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Efficient Iterative Method for Solving Korteweg-de Vries Equations

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Abstract

The Korteweg-de Vries equation plays an important role in fluid physics and applied mathematics. This equation is a fundamental within study of shallow water waves. Since these equations arise in many applications and physical phenomena, it is officially showed that this equation has solitary waves as solutions, The Korteweg-de Vries equation is utilized to characterize a long waves travelling in channels. The goal of this paper is to construct the new effective frequent relation to resolve these problems where the semi analytic iterative technique presents new enforcement to solve Korteweg-de Vries equations. The distinctive feature of this method is, it can be utilized to get approximate solutions for travelling waves of non-linear partial differential equations with small amount of computations does not require to calculate restrictive assumptions or transformation like other conventional methods. In addition, several examples clarify the relevant features of this presented method, so the results of this study are debated to show that this method is a powerful tool and promising to illustrate the accuracy and efficiency for solving these problems. To evaluate the results in the iterative process we used the Matlab symbolic manipulator.

Keywords: Korteweg-de Vries equations, Boussinesq equation, Solitary waves, Initial Value Problems, Semi Analytic Iterative Method.

كفاءة الطريقة التكرارية لحل معادلات كورتج_ديفرز

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الخلاصة

معادلة كورتج_ديفرز لها دوراً مهماً في فيزياء الموائع والرياضيات التطبيقية. وهي أداة مهمة في دراسة موجات المياه الضحلة. ونظراً لأن هذه المعادلة تنشأ في العديد من التطبيقات والظواهر الفيزيائية ، لقد اثبت أن هذه المعادلة لها موجات انفرادية كحلول ، تستخدم معادلة كورتج_ديفرز لوصف الموجات الطويلة التي تنتقل في القنوات.

الهدف من هذا البحث هو انشاء علاقة متكررة فعالة جديدة لحل هذه المسائل حيث تقدم التقنية التكرارية الشبه تحليلية تطبيقاً جديداً لحل معادلات كورتج_ديفرز. ان الخاصية المميزة لهذه الطريقة أنه يمكن استخدامها لأيجاد الحلول التقريبية للموجات المتحركة من المعادلات التفاضلية الجزئية غير الخطية مع كمية صغيرة من الحسابات ولا تتطلب حساب تحويلات أو فرضيات مقيدة مثل الطرق التقليدية الأخرى. بالإضافة إلى ذلك ، توضح العديد من الأمثلة الصفات المميزة للطريقة المقترحة ، لذا تمت مناقشة نتائج الدراسة لإظهار أن هذه الطريقة هي أداة قوية واعدة لبيان الدقة والكفاءة لحل مثل هذه المسائل. لحساب النتائج في العمليات التكرارية استخدم البرنامج الحاسوبي الماتلاب.

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Introduction

In the previous few decades, Non-linear phenomena play an important tool in physics, the solitary wave theory and applied mathematics. On the other hand, the study of non-linear partial differential equations in modelling physical phenomena become a significant gadget in a large class and vastly utilized in assorted fields of the nonlinear natural sciences, As for the behavior and properties of non-linear partial differential equations can be determined by exact solution ; therefore, non-linear equations play significant role in specifying such problems. Thus, their exact solution is more complicated to find as compared to the other solutions of linear equations. The Korteweg-de Vries equation(KdV) equation has been arisen in the study of shallow water waves, [1]. Particularly, it is utilized to characterize the diffusion of plasma waves in a dispersive medium and furthermore describes long waves traveling in canals too. The KdV equation has been widely studied and has gained a lot of interest. Many analytical and numerical techniques were used to study the solitary waves which outcome of such equation. Kruskal and Zabusky have researched in the interaction of repetition of the initial cases and solitary waves . They detected the solitary waves undergo non-linear interaction subsequent KdV equation. It is worth mentioning, waves emanate of this interaction retaining its amplitude and shape, also solitary waves maintain its character and identities resembles such as particle conduct, they stimulated to recall solitary wave solitons, [2]. Many research works have arisen in various of the scientific realm in worldwide to study the solution concept. It is renowned now that solutions show as the outcome of an equilibrium among dispersion and weak non-linearity and it has enticed formidable volume of studies due to its important part within different scientific fields like magneto-acoustic waves, plasma physics, fluid dynamics , astrophysics and others. The inquiry of the solutions of travelling wave perform an essential part in non-linear sciences, like Hirota’s Bilinear Method, [3]. Inverse Scattering Transform, [4]. Bäcklund Transformation,[5]. Sine-Cosine Method, [6]. Tanh-Function Method, [7]. Homotopy Perturbation Method, [8]. Homotopy Analysis Method, [9-10]. Variational Iteration Method(VIM), [11-12]. Exp-Function Method, [13–14]. Differential Transform Method, [15]. Tanh-Coth Method, [16–17]. (G'/G) -Expansion Method, [18–19]. Laplace Adomian Decomposition Method, [20]. Generalized Tanh-Coth Method and (G'/G) -Expansion Method, [21]. Recently, considerable attention has been given to some efficient analytic iterative methods, Temimi and Ansari have suggested a new iterative method, i.e Semi Analytical Iterative Method(SAIM) to resolve linear and non-linear functional equations, [22]. The SAIM has been successfully applied by many research works to solve some linear and non-linear partial, ordinary differential equations and higher order integro-differential equations, [23-27]. To the best of our knowledge, SAIM is not yet implemented to resolve the KdV equations. This method can be used to solve and get the analytical solutions of these problems. KdV equations were solved by Adomian Decomposition Method (ADM) and VIM, [28]. ADM had be utilized to solve the KdV equation to get a solitary wave solution, also the Modified Adomian Decomposition Method(MADM) used to solve non-linear dispersive waves via Boussinesq equation, [29]. The basic concept of this reasech is as follows. Section 2 An explanation for KdV equations is presented by the fundamental idea of SAIM. Several analytical outcomes are supplied in section 3 to clarify the method. Section 4 concludes the research.

2. Korteweg - deVries equation problems by the Basic idea of (SAIM):

The simplest form of the KdV equation, [30-31], for two independent variables x and t can be written as follows:

$$z_t + azz_x + z_{xxx} = 0 \tag{2.1}$$

And the Initial Value Problems (IVPs)

$$z(x, 0) = f_1(x) , 0 \leq x \leq 1 \text{ and } z_t(x, 0) = f_2(x), t > 0 \tag{2.2}$$

such that $z = z(x, t)$ at the some domain, zz_x the non-linear idiom and a is a parameter $\exists a > 0$. the basic idea of the reliable SAIM to solve Eq. (2.1) with IVPs (2.2) . The general form equation as:

$$L(z(x, t)) + N(z(x, t)) + h(x, t) = 0 \tag{2.3}$$

$$\text{with conditions } C(z, \frac{\partial z}{\partial t}) = 0 \tag{2.4}$$

So, thus $z(x, t)$ the unknown function, x and t indicate to the independent variables, while L, N represent linear and the non-linear operators, respectively. $h(x, t)$ represents the inhomogeneous term, which is a known function and C the conditions operator for the problem. Initial approximation is a primary step in the SAIM, by assuming that the initial guess $z_0(x, t)$ is a solution of the problem $z(x, t)$, the solution of the equation can be solving

$$L(z_0(x, t)) + h(x, t) = 0, C(z_0, \frac{\partial z_0}{\partial t}) = 0 \tag{2.5}$$

To generate the next iteration of the solution can be resolve the problem as follows

$$L(z_1(x, t)) + h(x, t) + N(z_0(x, t)) = 0, C(z_1, \frac{\partial z_1}{\partial t}) = 0 \tag{2.6}$$

After several simple iteration steps of the solution, the general form of this equation as

$$L(z_{n+1}(x, t)) + h(x, t) + N(z_n(x, t)) = 0, C(z_{n+1}, \frac{\partial z_{n+1}}{\partial t}) = 0 \tag{2.7}$$

Evidently each iteration of the function $z_n(x, t)$ represent effectively alone solution for Eq. (2.3).

We will implement the method steps at the Eq.(2.1), so Eq.(2.1) can be express as for:

$$L(z(x, t)) = -(h(x, t) + azz_x + z_{xxx}) \tag{2.8}$$

The differential operator $L(z(x, t))$ is the highest order derivative in the Eq.(2.8), by using the given IVPs in Eq.(2.2) and integrating both sides of Eq.(2.8) from 0 to t , we obtain the following equation:

$$z(x, t) = \psi_0 - \left(\int_0^t (h(x, t) + azz_x + z_{xxx}) d(t) \right) \tag{2.9}$$

Where the function ψ_0 is arising by integrating the source term from applying the given IVPs in Eq.(2.2) which are prescribed.

To find out how this method works, the following steps are as follows:

Step 1: to get $z_0(x, t)$ solving $L(z_0(x, t)) + h(x, t) = 0$ with IVPs in Eq.(2.2) and integrating both sides of Eq.(2.10) from 0 to t , we obtain

$$z_0(x, t) = \psi_0 - \int_0^t (h(x, t)) d(t)$$

Step 2 : The next iterate is

$$L(z_1(x, t)) - h(x, t) = 0 \text{ with IVPs in Eq.(2.2)} \tag{2.11}$$

Solving this equation and integrating both sides of Eq.(2.11) from 0 to t , leads to obtain

$$z_1(x, t) = \psi_0 - \left(\int_0^t (h(x, t) + az_0z_{0x} + z_{0xxx}) d(t) \right)$$

Step (3) : After several simple iterative steps of the solution, the general form of this equation which is

$$L(z_{n+1}(x, t)) - h(x, t) = 0 \text{ with IVPs in Eq.(2.2)} \tag{2.12}$$

Solving this equation and integrating both sides of Eq.(2.12) from 0 to t , leads to obtain $z_{n+1}(x, t)$

$$z_{n+1}(x, t) = \psi_0 - \left(\int_0^t (h(x, t) + az_nz_{nx} + z_{nxxx}) d(t) \right)$$

Evidently each iteration of the function $z_{n+1}(x, t)$ represent effectively alone solution for Eq. (2.8).

In the above steps, this method has merit to solve and apply KdV equation.

3. Numerical Examples:

We will be apply SAIM for solving several examples of the KdV equations.

Example 1:

We consider the KdV equation with the following IVP [28] :

$$z_t - 6zz_x + z_{xxx} = 0 \tag{3.1}$$

$$\text{with the IVP } z(x, 0) = 6x \tag{3.2}$$

Solution:

By implementing the same steps as described in the previous section, i.e. in section (2), we first begin by solving the following IVP to find the initial guess $z_0(x, t)$, SAIM start as follows

$$L(z) = z_t, N(z) = -6z z_x + z_{xxx} \text{ and } h(x, t) = 0 \tag{3.3}$$

So, the primary step is

$$L(z_0) = 0, \text{ with } z_0(x, 0) = 6x \tag{3.4}$$

Then, the general relation as follows

$$L(z_{n+1}) + N(z_n) + h(x, t) = 0, z_{n+1}(x, 0) = 6x \tag{3.5}$$

By solving the problem defined in Eq. (3.4) we have $z_0(x, t) = 6x$

The first iteration can be get as

$$(z_1)_t = (6z_0 z_{0x} - z_{0xxx}) \text{ with } z_1(x, 0) = 6x \tag{3.6}$$

Thus, the solution of Eq. (3.6) as

$$z_1(x, t) = 6x + 6^3tx$$

The second iteration is

$$(z_2)_t = (6z_1 z_{1x} - z_{1xxx}) \text{ with } z_2(x, 0) = 6x \tag{3.7}$$

Then, the solution of Eq. (3.7) as

$$z_2(x, 0) = 6x + 6^3xt + 6^5xt^2 + 2.6^6xt^3$$

The third iteration is

$$(z_3)_t = (6z_2 z_{2x} - z_{2xxx}) \text{ with } z_3(x, 0) = 6x \tag{3.8}$$

Then, the solution of Eq. (3.8) as

$$z_3(x, 0) = 6x + 6^3xt + 6^5xt^2 + 6^7xt^3 + \dots$$

Also, by the same steps, the other solutions can be generated from calculating these problems in the general form

$$(z_{n+1})_t = (6z_n z_{nx} - z_{nxxx}) \text{ with } z_n(x, 0) = 6x \tag{3.9}$$

Hence, in iteration steps, we have

$$z_n(x, t) = 6x(1 + 36t + (36t)^2 + (36t)^3 + (36t)^4 + \dots) \tag{3.10}$$

Thus, we obtained exact solution as follows

$$z(x, t) = \frac{6x}{1 - 36t}, \quad |36t| < 1.$$

Example2:

Consider the following the KdV equation in Eq. (3.1) with IVP [28]:

$$z(x, 0) = \frac{1}{6} (x - 1) \tag{3.10}$$

Solution:

Applying the same steps as in the previous example, we first begin by solving the following initial problem in order to find the initial guess $z_0(x, t)$, SAIM start by the same step in Eq. (3.3). So, the primary step is

$$L(z_0) = 0, \text{ with } z_0(x, 0) = \frac{1}{6} (x - 1) \tag{3.11}$$

Then, the general relation as follows

$$L(z_{n+1}) + N(z_n) + h(x, t) = 0, z_{n+1}(x, 0) = \frac{1}{6} (x - 1) \tag{3.12}$$

Via solving the primary problem defined in Eq. (3.11) we get

$$z_0(x, t) = \frac{1}{6} (x - 1)$$

The first iteration can be get as

$$(z_1)_t = (6 z_0 z_{0x} - z_{0xxx}) \quad \text{with } z_1(x, 0) = \frac{1}{6} (x - 1) \tag{3.13}$$

Thus, the solution of Eq. (3.13) as

$$z_1(x, 0) = \frac{1}{6} (x - 1) + \frac{1}{6} (x - 1)t$$

The second iteration is

$$(z_2)_t = (6 z_1 z_{1x} - z_{1xxx}) \quad \text{with } z_2(x, 0) = \frac{1}{6} (x - 1) \tag{3.14}$$

Then, the solution of Eq. (3.14) is

$$z_2(x, 0) = \frac{1}{6} (x - 1) + \frac{1}{6} (x - 1)t + \frac{1}{6} (x - 1)t^2 + \frac{1}{6 \times 3} (x - 1)t^3$$

The third iteration is

$$(z_3)_t = (6 z_2 z_{2x} - z_{2xxx}) \quad \text{with } z_3(x, 0) = \frac{1}{6} (x - 1) \tag{3.15}$$

Then, the solution of Eq. (3.15) is

$$z_3(x, 0) = \frac{1}{6} (x - 1) + \frac{1}{6} (x - 1)t + \frac{1}{6} (x - 1)t^2 + \frac{1}{6} (x - 1)t^3 + \dots$$

Hence, in iteration steps, The chain solution can be expressed as

$$z_n(x, t) = \frac{1}{6} (x - 1) (1 + t + t^2 + t^3 + \dots) \tag{3.16}$$

Thus, exact solution as the following

$$z(x, t) = \frac{1}{6} \left(\frac{x - 1}{1 - t} \right), \quad |t| < 1.$$

Example 3:

Consider the following the KdV equation in Eq. (3.1) with IVP [28]:

$$z(x, 0) = \frac{2}{(x-3)^2} \tag{3.17}$$

Solution:

Implementing the same procedure as in the previous examples, we begin by solving the following initial problem in order to find the initial guess $z_0(x, t)$, SAIM start as follows

So, the primary step is

$$L(z_0) = 0, \quad \text{with } z_0(x, 0) = \frac{2}{(x-3)^2} \tag{3.18}$$

Then, the general relation is as follows

$$L(z_{n+1}) + N(z_n) + h(x, t) = 0, \quad z_{n+1}(x, 0) = \frac{2}{(x-3)^2} \tag{3.19}$$

By solving the primary problem in Eq. (3.18) we obtain

$$z_0(x, 0) = \frac{2}{(x-3)^2}$$

The first iteration can be get as

$$(z_1)_t = (6 z_0 z_{0x} - z_{0xxx}) \quad \text{with } z_1(x, 0) = \frac{2}{(x-3)^2} \tag{3.20}$$

Thus, the solution of Eq. (3.20) as:

$$z_1(x, t) = \frac{2}{(x-3)^2}$$

The second iteration is

$$(z_2)_t = (6 z_1 z_{1x} - z_{1xxx}) \quad \text{with } z_2(x, 0) = \frac{2}{(x-3)^2} \tag{3.21}$$

Then, the solution of Eq. (3.21) as

$$z_2(x, t) = \frac{2}{(x-3)^2}$$

Hence, in iteration steps, we have

$$z_n(x, t) = \frac{2}{(x-3)^2} \tag{3.22}$$

Thus we can be get the exact solution to such problem is

$$z(x, t) = \frac{2}{(x - 3)^2}$$

The following examples, solved easier by the SAIM and the results are a good indicator of the exact solution and this solution is the same as the result obtained by ADM and VIM, [28].

Example 4:

In the following, SAIM be applied to get solitary wave solution of KdV equation [29]:

$$z_t = -6z^2z_x - z_{xxx} \tag{3.23}$$

$$\text{with the IVP } z(x, 0) = \frac{2ke^{kx}}{1+e^{2kx}} \tag{3.24}$$

Solution:

By implementing the same steps as described in the previous section, the SAIM algorithm will be applied at the following equations (3.23) and (3.24). We first begin by solving the following IVP to find the initial guess $z_0(x, t)$ as follows

$$L(z) = z_t, N(z) = 6z^2z_x + z_{xxx} \text{ and } h(x, t) = 0 \tag{3.25}$$

$$\text{So, the primary step is } L(z_0) = 0, \text{ with } z_0(x, 0) = \frac{2ke^{kx}}{1+e^{2kx}} \tag{3.26}$$

Then, the general relation as follows

$$L(z_{n+1}) + N(z_n) + h(x, t) = 0, z_{n+1}(x, 0) = \frac{2ke^{kx}}{1+e^{2kx}} \tag{3.27}$$

By solving the primary problem in Eq. (3.26) we obtain

$$z_0(x, 0) = \frac{2ke^{kx}}{1 + e^{2kx}}$$

The first iteration can be get as

$$(z_1)_t = (-6z_0^2z_{0x} - z_{0xxx}) \text{ with } z_1(x, 0) = \frac{2ke^{kx}}{1+e^{2kx}} \tag{3.28}$$

Thus, the solution of Eq. (3.28) as:

$$z_1(x, t) = \frac{2ke^{kx}}{1 + e^{2kx}} - \frac{2k^4e^{kx}(1 - e^{2kx})}{(1 + e^{2kx})^2} t$$

The second iteration is

$$(z_2)_t = (-6z_1^2z_{1x} - z_{1xxx}) \text{ with } z_2(x, 0) = \frac{2ke^{kx}}{1+e^{2kx}} \tag{3.29}$$

Then, the solution of Eq. (3.29) is

$$z_2(x, 0) = \frac{2ke^{kx}}{1 + e^{2kx}} - \frac{2k^4e^{kx}(1 - e^{2kx})}{(1 + e^{2kx})^2} t + \frac{k^7e^{kx}(1 - 6e^{2kx} + e^{4kx})}{(1 + e^{2kx})^3} t^2$$

Hence, in iteration steps, we have

$$z_n(x, t) = \frac{2ke^{kx}}{1+e^{2kx}} - \frac{2k^4e^{kx}(1-e^{2kx})}{(1+e^{2kx})^2} t + \frac{k^7e^{kx}(1-6e^{2kx}+e^{4kx})}{(1+e^{2kx})^3} t^2 - \dots \tag{3.30}$$

Thus,

$$z(x, t) = \mp \sqrt{c} \operatorname{sech} \sqrt{c} (x - ct) , \text{ noting that } c = k^2$$

Then, we get the exact solution of this problem readily.

Example 5:

A renowned model of non-linear diffusive waves was suggested via Boussinesq at the formula [29]:

$$z_{tt} = z_{xx} + 3(z^2)_{xx} + z_{xxxx} , \quad -80 \leq x \leq 80 \tag{3.31}$$

$$\text{with the IVP } z(x, 0) = \frac{2ak^2e^{kx}}{(1+ae^{kx})^2} , z_t(x, 0) = \frac{-2ak^3\sqrt{1+k^2}e^{kx}(ae^{kx}-1)}{(1+ae^{kx})^3} \tag{3.32}$$

Boussinesq equation (3.31) describes movements of a long wave in shoaly water beneath gravity and in a one-dimensional non-linear web.

Solution:

By implementing the same steps, we first begin by solving the following IVPs in order to find the initial guess $z_0(x, t)$, SAIM apply accordingly the same step in Eq. (3.31) and (3.32) as the following

$$L(z) = z_{tt}, N(z) = -3(z^2)_{xx} - z_{xx} - z_{xxx} \text{ and } h(x, t) = 0 \tag{3.33}$$

So, the primary step is $L(z_0) = 0$, with $z_0(x, 0) = \frac{2ak^2e^{kx}}{(1+ae^{kx})^2}, (z_0)_t(x, 0) = \frac{-2ak^3\sqrt{1+k^2}e^{kx}(ae^{kx}-1)}{(1+ae^{kx})^3}$

$$\tag{3.34}$$

Then, general relation as follows

$$L(z_{n+1}) + N(z_n) + h(x, t) = 0, z_{n+1}(x, 0) = \frac{2ak^2e^{kx}}{(1+ae^{kx})^2}, (z_{n+1})_t(x, 0) = \frac{-2ak^3\sqrt{1+k^2}e^{kx}(ae^{kx}-1)}{(1+ae^{kx})^3}$$

$$\tag{3.35}$$

Solving the problem defined in Eq. (3.34) we get

$$z_0(x, 0) = \frac{2ak^2e^{kx}}{(1+ae^{kx})^2} - \frac{2ak^3\sqrt{1+k^2}e^{kx}(ae^{kx}-1)}{(1+ae^{kx})^3} t$$

First iteration can be get as

$$(z_1)_{tt} = (z_0)_{xx} + 3(z_0^2)_{xx} + z_0)_{xxx}$$

with $z_1(x, 0) = \frac{2ak^2e^{kx}}{(1+ae^{kx})^2}, (z_1)_t(x, 0) = \frac{-2ak^3\sqrt{1+k^2}e^{kx}(ae^{kx}-1)}{(1+ae^{kx})^3}$

$$\tag{3.36}$$

Thus, solution of Eq. (3.36) as

$$z_1(x, 0) = (ak^2(36ae^{2kx}) + 90a^2e^{3kx} + 120a^3e^{4kx} + 90a^4e^{5kx} + 36a^5e^{6kx} + 6a^6e^{7kx} + 12ak^6t^4e^{2kx} + 12ak^8t^4e^{2kx} - 27a^2k^2t^2e^{3kx} - 27a^2k^4t^2e^{3kx} - 48a^3k^2t^2e^{4kx} - 48a^3k^4t^2e^{4kx} - 27a^4k^2t^2e^{5kx} - 120a^2k^6t^4e^{3kx} - 27a^4k^4t^2e^{5kx} - 120a^2k^8t^4e^{3kx} + 240a^3k^6t^4e^{4kx} + 3a^6k^2t^2e^{7kx} + 240a^3k^8t^4e^{4kx} - 120a^4k^6t^4e^{5kx} + 3a^6k^4t^2e^{7kx} - 120a^4k^8t^4e^{5kx} + 12a^5k^6t^4e^{6kx} + 12a^5k^8t^4e^{6kx} + \sqrt{k^2+1} \times 30a^2kte^{3kx} - \sqrt{k^2+1} \times 30a^4kte^{5kx} - \sqrt{k^2+1} \times 24a^5kte^{6kx} - \sqrt{k^2+1} \times 6a^6kte^{7kx} - \sqrt{k^2+1} \times 8ak^5t^3e^{2kx} + \sqrt{k^2+1} \times 24akte^{2kx} - \sqrt{k^2+1} \times 19a^2k^3t^3e^{3kx} - \sqrt{k^2+1} \times 19a^2k^5t^3e^{3kx} + \sqrt{k^2+1} \times 19a^4k^3t^3e^{5kx} + \sqrt{k^2+1} \times 19a^4k^5t^3e^{5kx} + \sqrt{k^2+1} \times 8a^5k^3t^3e^{6kx} + \sqrt{k^2+1} \times 8a^5k^5t^3e^{6kx} - \sqrt{k^2+1} \times a^6k^3t^3e^{7kx} - \frac{\sqrt{k^2+1} \times a^6k^5t^3e^{7kx}}{3(ae^{kx}+1)^8} + \frac{ak^2e^{kx}(3k^2t^2+3k^4t^2+k^3t^3\sqrt{k^2+1}+k^5t^3\sqrt{k^2+1}+6kt\sqrt{k^2+1}+6)}{3(ae^{kx}+1)^8}$$

Thus, exact solution as

$$z(x, t) = \frac{2ak^2e^{kx} + k\sqrt{1+k^2}t}{(1+ae^{kx} + k\sqrt{1+k^2}t)^2}$$

So as that equivalent

$$z(x, t) = \frac{ak^2}{2} \operatorname{sech}^2\left(\frac{k\sqrt{ax}}{2} + \frac{k\sqrt{a}}{2}\sqrt{1+k^2}t\right), \text{ where } c = ak^2.$$

We can be computed more iteration steps to get perfect results, but we stop at the first iteration since it is too long, so the few processes can represent the solution of Boussinesq equation. The essence of this method, the SAIM in comparison with the other analytical methods does not need large computations such as Lagrange multiplier in the VIM or any complex assumptions like nonlinear Adomian's polynomials in the ADM. It also does not need a long transformation or constructive homotopy polynomials in HPM. Furthermore, this method proved that it is efficient in overcoming the difficulties in calculating and solving KdV equation, a solitary wave for KdV equation and Boussinesq equation with easy steps.

4. Conclusion

This paper presents a technique method for solving the Korteweg-de Vries equations that gives faster and easier solutions. These solutions come accurate and in agreement with the exact solution

provided by analytical results. This method is characterized with significant analytical work that effectively reduced the amount of computational work compared with the classical methods, Adomian Decomposition Method, Modified Adomian Decomposition Method and Variation Iteration Method which require longer times. Hence, all examples showed that the results are reliable, efficient, compatible with the exact solution, and much better than those obtained by other methods. We hope that this work is a step towards using applications of the Semi Analytical Iterative Method to resolve non-linear problems emerging various physical phenomena, which may be debated in further work.

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