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Soft S_p-Continuous Functions

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Abstract
This paper introduces a novel concept of soft semi-continuous function known as a soft S_p-continuous function, as well as some of its properties. The interrelationships of this newly defined soft function with other types of soft continuous functions are investigated. We prove some important characterizations and derive some of the properties of these soft functions under the soft composition of soft functions.

Keywords: soft S_p-open set, soft semi-open set, soft pre-closed set, soft S_p-continuous functions.

الدوال المستمرة الناعمة من النوع S_p

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الخلاصة
يقدم هذا البحث مفهومًا جديدًا للدالة شبه المستمرة الناعمة المسمى بدالة المستمرة الناعمة من النوع S_p ، بالإضافة إلى بعض خصائصها. تم تحري في العلاقات المتبادلة لهذه الدالة الناعمة المحددة حديثًا مع الانواع الاخرى من الدوال المستمرة الناعمة. أثبتنا بعض المكافئات المهمة لهذه الدالة وبينا بعض خصائص هذه الدالة الناعمة مع الدوال الناعمة الاخرى باستخدام عملية التركيب الدوال الناعمة.

1.Introduction

Molodtsov [1] primarily introduced the concept of soft sets as an entirely new approach to dealing with imperfect information and has since been effectively implemented in many areas, including smoothness functions and other related theories. Formulation of soft operators was first attempted and presented by Maji et al. [2], describing null and absolute soft sets as complements of a soft set, a soft union and a soft intersection between two soft sets.

The main concepts of soft topologies were investigated by Shabir and Naz [3] for further formulations including soft closure operators, soft subspaces, and soft separation axioms, and

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followed by Hussain and Ahmad's [4] investigations of soft boundary operators and soft interior.

Athar Kharal and B. Ahmad [5] explained what soft mapping is in the context of soft classes and looked into a number of properties of soft set images and inverse images. Zorlutuna et al., [6] define soft points and soft continuous functions and [7] presented some new characterizations of soft continuity. Chen [8] introduced and investigated soft semi-open sets and their properties. Mahanta and Das [9] also introduced and defined soft semi-open sets and soft semi-continuous functions. The concept of soft semi-open sets and soft pre-closed sets was used by Mahmood et al., [10] to introduce and define new types of semi-open sets denoted as soft S_p-open sets.

However, this work introduces a new type of soft functions called soft S_p-continuous functions, which are strictly placed between the soft classes of $\tilde{S}S_c$-continuous functions and soft semi-continuous functions. Some of its basic properties and relationships with some other types of soft functions are given.

Throughout the present paper $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ or simply $\tilde{X}$ and $\tilde{Y}$ indicate soft topological spaces on which assume no separation axioms unless mentioned.

2.Preliminaries

Assume X is a universe set, $\mathfrak{P}(X)$ is the power of X, and $\mathcal{P}$ is a set of parameters. A pair $(A,\mathcal{P})=\{(e,A(e)): e\in\mathcal{P},\ A(e)\in\mathfrak{P}(X)\}$ is known as a soft set over X [1], where $A:\mathcal{P}\to\mathfrak{P}(X)$ is a function. The family of all soft sets over the universal set X with the set of parameters $\mathcal{P}$ is indicated by $\tilde{S}S(X,\mathcal{P})=\tilde{S}S(\tilde{X})$. In particular, $(X,\mathcal{P})$ is indicated by $\tilde{X}$. A soft point [6] $(A,\mathcal{P})$ is a soft set defined as $A(e)=\{x\}$ and $A(\acute{e})=\emptyset\,,\forall\,\acute{e}\in\mathcal{P}\backslash\{e\}$, we indicated by $\widetilde{e_x}$ such that $\widetilde{e_x}=(e,\{x\})$, where $x\in X$ and $e\in\mathcal{P}$. $\widetilde{e_x}\,\tilde{\in}\,(B,\mathcal{P})$ if for the element $e\in\mathcal{P}$, $\{x\}\subseteq B(e)$. The family of all soft points over X is indicated by $\tilde{S}P(\tilde{X})$. For $(A,\mathcal{P})\,\tilde{\in}\,\tilde{S}S(\tilde{X})$, the soft set $\tilde{X}\,\tilde{\backslash}(A,\mathcal{P})$ (or $(A,\mathcal{P})^c=(A^c,\mathcal{P})$) is the soft complement of $(A,\mathcal{P})$, where $A^c:\mathcal{P}\to\mathfrak{P}(X)$ is a function defined as $A^c(e)=X-A(e),\forall\,e\in\mathcal{P}$ [2]. The soft set $(A,\mathcal{P})$ is known as a null soft set, indicated by $\tilde{\emptyset}$, if $A(e)=\emptyset,\forall\,p\in\mathcal{P}$ and is known as an absolute soft set, indicated by $\tilde{X}$, if $A(e)=X,\forall\,p\in\mathcal{P}$. For $(A,\mathcal{P}_1),(B,\mathcal{P}_2)\,\tilde{\in}\,\tilde{S}S(\tilde{X})$ and $\mathcal{P}_1,\mathcal{P}_2\subseteq\mathcal{P}$, we say that $(A,\mathcal{P}_1)$ is a soft subset of $(B,\mathcal{P}_2)$, indicated by $(A,\mathcal{P}_1)\,\tilde{\subseteq}\,(B,\mathcal{P}_2)$, if $\mathcal{P}_1\subseteq\mathcal{P}_2$ and $A(e)\subseteq B(e),\forall\,e\in\mathcal{P}_1$ [2]. The soft union of $(A_\vartheta,\mathcal{P})\,\tilde{\in}\,\tilde{S}S(\tilde{X}),\forall\,\vartheta\in\aleph$ is a soft set $(A,\mathcal{P})\,\tilde{\in}\,\tilde{S}S(\tilde{X})$, where $A(e)=\tilde{\cup}_{\vartheta\in\aleph}A_\vartheta(e),\forall\,e\in\mathcal{P}$. Figuratively, we scribe $(A,\mathcal{P})=\tilde{\cup}_{n\in\aleph}(A_\vartheta,\mathcal{P})$, and the soft intersection of $(A_\vartheta,\mathcal{P})\,\tilde{\in}\,\tilde{S}S(\tilde{X}),\forall\,\vartheta\in\aleph$ is a soft set $(A,\mathcal{P})\,\tilde{\in}\,\tilde{S}S(\tilde{X})$, where $A(e)=\tilde{\cap}_{\vartheta\in\aleph}A_\vartheta(e),\forall\,e\in\mathcal{P}$. Figuratively, we scribe $(A,\mathcal{P})=\tilde{\cap}_{\vartheta\in\aleph}(A_\vartheta,\mathcal{P})$ [6].

Definition 2.1. [3] Let $\tilde{\tau}\,\tilde{\subseteq}\,\tilde{S}S(\tilde{X})$. Then $\tilde{\tau}$ is known as soft topology on $\tilde{X}$, if

(i) $\tilde{\emptyset},\tilde{X}\,\tilde{\in}\,\tilde{\tau}$,

(ii) If $(A,\mathcal{P}),(B,\mathcal{P})\,\tilde{\in}\,\tilde{\tau}$, then $(A,\mathcal{P})\,\tilde{\cap}\,(B,\mathcal{P})\,\tilde{\in}\,\tilde{\tau}$,

(iii) If $(A_\vartheta,\mathcal{P})\,\tilde{\in}\,\tilde{\tau},\forall\,\vartheta\in\aleph$, then $\tilde{\cup}_{\vartheta\in\aleph}(A_\vartheta,\mathcal{P})\,\tilde{\in}\,\tilde{\tau}$.

The triple $(\tilde{X},\tilde{\tau},\mathcal{P})$ (simply, $\tilde{X}$) is known as a soft topological space over X. The members of $\tilde{\tau}$ are referred to as soft open sets. The soft complements of every soft open or members of $\tilde{\tau}^c$ are known as soft closed sets [4]. A soft set $(A,\mathcal{P})$ that is both soft open and soft closed is referred to as a soft clopen set.

Definition 2.2. [3] Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ be a soft topological space and $Z \subseteq X$. Then $\tilde{\tau}_Z = \{(A_Z, \mathcal{P}) = \tilde{Z} \tilde{\cap} (A, \mathcal{P}); (A, \mathcal{P}) \tilde{\in} \tilde{\tau}\}$ is known as the soft relative topology on $\tilde{Z}$, where $A_Z(e) = \tilde{Z} \tilde{\cap} A(e)$, for all $e \tilde{\in} \mathcal{P}$. $(\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P})$ is a soft subspace of $(\tilde{X}, \tilde{\tau}, \mathcal{P})$. $\tilde{\tau}_{\tilde{Z}}$ is a soft topology on $\tilde{Z}$.

Definition 2.3. Let $(A, \mathcal{P})$ be a soft subset of $(\tilde{X}, \tilde{\tau}, \mathcal{P})$. Then,
(i) The soft closure of $(A, \mathcal{P})$ is $\tilde{s}cl(A, \mathcal{P}) = \tilde{\cap}\{(C, \mathcal{P}) : (C, \mathcal{P}) \tilde{\in} \tilde{\tau}^c, (A, \mathcal{P}) \tilde{\subseteq} (C, \mathcal{P})\}$ [3].
(ii) The soft interior of $(A, \mathcal{P})$ is $\tilde{s}int(A, \mathcal{P}) = \tilde{\cup}\{(O, \mathcal{P}) : (O, \mathcal{P}) \tilde{\in} \tilde{\tau}, (O, \mathcal{P}) \tilde{\subseteq} (A, \mathcal{P})\}$ [4].

Definition 2.4. A soft subset $(A, \mathcal{P})$ of $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is known as a soft semi-open [9] (resp., soft pre-open [11], soft α-open [12], soft b-open [13], soft β-open [14] and soft regular open [11]) set, if $(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(A, \mathcal{P}))$ (resp., $(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(A, \mathcal{P}))$, $(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{s}int(A, \mathcal{P})))$, $(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(A, \mathcal{P})) \tilde{\cup} \tilde{s}int(\tilde{s}cl(A, \mathcal{P}))$, $(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(A, \mathcal{P})))$, and $(A, \mathcal{P}) = \tilde{s}int(\tilde{s}cl(A, \mathcal{P}))$).

The family of all soft semi-open (resp., soft pre-open, soft α-open, soft b-open, soft β-open, and soft regular open) sets in $\tilde{X}$ is indicated by $\tilde{S}SO(\tilde{X})$ (resp., $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, $\tilde{S}bO(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$ and $\tilde{S}RO(\tilde{X})$).

Definition 2.5. A soft subset $(A, \mathcal{P})$ of $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is known as a soft S_p-open [10] (resp., soft ξ-open [15], $\tilde{S}S_c$-open [16], soft P_c-open [17], soft β_c-open [18] and soft b_c-open [19]) set, if $(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ (resp., $\tilde{\tau}$, $\tilde{S}SO(\tilde{X})$, $\tilde{S}PO(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$, and $\tilde{S}bO(\tilde{X})$) and for each $\widetilde{e_x} \tilde{\in} (A, \mathcal{P})$, there is a soft pre-closed (resp., soft semi-closed, soft closed, soft closed, soft closed and soft closed) subset $(B, \mathcal{P})$ of $\tilde{X}$ such that $\widetilde{e_x} \tilde{\in} (B, \mathcal{P}) \tilde{\subseteq} (A, \mathcal{P})$. The family of all soft S_p-open (resp., soft ξ-open, $\tilde{S}S_c$-open, soft P_c-open, soft β_c-open and soft b_c-open) subsets of $\tilde{X}$ is indicated by $\tilde{S}S_pO(\tilde{X})$ (resp., $\tilde{S}\xi O(\tilde{X})$, $\tilde{S}S_cO(\tilde{X})$, $\tilde{S}P_cO(\tilde{X})$, $\tilde{S}\beta_cO(\tilde{X})$ and $\tilde{S}b_cO(\tilde{X})$).

Definition 2.6. The soft complement of a soft S_p-open (resp., soft preopen, and soft regular open) set is known as soft S_p-closed (resp., soft pre-closed [11], regular closed [20]). The family of all soft S_p-closed (resp., soft pre-closed, and regular closed) sets in $\tilde{X}$ is indicated by $\tilde{S}S_pC(\tilde{X})$ (resp., $\tilde{S}PC(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$).

Definition 2.7. A soft topological space $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is known as:
(i) Soft extremally disconnected [21], if $\tilde{s}cl(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$, $\forall$ $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$.
(ii) Soft locally indiscrete [22], if every soft open set in $\tilde{X}$ is soft closed.
(iii) Soft submaximal [11], if each soft dense subset of $\tilde{X}$ is soft open set.
(iv) Soft T_1-space [23], if $\widetilde{e_x}, \widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\widetilde{e_x} \neq \widetilde{e_y}$, there are soft open sets $(A_1, \mathcal{P})$ and $(A_2, \mathcal{P})$ such that $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P})$, $\widetilde{e_y} \tilde{\notin} (A_1, \mathcal{P})$ and $\widetilde{e_y} \tilde{\in} (A_2, \mathcal{P})$, $\widetilde{e_x} \tilde{\notin} (A_2, \mathcal{P})$.
(v) Soft T_2-space [23], if $\widetilde{e_x}, \widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\widetilde{e_x} \neq \widetilde{e_y}$, there are soft open sets $(A_1, \mathcal{P})$ and $(A_2, \mathcal{P})$ such that $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P})$, $\widetilde{e_y} \tilde{\in} (A_2, \mathcal{P})$, and $(A_1, \mathcal{P}) \tilde{\cap} (A_2, \mathcal{P}) = \tilde{\emptyset}$.
(vi) Soft regular space [23], if $(C, \mathcal{P})$ is a soft closed set and $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\widetilde{e_x} \tilde{\notin} (C, \mathcal{P})$, there exist soft open sets $(A_1, \mathcal{P})$ and $(A_2, \mathcal{P})$ such that $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P})$, $(C, \mathcal{P}) \tilde{\subseteq} (A_2, \mathcal{P})$ and $(A_1, \mathcal{P}) \tilde{\cap} (A_2, \mathcal{P}) = \tilde{\emptyset}$.
(vii) Soft semi-regular space [24], if for each soft open set $(A, \mathcal{P})$ in $\tilde{X}$ and each $\widetilde{e_x} \tilde{\in} (A, \mathcal{P})$, there exists a soft regular open set $(O, \mathcal{P})$ in $\tilde{X}$ such that $\widetilde{e_x} \tilde{\in} (O, \mathcal{P}) \tilde{\subseteq} (A, \mathcal{P})$.

Definition 2.8. [5] Let $\tilde{S}S(\tilde{X})$, $\tilde{S}S(\tilde{Y})$ be the families of all soft sets over X and Y with $\mathcal{P}$ and $\acute{\mathcal{P}}$, respectively, let $u: X \to Y$ and $p: \mathcal{P} \to \acute{\mathcal{P}}$ be functions. Then, a soft function $\tilde{f}_{pu}: \tilde{S}S(\tilde{X}) \to \tilde{S}S(\tilde{Y})$ is defined as:
(i) If $(A, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$, the soft image of $(A, \mathcal{P})$ under $\tilde{f}_{pu}$, written as $\tilde{f}_{pu}(A, \mathcal{P}) = (\tilde{f}_{pu}(A), p(\mathcal{P})) \tilde{\in} \tilde{S}S(\tilde{Y})$, $\forall \beta \in \acute{\mathcal{P}}$ defined as:

$$\tilde{f}_{pu}(A)(\beta) = \begin{cases} u(\bigcup_{\alpha \in p^{-1}(\beta) \cap \mathcal{P}} A(\alpha)), & if\ p^{-1}(\beta) \cap \mathcal{P} \neq \emptyset \\ \tilde{\emptyset}, & otherwise \end{cases},$$

so if $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, then $\tilde{f}_{pu}(\widetilde{e_x}) = p(e)_{u(x)}$ [25].
(ii) If $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$, the soft inverse image of $(B, \acute{\mathcal{P}})$ under f_{pu}, written as $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = (\tilde{f}_{pu}^{-1}(B), p^{-1}(\acute{\mathcal{P}})) \tilde{\in} \tilde{S}S(\tilde{X})$, $\forall \alpha \in \mathcal{P}$ defined as:

$$\tilde{f}_{pu}^{-1}(B)(\alpha) = \begin{cases} u^{-1}\left(B\left(p(\alpha)\right)\right), & p(\alpha) \in \acute{\mathcal{P}} \\ \tilde{\emptyset}, & otherwise \end{cases},$$

so if $\widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$ and $\tilde{f}_{pu}$ is soft bijective, then $\tilde{f}_{pu}^{-1}(\widetilde{e_y}) = p^{-1}(\acute{e})_{u^{-1}(y)}$ [25].
The soft function $\tilde{f}_{pu}: \tilde{S}S(\tilde{X}) \to \tilde{S}S(\tilde{Y})$ is known as soft injective (resp., soft surjective, soft bijective) if u, p are both injective (resp., surjective, bijective) functions [26].

Theorem 2.9. [5, 6, 26] Let $\tilde{f}_{pu}: \tilde{S}S(\tilde{X}) \to \tilde{S}S(\tilde{Y})$ be a soft function, the following are true:
(i) If $(A_1, \mathcal{P}) \tilde{\subseteq} (A_2, \mathcal{P})$, then $\tilde{f}_{pu}(A_1, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(A_2, \mathcal{P})$, $\forall (A_1, \mathcal{P}), (A_2, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$.
(ii) If $(B_1, \acute{\mathcal{P}}) \tilde{\subseteq} (B_2, \acute{\mathcal{P}})$, then $\tilde{f}_{pu}^{-1}(B_1, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B_2, \acute{\mathcal{P}})$, $\forall (B_1, \acute{\mathcal{P}}), (B_2, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$.
(iii) $\tilde{f}_{pu}((A_1, \mathcal{P}) \tilde{\cup} (A_2, \mathcal{P})) = \tilde{f}_{pu}(A_1, \mathcal{P}) \tilde{\cup} \tilde{f}_{pu}(A_2, \mathcal{P})$, $\forall (A_1, \mathcal{P}), (A_2, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$.
(iv) $\tilde{f}_{pu}((A_1, \mathcal{P}) \tilde{\cap} (A_2, \mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A_1, \mathcal{P}) \tilde{\cap} \tilde{f}_{pu}(A_2, \mathcal{P})$, $\forall (A_1, \mathcal{P}),(A_2, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$, the equality holds if $\tilde{f}_{pu}$ is soft injective.
(v) $\tilde{f}_{pu}^{-1}((B_1, \acute{\mathcal{P}}) \tilde{\cup} (B_2, \acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B_1, \acute{\mathcal{P}}) \tilde{\cup} \tilde{f}_{pu}^{-1}(B_2, \acute{\mathcal{P}})$, $\forall (B_1, \acute{\mathcal{P}}), (B_2, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$.
(vi) $\tilde{f}_{pu}^{-1}((B_1, \acute{\mathcal{P}}) \tilde{\cap} (B_2, \acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B_1, \acute{\mathcal{P}}) \tilde{\cap} \tilde{f}_{pu}^{-1}(B_2, \acute{\mathcal{P}})$, $\forall (B_1, \acute{\mathcal{P}}), (B_2, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$.
(vii) $\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(A, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P}))$, $\forall (A, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$, the equality holds if $\tilde{f}_{pu}$ is soft surjective.
(viii) $\tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B, \acute{\mathcal{P}})) = \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$.
(ix) $\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})) \tilde{\subseteq} (B, \acute{\mathcal{P}})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y})$, the equality holds if $\tilde{f}_{pu}$ is soft surjective.
(x) $(A, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A, \mathcal{P}))$, $\forall (A, \mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X})$, the equality holds if $\tilde{f}_{pu}$ is soft injective.

Definition 2.10. Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be two soft topological spaces, a soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is known as soft continuous [6] (resp., soft semi-continuous [9], soft pre-continuous [27], soft α-continuous [27], soft β-continuous [28], and soft b-continuous [13]), if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}SO(\tilde{X})$, $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$, and $\tilde{S}bO(\tilde{X})$), $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.

Definition 2.11. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is known as:
(i) Soft perfectly continuous [29] if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})$ is a soft clopen set in $\tilde{X}$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(ii) Soft RC-continuous [29] if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}RC(\tilde{X})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(iii) Soft regular continuous [30] if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}RO(\tilde{X})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(iv) $\tilde{S}S_c$-continuous [16] if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(v) Soft β_c-continuous [18] if $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}\beta_cO(\tilde{X})$, $\forall (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.

(vi) Soft ξ-continuous [15] if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}\xi O(\tilde{X}), \forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(vii) $\tilde{s}p_c$-continuous [17] if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}P_c O(\tilde{X}), \forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.
(viii) Soft irresolute [9] $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X}), \forall (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$.
(ix) Soft open [31] if $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{\sigma}, \forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$.

Definition 2.12. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft almost continuous [32] (resp. soft almost semi-continuous [24], soft almost pre-continuous [33], soft almost α-continuous [34], and $\theta\tilde{S}S_c$-continuous [16]), if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}SO(\tilde{X})$, $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, and $\tilde{S}S_c O(\tilde{X})$), $\forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}RO(\tilde{Y})$.

Definition 2.13. [7] Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $\tilde{Z} \tilde{\subseteq} \tilde{X}$. Then, the restriction of $\tilde{f}_{pu}$ to $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ is the soft function $\tilde{f}_{pu}\lceil_{\tilde{Z}}$ from $(\tilde{Z},\tilde{\tau}_Z,\mathcal{P})$ to $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ which defined by the functions $u\lceil_{\tilde{Z}}: \tilde{Z} \to \tilde{Y}$, $p: \mathcal{P} \to \acute{\mathcal{P}}$.

Proposition 2.14. [7] Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $\tilde{Z} \tilde{\subseteq} \tilde{X}$. Then, $(\tilde{f}_{pu}\lceil_{\tilde{Z}})^{-1}(B,\acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\cap} \tilde{Z}, \forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proposition 2.15. [8] A soft set $(A,\mathcal{P})$ in $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft semi-open set iff there exists a soft open set $(O,\mathcal{P})$ such that $(O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(O,\mathcal{P})$.

Proposition 2.16. [7] Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then:
(i) $\tilde{f}_{pu}$ is soft open iff $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})), \forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.
(ii) $\tilde{f}_{pu}$ is soft open iff $\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})), \forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proposition 2.17. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$.
(i) If $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$ and $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then $\tilde{Z} \tilde{\cap} (A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Z})$ [35].
(ii) If $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$ and $(A,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$, then $\tilde{Z} \tilde{\cap} (A,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{Z})$ [36].

Lemma 2.18. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(A,\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Then, $\tilde{s}int(A,\mathcal{P}) = \tilde{s}int_{\tilde{Z}}(A,\mathcal{P})$, if $\tilde{Z} \tilde{\in} \tilde{\tau}$ [37].

Proposition 2.19. [10] Let $(A,\mathcal{P})$,$(B,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then:
(i) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$ iff $(A,\mathcal{P}) = \tilde{\cup}\, (B_\vartheta,\mathcal{P})$, where $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ and $(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X}), \forall\, \vartheta \in \aleph$.
(ii) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$ iff $\forall\, \widetilde{e_x} \tilde{\in} (A,\mathcal{P})$, there is $(B,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$ such that $\widetilde{e_x} \tilde{\in} (B,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$.
(iii) $(A,\mathcal{P}) \tilde{\cup} (B,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$, if $(A,\mathcal{P})$,$(B,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$.
(iv) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_p C(\tilde{X})$ iff $(A,\mathcal{P}) = \tilde{\cap}\, (B_\vartheta,\mathcal{P})$, where $(A,\mathcal{P}) \tilde{\in} \tilde{S}SC(\tilde{X})$ and $(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X}), \forall\, \vartheta \in \aleph$.

Remark 2.20. (i) Every soft S_p-open set is soft semi-open [10].
(ii) Every soft S_p-closed set is soft semi-closed.

Proposition 2.21. [10] For any soft subset $(A,\mathcal{P})$ of $(\tilde{X},\tilde{\tau},\mathcal{P})$, then:
(i) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$, if $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_c O(\tilde{X})$.
(ii) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$, if $(A,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$.

(i) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, if $(A,\mathcal{P})$ is a soft clopen set.

Proposition 2.22. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete (resp., soft T_1-space), then $\tilde{S}SO(\tilde{X}) = \tilde{S}S_cO(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

Corollary 2.23. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete, then
(ii) $\tilde{\tau} = \tilde{S}S_pO(\tilde{X})$.
(iii) $\tilde{S}\alpha O(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.
(iv) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$.
(v) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\beta_cO(\tilde{X})$ (resp., $\tilde{S}b_cO(\tilde{X})$).

Corollary 2.24. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft T_1-space, then:
(i) $\tilde{\tau} \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.
(ii) $\tilde{S}\alpha O(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.

Proposition 2.25. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft regular space, then $\tilde{\tau} \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.

Proposition 2.26. [10] $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft extremally disconnected iff $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$).

Proposition 2.27. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft extremally disconnected space, then:
(i) $\tilde{S}RO(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.
(ii) $\tilde{S}\xi O(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.

Proposition 2.28. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft submaximal space, then:
(i) $\tilde{S}S_cO(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.
(ii) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\beta_cO(\tilde{X})$ (resp., $\tilde{S}b_cO(\tilde{X})$).
(iii) $\tilde{S}P_cO(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$.

Proposition 2.29. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft p-Alexandroff space, then $\tilde{S}RC(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

Lemma 2.30. [10] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft locally indiscrete space, then $\tilde{S}SO(\tilde{X}) = \tilde{\tau}$.

Proposition 2.31. [10] Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a soft topological space and $(A,\mathcal{P})$,$(B,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. If $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $(B,\mathcal{P})$ is a soft clopen set, then $(A,\mathcal{P}) \tilde{\cap} (B,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Proposition 2.32. [10] Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(A,\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. If $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ and $\tilde{Z} \tilde{\in} \tilde{S}RC(\tilde{X})$ (resp., soft clopen in $\tilde{X}$), then $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

3. More Properties of Soft S_p-open and Some Other Results

Further properties of soft S_p-open sets are discussed in this section, as are some properties of various soft sets and soft space types.

Proposition 3.1. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$. If $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}PC(\tilde{Z})$.

Proof. Since $\tilde{Z} \tilde{\in} \tilde{\tau}$, by Lemma 2.18, then $\tilde{s}int_{\tilde{Z}}(A,\mathcal{P}) = \tilde{s}int(A,\mathcal{P})$, $\forall\ (A,\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Hence, we obtain $\tilde{s}cl_{\tilde{Z}}(\tilde{s}int_{\tilde{Z}}((C,\mathcal{P}) \tilde{\cap} \tilde{Z})) = \tilde{s}cl(\tilde{s}int((C,\mathcal{P}) \tilde{\cap} \tilde{Z})) \tilde{\cap} \tilde{Z} = \tilde{s}cl(\tilde{s}int(C,\mathcal{P}) \tilde{\cap} \tilde{s}int(\tilde{Z})) \tilde{\cap} \tilde{Z} \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(C,\mathcal{P})) \tilde{\cap} \tilde{s}cl(\tilde{s}int(\tilde{Z})) \tilde{\cap} \tilde{Z} = \tilde{s}cl(\tilde{s}int(C,\mathcal{P})) \tilde{\cap} \tilde{Z}$. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $\tilde{s}cl(\tilde{s}int(C,\mathcal{P})) \tilde{\subseteq} (C,\mathcal{P})$. Thus, $\tilde{s}cl_{\tilde{Z}}(\tilde{s}int_{\tilde{Z}}((C,\mathcal{P}) \tilde{\cap} \tilde{Z})) \tilde{\subseteq} (C,\mathcal{P}) \tilde{\cap} \tilde{Z}$. Therefore, $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}PC(\tilde{Z})$.

Proposition 3.2. Let a soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be soft continuous and soft open. If $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{X})$.

Proof. Since $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, then $\tilde{s}cl(\tilde{s}int(B,\acute{\mathcal{P}})) \tilde{\subseteq} (B,\acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{s}int(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft continuous, so $\tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{s}int(B,\acute{\mathcal{P}})))$ $\tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. By the soft openness of $\tilde{f}_{pu}$ and Proposition 2.16(ii), $\tilde{s}cl(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. Thus, $\tilde{s}cl(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. Therefore, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ is soft pre-closed in $\tilde{X}$.

Proposition 3.3 Let a soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be soft continuous and soft open. If $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$.

Proof. Since $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$, then by Proposition 2.15, there exists a soft open set $(O,\acute{\mathcal{P}})$ in $\tilde{Y}$ such that $(O,\acute{\mathcal{P}}) \tilde{\subseteq} (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}cl(O,\acute{\mathcal{P}})$. So, $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(O,\acute{\mathcal{P}}))$. Since $\tilde{f}_{pu}$ is soft continuous, $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}})$ is a soft open set in $\tilde{X}$ and also since $\tilde{f}_{pu}$ is a soft open function, then by Proposition 2.16(i), $\tilde{f}_{pu}^{-1}(\tilde{s}cl(O,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}))$. Hence, we obtain that $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}))$. Thus, by Proposition 2.15, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$.

Proposition 3.4. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., soft clopen). If $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(A,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$.

Proof. Since $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ and $(A,\mathcal{P}) = \tilde{\cup}_{\vartheta \in \aleph} (B_\vartheta,\mathcal{P})$ where $(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, $\forall\ \vartheta \in \aleph$. Then $(A,\mathcal{P}) \tilde{\cap} \tilde{Z} = \tilde{\cup}_{\vartheta \in \aleph} (B_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z} = \tilde{\cup}_{\vartheta \in \aleph} ((B_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z})$. Since $\tilde{Z} \tilde{\in} \tilde{\tau}$, then $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$, by Proposition 2.17(1), $(A,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{Z})$. Again, since $\tilde{Z}$ is a soft open soft subspace of $\tilde{X}$, so by Proposition 3.1, $(B_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}PC(\tilde{Z})$, $\forall\ \vartheta \in \aleph$. Then, by Proposition 2.19(i), $(A,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$.

Corollary 3.5. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$. If $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$ and $(A,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$, then $(A,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$.
Proof. Applying Proposition 2.17(ii) and Proposition 2.21(ii).

Definition 3.6. Let $(A,\mathcal{P})$ be a soft subset of $(\tilde{X},\tilde{\tau},\mathcal{P})$. Then,:
(i) The soft S_p-closure of $(A,\mathcal{P})$ is $\tilde{s}S_pcl(A,\mathcal{P}) = \tilde{\cap}\{(C,\mathcal{P}) : (C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X}),\ (A,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P})\}$. Clearly $\tilde{s}S_pcl(A,\mathcal{P})$ is the smallest soft S_p-closed set contains $(A,\mathcal{P})$.
(ii) The soft S_p-interior of $(A,\mathcal{P})$ is $\tilde{s}S_pint(A,\mathcal{P}) = \tilde{\cup}\{(O,\mathcal{P}) : (O,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}),\ (O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})\}$. Clearly $\tilde{s}S_pint(A,\mathcal{P})$ is the largest soft S_p-open set contained in $(A,\mathcal{P})$.
(iii) The soft S_p-boundary of $(A,\mathcal{P})$ is $\tilde{s}S_pBd(A,\mathcal{P}) = \tilde{s}S_pcl(A,\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(\tilde{X} \tilde{\setminus} (A,\mathcal{P}))$.

Proposition 3.7. Let $(A,\mathcal{P})$ be a soft subset of $(\tilde{X},\tilde{\tau},\mathcal{P})$. Then,

(i) $\widetilde{e_x} \tilde{\in} \tilde{s}S_pcl(A,\mathcal{P})$ if $(A,\mathcal{P}) \tilde{\cap} (O,\mathcal{P}) \neq \widetilde{\emptyset}, \forall \widetilde{e_x} \tilde{\in} (O,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.
(ii) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ if $\tilde{s}S_pBd(A,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$.

Proof. Obvious.

4.Soft S_p-continuous functions

In this section, we introduce the concept of soft S_p-continuous functions by using soft S_p-open sets.

Definition 4.1. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be two soft topological spaces and $u: X \to Y$, $p: \mathcal{P} \to \acute{\mathcal{P}}$ are functions. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft S_p-continuous at a soft point $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, if for each soft open set $(B,\acute{\mathcal{P}})$ in $\tilde{Y}$ containing $\tilde{f}_{pu}(\widetilde{e_x})$, there exists a soft S_p-open set $(A,\mathcal{P})$ in $\tilde{X}$ containing $\widetilde{e_x}$ such that $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} (B,\acute{\mathcal{P}})$.
If $\tilde{f}_{pu}$ is soft S_p-continuous at every soft point $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, then it's known as a soft S_p-continuous function.

Proposition 4.2. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous iff $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X}), \forall (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-continuous and since $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. To prove that $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \widetilde{\emptyset}$, implies that $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. If not, let $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, we have $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B,\acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, there is $\widetilde{e_x} \tilde{\in} (A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} (B,\acute{\mathcal{P}})$. Hence, $\widetilde{e_x} \tilde{\in} (A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ and therefore by Proposition 2.19(ii), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.
Conversely, let $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ and $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $(A,\mathcal{P}) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ such that $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} (B,\acute{\mathcal{P}})$. Therefore, $\tilde{f}_{pu}$ is soft S_p-continuous.

Proposition 4.3. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then,:
(i) $\tilde{f}_{pu}$ is soft semi-continuous, if $\tilde{f}_{pu}$ is soft S_p-continuous.
(ii) $\tilde{f}_{pu}$ is soft S_p-continuous, if $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous (resp., soft RC-continuous, and soft perfectly continuous).

Proof. Since $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}SO(\tilde{X})$, the proof (i) will follow, and by Proposition 2.21 and Proposition 4.2, the proof (ii) will follow.
As the next examples illustrates, the opposite of Proposition 4.3 is not always true:

Example 4.4. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $\mathcal{P} = \{e_1, e_2\}$, and $\acute{\mathcal{P}} = \{\acute{e}_1, \acute{e}_2\}$. Let $\tilde{\tau} = \{\widetilde{\emptyset}_X, \tilde{X}, (A_1,\mathcal{P}), (A_2,\mathcal{P}), (A_3,\mathcal{P}), (A_4,\mathcal{P}), (A_5,\mathcal{P}), (A_6,\mathcal{P}), (A_7,\mathcal{P})\}$ and $\tilde{\sigma} = \{\widetilde{\emptyset}_Y, \tilde{Y}, (B,\acute{\mathcal{P}})\}$ be soft topology on $\tilde{X}$ and $\tilde{Y}$ respectively, where $\widetilde{\emptyset}_X = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(A_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$, $(A_2,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\emptyset)\}$, $(A_3,\mathcal{P}) = \{(e_1,X),(e_2,\emptyset)\}$, $(A_4,\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_2\})\}$, $(A_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$, $(A_6,\mathcal{P}) = \{(e_1,\{x_2\}), (e_2,\{x_2\})\}$, $(A_7,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$, $\tilde{Y} = \{(\acute{e}_1,Y),(\acute{e}_2,Y)\}$, $\widetilde{\emptyset}_Y = \{(\acute{e}_1,\emptyset),(\acute{e}_1,\emptyset)\}$, and $(B,\acute{\mathcal{P}}) = \{(\acute{e}_1,\{y_2\}),(\acute{e}_2,\{y_1\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ are soft topological spaces over X and Y respectively. Now define $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$, where $p: \mathcal{P} \to \acute{\mathcal{P}}$ is a function defined by $p(e_1) = \{\acute{e}_1\}$, $p(e_2) = \{\acute{e}_2\}$ and $u: X \to Y$ is a function defined by $u(x_1) = \{y_2\}$, $u(x_2) = \{y_1\}$. The soft function $\tilde{f}_{pu}$ is soft semi-continuous, but is not soft S_p-continuous, since $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$, while $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = (A_5, \mathcal{P})$ is a soft semi-open set but is not soft S_p-open in $\tilde{X}$.

Example 4.5. Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $\mathcal{P} = \{e_1, e_2\}$, and $\acute{\mathcal{P}} = \{\acute{e}_1, \acute{e}_2\}$ with the soft topology $\tilde{\tau} = \{\widetilde{\emptyset}_X, \tilde{X}, (A_1, \mathcal{P}), (A_2, \mathcal{P}), (A_3, \mathcal{P}), (A_4, \mathcal{P}), (A_5, \mathcal{P})\}$ and $\tilde{\sigma} = \{\widetilde{\emptyset}_Y, \tilde{Y}, (B, \acute{\mathcal{P}})\}$ on $\tilde{X}$ and $\tilde{Y}$ respectively, where $\widetilde{\emptyset}_X = \{(e_1, \emptyset), (e_2, \emptyset)\}$, $\tilde{X} = \{(e_1, X), (e_2, X)\}$, $(A_1, \mathcal{P}) = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $(A_2, \mathcal{P}) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1, x_3\})\}$, $(A_3, \mathcal{P}) = \{(e_1, \{x_1, x_2\}) (e_2, \{x_1, x_2\})\}$, $(A_4, \mathcal{P}) = \{(e_1, X), (e_2, \{x_1, x_3\})\}$, $(A_5, \mathcal{P}) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}$, $\tilde{Y} = \{(\acute{e}_1, Y), (\acute{e}_2, Y)\}$, $\widetilde{\emptyset}_Y = \{(\acute{e}_1, \emptyset), (\acute{e}_1, \emptyset)\}$, and $(B, \acute{\mathcal{P}}) = \{(\acute{e}_1, \{y_1, y_2\}), (\acute{e}_2, \{y_3\})\}$. Thus, $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft topological spaces over X and Y respectively. Now define $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$, where $p: \mathcal{P} \to \acute{\mathcal{P}}$ is a function defined by $p(e_1) = \{\acute{e}_1\}$, $p(e_2) = \{\acute{e}_2\}$ and $u: X \to Y$ is a function defined by $u(x_1) = \{y_3\}$, $u(x_2) = \{y_1\}$, and $u(x_3) = \{y_2\}$. The soft function $\tilde{f}_{pu}$ is a soft S_p-continuous, but are not $\tilde{S}S_c$-continuous, soft RC-continuous, and soft perfectly continuous. Since $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$, while $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1\})\}$ is a soft S_p-open set but are not $\tilde{S}S_c$-open set, soft regular closed set, and soft clopen set in $\tilde{X}$.

Proposition 4.6. Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be two soft topological spaces. Then, a soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is soft semi-continuous, for each soft point $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, and for each soft open set $(B, \acute{\mathcal{P}})$ in $\tilde{Y}$ containing $\tilde{f}_{pu}(\widetilde{e_x})$, there is $\widetilde{e_x} \tilde{\in} (C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$.

Proof. Suppose that $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous. Let $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ and $(B, \acute{\mathcal{P}})$ is any soft open set in $\tilde{Y}$ containing $\tilde{f}_{pu}(\widetilde{e_x})$. By assumption, there is $\widetilde{e_x} \tilde{\in} (A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$. Since $(A, \mathcal{P})$ is a soft S_p-open set and $\widetilde{e_x} \tilde{\in} (A, \mathcal{P})$, there is $(C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\widetilde{e_x} \tilde{\in} (C, \mathcal{P}) \tilde{\subseteq} (A, \mathcal{P})$. Therefore, we have $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, then by Proposition 4.3(i), $\tilde{f}_{pu}$ is soft semi-continuous.
Conversely, let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft semi-continuous, so $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Let $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})$. Then, $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B, \acute{\mathcal{P}})$. By assumption, there is $\widetilde{e_x} \tilde{\in} (C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$, which implies that $\widetilde{e_x} \tilde{\in} (C, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})$. Therefore, $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Hence by Proposition 4.2, $\tilde{f}_{pu}$ is soft S_p-continuous.

Proposition 4.7. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft continuous, and soft open. If $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Proof. Since $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, then by Proposition 2.19(i), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$ and $(B, \acute{\mathcal{P}}) = \tilde{\bigcup} (D_\vartheta, \acute{\mathcal{P}})$, where $(D_\vartheta, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$ $\forall \vartheta \in \aleph$. Then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{\bigcup} (D_\vartheta, \acute{\mathcal{P}})) = \tilde{\bigcup} \tilde{f}_{pu}^{-1}(D_\vartheta, \acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft continuous and soft open, then by Proposition 3.3, $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$ and by Proposition 3.2, $\tilde{f}_{pu}^{-1}(D_\vartheta, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{X})$, $\forall \vartheta \in \aleph$. Hence by Proposition 2.19(i), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Corollary 4.8. Let a soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft continuous and soft open. If $(C, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(C, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. Applying Proposition 4.7 and Definition of a soft S_p-closed set.

Proposition 4.9. Let a soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft irresolute. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft locally indiscrete (resp., soft T_1-space) and $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Proof. Since $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, then $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft irresolute, so $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete (resp., soft T_1-space), then by Proposition 2.22, $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Proposition 4.10. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft locally indiscrete (resp., a soft T_1-space), then the following statements are equivalent:
(i) $\tilde{f}_{pu}$ is soft S_p-continuous.
(ii) $\tilde{f}_{pu}$ is soft semi-continuous.
(iii) $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous function.

Proof. (i) → (ii). Applying Proposition 4.3(i).
(ii) →(i) and (i) →(iii). Applying Proposition 4.2 and Proposition 2.22.
(iii) →(i). Applying Proposition 4.3(ii).

Corollary 4.11. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft locally indiscrete, then:
(i) $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is soft continuous (resp., soft α-continuous).
(ii) $\tilde{f}_{pu}$ is soft pre-continuous (resp., soft β_c-continuous, and soft b_c-continuous) if $\tilde{f}_{pu}$ is soft S_p-continuous.

Proof. By Proposition 4.2 and Corollary 2.23, the proof will follow them.

Corollary 4.12. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft continuous (resp., soft α-continuous) and $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is a soft T_1-space, then $\tilde{f}_{pu}$ is soft S_p-continuous.

Proof. By Proposition 4.2 and Corollary 2.24, the proof will follow them.

Corollary 4.13. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft continuous and $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is a soft regular space, then $\tilde{f}_{pu}$ is soft S_p-continuous.

Proof. By Proposition 4.2 and Proposition 2.25, the proof will follow them.

Proposition 4.14. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \rightarrow (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft submaximal, then:
(i) $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous.
(ii) $\tilde{f}_{pu}$ is soft β_c-continuous (resp., soft b_c-continuous) if $\tilde{f}_{pu}$ is soft S_p-continuous.
(iii) $\tilde{f}_{pu}$ is soft S_p-continuous if $\tilde{f}_{pu}$ is $\tilde{s}p_c$-continuous.
Proof. By Proposition 4.2 and Proposition 2.28, the proof will follow them.

Proposition 4.15. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function from soft extremally disconnected $(\tilde{X},\tilde{\tau},\mathcal{P})$ to $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$. Then, $\tilde{f}_{pu}$ is soft S_p-continuous if $\tilde{f}_{pu}$ is soft regular continuous (resp., soft ξ-continuous).

Proof. Let $(B,\acute{\mathcal{P}})$ be any soft open set in $\tilde{Y}$. Since $\tilde{f}_{pu}$ is soft regular continuous (resp., soft ξ-continuous), then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}RO(\tilde{X})$ (resp., $\tilde{S}\xi O(\tilde{X})$). Since $\tilde{X}$ is soft extremally disconnected, so by Proposition 2.27, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft S_p-continuous.

Corollary 4.16. If $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is a soft S_p-continuous function and $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft extremally disconnected , then $\tilde{f}_{pu}$ is soft pre-continuous (resp., soft α-continuous).

Proof. By Proposition 4.2 and Proposition 2.26, the proof will follow them.

Corollary 4.17. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft p-Alexandroff, then $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is soft RC-continuous.

Proof. By Proposition 4.2 and Proposition 2.29, the proof will follow them.

Proposition 4.18. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ are soft locally indiscrete, then $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is soft irresolute.

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{Y}$ is soft locally indiscrete, then by Lemma 2.30, $(B,\acute{\mathcal{P}})$ is a soft open set in $\tilde{Y}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. By Remark 2.20(i), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft irresolute.
Conversely, let $(B,\acute{\mathcal{P}})$ be any soft open set in $\tilde{Y}$. Then, $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft irresolute, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, so by Proposition 2.22, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft S_p-continuous.

Proposition 4.19. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function from soft locally indiscrete $(\tilde{X},\tilde{\tau},\mathcal{P})$ to soft semi-regular $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$. Then, $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}$ is $\theta\tilde{S}S_c$-continuous (resp., soft almost continuous, soft almost semi-continuous, and soft almost α-continuous).

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}RO(\tilde{Y})$. Then, $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 2.22 (resp., Corollary 2.23(i), Remark 2.20(i), and Corollary 2.23(ii)), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$ (resp., $\tilde{\tau}$, $\tilde{S}SO(\tilde{X})$, and $\tilde{S}\alpha O(\tilde{X})$). Thus, $\tilde{f}_{pu}$ is $\theta\tilde{S}S_c$-continuous (resp., soft almost continuous, soft almost semi-continuous, and soft almost α-continuous).
Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$ and $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, we have $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B,\acute{\mathcal{P}})$. By the soft semi-regularity of $\tilde{Y}$, there exists a soft regular open set $(O,\acute{\mathcal{P}})$ in $\tilde{Y}$ such that $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (O,\acute{\mathcal{P}}) \tilde{\subseteq} (B,\acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is $\theta\tilde{S}S_c$-continuous (resp., soft almost continuous, soft almost semi-continuous, and soft almost α-continuous), so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$ (resp., $\tilde{\tau}$, $\tilde{S}SO(\tilde{X})$, and $\tilde{S}\alpha O(\tilde{X})$), and $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. by Proposition 2.21(i) (resp., Corollary 2.23(i), Proposition 2.22, and Corollary 2.23(ii)), $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 2.19(ii), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft S_p-continuous.

Proposition 4.20. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function from soft extremally disconnected $(\tilde{X},\tilde{\tau},\mathcal{P})$ to $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$. Then, $\tilde{f}_{pu}$ is soft almost pre-continuous (resp., soft almost α-continuous) if $\tilde{f}_{pu}$ is soft S_p-continuous.

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}RO(\tilde{Y})$. Then, $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. By Proposition 2.26, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$). Thus, $\tilde{f}_{pu}$ is soft almost pre-continuous (resp., soft almost α-continuous).

5.Characterizations

In this section, we talk about soft S_p-continuous functions in terms of their properties and ways to describe them.

Theorem 5.1. For a soft function $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$, the following sentences are equivalent:

(i) $\tilde{f}_{pu}$ is soft S_p-continuous.

(ii) $\tilde{f}_{pu}^{-1}(C,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X}), \forall (C,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}^c$.

(iii) $\tilde{f}_{pu}(\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})), \forall\ (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

(iv) $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})), \forall\ (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

(v) $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})), \forall\ (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

(vi) $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}Bd(B,\acute{\mathcal{P}})), \forall\ (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

(vii) $\tilde{f}_{pu}(\tilde{s}S_pBd(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}Bd(\tilde{f}_{pu}(A,\mathcal{P})), \forall\ (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

Proof. (i) → (ii). Let $(C,\acute{\mathcal{P}})$ be any soft closed subset of $\tilde{Y}$. Then, $\tilde{Y}\tilde{\setminus}(C,\acute{\mathcal{P}})$ is soft open in $\tilde{Y}$, so by Theorem 2.9(viii) and Proposition 4.2, $\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(C,\acute{\mathcal{P}})) = \tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(C,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus, $\tilde{f}_{pu}^{-1}(C,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(ii) → (iii). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$. Since $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))$ and $\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))$ is a soft closed subset of $\tilde{Y}$. By (ii), $\tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})))$ is a soft S_p-closed set in $\tilde{X}$ and $(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})))$. But $\tilde{s}S_pcl(A,\mathcal{P}))$ is the smallest soft S_p-closed set containing $(A,\mathcal{P})$, so $\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})))$. Hence, $\tilde{f}_{pu}(\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))$.

(iii) → (iv). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$ and $\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{X}$. By (iii) and Theorem 2.9(viii) (ix), $\tilde{f}_{pu}(\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})))) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})))) \tilde{\subseteq} \tilde{s}cl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})) = \tilde{Y}\tilde{\setminus}\tilde{s}int(B,\acute{\mathcal{P}})$. Therefore, $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}\tilde{s}int(B,\acute{\mathcal{P}})) = \tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(\ \tilde{s}int(B,\acute{\mathcal{P}}))$. Since $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) = \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) = \tilde{X}\tilde{\setminus}\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$, so $\tilde{X}\tilde{\setminus}\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}}))$. Hence, $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$.

(iv) ↔ (v). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$ and $\tilde{f}_{pu}^{-1}(\tilde{s}int(\ \tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(\ \tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \leftrightarrow \tilde{f}_{pu}^{-1}(\ \tilde{Y}\tilde{\setminus}\tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\ \tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \leftrightarrow \tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(\ \tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\ \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \leftrightarrow \tilde{s}S_pcl(\ \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\ \tilde{s}cl(B,\acute{\mathcal{P}}))$.

(v) → (vi). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then by Definition 3.6(iii) and (v), $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) = \tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\cap} \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}\ \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\cap} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) = \tilde{f}_{pu}^{-1}(\tilde{s}cl\ (B,\acute{\mathcal{P}}) \tilde{\cap} \tilde{s}cl(\tilde{Y}\tilde{\setminus}\ (B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(\tilde{s}Bd(B,\acute{\mathcal{P}}))$. Hence, $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}Bd(B,\acute{\mathcal{P}}))$.

(vi) $\rightarrow$ (vii). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$, and by (vi), $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P})))$ $\tilde{\subseteq}\ \tilde{f}_{pu}^{-1}(\tilde{s}Bd(\tilde{f}_{pu}(A,\mathcal{P})))$. So, Theorem 2.9(x), $\tilde{s}S_pBd(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}Bd(\tilde{f}_{pu}(A,\mathcal{P})))$. Hence, $\tilde{f}_{pu}(\tilde{s}S_pBd(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}Bd(\tilde{f}_{pu}(A,\mathcal{P}))$.
(vii) $\rightarrow$ (vi). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$ and by (vii), we have $\tilde{f}_{pu}(\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}Bd(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})))$. So, by Theorem 2.9(ix), $\tilde{f}_{pu}(\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}Bd(B,\acute{\mathcal{P}})$. Hence, $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}Bd(B,\acute{\mathcal{P}}))$.
(vi) $\rightarrow$(i). Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}^c$ and by (vi), $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})\,)) \tilde{\subseteq}$ $\tilde{f}_{pu}^{-1}(\tilde{s}Bd(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})\,)) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))$ such that $\tilde{s}S_pBd(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))$. By Proposition 3.7(ii), $\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})) =$ $\tilde{X}\tilde{\setminus}\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Thus, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{f}_{pu}$ is soft S_p-continuous.

Theorem 5.2. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \rightarrow (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft bijective function. Then, $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(A,\mathcal{P}))$, $\forall\ (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

Proof. Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$, $\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$ and so $\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\in} \tilde{\sigma}$. By soft S_p-continuity of $\tilde{f}_{pu}$, then $\tilde{f}_{pu}^{-1}(\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))$. Since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{f}_{pu}^{-1}(\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} (A,\mathcal{P})$. But $\tilde{s}S_pint(A,\mathcal{P})$ is the largest soft S_p-open set contained in $(A,\mathcal{P})$, $\tilde{f}_{pu}^{-1}(\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{s}S_pint(A,\mathcal{P})$. Also, since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(A,\mathcal{P}))$.
Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $\tilde{s}int(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$. By assumption, we get $\tilde{s}int(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})))$. Since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{s}int(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}}) \tilde{\subseteq}$ $\tilde{f}_{pu}(\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})))$. Hence, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$. Thus, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{f}_{pu}$ is soft S_p-continuous.

Proposition 5.3. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \rightarrow (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $\tilde{S}\mathfrak{B}$ be any soft basis of $\tilde{Y}$. Then, $\tilde{f}_{pu}$ is soft S_p-continuous iff $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, $\forall\ (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}\mathfrak{B}$.

Proof. Assume that $\tilde{f}_{pu}$ is soft S_p-continuous. Since $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$, $\forall\ (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}\mathfrak{B}$, then by Proposition 4.2, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.
Conversely, let $\forall\ (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}\mathfrak{B}$, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $(O,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $(O,\acute{\mathcal{P}}) =$ $\tilde{\cup}_{\lambda\in\Lambda}(B_\lambda,\acute{\mathcal{P}})$ where $(B_\lambda,\acute{\mathcal{P}})$ is a soft member of $\tilde{S}\mathfrak{B}$ and Λ is a suitable index set. So, $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{\cup}_{\lambda\in\Lambda}(B_\lambda,\acute{\mathcal{P}})) = \tilde{\cup}_{\lambda\in\Lambda}\ \tilde{f}_{pu}^{-1}(B_\lambda,\acute{\mathcal{P}})$. By assumption, $\tilde{f}_{pu}^{-1}(B_\lambda,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}})$ is the soft union of a family of soft S_p-open sets in $\tilde{X}$, $\forall\ \lambda \in \Lambda$. Hence, $\tilde{f}_{pu}^{-1}(O,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{f}_{pu}$ is soft S_p-continuous.

Theorem 5.4. For a soft surjective function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \rightarrow (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$, the following sentences are equivalent:
(i) $\tilde{f}_{pu}$ is soft S_p-continuous.
(ii) $\forall\ (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$, $\tilde{s}int\tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}}))$, $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) = \tilde{\cap}_{\vartheta\in\aleph}(D_\vartheta,\mathcal{P})$ where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$.

(iii) $\forall\ (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$, $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) = \tilde{\bigcup}_{\vartheta \in \aleph}(C_\vartheta,\mathcal{P})$ where $(C_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$.
(iv) $\forall\ (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$, $\tilde{f}_{pu}(\tilde{s}int\tilde{s}cl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{f}_{pu}(A,\mathcal{P})$, $\tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{f}_{pu}(A,\mathcal{P})) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$ where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$.

Proof. (i) →(ii). Since $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$, then $\tilde{s}cl(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}^c$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, by Theorem 5.1(ii), then $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Therefore by Remark 2.20 and Proposition 2.19(iv), $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) \tilde{\in} \tilde{S}SC(\tilde{X})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Thus, $\tilde{s}int\tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}}))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$.
(ii) →(iii). Since $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$, then $\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. So, by (ii), $\tilde{s}int\tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}})))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Then, $\tilde{X}\tilde{\setminus}(\tilde{s}cl\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})))$ and $\tilde{X}\tilde{\setminus}(\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}}))) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Then, $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) = \tilde{\bigcup}_{\vartheta \in \aleph}(\tilde{X}\tilde{\setminus}(D_\vartheta,\mathcal{P})) = \tilde{\bigcup}_{\vartheta \in \aleph}(C_\vartheta,\mathcal{P})$, where $\tilde{X}\tilde{\setminus}(D_\vartheta,\mathcal{P}) = (C_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$.
(iii) →(i). Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $\tilde{s}int(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$ and thus by (iii), $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}cl\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}int(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \tilde{\bigcup}_{\vartheta \in \aleph}(C_\vartheta,\mathcal{P})$ where $(C_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$. Thus, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Hence, $\tilde{f}_{pu}$ is soft S_p-continuous.
(ii) →(iv). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$, and by (ii), $\tilde{s}int\tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Therefore, $\tilde{s}int\tilde{s}cl(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P})))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Hence, $\tilde{f}_{pu}(\tilde{s}int\tilde{s}cl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{f}_{pu}(A,\mathcal{P})$, $\tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{f}_{pu}(A,\mathcal{P})) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$.
(iv) →(ii). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$ and by (iv), $\tilde{f}_{pu}(\tilde{s}int\tilde{s}cl\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(B,\acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$ where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. This means that $\tilde{s}int\tilde{s}cl\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}}))$ and $\tilde{f}_{pu}^{-1}(\tilde{s}cl(B,\acute{\mathcal{P}})) = \tilde{\bigcap}_{\vartheta \in \aleph}(D_\vartheta,\mathcal{P})$, where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$.

6. Some properties of soft S_p-continuous functions

The restrictions of soft S_p-continuous functions to soft subspaces are soft S_p-continuous under some condition according to the following conclusions.

Proposition 6.1. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft S_p-continuous function. If $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., soft clopen), then $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous.

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. By soft S_p-continuity of $\tilde{f}_{pu}$ and Proposition 4.2, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., soft clopen), so by Proposition 3.4, $(\tilde{f}_{pu}\lceil_{\tilde{Z}})^{-1}(B,\acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\cap} \tilde{Z}$ is a soft S_p-open set in $\tilde{Z}$. Thus, $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z},\tilde{\tau}_Z,\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous.

Corollary 6.2. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft S_p-continuous function. If $\tilde{Z} \tilde{\in} \tilde{S}RO(\tilde{X})$, then $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous.

Proof. Since soft regular open is soft open, this is a direct result of Proposition 6.1.

Proposition 6.3. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous, if $\forall \widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, there is $\widetilde{e_x} \tilde{\in} \tilde{Z} \tilde{\in} \tilde{S}RC(\tilde{X})$ (resp., soft clopen) such that $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous.

Proof. Let $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$. and $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}\lceil_{\tilde{Z}}$ is soft S_p-continuous, there is $\widetilde{e_x} \tilde{\in} (A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ such that $\tilde{f}_{pu}\lceil_{\tilde{Z}}(A, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$. Also, since $\tilde{Z} \tilde{\in} \tilde{S}RC(\tilde{X})$ (resp., soft clopen). By Proposition 2.32, $(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and hence, $\tilde{f}_{pu}(A, \mathcal{P}) = \tilde{f}_{pu}\lceil_{\tilde{Z}}(A, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$. Thus, $\tilde{f}_{pu}$ is soft S_p-continuous.
We get the following results from Proposition 6.3:

Corollary 6.4. Let $\{\tilde{Z}_{\vartheta}: \vartheta \in \aleph\}$ be a soft regular closed (resp., soft clopen) cover of $\tilde{X}$. If $\tilde{f}_{pu}\lceil_{\tilde{Z}_{\vartheta}}: (\tilde{Z}_{\vartheta}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous for each $\vartheta \in \aleph$, then $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous.

Corollary 6.5. Let $\tilde{X} = \tilde{U} \tilde{\cup} \tilde{V}$, where $\tilde{U}$ and $\tilde{V}$ are soft regular closed (resp., soft clopen) sets in $\tilde{X}$, and both $\tilde{f}_{pu}\lceil_{\tilde{U}}: (\tilde{U}, \tilde{\tau}_{\tilde{U}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}\lceil_{\tilde{V}}: (\tilde{V}, \tilde{\tau}_{\tilde{V}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft S_p-continuous functions, then $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous.

Proposition 6.6. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft RC-continuous function. If $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous.

Proof. Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft RC-continuous, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}RC(\tilde{X})$. Since $\tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{X})$, so by Corollary 3.5, $(\tilde{f}_{pu}\lceil_{\tilde{Z}})^{-1}(B, \acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\cap} \tilde{Z}$ is a soft S_p-open set in $\tilde{Z}$. Thus, $\tilde{f}_{pu}\lceil_{\tilde{Z}}: (\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous.

Proposition 6.7. Let $\tilde{X} = \tilde{U} \tilde{\cup} \tilde{V}$, where $\tilde{U}$ and $\tilde{V}$ are soft regular closed (resp., soft clopen) sets in $\tilde{X}$ and both $\tilde{g}_{pu}: (\tilde{U}, \tilde{\tau}_U, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ and $\tilde{h}_{pv}: (\tilde{V}, \tilde{\tau}_V, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft S_p-continuous. If $\tilde{g}_{pu}(\widetilde{e_x}) = \tilde{h}_{pv}(\widetilde{e_x})$, $\forall \widetilde{e_x} \tilde{\in} \tilde{U} \tilde{\cap} \tilde{V}$, then the soft function $\tilde{f}_{ps}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ defined by $\tilde{f}_{ps}(\widetilde{e_x}) = \begin{cases} \tilde{g}_{pu}(\widetilde{e_x}) & if\ \widetilde{e_x} \tilde{\in} \tilde{U} \\ \tilde{h}_{pv}(\widetilde{e_x}) & if\ \widetilde{e_x} \tilde{\in} \tilde{V} \end{cases}$ is soft S_p-continuous.

Proof. Let $(B, \acute{\mathcal{P}})$ be any soft open subset of $\tilde{Y}$. Then, $\tilde{f}_{ps}^{-1}(B, \acute{\mathcal{P}}) = g_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\cup} \tilde{h}_{pv}^{-1}(B, \acute{\mathcal{P}})$. By soft S_p-continuity of $\tilde{g}_{pu}$ and $\tilde{h}_{pv}$ and Proposition 4.2, $\tilde{g}_{pu}^{-1}(B, \acute{\mathcal{P}})$ and $\tilde{h}_{pv}^{-1}(B, \acute{\mathcal{P}})$ are soft S_p-open sets in $\tilde{U}$ and $\tilde{V}$, respectively. Since $\tilde{U}$ and $\tilde{V}$ are soft regular closed (resp., soft clopen) sets in $\tilde{X}$, so by Proposition 2.32, $\tilde{g}_{pu}^{-1}(B, \acute{\mathcal{P}})$ and $\tilde{h}_{pv}^{-1}(B, \acute{\mathcal{P}})$ are soft S_p-open sets in $\tilde{X}$. So, by Proposition 2.19(iii), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{f}_{ps}$ is soft S_p-continuous.
Proposition 6.8. Let $\tilde{f}_{pu}, \tilde{g}_{qv}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft functions and $\tilde{Y}$ is a soft T_2-space. If $\tilde{f}_{pu}$ is soft S_p-continuous and $\tilde{g}_{pu}$ is soft perfectly continuous, then the soft set $(C, \mathcal{P}) = \{\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X}): \tilde{f}_{pu}(\widetilde{e_x}) = \tilde{g}_{qv}(\widetilde{e_x})\} \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. Let $\widetilde{e_x} \tilde{\notin} (C, \mathcal{P})$. Then, $\tilde{f}_{pu}(\widetilde{e_x}) \neq \tilde{g}_{qv}(\widetilde{e_x})$. Since $\tilde{Y}$ is a soft T_2-space, then there is soft open sets $(B_1, \acute{\mathcal{P}})$ and $(B_2, \acute{\mathcal{P}})$ in $\tilde{Y}$ such that $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B_1, \acute{\mathcal{P}})$, $\tilde{g}_{qv}(\widetilde{e_x}) \tilde{\in} (B_2, \acute{\mathcal{P}})$ and $(B_1, \acute{\mathcal{P}}) \tilde{\cap} (B_2, \acute{\mathcal{P}}) = \tilde{\emptyset}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, there is $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$

such that $\tilde{f}_{pu}(A_1,\mathcal{P}) \tilde{\subseteq} (B_1,\acute{\mathcal{P}})$. Since $\tilde{g}_{qv}$ is soft perfectly continuous, there exists a soft clopen set $(A_2,\mathcal{P})$ in $\tilde{X}$ containing $\widetilde{e_x}$ such that $\tilde{g}_{qv}(A_2,\mathcal{P}) \tilde{\subseteq} (B_2,\acute{\mathcal{P}})$. We put $(A,\mathcal{P}) = (A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P})$, then by Proposition 2.31, $\widetilde{e_x} \tilde{\in} (A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $(A,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$. Therefore, by Proposition 3.7(i), $\widetilde{e_x} \tilde{\notin} \tilde{s}S_pcl(C,\mathcal{P})$. This shows that, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proposition 6.9. Let $\tilde{f}_{pu}, \tilde{g}_{qv}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be soft S_p-continuous functions such that $\tilde{S}S_pO(\tilde{X})$ is a soft topology on $\tilde{X}$, and $\tilde{Y}$ is a soft T_2-space. Then, the soft set $(C,\mathcal{P}) = \{\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X}): \tilde{f}_{pu}(\widetilde{e_x}) = \tilde{g}_{qv}(\widetilde{e_x})\} \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. Let $\widetilde{e_x} \tilde{\notin} (C,\mathcal{P})$. Then, $\tilde{f}_{pu}(\widetilde{e_x}) \neq \tilde{g}_{qv}(\widetilde{e_x})$. Since $\tilde{Y}$ is a soft T_2-space, then there are soft open sets $(B_1,\acute{\mathcal{P}})$ and $(B_2,\acute{\mathcal{P}})$ in $\tilde{Y}$ such that $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (B_1,\acute{\mathcal{P}})$, $\tilde{g}_{qv}(\widetilde{e_x}) \tilde{\in} (B_2,\acute{\mathcal{P}})$ and $(B_1,\acute{\mathcal{P}}) \tilde{\cap} (B_2,\acute{\mathcal{P}}) = \tilde{\emptyset}$. Since $\tilde{f}_{pu}$ and $\tilde{g}_{qv}$ are soft S_p-continuous, there is $\widetilde{e_x} \tilde{\in} (A_1,\mathcal{P}), (A_2,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{f}_{pu}(A_1,\mathcal{P}) \tilde{\subseteq} (B_1,\acute{\mathcal{P}})$ and $\tilde{g}_{qv}(A_2,\mathcal{P}) \tilde{\subseteq} (B_2,\acute{\mathcal{P}})$. Then by hypothesis, the soft set $(A,\mathcal{P}) = (A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\widetilde{e_x}$ and $(A,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$. Therefore, by Proposition 3.7(i), $\widetilde{e_x} \tilde{\notin} \tilde{s}S_pcl(C,\mathcal{P})$. This shows that, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proposition 6.10. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma}_{\tilde{Y}},\acute{\mathcal{P}})$ be a soft S_p-continuous function and $\tilde{Y}$ is a soft subspace of $\tilde{Z}$. Then, $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Z},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous.

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Then, $(B,\acute{\mathcal{P}}) \cap \tilde{Y} \tilde{\in} \tilde{\sigma}_{\tilde{Y}}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, so by Proposition 4.2, $\tilde{f}_{pu}^{-1}((B,\acute{\mathcal{P}}) \cap \tilde{Y}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. But, $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} \tilde{S}P(\tilde{Y})$ for each $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$. Therefore, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}((B,\acute{\mathcal{P}}) \cap \tilde{Y}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Hence by Proposition 4.2, $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Z},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-continuous.
The soft composition of two soft S_p-continuous functions is usually not a soft S_p-continuous function as illustrated in the example:

Example 6.11. Let $X = \{x_1,x_2,x_3\}$, $\mathcal{P} = \{e\}$ with the soft topology $\tilde{\tau} = \{\tilde{\emptyset},\tilde{X},(A_1,\mathcal{P})\}$ and $\tilde{\sigma} = \{\tilde{\emptyset},\tilde{X},(B_1,\mathcal{P}),(B_2,\mathcal{P}),(B_3,\mathcal{P})\}$ on $\tilde{X}$, where $\tilde{\emptyset} = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e,X)\}$, $(A_1,\mathcal{P}) = \{(e,\{x_2\})\}$, $(B_1,\mathcal{P}) = \{(e,\{x_1\})\}$, $(B_2,\mathcal{P}) = \{(e,\{x_3\})\}$, and $(B_3,\mathcal{P}) = \{(e,\{x_1,x_3\})\}$. Then, $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{X},\tilde{\sigma},\mathcal{P})$ are soft topological spaces over X. So, $\tilde{S}S_pO(\tilde{X},\tilde{\tau}) = \{\tilde{\emptyset},\tilde{X},\{(e,\{x_2\})\},\{(e,\{x_2,x_3\})\}\}$ and $\tilde{S}S_pO(\tilde{X},\tilde{\sigma}) = \{\tilde{\emptyset},\tilde{X}, \{(e,\{x_1,x_2\})\}, \{(e,\{x_2,x_3\})\}\}$. Now define the soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{X},\tilde{\sigma},\mathcal{P})$, where $p: \mathcal{P} \to \mathcal{P}$ is a function defined by $p(e) = \{e\}$ and $u: X \to X$ is a function defined by $u(x_1) = \{x_2\}$, and $u(x_2) = u(x_3) = \{x_1\}$. The soft function $\tilde{f}_{pu}$ is soft S_p-continuous, and define the soft function $\tilde{h}_{pv}: (\tilde{X},\tilde{\sigma},\mathcal{P}) \to (\tilde{X},\tilde{\tau},\mathcal{P})$, where $v: X \to X$ is a function defined by $v(x_1) = \{x_1\}$, and $v(x_2) = v(x_3) = \{x_2\}$. The soft function $\tilde{h}_{pv}$ is soft S_p-continuous. But $\tilde{h}_{pv} o \tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{X},\tilde{\tau},\mathcal{P})$ is not soft S_p-continuous, since $vou(x_1) = \{x_2\}$, $vou(x_2) = vou(x_3) = \{x_1\}$ and $(A_1,\mathcal{P}) \tilde{\in} \tilde{\tau}$, then $(\tilde{h}_{pv} o \tilde{f}_{pu})^{-1}(A_1,\mathcal{P}) = \{(e,\{x_1\})\} \tilde{\notin} \tilde{S}S_pO(\tilde{X},\tilde{\tau})$.

Theorem 6.12. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}}) \to (\tilde{W},\tilde{\rho},\ddot{\mathcal{P}})$ be two soft functions. Then, $\tilde{g}_{qv} o \tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{W},\tilde{\rho},\ddot{\mathcal{P}})$ is soft S_p-continuous, if one of the below is held:
(i) $\tilde{f}_{pu}$ is soft S_p-continuous and $\tilde{g}_{qv}$ is soft continuous.

(ii) $\tilde{f}_{pu}$ is soft continuous and soft open, $\tilde{g}_{qv}$ is soft S_p-continuous.
(iii) $\tilde{f}_{pu}$ is soft irresolute and $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft locally indiscrete, $\tilde{g}_{qv}$ is soft S_p-continuous.
(iv) $\tilde{f}_{pu}$ and $\tilde{g}_{qv}$ are soft S_p-continuous and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete.

Proof. (i) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\rho}$. Since $\tilde{g}_{qv}$ is a soft continuous function, then $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, so by Proposition 4.2, $(\tilde{g}_{qv} o \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}})$=$\tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{g}_{qv} o \tilde{f}_{pu}$ is soft S_p-continuous.
(ii) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\rho}$. Since $\tilde{g}_{qv}$ is soft S_p-continuous, then by Proposition 4.2, $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft continuous and soft open, then by Proposition 4.7, $(\tilde{g}_{qv} o \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{g}_{qv} o \tilde{f}_{pu}$ is soft S_p-continuous.
(iii) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\rho}$. Since $\tilde{g}_{qv}$ is soft S_p-continuous, then by Proposition 4.2, $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ and so $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft irresolute, then $(\tilde{g}_{qv} o \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft locally indiscrete, then by Proposition 2.22, $(\tilde{g}_{qv} o \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{g}_{qv} o \tilde{f}_{pu}$ is soft S_p-continuous.
(iv) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\rho}$. Since $\tilde{g}_{qv}$ is soft S_p-continuous, by Proposition 4.2, $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Corollary 2.23(i), $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. By soft S_p-continuity of $\tilde{f}_{pu}$ and Proposition 4.2, $(\tilde{g}_{qv} o \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Proposition 4.2, $\tilde{g}_{qv} o \tilde{f}_{pu}$ is soft S_p-continuous.

7.Conclusion.

Through the current research work, we have continued to investigate the properties of soft semi- continuous functions in soft topological spaces. By using the soft S_p-open (resp., soft S_p-closed) set, a new type of soft semi-continuous function is defined as a strong form, named a soft S_p-continuous function, which is weaker than both soft S_c-continuous and soft RC-continuous functions. A number of descriptors and some of their characteristics have been obtained. Also, its interactions with other soft continuous functions were investigated. Some supportive examples were presented to demonstrate that these functions do not coincide. Furthermore, some conditions were provided that renders soft S_p-continuity equivalent to some other types of soft continuity and introduce a soft restriction on soft S_p-continuous function. Researchers might be able to use the results of this work to conduct more research in the field of soft topology and the practical applications within this field.

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