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The Orbital Elements Variation of the Moon Through 2000-2100

Abdul Rahman H. Salih[1]*, Majed M. Jarad[2], Fouad M. Abdulla[1]
[1]Department of Astronomy and Space, College of Science, University of Baghdad, Baghdad, Iraq
[2]Department of Physics, College of Science, University of Anbar, Al- Anbar, Iraq

Abstract

The locations of the Moon, velocity and distance were determined through hundred years using a modified formula Meeus 1998, which is used to calculate the orbit's elements, Additionally which allows us to specify the possible date for monitoring the crescent moon. In this project we describe the orbits, orbit types and orbital elements than describe the orbit of the Moon and the perturbations effect on shape and direction of the Moon's orbit, the orbital elements effect by the all perturbation were calculated directly using empirical formula. The orbital elements of the Moon's orbit for 1326 anomalies months are calculated by our Q. Basic programs and the time variation of the Moon's orbital element with perturbations can be computed by development these programs. The results get the values of the eccentricity, semi-major axis, inclination, longitude of ascending node, longitude of perigee and the anomalistic period and there variation through many years with all perturbations. The results appear that the Moon moves under balance perturbation forces in other word, with constant mean value through many hundred years.

Keywords: Moon, orbit, Moon period, anomalistic month.

تغير العناصر المدارية للقمر خلال السنوات 2000-2100م

عبدالرحمن حســين صالح[1]*،مجيد محمود جراد[2]، فؤاد محمود عبدالله[1]
[1] قسم الفلك والفضاء، كلية العلوم، جامعة بغداد، بغداد، العراق
[2] قسم الفيزياء، كلية العلوم، جامعة الانبار، الانبار، العراق

الخلاصة

تم حساب إحداثيات الموضع والسرعة للقمر وبعده خلال مئة عام باستخدام العلاقات الفلكية المحدثة من قبل Meeus عام 1998م. وقد استخدمت لحساب العناصر المدارية للقمر. كذلك تم استخدام علاقات أخرى لإيجاد تأريخ ووقت ولادة الهلال. في هذا المشروع تم وصف المدارات وأنواعها والعناصر المدارية ، تم وصف مدار القمر ومدى تغير عناصر مداره التي تم حسابها من قبل الباحثين لحد تأريخ هذا البحث ، وتم توضيح الاضطرابات التي تؤثر على شكل مسار القمر واتجاهه. وقد حسبت عناصر مدار القمر مباشرة من قيم التأريخ الجولياني من العلاقات التجريبية أي بوجود جميع الاظطرابات. تم حساب إحداثيات مدار للقمر وتغيرها في مساره لـ 1326 شهر حضيضي أي مئة سنة قمرية باستخدام برامجنا التي تم اعدادها وتطويرها بلغة البيسك السريعة لحساب العناصر المدارية بوجود جميع الاضطرابات . النتائج التي تم الحصول عليها بينت انه يوجد تغير دوري في عناصر مدار القمر وهي الشذوذ المركزي ونصف المحور الرئيسي والميل وخطي طول العقدة الصاعدة والحضيض وكذلك مدة الشهر الحضيضي بوجود كل الاضطرابات. كم أظهرت النتائج أن القمر يتحرك نحت تأثير قوى اضطراب متوازنة أي أن معدلها ثابت مع مرور مئات السنين.

*Email: Abdrahman29@yahoo.com

1. Introduction

The Moon rotate around Earth Moon mass center in an elliptical orbit with a mean eccentricity of 0.0549, this orbit is unclosed orbit because the perturbation effects. Thus, the Moon- Earth distance varies between 363000 km. to 405000 km. as a mean values. The lunar orbital period with respect to the stars (as a mean sidereal month) is 27.32166 days (27d 07h 43m 12s). However, there are three other orbital periods or months that are crucial to the understanding and prediction of eclipses. These three cycles and the harmonics between them determine when, where, and how solar and lunar eclipses occur [1].

The mutual gravitational force between the Sun and Moon is over twice as large as between the Moon and Earth. For this reason, the Sun plays a dominant role in perturbing the Moon's motion. The ever changing distances and relative positions between the Sun, Moon, and Earth, the inclination of the Moon's orbit, the obletness of Earth, and the gravitational attraction of the other planets all act to throw the Moon's orbital parameters into a constant state of change. The Solar radiation pressure on the Moon, the oblations of Earth less than the gravitational attraction of the other planets all act to the Moon's orbital parameters as a constant state of change which are less than the solar attraction. The Moon's position and velocity can be described by *the* classic Keplerian orbital elements; such osculating elements are only valid for a single instant in time [1, 2].

There are five periods of time that it takes for the Moon to compete cycles, traveling from one 'observable' reference point, back to the same point [3]. The sidereal month (27 days, 7 h, 43 min, and 11.6 sec), the syndic (29 days, 12 hours, 44 minutes, and 2.8 seconds), draconic month (27.212 221days), Tropical month (27.321 582 days) and the anomalistic month is defined as the revolution of the Moon around its elliptical orbit as measured from perigee to perigee. The length of this period can vary by several days from its mean value of 27.55455 days (27d 13h 18m 33s) [4, 5].
The Moon orbital elements with effect of all perturbations by using a modified theoretical model were computed through 100 years to determine the Moon periods.

2. The Moon's orbit

The orbit of the Moon is ellipse, the elliptic orbits defines as the path of a celestial body or an artificial satellite as it revolves around another body with negative energy and eccentricity $0 < e < 1$. The six orbital elements which determine the orbit of satellite these element called Keplerian element and there are (semi major axis a, Eccentricity e, Inclination i, longitude of the ascending node, Argument of the perihelion ω, and Mean anomaly at epoch (M_o). The Orbital elements are the parameters required to explain a specific orbit. In celestial mechanics these elements are generally considered in classical two-body systems, where a Kepler orbit is used (derived from Newton's laws of motion and Newton's law of universal gravitation [6].

The orbital elements changes over time due to gravitational perturbations by other objects and the effects of relativity. The main two elements define the shape and sizes of the ellipse are the semi major axis and Eccentricity (a, e) [6, 7]:

Where: a = ($r_a + r_p$ **) / 2** **(1)**

$$e = \frac{r_a - r_p}{r_a + r_p} \quad \textbf{(2)}$$

r_a : is radius at apogee point.
r_p : is radius at perigee point.

Mean anomaly at epoch defines the position of the orbiting body along the ellipse at a specific time (the epoch).The mean anomaly is a mathematically convenient "angle" which varies linearly with time, but which does not correspond to a real geometric angle. It can be converted into the true anomaly ν, which does represent the real geometric angle in the plane of the ellipse, between perigee and the position of the orbiting object at any given time. The angles of inclination, longitude of the ascending node, and argument of perigee call the Euler angles defining the orientation of the orbit relative to equator [6].

These three angles of the Moon orbit can calculate directly using empirical formulas depending on the Julian date.

3. Calculate the Moon coordinates and the orbital elements:

There are many methods can use to calculate the orbit element of the moon from one of them is calculated the position and the velocity of the moon by observing or empirical equations (first method) [7-9].

Also these elements can be calculated from the mean motion by solving Kepler equation to find the eccentric anomaly which is used to calculate the position and velocity coordinates of the Moon at instant time. These coordinates used to calculate the momentum and Euler angles (second method) as [10, 11].

The first thing to do is define the epoch on which we shall base our calculation consider:

$$U.T=U.T+\Delta t \quad (3)$$

And input the date (day, month, and year) to calculate the Julian date (J.D)

$$JD = B + C + D + d + 1720994.5 \quad (4)$$

Where:

B: the correction later than 1582 October 15

$$B= 2- A+ \text{integer part of } (A/ 4)\ldots,.. \text{ (Otherwise B=0)} \quad (5)$$

$$C = \text{integer part of } (365.25*Y)$$

$$D = \text{integer part of } (30.001*(M+1))$$

Then calculate Julian century from beginning of 1st January 1900 [7, 10]

$$T1 = (JD - 2415020) / 36525 \quad (6)$$

And after year 2000 the following formula can be used [10]

$$T2 = (JD - 2451545) / 36525 \quad (7)$$

By using these values it can determine the geocentric ecliptic coordinates (longitude and latitude) for the Moon at that time as in Meeus 1998[11]:

The position of the Moon as can be computed by an empirical formula.

The Moon's ecliptic longitude (λm) and latitude (βm) is given by [11]:

$$\lambda m = 218.316 + 481267.881T_2 + 6.29 \sin (134.9 + 477198.85\ T_2) - 1.27 \sin (259.2 - 413335.38\ T_2) + 0.66 \sin (235.7 + 890534.23\ T_2) + 0.21 \sin (269.9 + 954397.7\ T_2) - 0.19 \sin (357.5 + 35999.05\ T_2) - 0.11 \sin (186.6 + 966404.05\ T_2) \quad (8)$$

$$\beta m = 5.13 \sin (93.3 + 483202.03\ T_2) + 0.28 \sin (228.2 + 960400.87\ T_2) - 0.28 \sin (318.3 + 6003.18\ T_2) - 0.17 \sin (217.6 - 407332.2\ T_2) \quad (9)$$

Conversion of Elliptical to Equatorial and to horizontal coordinates [6]:

$$\tan(\alpha) = [\sin(\lambda) \cos(\varepsilon) - \tan (\beta) \sin(\varepsilon)] / \cos (\lambda) \quad (10)$$

$$\sin(\delta) = \sin(\beta) \cos (\varepsilon) + \cos (\beta) \sin(\varepsilon) \sin (\lambda) \quad (11)$$

Where the obliquity angle (ε) get as:

$$\varepsilon = 23.452294-0.0130125T1- 0.00000164T1^{2\ +}5.03\times10^{-7}\ T1^3 - \quad (12)$$

The horizontal coordinate (altitude and azimuth) calculated as [10]:

$$\tan(A) = \sin(H) / (\cos(H) \sin (\varphi) - \tan (\delta) \cos(\Phi) \quad (13)$$

$$\sin(a) = \sin(\varphi) \sin(\delta) + \cos(\Phi) \cos(H\ ) \cos(\delta)) \quad m \quad (14)$$

Where: (ϕ) is the observer geographical latitude, (H): Hour angle.

The Julian day for crescent Moon can be calculate by the following equation [8]:

$$JD = 2415020.5933+29.53058868\ T1+ 0.0001178\ T1^2 + 0.00033 \sin (166.56+ 132.87\ T1 - 0.009173T1^2)-1.55*10^{-7}*T1^3 \quad (15)$$

The Moon's distance from the centre of the Earth can be calculated as the following [8]:

The equatorial horizontal Parallax of the Moon (JI) can be calculated from Moon distance (Rm) by formula [12]:

$$\sin (JI)= 6378.14 / Rm \quad (16\text{-a})$$

Or as ref. [10]:

$$Rm= 385000 - 20905 \cos l - 3699 \cos (2D - l) - 2956 \cos (2D) - 570 \cos (2l) + 246 \cos (2l - 2D) - 152 \cos (l + l' - 2D) \quad \text{(in unit km.)} \quad (16\text{-b})$$

Where: the Moon's mean anomaly is (l), the Sun's mean anomaly is (l'), the difference between the mean longitudes of the Sun and the Moon is (D), and the Moon's argument of latitude (f) which is functions of Julian centuries ($T2=T_{2000}$) and calculated as [9, 12]:

$$l = 134°.96292 + 477198°.86753\ T_2 + 0.0087414 \quad (17)$$

$$l' = = 357°.52543 + 35999°.04944\ T_2 - 0.0001536T_2^2 \quad (18)$$

$$D = 297°.85027 + 445267°.11135\ T_2 - 0.0018819T_2^2 \quad (19)$$

$$f = 93.27209 + 483202.01752\ T_2 - 0.0036539\ T_2^2 \quad (20)$$

Results and discussion:

The orbit of the Moon have small change in distance due to the sun attraction and the other is attraction by the nearest planet as Jupiter this attraction forces depend on the distances from Moon to them and it's masses, the perturbation acceleration component can be computed by complex equations and solve the equation of motion for the moon.

The Moon coordinates, distances from the Earth and velocity through 100 years were calculated and some of these results were plotted in Figures-1a, 1b respectively. These figs show that the distance various between the minimum and maximum through one anomalistic month, through many periods the minimum distances values is various between (356404 – 370346) km. Also the maximum value is between (404050 - 406708) km through 100 year (2000-2100). The mean distances of the Moon were (383988.8) km this value is agreed with the Moon orbit semi-major axis.

The maximum declination of the Moon represents the Moon orbit inclination. The inclination of the Moon orbit was varies between 18.139° and 28.724° through 18.6 years, as in Figure-2, The same variation was showed through 100 years, The Moon inclination variation equal the Serous period (18.6 years). Figure-3 show the variation of semi-major axis of the Moon orbit through 100 years, it's clear that its value was various between (381493 - 387421) Km, and the mean value was (383988.8) Km., The mean value was constant with time.

Figure-4 show the perigee, Apogee and semi-major axis variation through three years (2015-2018), the variation period near seven month and the secular variation was zero ,or the mean value of these three parameters are constant. Figure-5show the anomalistic period of the Moon varies between 25.0146 day and 28.81106 day and the mean period value gated from our result is 27.55571 days. The anomalistic month is various than the mean by about 2 days. When the Moon perigee nears the Sun direction the anomalistic months are shortest correlated with values of 90° and 270°, when the line of upsides is perpendicular to the Sun's direction. The anomalistic month of the Moon's elliptical orbit various because one must first consider Earth's elliptical orbit around the Sun, which has a mean eccentricity of 0.0167 and the Sun-Earth distance varies with mean values of 147,098,074 km at perihelion to 152,097,701 km at aphelion.

The first method to calculate Euler angles of the Moon orbit take a better result from the second method, when the results compared with some references and observations as [8 ,9], therefore the first method used hear . The results for the orbital elements variations with time as in Table-1 and Figures-6, 7, show the changing of the longitude of ascending node, node latitude with Julian date, there are a long period contain four short period any one through 12 anomalistic month. Figures-8,9 show the eccentricity changing with date it's values between 0.0439, 0.0659, the mean value equal 0.0556. The results prove that the Moon orbit was balance through many hundred years.

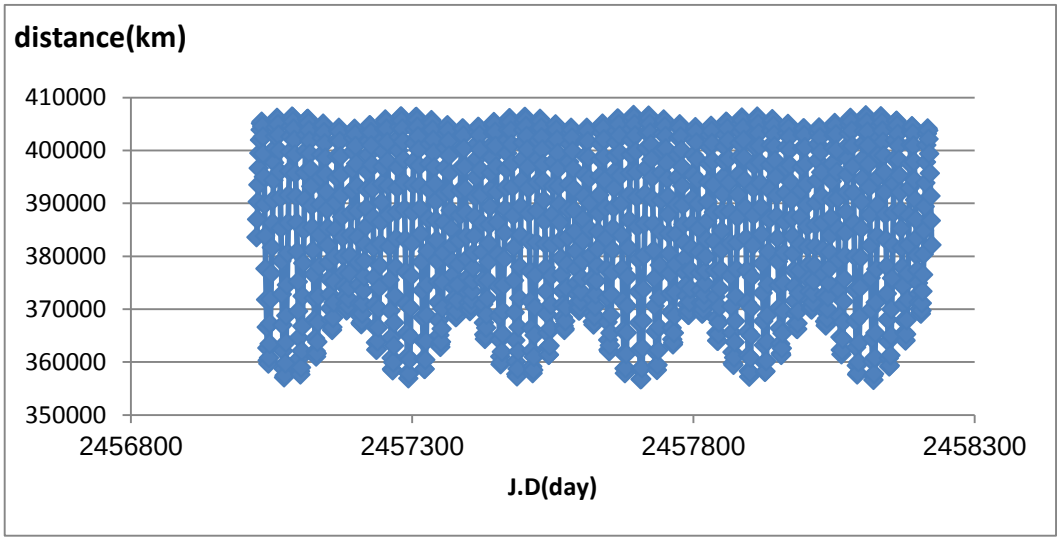

Figure 1a- The Moon distance (km) with J.D through three years

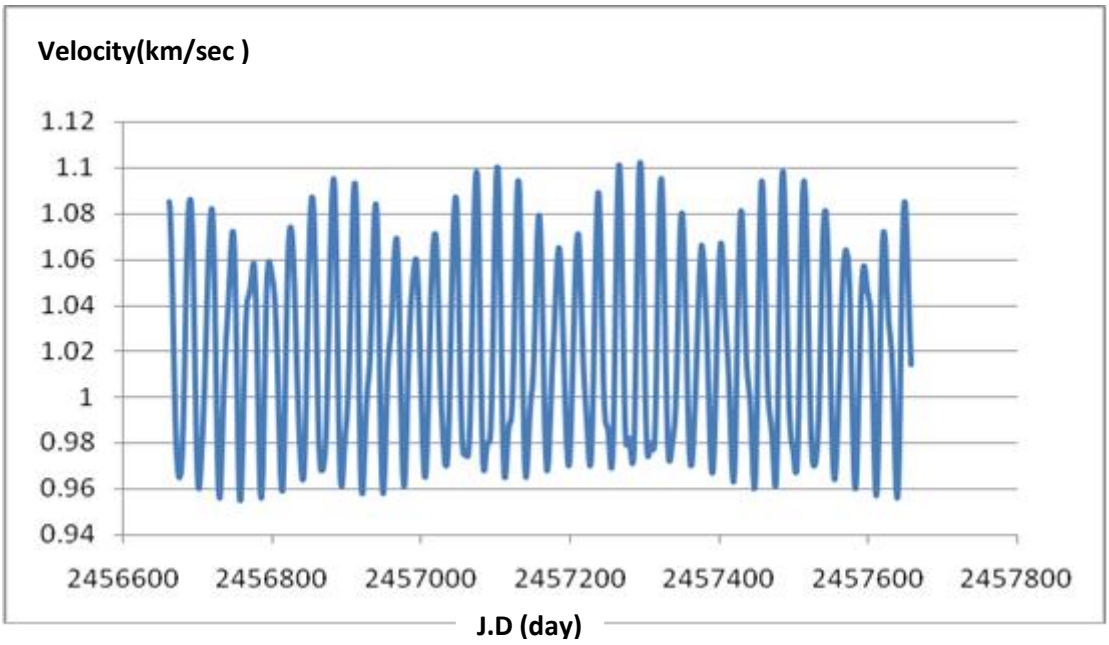

Figure 1b- The velocity (Km/sec) for the Moon with J.D through three years.

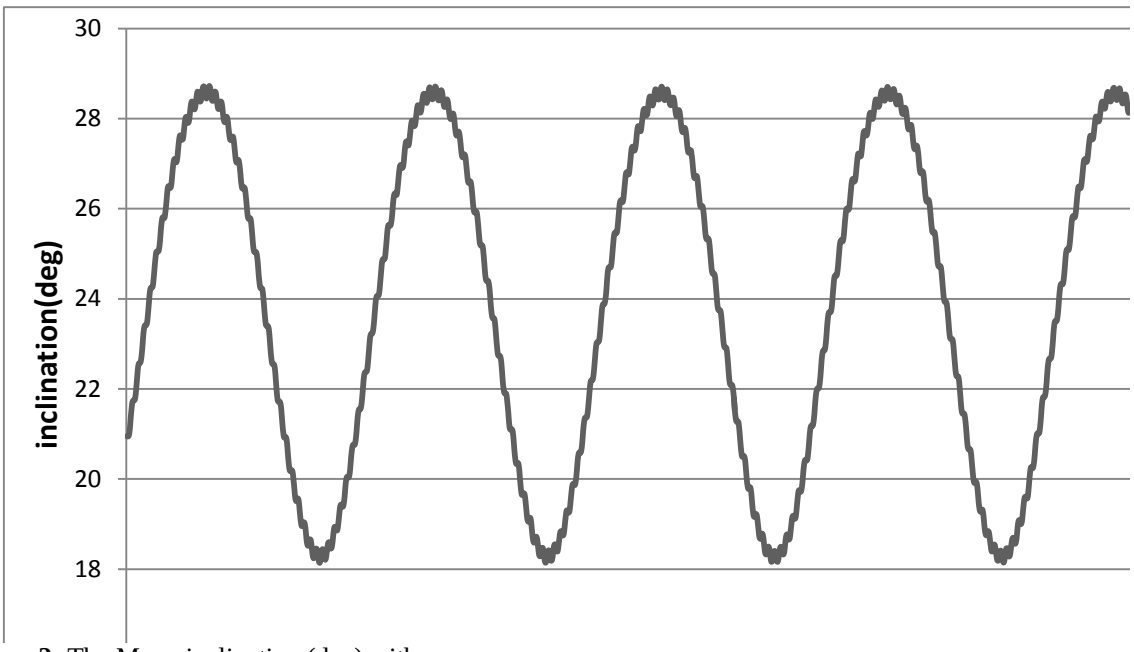

Figure 2- The Moon inclination (deg) with years.

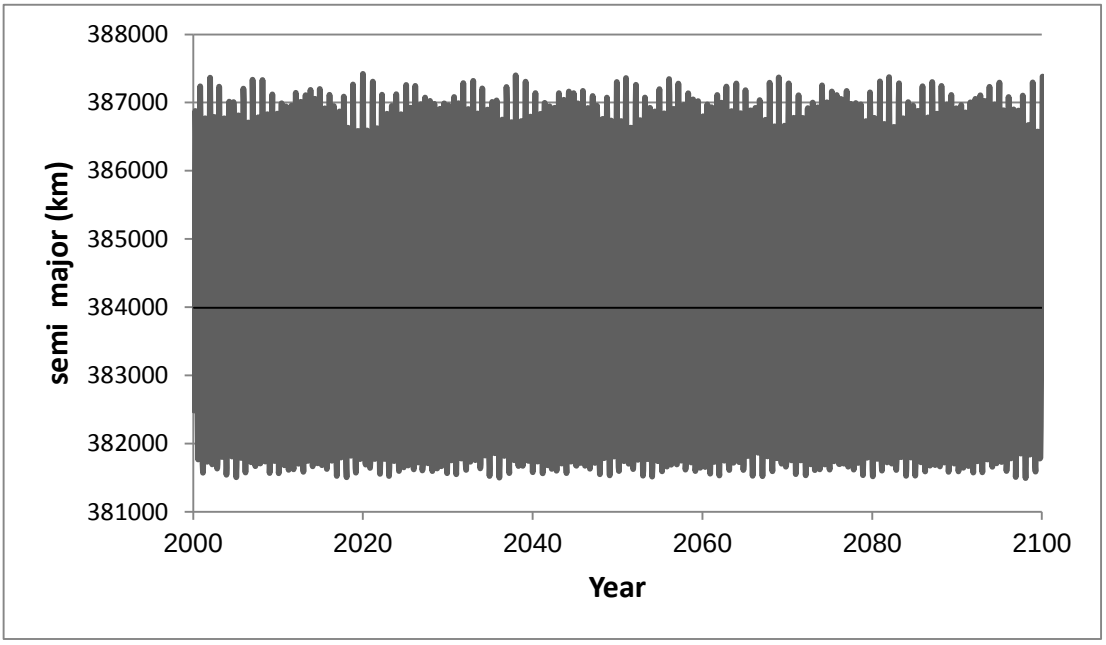

Figure 3- The semi major axis (km) with date through 100 years.

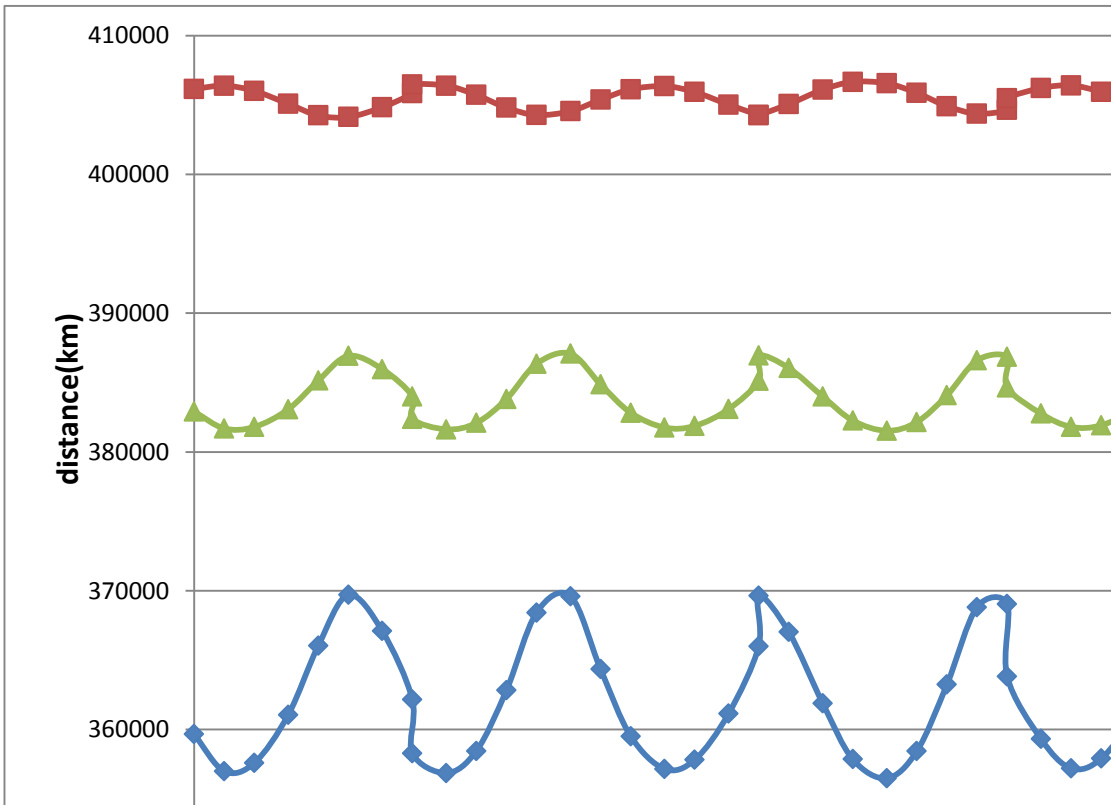

Figure 4- The perigee, Apogee and semi major axis variation in (km) through three years (2015-2018).

Table 1- Anomalistic month Length in 2015 (sample of results through 100 years)

Date of New Moon (U.T)			Length of anomalistic month	Difference From Mean anomalistic month
month	**day**	**h m s**	**(day)**	**(hour)**
1	21	18 16 39	28.09642	0.541867
2	19	10 45 60	28.68705	1.132497
3	19	15 7 42	28.18174	0.627191
4	17	7 12 20	28.66988	1.115326
5	14	23 25 44	27.67597	0.121423
6	10	4 15 33	26.20127	-1.35328
7	5	22 20 6	25.75316	-1.80139
8	2	7 13 28	27.37039	-0.18416
8	30	18 23 30	28.4653	0.910753
9	28	1 20 45	28.28975	0.735196
10	26	10 47 21	28.39348	0.838926
11	23	23 29 14	28.52909	0.974538
12	21	3 46 11	27.17844	-0.37611

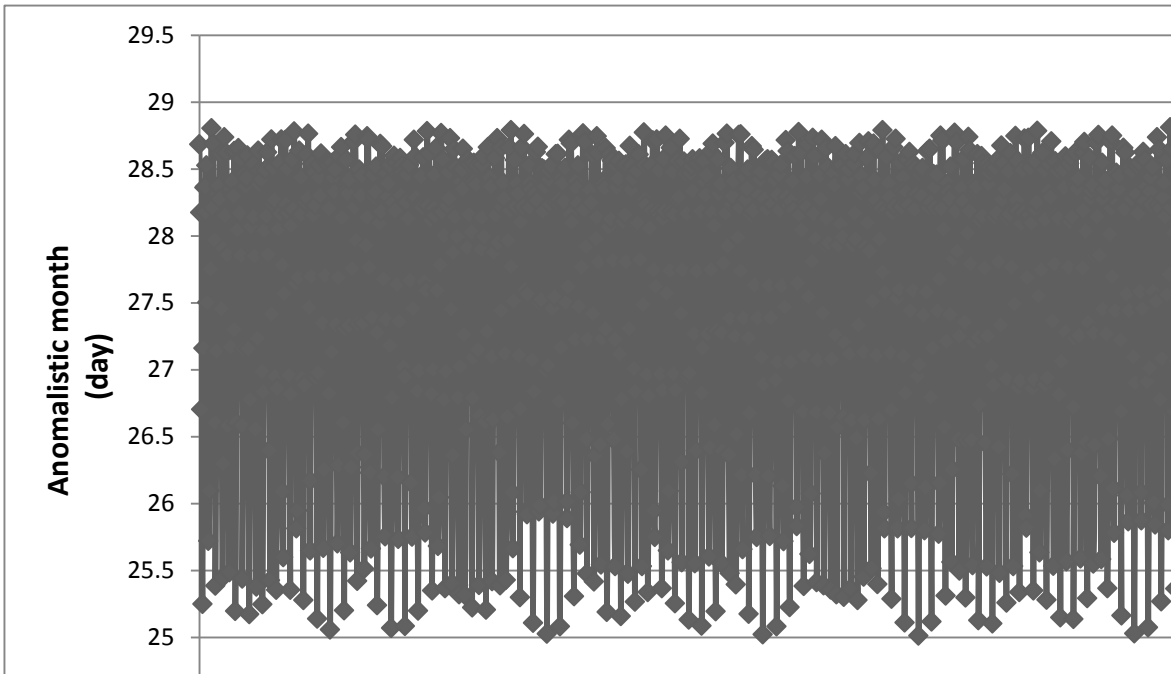

Figure 5- Anomalistic period with time through 100 years.

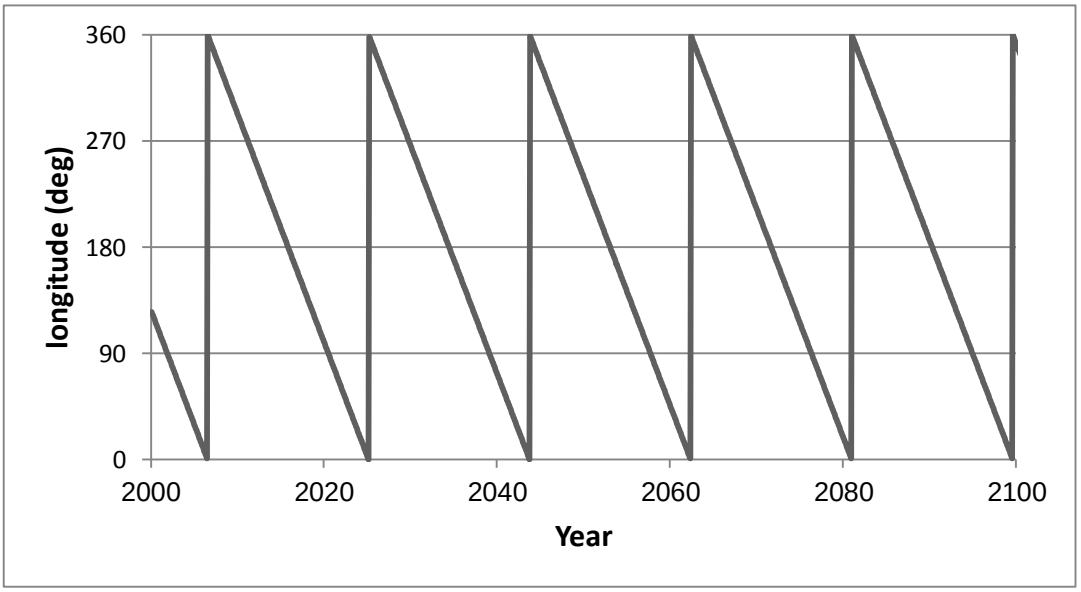

Figure 6- The longitude of ascending node with time through 100 years.

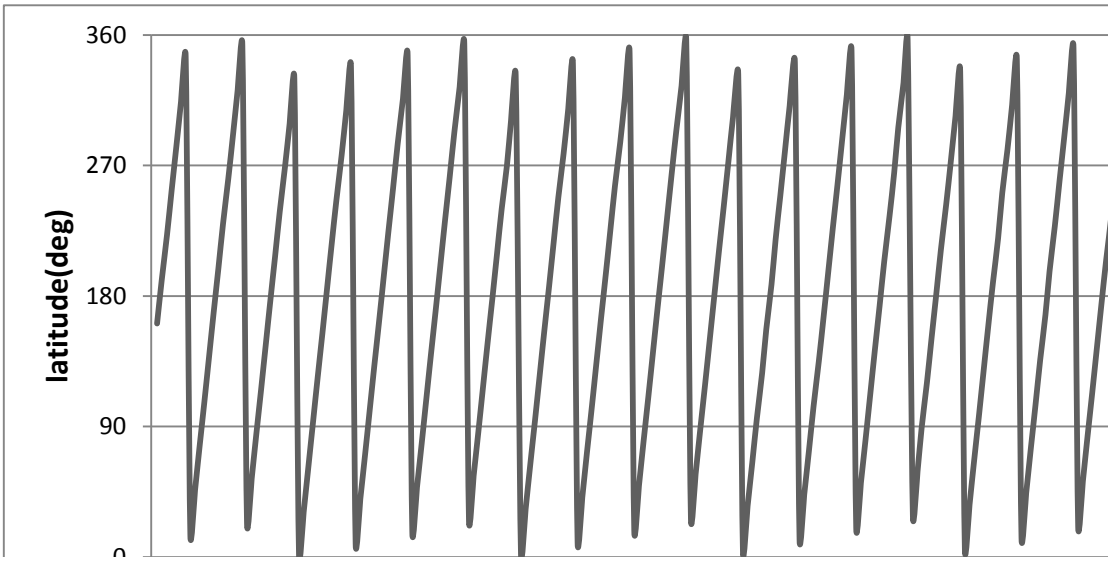

Figure 7- Moon node latitude through 20 years.

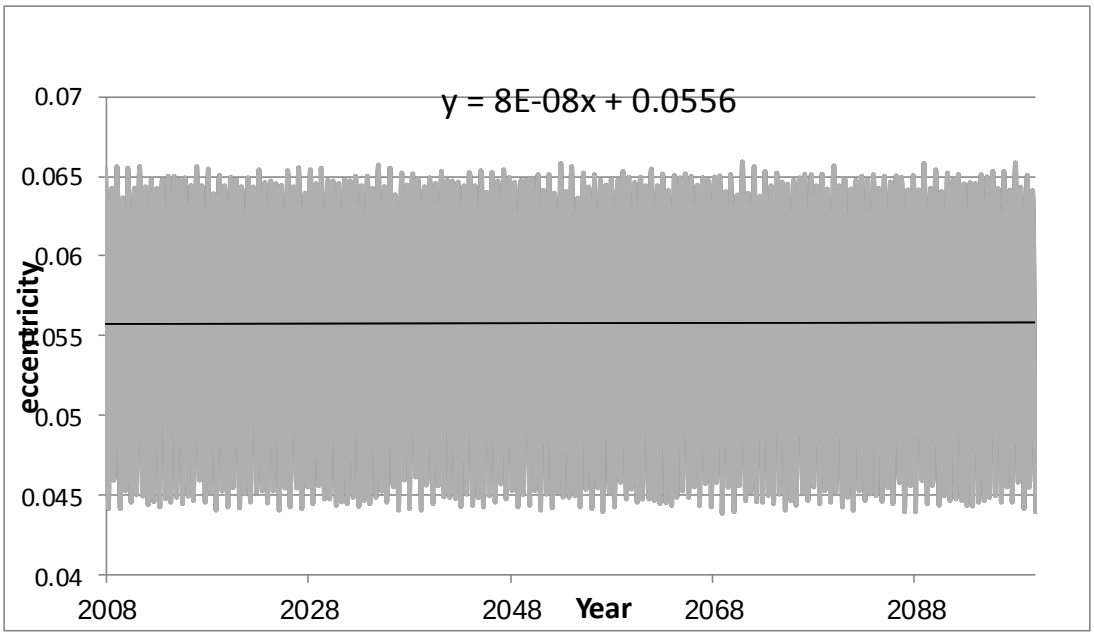

Figure 8- The eccentricity variation with time through 100 years.

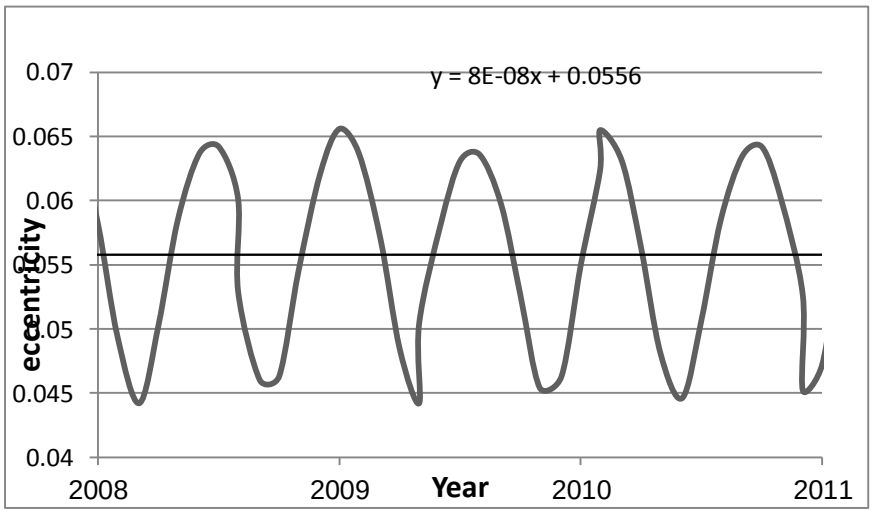

Figure 9- The eccentricity variation with time through 3 years.

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