M}$ is denoted by $cl\mathcal{M}$ ($resp.\ int\mathcal{M}$).

A subset $\mathcal{M}$ of $(\mathcal{X}, \zeta)$ is said to be θ-open [10] ($resp.$ θ_{ω}-open [11]), if for each $x \in \mathcal{M}$, there is an open set $\mathcal{F}$ such that $x \in \mathcal{F} \subseteq cl\mathcal{F} \subseteq \mathcal{M}$ ($resp.$ $x \in \mathcal{F} \subseteq cl_{\omega}\mathcal{F} \subseteq \mathcal{M}$). $\mathcal{M}$ is said to be b-open [12] ($resp.$ β-open [13]), if $\mathcal{M} \subseteq int(cl(\mathcal{M})) \cup cl(int(\mathcal{M}))$ ($resp. \mathcal{M} \subseteq cl\left(int\left(cl(\mathcal{M})\right)\right)$). ω-open [14] ($resp.$ ω^*-open [15], $\mathcal{J}$-ω^*-open [16], ωb-open [17], $\omega\beta$-open [18])) refers to a subset $\mathcal{M}$ of $(\mathcal{X}, \zeta, \mathcal{J})$ in which for any $x \in \mathcal{M}$, there is an open ($resp.$ open ,open, b-open, β-open) set $\mathcal{F}$ containing x such that $\mathcal{F} \setminus \mathcal{M}$($resp. cl\mathcal{F} \setminus \mathcal{M}$, $cl^*\mathcal{F} \setminus \mathcal{M}, \mathcal{F} \setminus \mathcal{M}, \mathcal{F} \setminus \mathcal{M}$) is a countable subset of $\mathcal{X}$. The complement of ω-open ($resp.$ ω^*-open, $\mathcal{J}$-ω^*-open, ωb-open, $\omega\beta$-open) is called ω-closed ($resp.$ ω^*-closed, $\mathcal{J}$-ω^*-closed, ωb-closed, $\omega\beta$-closed). The intersection of all $\mathcal{J}$-ω^*- closed sets containing $\mathcal{M}$ is called $\mathcal{J}$-ω^*-closure of $\mathcal{M}$ and denoted by $cl_{\mathcal{J}\omega^*}(\mathcal{M})$. The $\mathcal{J}$-ω^*- interior of $\mathcal{M}$ is the union of all $\mathcal{J}$-ω^*-open sets contained in $\mathcal{M}$ and denoted by $int_{\mathcal{J}\omega^*}(\mathcal{M})$. The family of all $\mathcal{J}$-ω^*-open ($resp.$ ω-open, ω^*-open, θ-open) sets of ideal space $(\mathcal{X}, \mathcal{T}, \mathcal{J})$ denoted by $\zeta_{I\omega^*}$($resp.$ $\zeta_\omega, \zeta_{\omega^*}, \zeta_\theta$). [3]. A set $\mathcal{M}$ of $(\mathcal{X}, \zeta, \mathcal{J})$ is said to be α-$\mathcal{J}_\omega$-open ($resp.$ pre-$\mathcal{J}_\omega$-open, β-$\mathcal{J}_\omega$-open), if $\mathcal{M} \subseteq int_\omega\left(cl^*(int_\omega \mathcal{M})\right)$ ($resp. \mathcal{M} \subseteq int_\omega(cl^*\mathcal{M}), \mathcal{M} \subseteq cl^*(int_\omega(cl^*\mathcal{M}))$).

A function $f: (X, \zeta) \mapsto (\Upsilon, ð)$ is said to be ωb-continuous [17] ($resp.$ ωβ-continuous [5], θ-continuous [19], θ_ω-continuous [20], $faintly$ ω-continuous [21], ω-continuous [14]), at a point $x \in \mathcal{X}$, if for every open ($resp.$ open, open, open, θ-open, open) set $\mathcal{M}$ in Υ containing $f(x)$, there is an ωb-open ($resp.$ $\omega\beta$-open, open, open, ω-open, ω-open) set L containing x in which $f(L) \subseteq \mathcal{M}$ ($resp.$ $f(L) \subseteq \mathcal{M}$, $f(clL) \subseteq cl\mathcal{M}$, $f(clL) \subseteq cl_\omega(\mathcal{M}), f(L) \subseteq \mathcal{M}, f(L) \subseteq \mathcal{M}$). A function $f: (X, \zeta) \mapsto (\Upsilon, ð)$ will be $faintly$ continuous [22]($resp. quasi$-θ-continuous [23] , $slightly$ ω-continuous [24], b-continuous [17], β-continuous [13]), if the inverse image of every θ-open ($resp.$ θ-open, clopen, open, open) set is open ($resp.$ θ-open, ω-open, b-open, β-open).

Definition 1 [4] A subset $\mathcal{M}$ of an ideal space $(\mathcal{X}, \zeta, \mathcal{J})$ is said to be locally $\mathcal{J}_{\omega^*}$-closed if $\mathcal{M} = L \cap \mathcal{K}$, where L is $*$-open and $\mathcal{K}$ is ω-closed.

Definition 2 A subset $\mathcal{M}$ of an ideal space $(\mathcal{X}, \zeta, \mathcal{J})$ is said to be:
1- $semi$ -$\mathcal{J}_s$- open [25] if $\mathcal{M} \subseteq cl^{*s}(int\mathcal{M})$.
2- $semi$-$\mathcal{J}$-open [26] if $\mathcal{M} \subseteq cl^*(int\mathcal{M})$.
3- $\mathcal{J}$-open set [2] if $\mathcal{M} \subseteq int\mathcal{M}^*$.

Definition 3 A space $\mathcal{X}$ is said to be:
1- locally indiscrete [27] if every open subset of $\mathcal{X}$ is closed.
2- locally countable [28] if every point of $\mathcal{X}$ has a countable open neighbourhood.
3- ω-space [29] if every ω-open set is open.

The following results have been found in [16]:
Theorem 1 The union (intersection) of any family of $\mathcal{J}$-ω^*-open ($\mathcal{J}$-ω^*-closed) sets in any ideal space is $\mathcal{J}$-ω^*-open ($\mathcal{J}$-ω^*-closed).

Proposition 1 Every ω^*-open set in ideal space $(\mathcal{X}, \zeta, \mathcal{J})$ is an $\mathcal{J}$-ω^*-open.

Proposition 2 In any ideal space $(\mathcal{X}, \zeta, \mathcal{J})$, $\zeta_\theta \subseteq \zeta_{\omega^*} \subseteq \zeta_{I\omega^*} \subseteq \zeta_\omega$.
Corollary 1 Every $\mathcal{J}$ -ω^*-open set is α-$\mathcal{J}_\omega$- open ($resp.$ pre-$\mathcal{J}_\omega$–open and β-$\mathcal{J}_\omega$–open).

Theorem 2 Let $(\mathcal{X}, \zeta, \mathcal{J})$ be an ideal space and Υ is a clopen subset of $\mathcal{X}$, then $(\zeta_\Upsilon)_{\mathcal{J}\omega^*} = (\zeta_{\mathcal{J}\omega^*})_\Upsilon$.

Recall that an ideal space $(\mathcal{X}, \zeta, \mathcal{J})$ is $R\mathcal{J}$-space [30] if for each $x \in \mathcal{X}$ and each open set $\mathcal{M}$

containing x, there exists an open set L such that $x \in L \subseteq cl^*L \subseteq \mathcal{M}$.

Theorem 3 Let $(\mathcal{X}, \zeta, \mathcal{I})$ be an $R\mathcal{I}$-space, then $\zeta_\omega = \zeta_{\mathcal{I}\omega^*}$.
Theorem 4 If $(\mathcal{X}, \zeta, \mathcal{I})$ is an ideal topological space and $\mathcal{X}$ is an ω-space, then $\zeta_{\mathcal{I}\omega^*} \subseteq \zeta$.
Proof
From Proposition 2, we have $\zeta_{\mathcal{I}\omega^*} \subseteq \zeta_\omega$ and $(\mathcal{X}, \zeta)$ is an ω-space; then, $\zeta_\omega = \zeta$. Hence, $\zeta_{\mathcal{I}\omega^*} \subseteq \zeta$.

The results from [15] are as follows:
Theorem 5 A space $(\mathcal{X}, \zeta)$ is regular if and only if $\zeta_\theta = \zeta$.

Theorem 6 If $(\mathcal{X}, \zeta)$ is a regular space, then $\zeta \subseteq \zeta_{\omega^*}$ and $\zeta_{\omega^*} = \zeta_\omega$.

Definition 4 function $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, ð)$ is said to be $semi$-$\mathcal{I}_S$- continuous [25] ($resp.$ $semi$-$\mathcal{I}$-continuous [25], $\mathcal{I}$-continuous [2] , α-$\mathcal{I}_\omega$-continuous [3], pre-$\mathcal{I}_\omega$-continuous [3], β-$\mathcal{I}_\omega$-continuous [3], $\mathcal{I}_\omega$-c-continuous [4]) if $f^{-1}(\mathcal{V})$ is $semi$-$\mathcal{I}_S$-open ($resp.$ $semi$-$\mathcal{I}$-open, $\mathcal{I}$-open, α-$\mathcal{I}_\omega$- $open$, pre-$\mathcal{I}_\omega$-open, β-$\mathcal{I}_\omega$-open, locally $\mathcal{I}_{\omega^*}$-closed) in $(\mathcal{X}, \zeta, \mathcal{I})$ for each $\mathcal{V} \in ð$.

Theorem 7 [31] If function $f : (\mathcal{X}, \zeta) \longmapsto (\Upsilon, ð)$ is ω-continuous, then it is $faintly$ ω-continuous.

Theorem 8 [31] If $f: (\mathcal{X}, \zeta) \longmapsto (\Upsilon, ð)$ is a $faintly$ ω -continuous function, then it is $slightly$ ω - continuous.

The following results are from [5]:
Theorem 9 Let $f: (X, \zeta) \longmapsto (\Upsilon, ð)$ be a function then, every ω-continuous is ωb-continuous function.

Theorem 10 Let $f: (X, \zeta) \longmapsto (\Upsilon, ð)$ be a function then, every ωb-continuous is $\omega\beta$-continuous function.

Corollary 2 [32] Every θ-continuous function is $quasi$- θ-continuous.

Theorem 11 [14] function $f : (X, \zeta) \longmapsto (\Upsilon, ð)$ is ω-continuous function if and only if $f^{-1}(L) \in \zeta_\omega$, for each $L \in ð$.

3. $\mathcal{I}$-ω*-continuous functions:

We introduce a new class of function called $\mathcal{I}$-ω^*- continuous functions in ideal topological space. Moreover, we investigate the relationships of $\mathcal{I}$-ω^*- continuous functions and some other classes of functions. We started with the following results we needed for this work:

Corollary 3 Let $(\mathcal{X}, \zeta, \mathcal{I})$ be an ideal space such that $(\mathcal{X}, \zeta)$ is locally indiscrete. Then, $\zeta_{\omega^*} = \zeta_{\mathcal{I}\omega^*} = \zeta_\omega$.
Proof
From Proposition 1 and Proposition 2, we have $\zeta_{\omega^*} \subseteq \zeta_{\mathcal{I}\omega^*} \subseteq \zeta_\omega$. Now, let $\mathcal{M} \in \zeta_\omega$. If $\mathcal{M} = \emptyset$, then $\mathcal{M} \in \zeta_{\omega^*}$. Otherwise, for each $x \in \mathcal{M}$, there exists an open set $\mathcal{F}$ in $\mathcal{X}$ containing x such that $\mathcal{F} \backslash \mathcal{M}$ is countable. Since $(\mathcal{X}, \zeta)$ is a locally indiscrete space, $\mathcal{F} = cl\mathcal{F}$. Hence, $cl\mathcal{F} \backslash \mathcal{M}$ is countable, so $\mathcal{M} \in \zeta_{\omega^*}$. Thus, $\zeta_\omega \subseteq \zeta_{\omega^*} \subseteq \zeta_{\mathcal{I}\omega^*} \subseteq \zeta_\omega$. Therefore, they are the same.

Theorem 12 Let $\mathcal{M} \subseteq \mathcal{X}$. Then, $\mathcal{M}$ is $\mathcal{I}$-ω^*- open if and only if $\mathcal{M} = int_{\mathcal{I}\omega^*}(\mathcal{M})$.
Proof
It is clear.

Theorem 13 For any subset $\mathcal{M}$ and $\mathcal{H}$ of an ideal space $(\mathcal{X}, \zeta, \mathcal{I})$ we have:
1- $\mathcal{M} \subseteq cl_{\mathcal{I}\omega^*}(\mathcal{M})$.
2- $x \in cl_{\mathcal{I}\omega^*}(\mathcal{M})$ if and only if $\mathcal{M} \cap L \neq \emptyset$ for all $L \in \zeta_{\mathcal{I}\omega^*}$.
3- $\mathcal{M}$ is $\mathcal{I}$-ω^*-closed if and only if $cl_{\mathcal{I}\omega^*}(\mathcal{M}) = \mathcal{M}$.
4- $cl_{\mathcal{I}\omega^*}(\mathcal{X}) = \mathcal{X}$ and $cl_{\mathcal{I}\omega^*}(\emptyset) = \emptyset$.
5- $cl_{\mathcal{I}\omega^*}(cl_{\mathcal{I}\omega^*}(\mathcal{M})) = cl_{\mathcal{I}\omega^*}(\mathcal{M})$.
6- If $\mathcal{M} \subseteq \mathcal{H}$, then $cl_{\mathcal{I}\omega^*}(\mathcal{M}) \subseteq cl_{\mathcal{I}\omega^*}(\mathcal{H})$.
Proof
It is clear.

Corollary 4 For any subset $\mathcal{M}$ of an ideal space $(\mathcal{X}, \zeta, \mathcal{I})$. The statements are correct:
1- $\mathcal{X} \backslash int_{\mathcal{I}\omega^*}(\mathcal{M}) = cl_{\mathcal{I}\omega^*}(\mathcal{X} \backslash \mathcal{M})$.
2- $\mathcal{X} \backslash cl_{\mathcal{I}\omega^*}(\mathcal{M}) = int_{\mathcal{I}\omega^*}(\mathcal{X} \backslash \mathcal{M})$.
3- $int_{\mathcal{I}\omega^*}(\mathcal{M}) = \mathcal{X} \backslash cl_{\mathcal{I}\omega^*}(\mathcal{X} \backslash \mathcal{M})$.
4- $cl_{\mathcal{I}\omega^*}(\mathcal{M}) = \mathcal{X} \backslash int_{\mathcal{I}\omega^*}(\mathcal{X} \backslash \mathcal{M})$.
Proof
It is clear.

Definition 5 A function $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$ is said to be $\mathcal{I}$-ω^*- continuous at a point $x \in \mathcal{X}$ if for each open set $\mathcal{M}$ in $(\Upsilon, \eth)$ containing $f(x)$ there is $L \in \zeta_{\mathcal{I}\omega^*}$ containing x such that $f(L) \subseteq \mathcal{M}$. If f is $\mathcal{I}$-ω^*- continuous at each point of $\mathcal{X}$, then f is said to be $\mathcal{I}$-ω^*- continuous on $\mathcal{X}$.

Theorem 14 For a function $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$, the following statements are equivalent:
1- f is $\mathcal{I}$-ω^*-continuous;
2- $f^{-1}(\mathcal{M})$ is $\mathcal{I}$-ω^*-open in $\mathcal{X}$, for each open set $\mathcal{M} \subseteq \Upsilon$;
3- $f^{-1}(\mathcal{H})$ is $\mathcal{I}$-ω^*-closed in $\mathcal{X}$, for each closed set $\mathcal{H} \subseteq \Upsilon$;
4- $f(cl_{\mathcal{I}\omega^*}(\mathcal{M})) \subseteq cl(f(\mathcal{M}))$, for each $\mathcal{M} \subseteq \mathcal{X}$;
5- $cl_{\mathcal{I}\omega^*}(f^{-1}(\mathcal{P})) \subseteq f^{-1}(cl(\mathcal{P}))$, for each $\mathcal{P} \subseteq \Upsilon$;
6- $f^{-1}(int\mathcal{P}) \subseteq int_{\mathcal{I}\omega^*}(f^{-1}(\mathcal{P}))$, for each $\mathcal{P} \subseteq \Upsilon$.
Proof
(1) ⇒ (2) Let $\mathcal{M}$ be open set in Υ. If $f^{-1}(\mathcal{M}) = \emptyset$, then there is nothing to prove. Otherwise, for $x \in f^{-1}(\mathcal{M})$,we have $f(x) \in \mathcal{M}$. Since f is $\mathcal{I}$-ω^*- continuous, then there is $L_x \in \zeta_{\mathcal{I}\omega^*}$ containing x such that $f(L_x) \subseteq \mathcal{M}$. Thus, $x \in L_x \subseteq f^{-1}(\mathcal{M})$ consequently $f^{-1}(\mathcal{M}) = \bigcup_{x \in f^{-1}(\mathcal{M})}\{x\} \subseteq \bigcup_{x \in f^{-1}(\mathcal{M})} L_x \subseteq f^{-1}(\mathcal{M})$. Hence, $f^{-1}(\mathcal{M}) = \bigcup_{x \in f^{-1}(\mathcal{M})} L_x$, where $L_x \in \zeta_{\mathcal{I}\omega^*}$ for each x. Then $f^{-1}(\mathcal{M})$ is union of any family of $\mathcal{I}$-ω^*- open sets. Subsequently by Theorem 1, $f^{-1}(\mathcal{M}) \in \zeta_{\mathcal{I}\omega^*}$.
(2) ⇒ (3) Let $\mathcal{H}$ be any closed subset of Υ. Then, $\Upsilon \backslash \mathcal{H}$ is open in Υ. Consequently by (2), $f^{-1}(\Upsilon \backslash \mathcal{H}) = \mathcal{X} \backslash f^{-1}(\mathcal{H})$ is $\mathcal{I}$-ω^*-open in $\mathcal{X}$. Thus, $f^{-1}(\mathcal{H})$ is $\mathcal{I}$-ω^*-closed in $\mathcal{X}$.
(3) ⇒ (4) Let $\mathcal{M} \subseteq \mathcal{X}$. Then, $f(\mathcal{M}) \subseteq \Upsilon$. Since $cl(f(\mathcal{M}))$ being a closed set in Υ so by (3), $f^{-1}(cl(f(\mathcal{M})))$ is $\mathcal{I}$-ω^*- closed in $\mathcal{X}$ and $\mathcal{M} \subseteq f^{-1}(cl(f(\mathcal{M})))$, then $cl_{\mathcal{I}\omega^*}(\mathcal{M}) \subseteq f^{-1}(cl(f(\mathcal{M})))$. Hence, $f(cl_{\mathcal{I}\omega^*}(\mathcal{M})) \subseteq cl(f(\mathcal{M}))$.
(4) ⇒ (5) Let $\mathcal{P} \subseteq \Upsilon$. Then, $f^{-1}(\mathcal{P}) \subseteq \mathcal{X}$ and by (4) $f\left(cl_{\mathcal{I}\omega^*}(f^{-1}(\mathcal{P}))\right) \subseteq clf(f^{-1}(\mathcal{P}))$.

Thus, $f\left(cl_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{P})\right)\right) \subseteq cl(\mathcal{P})$. Therefore, $cl_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{P})\right) \subseteq f^{-1}(cl(\mathcal{P}))$.
(5)⇒ (6) Let $\mathcal{P} \subseteq \Upsilon$. Then, $\Upsilon\backslash\mathcal{P} \subseteq \Upsilon$. Therefore, by (5) $cl_{\mathcal{J}\omega^*}(f^{-1}(\Upsilon\backslash\mathcal{P})) \subseteq f^{-1}(cl(\Upsilon\backslash\mathcal{P}))$. That is, $cl_{\mathcal{J}\omega^*}(\mathcal{X}\backslash f^{-1}(\mathcal{P})) \subseteq f^{-1}(\Upsilon\backslash int(\mathcal{P}))$. Thus by (1) of Corollary 4 implies that $\mathcal{X}\backslash int_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{P})\right) \subseteq \mathcal{X}\backslash f^{-1}(int(\mathcal{P})))$. Hence, $f^{-1}(int\mathcal{P}) \subseteq int_{\mathcal{J}\omega^*}(f^{-1}(\mathcal{P}))$.
(6) ⇒ (1) Let L be any open subset of Υ. Then, by (5), $f^{-1}(intL) \subseteq int_{\mathcal{J}\omega^*}\left(f^{-1}(L)\right)$. This implies that $f^{-1}(L) \subseteq int_{\mathcal{J}\omega^*}\left(f^{-1}(L)\right)$ and always $int_{\mathcal{J}\omega^*}\left(f^{-1}(L)\right) \subseteq f^{-1}(L)$. Hence, $f^{-1}(L) = int_{\mathcal{J}\omega^*}\left(f^{-1}(L)\right)$. So by (3) of Theorem 12, $f^{-1}(L) \in \zeta_{\mathcal{J}\omega^*}$. Therefore, f is $\mathcal{J}$-ω^*-continuous function.

Proposition 3 Every $\mathcal{J}$-ω^*-continuous function is ω-continuous.
Proof
It follows directly through Proposition 2, $\zeta_{\mathcal{J}\omega^*} \subseteq \zeta_\omega$.

The converse of Proposition 3 is not true in general, as illustrated by the example below:

Example 1 Consider the ideal topological spaces $(R, \zeta, \mathcal{J})$ and $(\Upsilon, ð)$ where $\zeta = \{\emptyset, R, Q\}, \Upsilon = \{0,1\}, ð = \{\emptyset, \Upsilon, \{1\}\}$ and $\mathcal{J} = \{\emptyset\}$. We define a function $f: (R, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ as:

$$f(x) = \begin{cases} 1 & x \in Q \\ 0 & x \in Irr. \end{cases}$$

Then, f is an ω-continuous function, but it is not $\mathcal{J}$-ω^*-continuous.

Corollary 5 Every $\mathcal{J}$-ω^*-continuous function is $faintly$ ω-continuous.
Proof
It arises directly from Proposition 3 and Theorem 7.
However, the converse of Corollary 5 is not true in general, as shown by the example below:

Example 2 Consider the ideal space $(R, \zeta, \mathcal{J})$, where $\zeta = \{\emptyset, R, Q\}$, $\mathcal{J} = \{\emptyset\}$ and $(\Upsilon, ð)$, where $\Upsilon = \{1, \sqrt{2}, 2\}$ with $ð = \{\emptyset, \{1\}, \{\sqrt{2}\}, \{1, \sqrt{2}\}, \Upsilon\}$. Define the function $f: (R, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ by:

$$f(x) = \begin{cases} 1 & 1 \in Q \\ \sqrt{2} & \sqrt{2} \in Irr. \end{cases}$$

Then, f is $faintly$ ω-continuous function, but it is not $\mathcal{J}$-ω^*-continuous.

Proposition 4 Every $\mathcal{J}$-ω^*-continuous function is $slightly$ ω-continuous.
Proof
It follows from Corollary 5 and Theorem 8.
The example below explains that the opposite of Proposition 4 is untrue in general:

Example 3 Let $\mathcal{X} = R$ with $\zeta = \{\emptyset, R, Irr\}$ and $\mathcal{J} = \{\emptyset\}$. Let $\Upsilon = R$ with $ð = \{\emptyset, R, Q\}$. Then, the identity function $f: (\mathcal{X}, \zeta, I) \longmapsto (\Upsilon, ð)$ is slightly ω-continuous function but it is not $\mathcal{J}$-ω^*-continuous.

Corollary 6 Every $\mathcal{J}$-ω^*-continuous function is an ωb-continuous function.
Proof
It follows directly through Proposition 3 and Theorem 9.

The following example illustrates that the converse of Corollary 6 is incorrect:

Example 4 In the ideal space $(R, \zeta, \mathcal{J})$, where $\zeta = \{\emptyset, R, Q\}$ with $\mathcal{J} = \{\emptyset\}$ and space $(\Upsilon, ð)$, where $\Upsilon = \{0,1\}$ and $ð = \{\emptyset, \{0\}, \Upsilon\}$. Let $f: (R, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be a function defined as:

$$f(x) = \begin{cases} 0 & x \in Q \\ & \\ 1 & x \in Irr . \end{cases}$$

Then, f is ωb-continuous function, but it is not $\mathcal{J}$-ω^*-continuous.

Corollary 7 Every $\mathcal{J}$-ω^*-continuous function is an $\omega\beta$-continuous function.
Proof
It follows from Corollary 6 and Theorem 10.

However, the opposite of Corollary 7 is untrue, as shown by the following example:

Example 5 Let $\mathcal{X} = R$ with the usual ideal topology $\zeta, \mathcal{J} = \{\emptyset\}$ and $\Upsilon = \{0,1\}$ with the $ð = \{\emptyset, \Upsilon, \{0\}\}$. The function $f: (X, \zeta, I) \longmapsto (\Upsilon, ð)$ is defined as:

$$f(x) = \begin{cases} 0 & x \in Q \\ & \\ 1 & x \in Irr. \end{cases}$$

Then, f is $\omega\beta$-continuous, but it is not $\mathcal{J}$-ω^*-continuous.

Corollary 8 Every $\mathcal{J}$-ω^*-continuous function is α- $\mathcal{J}_\omega$-continuous ($resp.\ pre$ - $\mathcal{J}_\omega$-continuous and β- $\mathcal{J}_\omega$-continuous).
Proof
Proposition 3 and Corollary 1 provide the proof.

The following example shows that the converse of Corollary 8 is untrue in general:

Example 6 1- Let $(R, \zeta, \mathcal{J})$ and $(\Upsilon, ð)$ be two ideal spaces where ζ is usual topology, $\mathcal{J} = F$ (the ideal of all finite subsets of R, $\Upsilon = R$ and $ð = \{\emptyset, \Upsilon, Q\}$. Then, the identity function $f: (R, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is pre - $\mathcal{J}_\omega$-continuous and β- $\mathcal{J}_\omega$-continuous, but it is not an $\mathcal{J}$-ω^*-continuous function.

2- Consider $(\mathcal{X}, \zeta, \mathcal{J})$ and $(\Upsilon, ð)$, where $\mathcal{X} = R$, $\zeta = \{\emptyset, \mathcal{X}, Q\}, \mathcal{J} = \{\emptyset\}$, $\Upsilon = R$ and $ð = \{\emptyset, R, Q\}$. Then, the identity function $f(R, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is an α- $\mathcal{J}_\omega$-continuous function but it is not $\mathcal{J}$-ω^*-continuous.

Corollary 9 Every $\mathcal{J}$-ω^*-continuous function is $\mathcal{J}_\omega$-c-continuous.
Proof
Proposition 3 and [4, Remark 6.2] provide the proof.
The converse of Corollary 9 is incorrect as shown by the example below:

Example 7 Consider the usual ideal space $(R, \zeta_u, \mathcal{J})$ and the space $(\Upsilon, ð)$ where $\mathcal{J} = \{\emptyset\}$, $\Upsilon = R$ with $ð = \{\emptyset, R, Q\}$. Then, the identity function $f: (R, \zeta, \mathcal{J}) \longmapsto (R, ð)$ is $\mathcal{J}_\omega$-c-continuous. To see this, $Q \in ð$ and Q are ω-closed. Since every ω-closed is locally $\mathcal{J}_\omega$-closed [4, Remark 6.2]. Hence, f is $\mathcal{J}_\omega$-c-continuous, but it is not $\mathcal{J}$-ω^*-continuous function.

Example 8 1- The spaces $\mathcal{X} = \Upsilon = R$ with the topologies $\zeta = ð = \{\emptyset, R, Q\}$ and $\mathcal{J} = \{\emptyset\}$. Then, the identity function $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is continuous, $quasi$-θ-continuous, b-continuous, β-continuous, $faintly$ continuous but it is not $\mathcal{J}$-ω^*-continuous function.
2- Let $\mathcal{X} = \{1,2,3,4\}$, $\zeta = \{\emptyset, \mathcal{X}, \{1\}, \{2\}, \{1,2\}\}$, $ð = \{\emptyset, \mathcal{X}, \{1,3\}, \{2,4\}\}$ and $\mathcal{J} = \{\emptyset\}$. Then, the identity function $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\mathcal{X}, ð)$ is an $\mathcal{J}$-ω^*-continuous function, but it is not continuous, $faintly$ continuous, θ-continuous and $quasi$-θ-continuous.
3- Let $(\mathcal{X}, \zeta, \mathcal{J})$ be ideal space where $\mathcal{X} = \{1,2,3\}$ with the topology $\zeta = \{\emptyset, \mathcal{X}, \{1\}, \{2\}, \{1,2\}\}, \mathcal{J} = \{\emptyset\}$ and $(\Upsilon, ð)$ be a space where $\Upsilon = \{m, n\}$ with the topology $ð = \{\emptyset, \Upsilon, \{n\}\}$. The function $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is defined by:

$$f(x) = \begin{cases} m & x \in \{1,2\} \\ n & x = 3. \end{cases}$$

Then, clearly f is an $\mathcal{J}$-ω^*-continuous function, but it is neither b-continuous nor β-continuous.
4- Consider the identity function $f: (R, \zeta, \mathcal{J}) \longmapsto (R, ð)$, where $\zeta = \{\emptyset, R\}$, $\mathcal{J} = \{\emptyset\}$ and $ð = \{\emptyset, R, Q\}$. Then, f is θ-continuous ($resp.$ θ_ω-continuous),but it is not $\mathcal{J}$-ω^*-continuous function.
5- Consider the identity function $f: (N, \zeta, \mathcal{J}) \longmapsto (N, ð)$, where $\zeta = ð = \{\emptyset, N, \{2\}\}$. Then, f is an $\mathcal{J}$-ω^*-continuous function but it is not θ_ω-continuous. Therefore, $x = 2$ and $\mathcal{K} = \{2\}$ are placed in ð with $f(2) = 2 \in \mathcal{K}$ and $cl_\omega(\mathcal{K}) = \{2\}$. There exists $L = \{2\}$ or $L = N$ in ζ containing x and $clL = N$ if $L = \{2\}$ or $L = N$ such that $f(clL) = N \nsubseteq cl_\omega(\mathcal{K}) = \{2\}$. Hence, f is not θ_ω-continuous.

Example 9 1- Let $(\mathcal{X}, \zeta, \mathcal{J})$ be ideal space where $\mathcal{X} = \{2,3,4\}$ with $\zeta = \{\emptyset, \mathcal{X}, \{2\}, \{3\}, \{2,3\}\}$, $\mathcal{J} = P(R)$ and $(\Upsilon, ð)$ be a space where $\Upsilon = \{m, n\}$ with the topology $ð = \{\emptyset, \Upsilon, \{m\}\}$. Then, the function $f: (X, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is defined as:

$$f(x) = \begin{cases} m & x = \{4\} \\ n & x \in \{2,3\}. \end{cases}$$

Then, f is an $\mathcal{J}$-ω^*-continuous function, but it is neither $semi$-$\mathcal{J}_s$-continuous nor $semi$-$\mathcal{J}$-continuous.
2- Let $\mathcal{X} = \Upsilon = R$ with $\zeta = ð = \{\emptyset, R, Q\}$ and $\mathcal{J} = \{\emptyset\}$. Then, the identity function $f: (X, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ is $semi$-$\mathcal{J}_s$-continuous and $semi$-$\mathcal{J}$- continuous, but it is not $\mathcal{J}$-ω^*-continuous.
3- Consider the identity function $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$, where $\mathcal{X} = \Upsilon = \{1,2,3,4\}, \tau = \{\emptyset, \mathcal{X}\}$, $ð = P(\Upsilon)$ and $\mathcal{J} = \{\emptyset, \{3\}\}$. Then, f is an $\mathcal{J}$-ω^*-continuous function but it is not $\mathcal{J}$-continuous since $\{3\} \in ð$, and $f^{-1}(\{3\}) = \{3\}$ is not $\mathcal{J}$-open in $(\mathcal{X}, \zeta)$.
4- Consider the spaces $(R, \zeta_u, \mathcal{J})$ and $(R, ð)$ where $\mathcal{J} = F$ (the ideal of all finite subsets of R) and $ð = \{\emptyset, R, Q\}$. Then, the identity function f: $(R, \zeta_u, \mathcal{J}) \longmapsto (R, ð)$ is $\mathcal{J}$-continuous since $Q \in ð$ and $f^{-1}(\{Q\}) = \{Q\}$, $Q^* = R$. Implies that $Q \subseteq int(Q^*) = intR = R$. However, f is not $\mathcal{J}$-ω^*-continuous function.

Remark 1 1- From Example 8, we notice that the concept of $\mathcal{J}$-ω^*-continuous function with each of the classes continuous, b-continuous, β-continuous, $faintly$ continuous, θ-continuous, θ_ω-continuous and $quasi$-θ-continuous are independent.
2- From Example 9, we note that the concepts of $\mathcal{J}$-ω^*-continuous function with each of the classes $semi$-$\mathcal{J}_s$-continuous, $semi$-$\mathcal{J}$- continuous and $\mathcal{J}$-continuous are independent.

From Proposition 3, Corollary 5, Proposition 4, Corollary 6, Corollary 8 and Corollary 9 we have the Diagram below:

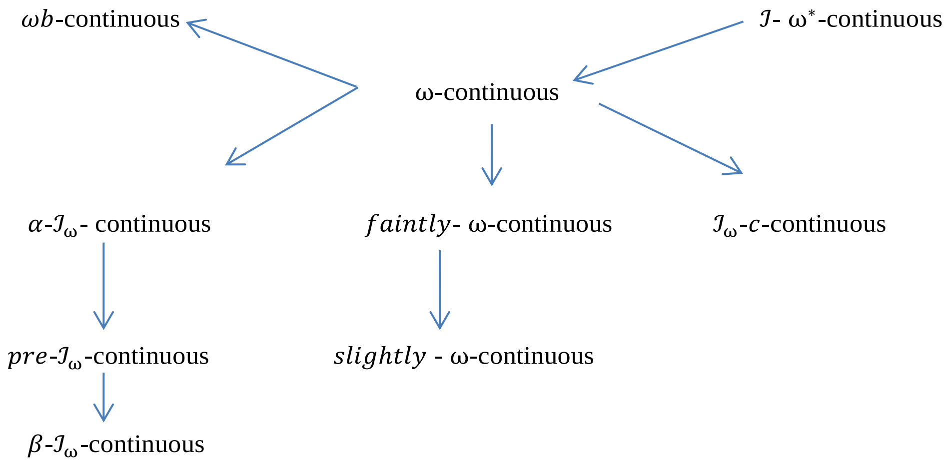

Diagram-1

Proposition 5 Let $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$ be an $\mathcal{I}$-ω^*-continuous function. If $\mathcal{M}$ is clopen subset of $\mathcal{X}$, then $f_{\mathcal{M}}: (\mathcal{M}, \zeta_{\mathcal{M}}, \mathcal{I}_{\mathcal{M}}) \to (\Upsilon, \eth)$ is $\mathcal{I}$-ω^*-continuous.
Proof
Let L be any open subset of Υ. Since f is an $\mathcal{I}$-ω^*-continuous function so by Theorem 14, $f^{-1}(L) \in \zeta_{\mathcal{I}\omega^*}$. Since $\mathcal{M}$ is clopen subset in $\mathcal{X}$ by Theorem 2, $(f_{\mathcal{M}})^{-1}(L) = f^{-1}(L) \cap \mathcal{M}$ is an $\mathcal{I}$-ω^*-open subspace of $\mathcal{M}$. This shows that $f_{\mathcal{M}}: \mathcal{M} \to \Upsilon$ is $\mathcal{I}$-ω^*-continuous.

Proposition 6 Let $f: (X, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$ be a function and let $\{\mathcal{M}_{\leftthreetimes} : \leftthreetimes \in \Delta\}$ be a clopen cover of $\mathcal{X}$. If the restriction function $f_{\mathcal{M}}: (\mathcal{M}_{\leftthreetimes}, \zeta_{\mathcal{M}_{\leftthreetimes}}, \mathcal{I}_{\mathcal{M}_{\leftthreetimes}}) \to (\Upsilon, \eth)$ is $\mathcal{I}$-ω^*-continuous, for any $\leftthreetimes \in \Delta$, then f is $\mathcal{I}$-ω^*-continuous.
Proof
Let $f_{\mathcal{M}_{\leftthreetimes}}: (\mathcal{M}_{\leftthreetimes}, \zeta_{\mathcal{M}_{\leftthreetimes}}, \mathcal{I}_{\mathcal{M}_{\leftthreetimes}}) \longmapsto (\Upsilon, \eth)$ be an $\mathcal{I}$-ω^*-continuous function for each $\leftthreetimes \in \Delta$ and let L be any open subset of Υ. Consequently by Theorem 14, $(f_{\mathcal{M}_{\leftthreetimes}})^{-1}(L)$ is an $\mathcal{I}$-ω^*-open set in $\mathcal{M}_{\leftthreetimes}$ for each $\leftthreetimes \in \Delta$. However, $(f_{\mathcal{M}_{\leftthreetimes}})^{-1}(L) = f^{-1}(L) \cap \mathcal{M}_{\leftthreetimes}$ is an $\mathcal{I}$-ω^*-open set in $\mathcal{M}_{\leftthreetimes}$ for each $\leftthreetimes \in \Delta$. $\mathcal{M}_{\leftthreetimes}$ is clopen for each $\leftthreetimes \in \Delta$. Then by Theorem 2, $f^{-1}(L) \cap \mathcal{M}_{\leftthreetimes}$ is $\mathcal{I}$-ω^*-open in $\mathcal{X}$ $\forall$ $\leftthreetimes \in \Delta$. Therefore, $f^{-1}(L) = \cup_{\leftthreetimes \in \Delta}(f^{-1}(L) \cap \mathcal{M}_{\leftthreetimes})$ is an $\mathcal{I}$-ω^*-open set in $(\mathcal{X}, \zeta, \mathcal{I})$. Therefore, $f^{-1}(L) \in \zeta_{\mathcal{I}\omega^*}$. Hence by Theorem 14, f is an $\mathcal{I}$-ω^*-continuous function.

Corollary 10 Let $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$ be function and let $\{\mathcal{M}_{\leftthreetimes} : \leftthreetimes \in \Delta\}$ be clopen cover of $\mathcal{X}$. If the restriction function $f_{\mathcal{M}_{\leftthreetimes}}: (\mathcal{M}_{\leftthreetimes}, \zeta_{\mathcal{M}_{\leftthreetimes}}, \mathcal{I}_{\mathcal{M}_{\leftthreetimes}}) \longmapsto (\Upsilon, \eth)$ is an $\mathcal{I}$-ω^*-continuous function for each $\leftthreetimes \in \Delta$, then f is ω-continuous.
Proof
It directly arises from Proposition 6 and Proposition 3.

Corollary 11 Let $(\mathcal{X}, \zeta, \mathcal{I})$ be an $R\mathcal{I}$-space. Then, a function $f: (\mathcal{X}, \zeta, \mathcal{I}) \longmapsto (\Upsilon, \eth)$ is $\mathcal{I}$-ω^*-continuous if and only if it is ω-continuous.
Proof
Theorem 3 and Proposition 3 provide the proof.

Corollary 12 Let $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be a function and $(\mathcal{X},\zeta)$ be locally indiscrete space. Then, the following properties are equivalent:
1- f is $\mathcal{J}$-ω^*-continuous;
2- f is ω-continuous.
Proof
Proof
(1) ⇒ (2) It arises from Proposition 3.
(2) ⇒ (1) It arises from Corollary 3.

Proposition 7 Let $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be an $\mathcal{J}$-ω^*-continuous function and $\mathcal{X}$ be an ω-space. Then, f is continuous.
Proof
It follows Theorem 4.

Theorem 15 Let $(\mathcal{X},\zeta)$ be a regular space and $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be function. Then,
1- If f is continuous, then it is $\mathcal{J}$- ω^*-continuous.
2- f is $\mathcal{J}$- ω^*-continuous if and only if it is ω-continuous.
Proof
1- Let L be any open set in Υ. Since f is a continuous function, then $f^{-1}(L) \in \zeta$. However, $\mathcal{X}$ is a regular space then by Theorem 6, $f^{-1}(L) \in \zeta_{\omega^*}$. So by Proposition 1, $f^{-1}(L) \in \zeta_{\mathcal{J}\omega^*}$. Therefore, by Theorem 14, f is $\mathcal{J}$- ω^*-continuous.
2- Let L be any open set in Υ and let f be $\mathcal{J}$- ω^*-continuous. As a consequently of Theorem 14, $f^{-1}(L) \in \zeta_{\mathcal{J}\omega^*}$. According to Proposition 2, $f^{-1}(L) \in \zeta_{\omega}$. Hence by Theorem 11, f is ω-continuous.
Conversely, let L be any open subset of Υ and f be an ω-continuous function. Then, $f^{-1}(L) \in \zeta_{\omega}$. Since $\mathcal{X}$ is regular space according to Theorem 6, $f^{-1}(L) \in \zeta_{\omega^*}$. Since, every ω^*-open set is $\mathcal{J}$- ω^*-open so $f^{-1}(L) \in \zeta_{\mathcal{J}\omega^*}$. Therefore by Theorem 14, f is $\mathcal{J}$- ω^*-continuous.

Proposition 8 Let $(\mathcal{X},\zeta)$ be a regular space and $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be a $quasi$ - θ – continuous function. Then, f is $\mathcal{J}$-ω^*-continuous ($resp.$ ω-continuous).
Proof
Let $\mathcal{H}$ be any open set in Υ. Since $\mathcal{X}$ is a regular space, Theorem 5 implies that $\mathcal{H}$ is a θ-open set in Υ. However, f is a $quasi$ - θ – continuous function, so $f^{-1}(\mathcal{H})$ is a θ-open set in $\mathcal{X}$. Hence by Proposition 2, $f^{-1}(\mathcal{H}) \in \zeta_{\mathcal{J}\omega^*}$ ($resp.$ $f^{-1}(\mathcal{H}) \in \zeta_{\omega}$). Thus, f is $\mathcal{J}$-ω^*-continuous ($resp.$ ω-continuous).

Corollary 13 Let $(\mathcal{X},\zeta)$ be a regular space and $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be a θ-continuous function. Then, f is $\mathcal{J}$-ω^*-continuous ($resp.$ ω-continuous).
Proof
It arises from Corollary 2 and Proposition 8.

Theorem 16 Let $f:\mathcal{X} \longmapsto \Upsilon$ be an $\mathcal{J}$-ω^*-continuous function and Υ be a subspace of $\mathcal{Z}$. Then, $f:\mathcal{X} \to \mathcal{Z}$ is $\mathcal{J}$-ω^*-continuous.
Proof
Let L be any open set in $\mathcal{Z}$. Then, $L \cap \Upsilon$ is open in Υ. Since f being $\mathcal{J}$-ω^*-continuous so by Theorem 14, $f^{-1}(L \cap \Upsilon) \in \zeta_{\mathcal{J}\omega^*}$. However, $f(x) \in \Upsilon$ for each $x \in \mathcal{X}$. Therefore, $f^{-1}(L) = f^{-1}(L \cap \Upsilon) \in \zeta_{\mathcal{J}\omega^*}$. Hence by Theorem 14, $f:\mathcal{X} \to \mathcal{Z}$ is $\mathcal{J}$-ω^*-continuous.

The next example shows that the composition of two $\mathcal{J}$-ω^*-continuous functions are not necessary to be $\mathcal{J}$-ω^*-continuous:

Example 10 Let $\mathcal{X} = R$ with topology $\zeta = \{\emptyset, R, Irr\}$ and $\mathcal{J} = \{\emptyset\}$. $\Upsilon = \{1,2\}$ with the topology $ð = \{\emptyset, \Upsilon, \{1\}\}$ and the ideal $\mathcal{J} = \{\emptyset\}$. Let $\mathcal{Z} = \{a, b\}$ with $\mathcal{P} = \{\emptyset, \mathcal{Z}, \{a\}\}$. Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be the function defined by:

$$f(x) = \begin{cases} 1 & x \in Irr \\ 2 & x \in Q \end{cases}$$

and $\Phi: (\Upsilon, ð, \mathcal{J}) \to (\mathcal{Z}, \mathcal{P})$ be the function defined by:

$$\Phi(x) = \begin{cases} a & x = 2 \\ b & x = 1. \end{cases}$$

Therefore,

$$(\Phi o f)(x) = \begin{cases} b & x \in Irr \\ a & x \in Q \end{cases}$$

Then, f is $\mathcal{J}$-ω^*-continuous, and Φ is $\mathcal{J}$-ω^*-continuous. However, $\Phi o f$ is not $\mathcal{J}$-ω^*-continuous since $(\Phi o f)^{-1}(\{a\}) = Q$ which is uncountable.

Theorem 17 Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be $\mathcal{J}$-ω^*-continuous and $\Phi: (\mathcal{X}, \Upsilon, ð) \longmapsto (\mathcal{Z}, \mathcal{P})$ be continuous. Then, $\Phi o f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\mathcal{Z}, \mathcal{P})$ is $\mathcal{J}$-ω^*-continuous.
Proof
Let $\mathcal{M}$ be any open subset of $\mathcal{Z}$. Since Φ being continuous, then $\Phi^{-1}(\mathcal{M}) \in ð$. Since f is $\mathcal{J}$-ω^*-continuous, then by Theorem 14, $(\Phi o f)^{-1}(\mathcal{M}) = f^{-1}\left(\Phi^{-1}(\mathcal{M})\right) \in \zeta_{\mathcal{J}\omega^*}$. As a consequence of Theorem 14, $\Phi o f$ is $\mathcal{J}$-ω^*-continuous.

Theorem 18 Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be $\mathcal{J}$-ω^*-continuous, $\Phi: (\Upsilon, ð) \longmapsto (\mathcal{Z}, \rho)$ be ω-continuous and $(\Upsilon, ð)$ an ω-space. Then, $\Phi o f: \mathcal{X} \longmapsto \mathcal{Z}$ is $\mathcal{J}$-ω^*-continuous.
Proof
Let $\mathcal{H}$ be any open subset of $\mathcal{Z}$. Since Φ is an ω-continuous function, then $\Phi^{-1}(\mathcal{H}) \in ð_{\omega}$. Since Υ is ω-space, $\Phi^{-1}(\mathcal{H})$ is open in Υ. Since f is $\mathcal{J}$-ω^*-continuous subsequently by Theorem 14, $(\Phi o f)^{-1}(\mathcal{H}) = f^{-1}(\Phi^{-1}(\mathcal{H})) \in \zeta_{\mathcal{J}\omega^*}$. According to Theorem 14, $\Phi o f$ is $\mathcal{J}$-ω^*-continuous.

Corollary 14 If $\mathcal{X} = \mathcal{M} \cup \mathcal{H}$, where $\mathcal{M}$ and $\mathcal{H}$ are clopen sets and $f: \mathcal{X} \longmapsto \Upsilon$ is a function in which both $f_{\mathcal{M}}: \mathcal{M} \longmapsto \Upsilon$ and $f_{\mathcal{H}}: \mathcal{H} \longmapsto \Upsilon$ are $\mathcal{J}$-ω^*-continuous, then f is $\mathcal{J}$-ω^*-continuous.
Proof
It follows Proposition 6.

Theorem 19 Let $f, \Phi: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be $\mathcal{J}$-ω^*-continuous functions and Υ be a Hausdorff space. Then, the set $\mathcal{H} = \{x \in X: f(x) = \Phi(x)\}$ is $\mathcal{J}$-ω^*-closed in $\mathcal{X}$.
Proof
Suppose that $x \notin \mathcal{H}$. Then, $f(x) \neq \Phi(x)$. Since Υ is Hausdorff, there exist open sets L_1 and L_2 of Υ such that $f(x) \in L_1$, $\Phi(x) \in L_2$ and $L_1 \cap L_2 = \emptyset$. Since f and Φ are $\mathcal{J}$-ω^*-continuous, then there exist $\mathcal{J}$-ω^*-open sets $\mathcal{F}_1$ and $\mathcal{F}_2$ in $\mathcal{X}$ containing x so that $f(\mathcal{F}_1) \subseteq \mathcal{F}_1$ and $\Phi(\mathcal{F}_2) \subseteq L_2$. The set $\mathcal{M} = \mathcal{F}_1 \cap \mathcal{F}_2 \in \zeta_{\mathcal{J}\omega^*}$ contains x where $\mathcal{M} \cap \mathcal{H} = \emptyset$. Therefore by

Theorem 13, we obtain that $x \notin cl_{\mathcal{J}\omega^*}(\mathcal{H})$. This indicated that $\mathcal{H}$ is $\mathcal{J}$-ω^*-closed in $\mathcal{X}$.

Theorem 20 Let $f: \mathcal{X} \longmapsto \Upsilon$ be a function and $\mathcal{B}$ be any basis of Υ. Then, f is $\mathcal{J}$-ω^*-continuous if and only if for each $\mathcal{B} \in \beta$, $f^{-1}(\mathcal{B}) \in \zeta_{\mathcal{J}\omega^*}$.

Proof

Let f be $\mathcal{J}$- ω^*-continuous. Since each $\mathcal{B} \in \beta$ is open subset of Υ and f is $\mathcal{J}$-ω^*-continuous consequently by Theorem 14, $f^{-1}(\mathcal{B}) \in \zeta_{\mathcal{J}\omega^*}$.

Conversely, let for each $\mathcal{B} \in \beta$, $f^{-1}(\mathcal{B}) \in \zeta_{\mathcal{J}\omega^*}$. Let $\mathcal{M}$ be any open set in Υ. Then, $\mathcal{M} = \cup \mathcal{B}_i, i \in \Delta$, where $\mathcal{B}$ is a member of β and Δ is a suitable index set. It follows that $f^{-1}(\mathcal{M}) = \{f^{-1}(\cup \mathcal{B}_i), i \in \Delta\} = \{\cup (f^{-1}\mathcal{B}_i): i \in \Delta\}$. However, $f^{-1}(\mathcal{B}_i) \in \zeta_{\mathcal{J}\omega^*}$ for each $i \in \Delta$. Therefore, $f^{-1}(\mathcal{M})$ is the union of a family of $\mathcal{J}$-ω^*-open subsets of $\mathcal{X}$. Hence, $f^{-1}(\mathcal{M}) \in \zeta_{\mathcal{J}\omega^*}$. Accordingly Theorem 14, f is $\mathcal{J}$-ω^*-continuous.

Theorem 21 Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be a function and $\Phi: (\mathcal{X}, \zeta, \mathcal{J}) \rightarrow (\mathcal{X} \times \Upsilon, \zeta \times ð)$ the graph function of f, defined by $\Phi(x) = (x, f(x))$ for every $x \in \mathcal{X}$. If Φ is $\mathcal{J}$-ω^*-continuous, then f is $\mathcal{J}$-ω^*-continuous.

Proof

Let $\mathcal{M}$ be an open set in Υ. Then, $\mathcal{X} \times \mathcal{M}$ is an open set in $\mathcal{X} \times \Upsilon$. Since Φ is $\mathcal{J}$-ω^*-continuous, subsequently by Theorem 14, $\Phi^{-1}(\mathcal{X} \times \mathcal{M} \in \zeta_{\mathcal{J}\omega^*}$. However, $f^{-1}(\mathcal{M}) = \Phi^{-1}(\mathcal{X} \times \mathcal{M})$. Hence, f is $\mathcal{J}$-ω^*-continuous.

Theorem 22 Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be a function. Then, $\mathcal{X} \backslash \omega^*{}_{\mathcal{J}}\mathrm{C}(f) = \{B_{\mathcal{J}\omega^*}(f^{-1}(\mathcal{M}): \mathcal{M} \in ð, f(x) \in \mathcal{M}, x \in \mathcal{X}\}$ at which $\omega^*{}_{\mathcal{J}}\mathrm{C}(f)$ means the set of all points make f to be $\mathcal{J}$-ω^*-continuous.

Proof

Let $x \in \mathcal{X} \backslash \omega^*{}_{\mathcal{J}}\mathrm{C}(f)$. Then, $\exists\, \mathcal{M} \in ð$ containing $f(x)$, for each $x \in \mathcal{H} \in \zeta_{\mathcal{J}\omega^*}$ such that $f(\mathcal{H}) \nsubseteq \mathcal{M}$. Thus, $\mathcal{H} \cap \left(X \backslash f^{-1}(\mathcal{M})\right) \neq \emptyset$ for any $\mathcal{J}$-ω^*-open sets $\mathcal{M}$ containing x. As a consequence of Theorem 13, $x \in cl_{\mathcal{J}\omega^*}\left(\mathcal{X} \backslash f^{-1}(\mathcal{M})\right)$. Therefore, $x \in (f^{-1}(\mathcal{M}) \cap cl_{\mathcal{J}\omega^*}\left(\mathcal{X} \backslash f^{-1}(\mathcal{M})\right)) \subseteq B_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{M})\right)$. Hence, $\mathcal{X} \backslash \omega^*{}_{\mathcal{J}}\mathrm{C}(f) \subseteq \cup \{B_{\mathcal{J}\omega^*}(f^{-1}(\mathcal{M}): \mathcal{M} \in \sigma, f(x) \in \mathcal{M}, x \in \mathcal{X}\}$.

Conversely, let $x \notin \mathcal{X} \backslash \omega^*{}_{\mathcal{J}}\mathrm{C}(f)$. Then, for each $\mathcal{M} \in ð$ containing $f(x)$, $f^{-1}(\mathcal{M}) \in \zeta_{\mathcal{J}\omega^*}$ containing x thus, for every $\mathcal{M} \in ð$ containing $f(x)$, $x \in int_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{M})\right)$ and hence $x \notin B_{\mathcal{J}\omega^*}\left(f^{-1}(\mathcal{M})\right)$. Therefore, $\cup \{B_{\mathcal{J}\omega^*}(f^{-1}(\mathcal{M}): \mathcal{M} \in ð, f(x) \in \mathcal{M}, x \in \mathcal{X}\} \subseteq \mathcal{X} \backslash \omega^*{}_{\mathcal{J}}\mathrm{C}(f)$.

Theorem 23 Let $f: (\mathcal{X}, \zeta, \mathcal{J}) \longmapsto (\Upsilon, ð)$ be a function from locally indiscrete $(\mathcal{X}, \zeta)$ to locally indiscrete $(\Upsilon, ð)$. Then the following are equivalent:

1-f is $\mathcal{J}$-ω^*-continuous;

2- f is ω-continuous;

3- For each $x \in \mathcal{X}$ and each open set $\mathcal{F}$ in Υ containing $f(x)$, there is $\mathcal{M} \in \zeta_\omega$ contains x such that $f(\mathcal{M}) \subseteq int_\sigma cl_\sigma(\mathcal{F})$;

4- For each $x \in \mathcal{X}$ and each open set $\mathcal{F}$ in Υ containing $f(x)$, there is $\mathcal{M} \in \zeta_\omega$ contains x such that $f(\mathcal{M}) \subseteq int_{\omega_\sigma}(cl_\sigma(\mathcal{F}))$;

5- For each $x \in \mathcal{X}$ and each open set $\mathcal{F}$ in Υ containing $f(x)$, there is $\mathcal{M} \in \zeta_\omega$ contains x such that $f(\mathcal{M}) \subseteq cl_\sigma(\mathcal{F})$.

Proof

It arises from the definitions and Corollary 3.

Proposition 9 The open image of $\mathcal{J}$-ω^*-open set in locally indiscrete space to locally indiscrete space is $\mathcal{J}$-ω^*-open.
Proof
Let $f:(\mathcal{X},\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð)$ be an open function where $(\mathcal{X},\zeta),(\Upsilon,ð)$ are locally indiscrete spaces and let $\mathcal{H} \in \zeta_{\mathcal{J}\omega^*}$. Let $y \in f(\mathcal{H})$. Then, there exists $x \in \mathcal{H}$ such that $y = f(x)$ is satisfied. We choose $x \in \mathcal{F} \in \zeta$ at which $cl^*\mathcal{F}\backslash\mathcal{H} = C$ is countable; because $\mathcal{X}$ is locally indiscrete subsequently by Corollary 3, $cl^*\mathcal{F}\backslash\mathcal{H} = \mathcal{F}\backslash\mathcal{H} = C$ is countable. Since f is open, then $f(\mathcal{F}) \in ð$ such that $f(x) \in f(\mathcal{F})$ and $f(\mathcal{F})\backslash f(\mathcal{H}) \subseteq f(\mathcal{F}\backslash\mathcal{H}) = f(C)$ are countable. Therefore, $f(\mathcal{H})$ is ω-open, but Υ is locally indiscrete according to Corollary 3, $f(\mathcal{H}) \in ð_{J\omega^*}$.

4. $\mathcal{J}$-ω^*-open function:

In this section, we present a new class of functions called $\mathcal{J}$-ω^*-open functions that are defined by the notion $\mathcal{J}$-ω^*-open in ideal topological space.

Definition 6 A function $f:(\mathcal{X},\zeta) \longmapsto (\Upsilon,ð,\mathcal{J})$ is called $\mathcal{J}$-ω^*-open ($resp.$ $\omega\beta$-open [5]) , if $f(\mathcal{H})$ is $\mathcal{J}$-ω^*-open ($resp.$ $\omega\beta$-open) in Υ, for each open set $\mathcal{H}$ in $\mathcal{X}$.

Proposition 10 Every $\mathcal{J}$-ω^*-open is $\omega\beta$-open function.
Proof
It arises from Proposition 2 and every ω-open set is ωβ-open sets see [5].

The example below demonstrated that the converse of Proposition 10, does not hold:

Example 11 Consider the space $(\mathcal{X},\zeta)$ where $\mathcal{X} = \{0,1\}$, $\zeta = \{\emptyset,\mathcal{X},\{0\}\}$ and the usual ideal space $(R,\zeta_u,\mathcal{J})$ where $\mathcal{J} = \{\emptyset\}$. The function $f:(\mathcal{X},\zeta) \longmapsto (R,\zeta_u,\mathcal{J})$ defined by:

$$f(x) = \begin{cases} 0 & x \in \mathrm{Q} \\ 1 & x \in \mathrm{Irr.} \end{cases}$$

Then f is $\omega\beta$-open but it is not $\mathcal{J}$-ω^*-open.

The conception of $\mathcal{J}$-ω^*-open function is independent with the class $\mathcal{J}$-ω^*-continuous function the following examples show that:

Example 12 Let $f:(R,\zeta,\mathcal{J}) \longmapsto (\Upsilon,ð,\mathcal{K})$ be function, where $(R,\zeta,\mathcal{J})$ and $(\Upsilon,ð,\mathcal{K})$ are ideal spaces with $\zeta = ð = \{\emptyset,R,Q\}$, $\mathcal{J} = \mathrm{P}(R)$ and $\mathcal{K} = \{\emptyset\}$. Then, identity function f is $\mathcal{J}$-ω^*-continuous but it is not $\mathcal{J}$-ω^*-open.

Example 13 Let the ideal spaces $(R,\zeta,\mathcal{J}),(R,ð,\mathcal{K})$, where $\zeta = \{\emptyset,R,Q\}, \mathcal{J} = \{\emptyset\}$,$ð = \{\emptyset,R,Q\}$, and $\mathcal{K} = \mathrm{P(R)}$. Then, the identity function $f:(R,\zeta,I) \longmapsto (R,ð,\mathcal{K})$ is $\mathcal{J}$-ω^*-open but it is not $\mathcal{J}$-ω^*-continuous.

Theorem 24 Let $f:(\mathcal{X},\zeta) \longmapsto (\Upsilon,ð,\mathcal{J})$ be a function. Then the following are equivalent:
1- f is an $\mathcal{J}$-ω^*-open function ;

2- For each $\mathcal{A} \subseteq \mathcal{X}$, $f(int\,\mathcal{A}) \subseteq int_{\mathcal{J}\omega^*}(f(\mathcal{A}))$;
3- For each $x \in \mathcal{X}$ and each neighbourhood $\mathcal{U}$ of x in $\mathcal{X}$, there exists an $\mathcal{J}$-ω^*-open set $\mathcal{V}$ of $f(x)$ in Υ such that $\mathcal{V} \subseteq f(\mathcal{U})$.

Proof
(1) ⇒ (2) Suppose that f is an $\mathcal{J}$-ω^*-open function, and let $\mathcal{A}$ be an arbitrary subset of $\mathcal{X}$. Therefore, int$\mathcal{A}$ is open subset of $\mathcal{X}$.Since f is $\mathcal{J}$-ω^*-open, then $f(int\mathcal{A})$ is $\mathcal{J}$-ω^*-open subset of Υ. Since $int\mathcal{A} \subseteq \mathcal{A}$, so $f(int\mathcal{A}) \subseteq f(\mathcal{A})$ but $int_{\mathcal{J}\omega^*}(f(\mathcal{A}))$ is the largest $\mathcal{J}$-ω^*-open set contained in $f(\mathcal{A})$. That is, $f(int\mathcal{A}) \subseteq int_{\mathcal{J}\omega^*}(f(\mathcal{A}))$.
(2) ⇒ (3) We suppose that (2) is hold and let $x \in \mathcal{X}$ and $\mathcal{U}$ be an arbitrary neighbourhood of x in $\mathcal{X}$. Implies that, there exists an open set $\mathcal{O}$ in $\mathcal{X}$ such that $x \in \mathcal{O} \subseteq \mathcal{U}$. Since $\mathcal{O} = \text{int}\mathcal{O}$, subsequently by assumption, $f(\mathcal{O}) = f(int\mathcal{O}) \subseteq int_{\mathcal{J}\omega^*}(f(\mathcal{O}))$. Thus, $f(\mathcal{O}) \subseteq int_{\mathcal{J}\omega^*}(f(\mathcal{O}))$. Then, $f(\mathcal{O})$ is $\mathcal{J}$-ω^*-open in Υ. Therefore, $f(x) \in f(\mathcal{O}) \subseteq f(\mathcal{U})$, hence $\mathcal{V} = f(\mathcal{U})$ is the requisite $\mathcal{J}$-ω^*-neighbourhood of $f(x)$.
(3) ⇒ (1) Let (3) be hold. To show f is an $\mathcal{J}$-ω^*-open function. let $\mathcal{U}$ be any open subset of $\mathcal{X}$. Consequently, by hypothesis for each $x \in \mathcal{U}$, there exists an $\mathcal{J}$-ω^*-neighbourhood $\mathcal{V}_{f(x)}$ of $f(x)$ in Υ such that $\mathcal{V}_{f(x)} \subseteq f(\mathcal{U})$. Now, since $\mathcal{V}_{f(x)}$ is an $\mathcal{J}$-ω^*-neighbourhood of $f(x)$, then there exists an $\mathcal{J}$-ω^*-open set $\mathcal{A}_{f(x)}$ in Υ such that $f(x) \subseteq \mathcal{A}_{f(x)} \subseteq \mathcal{V}_{f(x)} \subseteq f(\mathcal{U})$. So, $f(\mathcal{U}) = \bigcup_{f(x) \in f(\mathcal{U})} \mathcal{A}_{f(x)}$. Because of $\bigcup_{f(x) \in f(\mathcal{U})} \mathcal{A}_{f(x)}$ is $\mathcal{J}$-ω^*-open, then $f(\mathcal{U})$ is $\mathcal{J}$-ω^*-open set in Υ. Hence, $f$is $\mathcal{J}$-ω^*-open.

Theorem 25 A function $f: (\mathcal{X}, \zeta) \mapsto (\Upsilon, ð, \mathcal{J})$ is $\mathcal{J}$-ω^*-open if and only if for every $\mathcal{B} \subseteq \Upsilon$, $f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq clf^{-1}(\mathcal{B})$.

Proof
Let $\mathcal{B} \subseteq \Upsilon$. To show $f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq clf^{-1}(\mathcal{B}))$. Let $\in f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B}))$. Then $f(x) \in cl_{\mathcal{J}\omega^*}(\mathcal{B})$. Assume $\mathcal{H}$ be any open subset of $\mathcal{X}$ such that $x \in \mathcal{H}$ so $f(x) \in f(\mathcal{H})$. Since f is an $\mathcal{J}$-ω^*-open function, then $f(\mathcal{H})$ is an $\mathcal{J}$-ω^*-open subset of Υ. According to Theorem 13, $f(\mathcal{H}) \cap \mathcal{B} \neq \emptyset$. This indicates that, $\mathcal{H} \cap f^{-1}(\mathcal{B}) \neq \emptyset$. Hence, $x \in cl\, f^{-1}(\mathcal{B})$. Therefore, $f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq cl\, f^{-1}(\mathcal{B})$.
Conversely, let, $f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq clf^{-1}(\mathcal{B})$ for each $\mathcal{B} \subseteq \Upsilon$. To show f is an $\mathcal{J}$-ω^*-open function. Let $\mathcal{A}$ be any open subset of $\mathcal{X}$. Then, $\mathcal{B} = \Upsilon \backslash f(\mathcal{A}) \subseteq \Upsilon$. Subsequently by assumption, $f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq clf^{-1}(\mathcal{B})$, then $\mathcal{A} \cap f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) \subseteq \mathcal{A} \cap clf^{-1}(\mathcal{B})$. Since, $\mathcal{A} \cap f(\mathcal{B}) = \emptyset$, then $\mathcal{A} \cap \text{cl}f^{-1}(\mathcal{B}) = \emptyset$. Therefore, $\mathcal{A} \cap f^{-1}(cl_{\mathcal{J}\omega^*}(\mathcal{B})) = \emptyset$. Implies that, $f(\mathcal{A}) \cap cl_{\mathcal{J}\omega^*}(\mathcal{B}) = \emptyset$. Hence, $cl_{\mathcal{J}\omega^*}(\mathcal{B}) \subseteq \Upsilon \backslash f(\mathcal{A}) = \mathcal{B}$. This means that $\mathcal{B}$ is $\mathcal{J}$-ω*-closed, so $\Upsilon \backslash \mathcal{B} = f(\mathcal{A})$ is $\mathcal{J}$-ω^*-open in Υ. Thus, f is an $\mathcal{J}$-ω^*-open function.

Theorem 26 [14] Let Υ be a subset of space $(\mathcal{X}, \zeta, \mathcal{J})$. Then, $(\zeta_{\mathcal{J}\omega^*})_\Upsilon \subseteq (\zeta_\Upsilon)_{\mathcal{J}\omega^*}$.

Theorem 27 Let $f: (\mathcal{X}, \zeta) \mapsto (\Upsilon, ð, \mathcal{J})$ be a bijective $\mathcal{J}$-ω^*-open function. Then $f_{f^{-1}(\mathcal{A})}: (f^{-1}(\mathcal{A}), \tau_{f^{-1}(\mathcal{A})}) \mapsto (\mathcal{A}, ð_\mathcal{A}, \mathcal{J}_\mathcal{A})$ is $\mathcal{J}$-ω^*-open, for each $\mathcal{A} \subseteq \Upsilon$.

Proof
Let $\mathcal{U}_1$be an open subset of $f^{-1}(\mathcal{A})$. Then, there exists an open set $\mathcal{U}$ in $\mathcal{X}$ such that $\mathcal{U}_1 = \mathcal{U} \cap f^{-1}(\mathcal{A})$. Since f is bijective, then $(f_{f^{-1}(\mathcal{A})})(\mathcal{U}_1) = f(\mathcal{U} \cap f^{-1}(\mathcal{A})) = f(\mathcal{U}) \cap \mathcal{A}$. Since f is an $\mathcal{J}$-ω^*-open function, implies $f(\mathcal{U})$ is $\mathcal{J}$-ω^*-open in Υ. Then $f(\mathcal{U}) \cap \mathcal{A}$ is $\mathcal{J}$-ω^*-open in $(ð_{\mathcal{J}\omega^*})_\mathcal{A}$ so by Theorem 26, we obtain $f(\mathcal{U}) \cap \mathcal{A}$ is $\mathcal{J}$-ω^*-open in $(\mathcal{A}, ð_\mathcal{A}, \mathcal{J}_\mathcal{A})$. Accordingly, $(f_{f^{-1}(\mathcal{A})})(\mathcal{U}_1)$ is $\mathcal{J}$-ω^*-open in $\mathcal{A}$. Thus, $f_{f^{-1}(\mathcal{A})}$ is $\mathcal{J}$-ω^*-open.

5. Conclusions:

The continuity of functions is one of the crucial and fundamental concepts that many authors have studied in the theory of classical point set topology and other branches of mathematics. In this study, we introduce the $\mathcal{J}$-ω^*-continuous function, a new class of continuous functions in ideal topological spaces. Additionally, this function is stronger than each of the class (ω-continuous, ωb-continous, α-$\mathcal{J}_\omega$-continuous, pre-$\mathcal{J}_\omega$-continuous, β-$\mathcal{J}_\omega$-continuous, $faintly$- ω-continuous, $slightly$ - ω-continuous, $\mathcal{J}_\omega$-c-continuous). Furthermore, relationships between these classes and other relevant classes are investigated, and some characterizations of this new class of functions are studied. Finally, we introduce a new class of functions via the notion $\mathcal{J}$-ω^*-open sets called $\mathcal{J}$-ω^*-open functions are stronger than $\omega\beta$-open with some results given. In future research, we will define topological structures, including separation axioms, compactness, and connectedness, for practical application via $\mathcal{J}$-ω^*-open sets.

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