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The Effect of Solar and Lunar Attraction and SRP on the HEO of Satellite

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Abstract
The perturbation of a satellite high orbit due to the presence of other gravitational bodies (such as the Sun and the Moon) and SRP from the conservative perturbing forces were studied, using our modified model. A precise calculation of the perturbations is possible only if the initial orbit is sufficiently well known. Orbital elements that have been entered hp=3000Km., inclination= 63°, 23° and eccentricity= 0.1, longitude of ascending node 30°, argument of perigee 40° where the orbital elements will deviate from initial values with time through 3000 days. Newton-Rapson method was used to calculate the position and velocity with out perturbation . The perturbed equation of motion solved numerically using the fourth order Runge-Kutta method .The results appear that all elements are varies non linear with time for all inclination 63,23 deg ,the time period was increased, It's mean value 22.55 hours with i=63° and 22.52 hours with i=23° deg. and the orbit approach to circle .

Keywords: Satellite, other body attraction, perturbation, orbital elements.

تأثير الشمس والقمر الجذبي وضغط الإشعاع الشمسي على مدارات الأقمار الصناعية العالية

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الخلاصة
تم دراسة الاضطرابات على المدارات العالية للأقمار الصناعية التي يسببها جذب الشمس والقمر وكذلك ضغط الإشعاع الشمسي وهي من القوى المحفوظة ، باستخدام نموذج مطور. الحسابات الحالية تكون مناسبة عندما يكون المدار الابتدائي معروف لدينا. القيم الابتدائية للعناصر المدارية المستخدمة :

hp=3000Km., inc= 63°and 23° , e= 0.1, Ω= 30°, ω= 40°

حيث تنحرف هذه العناصر عن قيمها الابتدائية خلال ٣٠٠٠ يوم . تم استخدام طريقة نيوتن- رفسن لحساب الموضع والسرعة بدون اضطراب. تم حل معادلة الحركة بوجود الاضطراب باستخدام طريقة رانج-كوتا من المرتبة الرابعة. أظهرت النتائج أن جميع قيم العناصر المدارية تتغير بشكل غير خطي عند كلا الميلين ٦٣ و٢٣ درجة ، زمن الدورة يزداد وكان معدله ٢٢,٥٥ ساعة عندما الميل ٦٣ درجة و ٢٢,٥٢ ساعة عندما الميل ٢٣ درجة. والمدار يقترب من الشكل الدائري.

Introduction

The satellite orbits are classified as many types according to high and inclination as well as the aims of the satellite work. The Earth and satellite are two body in space moving around the center of mass which is consider on the center of the Earth where mass of the earth was more greater than mass of satellite $M_e \gg m_{sat}$,there are many affects on the satellite orbit by the other body in space or by solar radiation ,these effect call a perturbations which were studied by many astronomers as: Kozai

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(1959) developed the main secular and long-period terms of the disturbing function due to the lunisolar perturbations in terms of the orbital elements [1]. Musen (1960) derived first order expressions for the rates of change in the osculating elements caused by direct solar radiation pressure [2]. This research would be later expanded by Musen, Bailie and Upton (1961) to include the Paralactic term in the disturbing function [3]. In 1961 Kozai developed the main secular and long period terms of the disturbing function due to the lunar and solar perturbations in terms of the orbital elements of the satellite [4]. Kaula (1962) developed the Lunar and Solar disturbing function for a close satellite and developed a quasi potential for the radiation pressure effects to be used in the equation of motion. He did not obtain the solution [5]. Radzievskii and Artem've (1962) studied the effect of solar radiation pressure on the motion of artificial Earth satellites [6]. Adams, Jr., and Hodge(1965) calculate the effect of SRP on orbital eccentricity. [7]. Buffet (1985) studied the perturbations of orbital elements of the GPS satellites [8].

In 1992 Broucke developed the general form of the disturbing function of the third body which was truncated after the term of second order in the expansion of Legendre polynomials [9].
Su (2000) studied the GEO, MEO satellites and like GPS, GLONAS [10].

The effect of the third body perturbation studied by Solórzano and Prado (2004) [11] and [12,13]. Eshagh and Najafi, (2007), Calculate the Perturbations affect on orbital elements of a low earth orbiting satellite"[14]. F. Toshio (2008) published an orbital element formulation without solving Kepler's equation[15]. Costa and Prado (2010) developed a semi-analytical study of the perturbation effected on a spacecraft by a third body involved in the dynamics [16].

Khalil and Ismail (2011) studied Effects of radiation pressure and Earth's obletness on high altitude artificial satellite orbit[17]. Rahoma and Metris (2012) presented Invariant Relative Orbits Taking into Account Third-Body Perturbation[18] . Lara , San-Juan , López and Cefola (2012) published on the third-body perturbations of high-altitude orbits[19].

The aim of this work were to calculate the variation of orbital elements due to solar and lunar gravity for the height Earth orbit of satellites at differences height, inclination, and eccentricity through 3000 periods or 2818 days using a new modified model.

The Satellite Orbit and Solution:

In celestial mechanics one is concerned with the motions of celestial bodies under the influence of mutual mass attraction. The simplest form is the motion of two bodies (Two-body problem). For artificial satellites the mass of the smaller body (satellite) usually is neglected, compared with the mass of the central body (the Earth).

Under the assumption that the mass distribution of bodies is homogeneous, and thus generates the gravitational field effect of a point mass the orbital motion for the two-body problem can be described empirically by Kepler's laws and can also be derived analytically from Newtonian Mechanics [6]. We can find the mean motion (n) :

$$n = \sqrt{\frac{\mu}{a^3}} \quad , \qquad (1)$$

Where:
$\mu = G(M + m)$,
$G = 6.673 \times 10^{-11}\, m^3 kg^{-1} S^{-2}$ is the gravitational constant,
M and m are the masses of the Earth and satellite respectively.
The mean anomaly in any time which used to describing the location of the satellite in an orbit

$$M_{i+1} = M_i + n \times \Delta t \ , \qquad (2)$$

And we can calculate the eccentric anomaly for the orbit as the equation of Kepler, although its looks as simple equation but its solved by using numerical methods(iterative) ,and one of these formulas which gives a good approximate results can be found by following equation in ref. [16]:

$$E_{i+1} = E_i - \frac{E_i\, e\sin E_i - M}{1 - e1\cos(E_i} \qquad (3)$$

$$e1 = e \times \frac{180}{\pi}$$

To find the Cartesian coordinate (x_w and y_w) to the satellite in his orbit

$$x_w = a(\cos E - e),$$

$$y_w = a\sqrt{1-e^2}\sin E\,,$$
$$z_w = 0$$

And the displacement radius (r) will be

$$r = a(1 - e\cos E) \qquad (4)$$

By direct differentiation for (x_w and y_w) one obtains

$$\dot{x}_w = \frac{\sqrt{\mu a}}{r}\sin E$$

$$\dot{y}_w = \frac{\sqrt{\mu a\,(1-e^2)}}{r}\cos E$$
$$\dot{Z}_w = 0$$
$$\dot{r} = \frac{\sqrt{\mu a}}{r}\, e\sin E \qquad (5)$$

The conversion of position and velocity of the satellite from this orbital plane to the Earth equatorial plane can be utilized by Gaussian vector (3*3-conversion matrix), which content Euler angle (I, Ω, ω) .[20,21]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{-1}\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \qquad (6)$$

Where R^{-1} is the inverse of Gauss matrix $R^{-1} = \begin{bmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{bmatrix}$

And the velocity components ($\dot{x}$, $\dot{y}$, $\dot{z}$) can be gated by the same way:

$$\dot{r} = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \qquad (7)$$

The basic equation of satellite motion is:

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\,\boldsymbol{r} \qquad (8)$$

Perturbed Equation of Motion and Solutions Methods:

To distinguish between perturbing forces and the central force (central body acceleration) these are generally referred to the satellite experiences additional accelerations < 0.001 main acceleration $\ddot{\vec{r}}$. The extended equations of motion are[9]:

$$\ddot{r} = -\frac{\mu}{r^3}\,r + \ddot{r}_p \qquad (9)$$

Perturbing forces are in particular responsible for:

1. Accelerations due to the non-spherically and inhomogeneous mass distribution within Earth (central body),$\ddot{\vec{r}}_E$.
2. Accelerations due to other celestial bodies (Sun, Moon and planets), mainly $\ddot{\vec{r}}_S$, $\ddot{\vec{r}}_M$.
3. Accelerations due to Earth and oceanic tides, $\ddot{\vec{r}}_e$, $\ddot{\vec{r}}_o$.
4. Accelerations due to atmospheric drag, $\ddot{\vec{r}}_D$.
5. Accelerations due to direct and Earth-reflected solar radiation pressure, $\ddot{\vec{r}}_{SP}$, $\ddot{\vec{r}}_A$.

$$\ddot{\boldsymbol{r}}_p = \ddot{\boldsymbol{r}}_E + (\ddot{\boldsymbol{r}}_S + \ddot{\boldsymbol{r}}_M) + (\ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_o) + \ddot{\boldsymbol{r}}_D + (\ddot{\boldsymbol{r}}_{SP} + \ddot{\boldsymbol{r}}_A).$$

Integration of Equation (9) with $\ddot{\boldsymbol{r}}_p = (\ddot{\boldsymbol{r}}_S + \ddot{\boldsymbol{r}}_M)$ by using numerical integration method and the six variables required from equation (9) are composed of three positional components and three velocity components.

The fourth-order Runge Kutta integration method has been used hear to find the accurate components of position and velocity which are used to calculate the new elements for the perturbed orbit.

The Modified method to Calculate the Solar and Lunar Gravity Perturbation:

The effects of the Moon will be treated as a third body acting on the satellite. Although the mass of the Moon is much lower than that of the Sun the reduced distance between perturbing body and satellite makes the Lunar perturbation about equal to the Solar perturbation.

Assuming that the Sun and the Moon can be considered to be point-masses, as with the satellite, the basic equation of motion (8). The following equations are valid for the accelerations acting on a satellite with negligible mass. The axes of an arbitrary are *x, y, z*, where x,y are inertial reference plane (Equatorial plane).

Using the notation of Figure-1 the acceleration of the satellite caused by the mass M_l of the Moon and the mass M_e of Earth [4,18]

$$\ddot{r}_0 = G\left(-\frac{M_e}{|r|^3}\vec{r} + \frac{M_l}{|r_{satl}|^3}\vec{r}_{satl}\right) \qquad (10)$$

Furthermore, the acceleration of *the Earth* caused by the Moon (M_l)

$$\ddot{r}_{0E} = G\frac{M_l}{|r_l|^3}\vec{r}_l \qquad (11)$$

The relative acceleration of the satellite with respect to Earth is

$$\ddot{r}_0 - \ddot{r}_{0E} = \ddot{r} = G\left\{\frac{M_l}{|r_{satl}|^3}\vec{r}_{satl} - \frac{M_l}{|r_l|^3}\vec{r}_l - \frac{M_e}{|r|^3}\vec{r}\right\} \qquad (12)$$

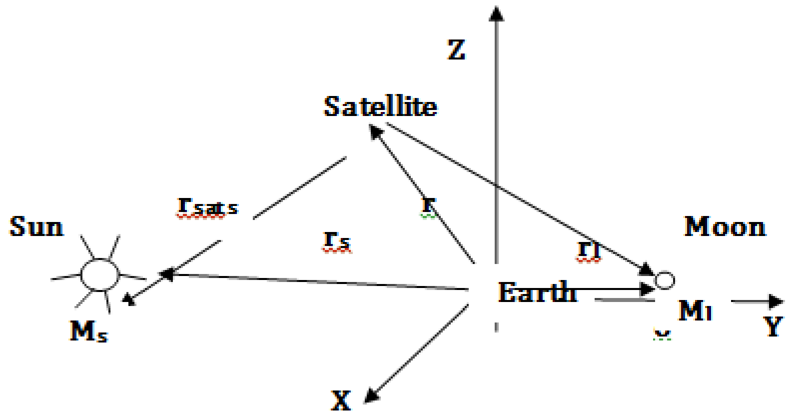


Figure 1- Vectors and gravitational force of the Sun and the Moon on the Earth satellite

With $|\boldsymbol{r}_{sat}| = r_{sat}$, $|\boldsymbol{r}| = r$ and with the origin of the coordinate system at Earth's center of mass, founded

$$\ddot{r} = -\frac{GM_e}{r^3}\vec{r} + GM_l\left(\frac{\vec{r}_{satl}}{r_{satl}{}^3} - \frac{\vec{r}_l}{r_l{}^3}\right) \qquad (13)$$

The first term is due to the acceleration caused by Earth (central term). The additional perturbing acceleration, caused by the gravitational attraction of the Moon acting on the satellite and on the Earth. Where $\bar{r}_{satl} = \bar{r}_l - \bar{r}$, $r_{satl}{}^3 = |r_l - r|^3$

The modified in this model that is the second term in equation (12) must be represent the variation of Moon position by Earth gravity because the $M_e > M_l$ and the Moon and satellite rotate around the Earth. Must be different for the case of the Sun, therefore equation (13) must be written as the following form:

$$\ddot{r} = -\frac{GM_e}{r^3}\vec{r} + G\left(M_l\frac{\vec{r}_l - \vec{r}}{|r_l - r|^3} - M_e\frac{\vec{r}_l}{r_l{}^3}\right) \qquad (14)$$

The equations (12) and (14) are used to find satellite position with the Moon attraction and have been getting a results shown in .

The corresponding of eq (13) influence $\ddot{r}_s$, due to the Sun, is [10,24]

$$\ddot{r}_s = -\frac{GM_e}{r^3}\vec{r} + GM_s\left(\frac{\vec{r}_s - \vec{r}}{|r_s - r|^3} - \frac{\vec{r}_s}{r_s{}^3}\right) \qquad (15)$$

The masses of the disturbing bodies and their locations within a geocentric reference frame have to be known for numerical computations. This formulas can also be used to calculate the perturbations on artificial satellites caused by the planets.

The position of the Moon and the Sun as geocentric ecliptical coordinates $(\lambda_l,\beta_l),(\lambda_s,\beta_s)$ The ecliptic longitude is given by [22,23]. Transfer the ecliptic coordinates into equatorial coordinates (α_l,δ_l) , (α_s,δ_s) [22], The position components distance in Cartesian coordinate can be calculated using equatorial coordinate as the following [23]:

$$x_l = r_l \cos\delta_l \cos\alpha_l \ , \ y_l = r_l \cos\delta_l \sin\alpha_l \ , \ z_l = r_l \sin\delta_l$$

The Moon's distance from the centre of the Earth can be calculated as the following:

$$r_l = 385000 - 20905\cos l - 3699\cos(2D' - l) - 2956\cos(2D') - 570\cos(2l) + 246\cos(2l - 2D') - 152\cos(l + l' - 2D') \qquad (16)$$

The distance of the Sun from the Earth center can be also found as [22]:

$$r_s = \frac{a_e(1 - e_e{}^2)}{1 + e_e\cos(\lambda_s - 282.596403)} \qquad (17)$$

a_e is the semi major axis of Earth orbit equal 1.495985×10^8 km.

e_e is the eccentricity of Earth orbit and equal to 0.016718.

Calculating the Orbital Elements

The elliptical orbital elements in general are (i, Ω, ω, a, e, M) can be calculated from the component of position, velocity and angular momentum components (h_x, h_y, h_z) as follows [20, 21, 23] :

The inclination (i) of the orbit from the equatorial plane is given by

$$\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z} \qquad (18)$$

The longitude of ascending node (Ω) is calculated as

$$\tan \Omega = \frac{h_x}{-h_y} \qquad (19)$$

The argument of perigee w : $$\tan\omega = \left(\frac{zh}{-xh_y + yh_x}\right) \qquad (20)$$

the semi-major axis of the orbit calculated as:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1} \qquad (21)$$

For elliptic orbits a will always be positive. The eccentricity (e) of the orbit is calculated as:

$$e = \sqrt{1 - \frac{h^2}{\mu a}} \qquad (22)$$

The eccentric anomaly (E) is calculated as :

$$\tan E = \left(\frac{1 - \frac{r}{a}}{x\dot{x} + y\dot{y} + z\dot{z}}\right)\sqrt{a\mu} \qquad (23)$$

The mean anomaly (M) is calculated from Kepler equation:

The Programs Algorithm:

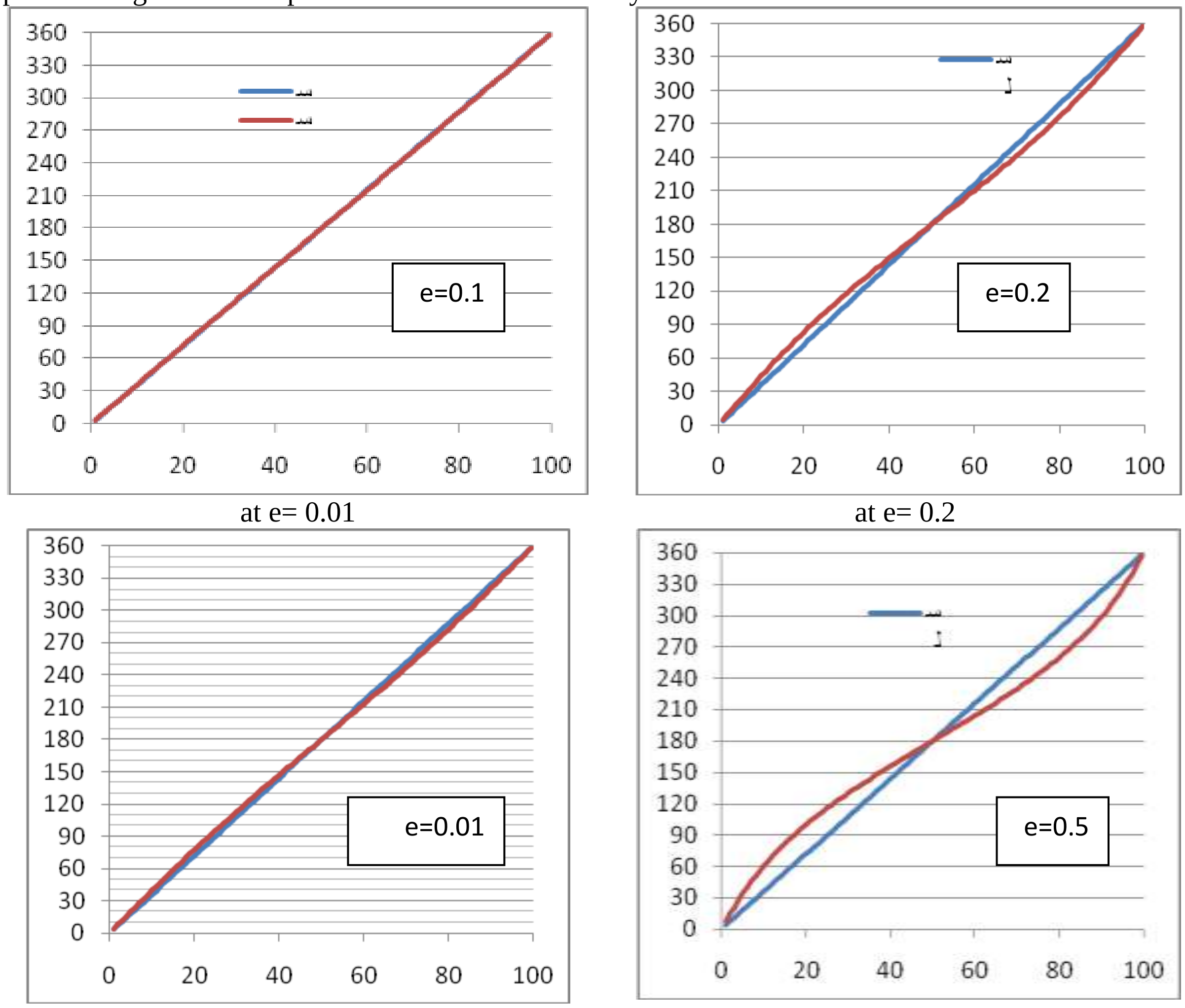
1. Calculation of the Julian date at perigee by calculation Julian date and the time of period .
2. Calculation of position and velocity of satellite by solving kepler's equation using Newton Rapson's method.
3. Calculation of the central acceleration and solar radiation pressure perturbation and lunar attraction perturbation and find the position and velocity components by using fourth order Runge-Kutta method trough time T/100.
4. Calculation of the satellite orbital elements due to each perturbation through 1000 periods.

The semi major axis of the orbit $a = \frac{r_p}{(1-e)}$

And the period calculation by kepler's 3th law $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

Results and Discussion

The solving of Kepler equation to calculate the eccentric anomaly for any mean anomaly trough any period using Newton-Rapson method with un-accuracy = 0.000000001.



at e= 0.01 at e= 0.2

At e= 0.5

Figure 2-The eccentric anomaly and the mean anomaly through one period for satellite at deferent e (0.01, 0.1, 0.2, 0.5) using Newton Raphson method.

Figure-2 shows the eccentric anomaly and the mean anomaly through one period have a same value at 0,50,100 step no. or 0,180,360 degree. Also they are approach each other at small eccentricity . the orbit which studied is e=0.1 where the most satellites have near this value.

Figures-3 shows that the periodic variation in position and velocity of satellite without perturbation through 20 and one periods between perigee (r = 26378.165 , v= 4.077) and apogee (r = 32239.980 , v = 3.336). Figures-4,5-a illustrate the change in position and velocity through 3000 days (300000) locations with solar and lunar attraction as a Fig (5-b) the scope of the velocity variation through 20 periods it's show that the perigee and apogee slowly approach to other with time because the orbit approach to circle. Figures from (6-11) shows the effect of both perturbing bodies (Sun and Moon) and SRP on the satellite orbit (orbital elements) as the following:

Figure-6 shows that the eccentricity (*e*) curvature convex decrease where the orbit approach a circle after nearly 2000 days at *both i*= 63,23°.

Figure-7 shows that the inclination (*i*) have a small linear increase at both cases *i*= 63, 23° and more percentage increase *i*= 23°.

Figure-8 shows that the time period (*Tp*) was concave increases from 80870 to 81470 sec. at *i*=63°, and concave increases from 80870 to 81210 sec. at *i*=23° and the orbit approach to circle.

Figure-9 shows the semi-major axis curvature increase ,the increases is to 40625Km. at *i*= 63° , but 40525 at *i*= 23° that means the Solar and Lunar perturbations make a circle orbits.

Figure-10 longitude of ascending node (Ω) for h_p=30000 Km is curvature increases and decrease through 3000 days the peak (30.0045 at 7900 MJD) at i=63 deg. put the peak (30.015 at 7900MJD) at i=23 deg.

Figure-11 shows a curveted decreasing relation for values of the argument of perigee at h_p=30000 Km due to Solar and Lunar attraction and Solar radiation pressure , it's clear the same variation at *i*= 23° and at *i*=63°.

The results are agree with many references as [20, 23, 24].

The Important Conclusions:

1. The semi major axis of the orbit and the time period are increase with time, but the eccentricity of the orbit is decrease.
2. The results indicate to secular variation in all orbital elements.
3. The solar attraction makes the inclination and right ascension of ascending node are fixed at *i* = 23.5° and decreases at *i* = 63°, That means the satellite orbits are more stable near ecliptic plane for MEO and HEO.
4. The lifetime of the satellite is long for HEO with all perturbations.

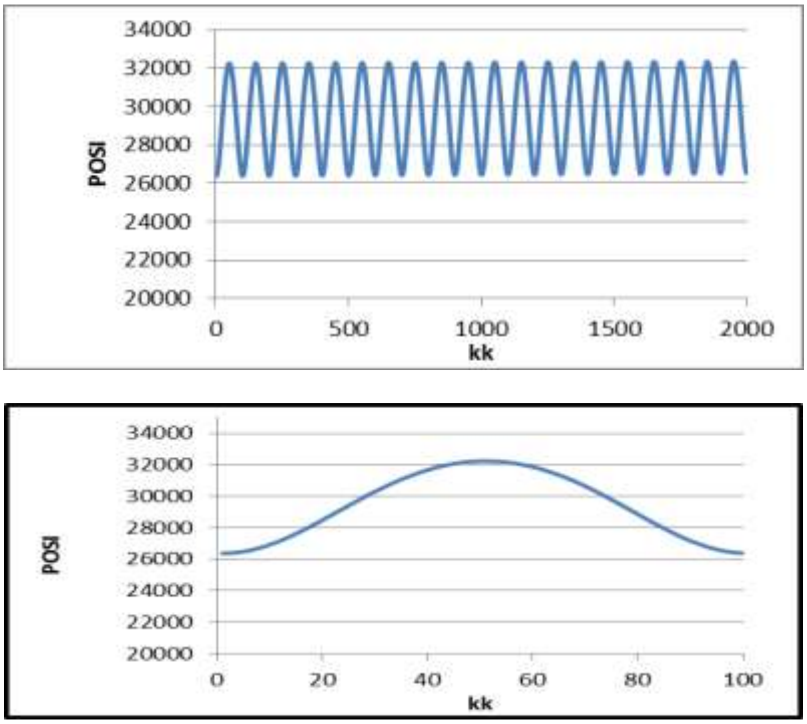


Figure 3- Satellite's position as a function of time during 20 ,one period .

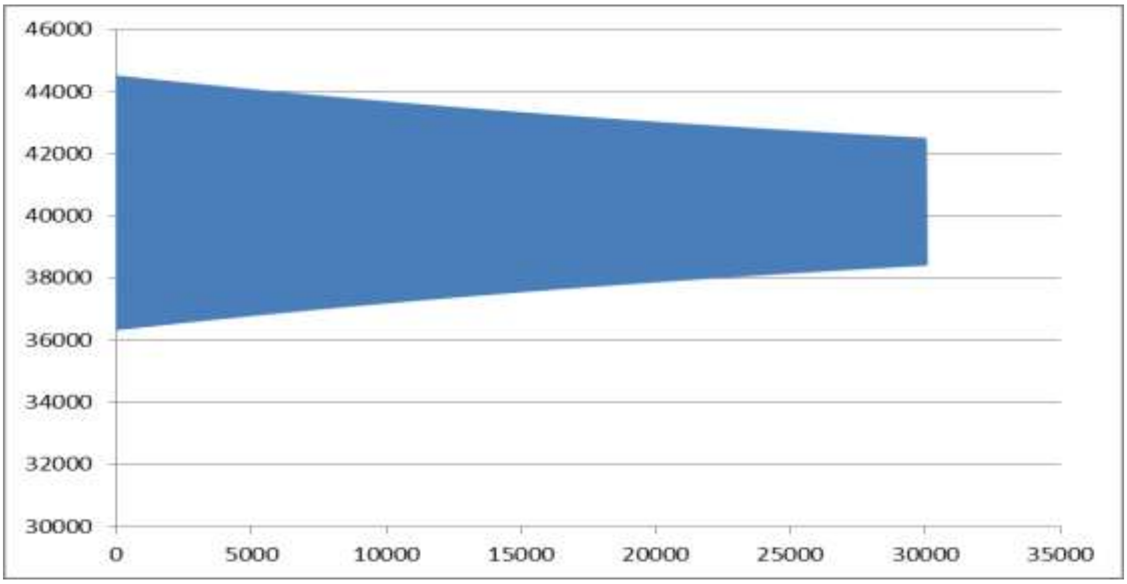


Figure 4- Position of satellite with time step no. through 30000 step with sun and moon attraction perturbations i=63, hp=30000Km.

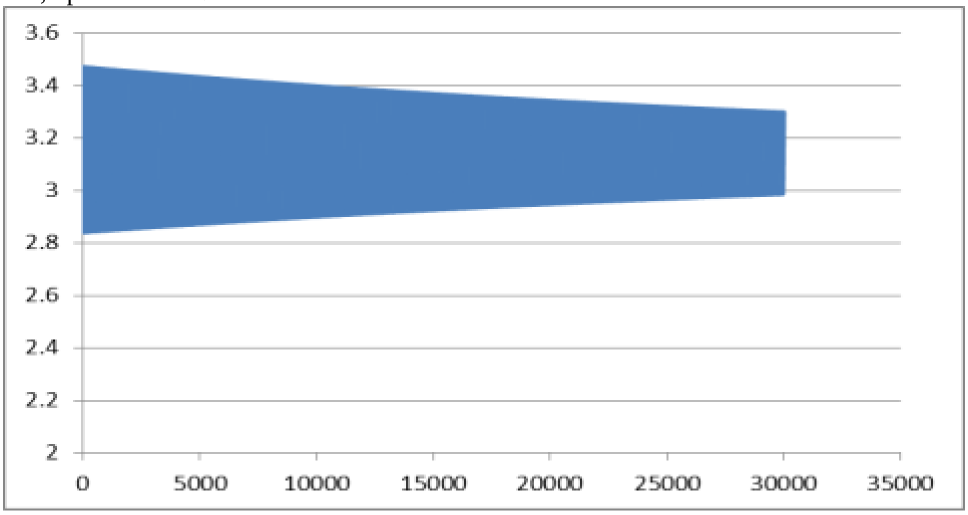


Figure 5a- Velocity(m/sec.) of satellite with time step no. through 300 periods with sun and moon attraction perturbations i=63, hp=30000Km.

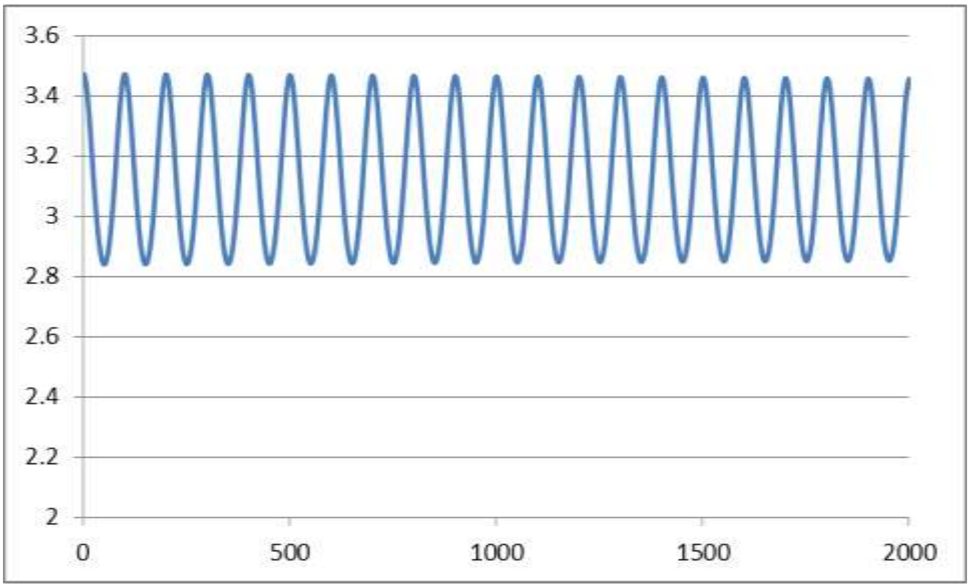


Figure 5b- Scope from fig (5.a) through 20 periods only.

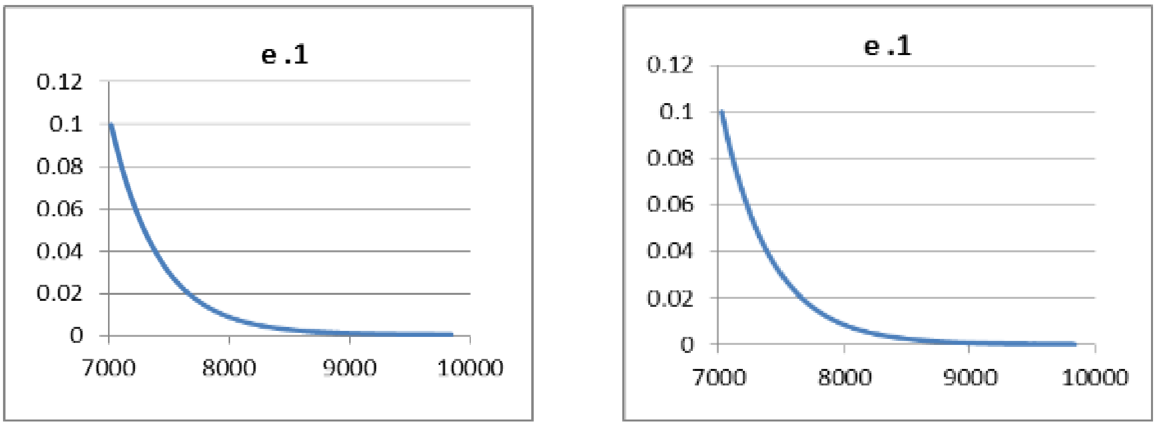


Figure 6-The eccentricity variation at hp =30000 Km. i=63, 23 deg. with Solar and Lunar attraction and SRP.

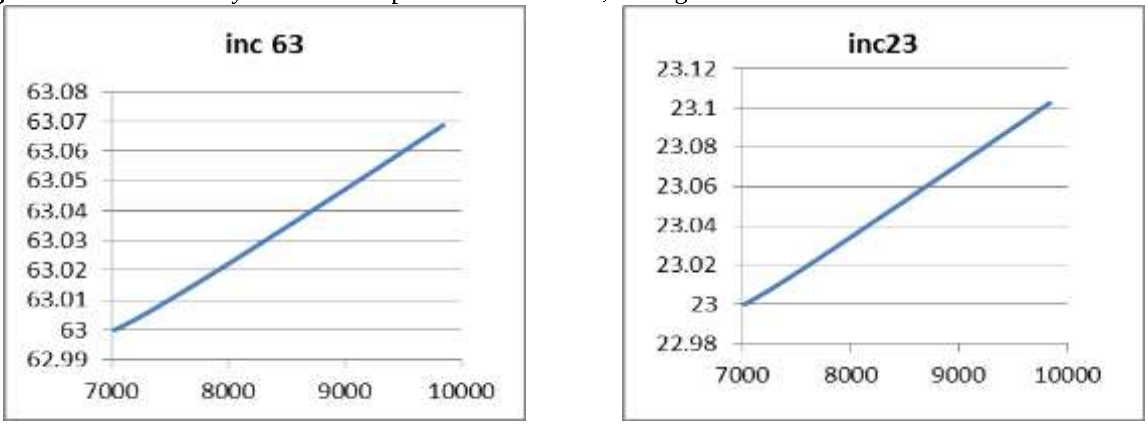


Figure 7- The Inclination variation at hp =30000 Km. i=63, 23 deg. with Solar and Lunar attraction and SRP.

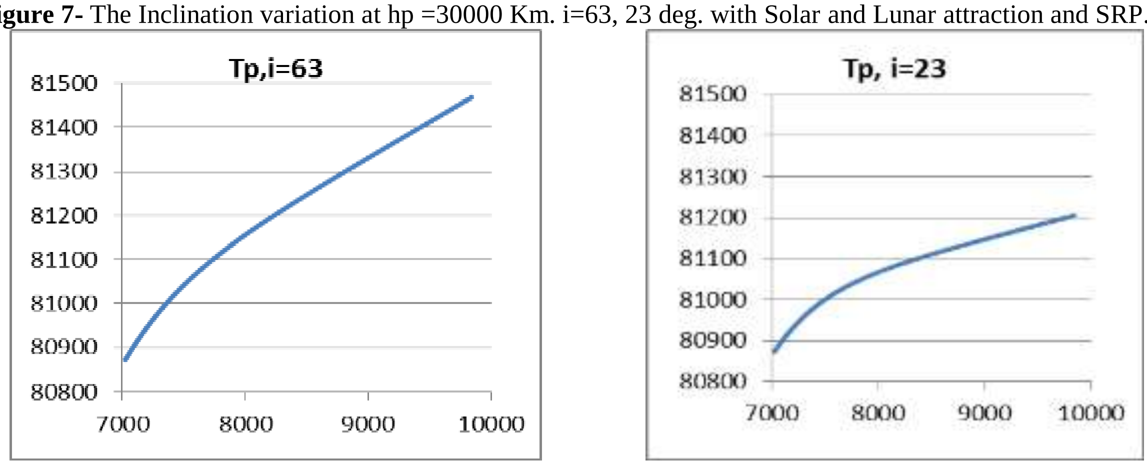


Figure 8- The period variation with J.D. at i=63,23 deg. with solar and lunar attraction and SRP.

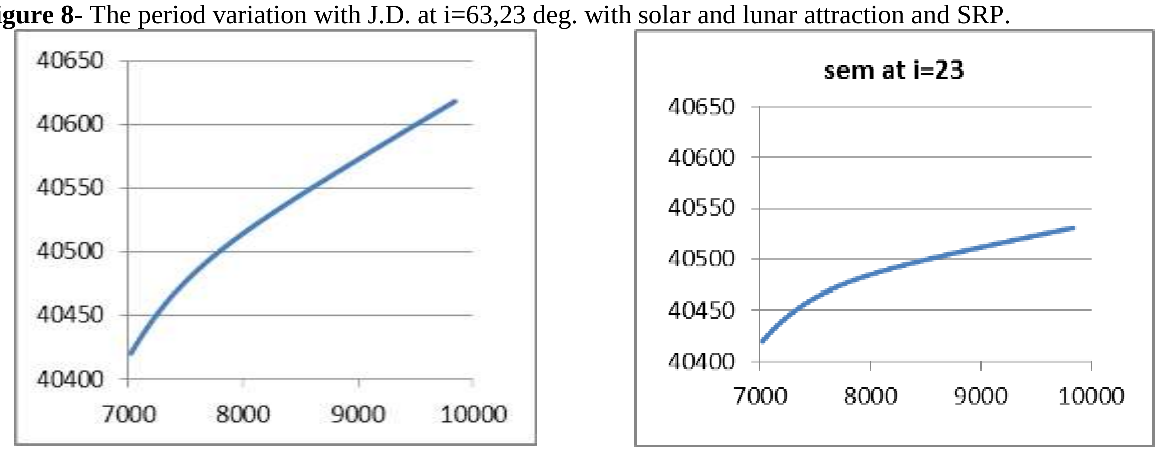


Figure 9-The Semi-major axis variation at hp =30000 Km. i=63 ,23 deg. with Solar and Lunar attraction and SRP.

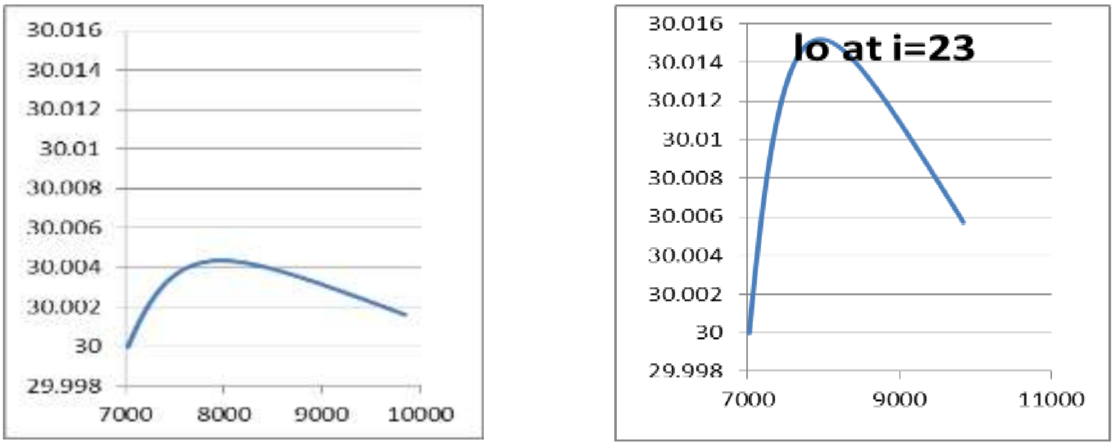


Figure 10-Variation of longitude of ascending node with time due toSolar and lunar attraction and SRP. at, *i*=63,23°

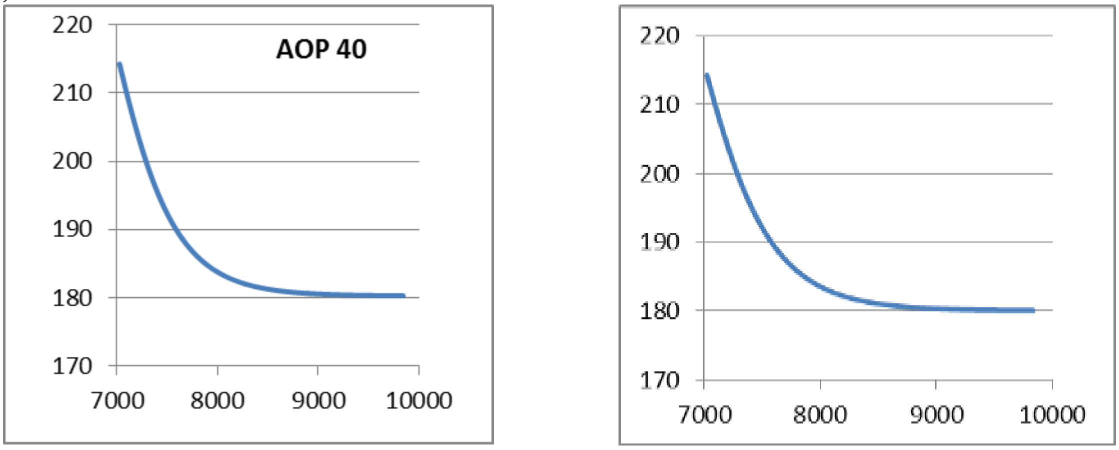


Figure 11- Aop variation at hp =30000 Km. i=63,23 deg. with Solar and Lunar attraction and SRP..

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