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Bornological Semigroups with Respect to Semi-Bounded Sets

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Abstract

The main goal of this work, is to study new classes of bornological semi group with respect to S-bounded maps, S*-bounded maps and S**-bounded maps. This manuscript would be useful to give the theoritical solution for problems in bounded and limatation by restrict the condition of boundedness. So, the motivation of this work is to require less restrictive condition on the semigroup operations neither of the operation is required to be bounded. The main important results, we prove that every bornological semigroup is an S-bornological semigroup, S*-bornological semigroup and S**-bornological semigroup. Furthermore, the certain condition for any codomain of S*-bornological semigroup to be S*-bornological semigroup was given. In addition, every left (right) translation is S-bornological isomorphism and S*-bornological isomorphism.

Key words: Bornological Group, Semi Bounded Set, Semi Bounded Map, Bounded Map.

شبه الزمر البرنولوجية فيما يتعلق بشبه المجاميع المحدودة

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الخلاصة

الهدف الرئيسي من هذا العمل، هو دراسة فئات جديدة من الزمر شبه البرنولجيه فيما يتعلق بالدوال المقيدة S و الدوال المقيدة S * والدوال المقيدة S **. سيكون هذه البحث مفيدة لإعطاء الحل النظري لمشاكل الحدود والتقييد بتقليل شرط التقيد. لذلك، فإن الدافع من هذا العمل هو وضع شرط أقل تقييدًا على عمليات شبه الزمرة. النتائج المهمة الرئيسية ، لقد أثبتنا أن كل شبه الزمرة البرنولوجيه هي شبه زمرة البرنولوجيه – S، S *– شبه زمرة البرنولوجيه و S ** – شبه الزمرة البرنولوجيه. علاوة على ذلك، تم إعطاء شرط معين لأي مجال مقابل لمجموعة S * –bornological semigroup لتكون S * –bornological semigroup.

1-Introduction:

There is a very important reason to construct the structure of bornology. Previously, when the researchers want to solve the problem of boundedness for any set or structure, they gave the concept of bounded set in that set or structurer. For example, a set B in ℝ is bounded set if it is absorbent in an interval and in metric space a set B is bounded if it is absorbent in a ball

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[1]. This observation motivated the researchers to construct such kind of structures in functional analysis, which have minimum condition to solve the problem of boundedness for any set in general way, which it is called bornological structure. The bornology concept was first introduced in [1]and [2]. In [3]the construction of a new structure has been given, which it is called bornological group to solve the problem of boundedness for group. In 2012, the fundamental constructions of bornological group has been studied [4]. Bombozzi [5] studied the category theory of algebraic bornological structures. Anwar in [6] started to solve many existence problems in algebraic bornology. Also, in [7] Anwar studied kind of groups, which cannot bornologies because the inverse map is not bounded by introducing new structure bornological semigroup.

In this paper, the new classes of bornological semigroup are introduced. In fact, that new structures are called S-bornological semigroup, S*-bornological semigroup and S**-bornological semigroup. The motivation of this work is to require less restrictive the condition on the semigroup operations neither of the operation is required to be bounded.

2. Bornological Structures

We recall some basic concepts of bornological structures. At the beginning we explain and clarify the structure of bornological set.

Definition 2.1: [3] **Bornological set**

A bornology on a set X is a family $\beta \subseteq \mathcal{P}(X)$ such that:

(i) β covers X, i.e. $X = \cup_{B \in \beta} B$

(ii) β is hereditary under inclusion, i.e. if $A \subseteq B$ and $B \in \beta$ then $A \in \beta$;

(iii) β stable under a finite union, i.e. if $B_1, B_2 \in \beta$, then $B_1 \cup B_2 \in \beta$.

A pair (X, β) consisting of a set X and a bornology β on X is called a ***bornological set***, and the elements are called bounded sets. In other words, if we have a set X, then the collection of subsets of X is a bornology on X if it is satisfying three conditions. This family should covers the set, we can satisfy the first condition in different ways, if the whole set belong to the bornology then, β covers X or $\forall\, x \in X, \{x\} \in$ β or X=

The second condition β has hereditary property, i.e. if B ∈ β and A ⊆ B, then A ∈ β. Finally, β is stable under finite union.

Some types of a bornology 2.2 [5]

i. The **discrete bornology** (β_{dis}) is the collection of all subsets of *X*, i.e. discrete bornology P(X) =2^X.

ii. The **usual bornology** (β_u) is the collection of all usual bounded subsets of *X*, i.e. usual bornology = {B⊆*X*: B is usual bounded, with respect to *X*}.

iii. The **finite bornology** (β_{fin}) is the collection of all finite bounded subsets of *X*.

After that in [3] , they decided to solve the problem of bounded for group in general way by introducing bornololgical group. In order to address the issues of boundedness for groups, we will review the idea of bornological group in this section.

Definition 2.3: [4] **Bornological group**

A bornological group (G, β) is a set with two structures, (G, *) is a group and which is β a bornology on G, such that:

i. the product map ψ:(G, β) × (G, β) ⟶ (G, β) is bounded.

ii. the map ψ^{-1}:(G, β) ⟶ (G, β) is bounded.

In the other words, a bornological group is a group G together with a bornology on G such that the group product and the inverse maps are bounded with respect to the bornology. The idea of the bornological group (BG) came from starting to solve the problem of boundedness of the group. For more information can see [8]and [9].

As we know, every group can be turned into bornological groups by providing it with the discreet bornology. But the problem it is with indiscreet bornology, there are such kind of group cannot be bornological group because the inverse map is not bounded that means, cannot solve the problem of boundedness. As every group is a semigroup, so the problem in [7] was solved by introducing a related structure to bornological group which is bornological semigroups.

Definition 2.4:[[7] **Bornological semigroup**

A ***bornological semigroup*** is a non-empty set S with two structures:

(1) S is a semigroup with a binary operation f: $S \times S \rightarrow S$;

(2) (S, β) is a bornological set and f: $(S, \beta)\times(S, \beta)\rightarrow(S, \beta)$ is bounded map.

Every bornological group is a bornological semigroup but the converse is not true.

Example 2.5 [6]Consider the group (Z, +) and the bornology $\beta=\{B \subset Z : B \subset(-\infty, b),$ for $b\in Z\}$, on Z. Then (Z, +, β) is a bornological semigroup. However, (Z,β) is not a bornological group since, the image $-B=[-b,+\infty)$ of $B=(-\infty,b]\in\beta$ under the inverse map is not bounded in β.

Note that, usual bornology or finite bornology on infinite total order set cannot belong to the collection of bornologies. So, to solve this problem the authors in [8] introduced the concept of semi bounded set.

Definition 2.6: [8]**Semi bounded set**

A subset S of a bornological set (X, β) is said to be a semi bounded set if there is a bounded subset B of (X, β) such that, $B \subseteq S \subseteq \overline{B}$, where $\overline{B}$= {all upper and lower bounds of B } $\cup B$.

Note that, every bounded set is semi bounded set, but the converse is not true just in discreet bornology on infinite set. $SB(X)$ is the collection of all semi bounded subsets of (X, β).

A map f from a bornological set (X, β) into a bornological set (Y, β) is said to be: [8] .

1-bounded map if the image of any bounded set of (X, β) is bounded set in (Y, β).

2-S-bounded map if the image of any bounded set in (X, β) is semi bounded set in (Y, β).

3-S*-bounded map if the image of any semi bounded set in (X, β) is bounded set in (Y, β) .

4-S**-bounded map if the image of any semi bounded set in (X, β) is semi bounded set in (Y, β).

Example 2.7: [6] Let $(\mathbb{R},\beta)$ be the usual bornology induced by the norm, and the sets $A=[1,\infty)$ where 1 in $\mathbb{R}$. In this case A is semi bounded sets but not bounded, $\exists\, B=[1,5]\subset[1,\infty)\subseteq\overline{B}$.

Proposition 2.8: [8]

1. Every bounded map is an S-bounded map.
2. Every bounded map is an S*-bounded map.
3. Every bounded map is an S**-bounded map.
4. Every S-bounded map is an S*-bounded map.
5. Every S-bounded map is an S**-bounded map.

3. New Classes of Bornological Semigroup

Now, we give affined structures to bornological semigroup which are called S-bornological semigroups, S*-bornological semigroups and S**-bornological semigroups, these new classes were defined by S- bounded map, S*- bounded map and S**-bounded map, respectively. The motivation for this work is come from, the idea of lessening the condition of bounded set in to semi bounded set.

3.1. S- Bornological Semigroup

In this section new class of bornological semigroup is shown, which is called an ***S-bornological semigroup***.

Definition 3.1.1:

A bornological semigroup $(\mathcal{S},*,\beta)$ is called an ***S- bornological semigroup*** if for every bounded sets B_1, B_2 contain $g_1, g_2 \in \mathcal{S}$, there exists $g_1 * g_2$ in $\mathcal{S}$, where S is a semi bounded set, such that $B_1 * B_2 \subset S$.
This an equivalent concept to S-bornological semigroup.

Definition 3.1.2:

A semigroup and bornology are called an ***S-bornological semigroup*** if $f : \mathcal{S} \times \mathcal{S} \to \mathcal{S}$ is S-bounded map.

Proposition 3.1.3: Every bornological semigroup $(\mathcal{S},*,\beta)$ is an S-bornological semigroup.
Proof: Since $(\mathcal{S},*,\beta)$ is a bornological semigroup, then $f:(\mathcal{S},\beta) \times (\mathcal{S},\beta) \to (\mathcal{S},\beta)$ is bounded map. Since every bounded map is an S-bounded map, Then, the proved is obvious.

Definition 3.1.4:

A function f from a bornology β to β' is an ***S-bornological isomorphism*** if it is a bijective and f, f^{-1} are an S-bounded maps.

Proposition 3.1.5: Every bornological isomorphism map is an S-bornological isomorphism.
Proof: Let f be a function from a bornological semigroup $(\mathcal{S},*,\beta)$ to $(\mathcal{S},*,\beta')$. A function f is a bornological isomorphism if it is bijective and f, f^{-1} are bounded. Since every bounded map is an S-bounded map. Then, the result it is clear.

Example 3.1.6:
Let $X = \{10,11\}$, $Y = \{5,6\}$ with two discrete bornological set, respectively. Suppose that $\beta = \{\emptyset, X, \{10\}, \{11\}\}$ and $\beta' = \{\emptyset, Y, \{5\}, \{6\}\}$. We define $f:(X,\beta) \to (Y,\beta')$ by $f(B) = B'$, $\forall\, B \in \beta,\ B' \in \beta'$. It is clear that f is bijective. Since, as $f(\emptyset) = \emptyset$, $f(X) = Y$, $f(\{10\}) = \{5\}$, $f(\{11\}) = \{6\}$. Then f is bounded map. Since every bounded map S-bounded map. So, f is S-bounded map.
Also, as $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$, $f^{-1}(\{5\}) = \{10\}$, $f^{-1}(\{6\}) = \{11\}$. Furthermore, f^{-1} is an S-bounded map. Thus, f is an S-bornological isomorphism.

Theorem 3.1.7: Let $(\mathcal{S},*,\beta)$ be an S-bornological semigroup and S be a semi - bounded set in $\mathcal{S}$. Then the set $L = \bigcup_{n=1}^{\infty} S^n$ is a semi bounded set in $\mathcal{S}$.
Proof: Since S is a semi-bounded in an S -bornological semigroup $(\mathcal{S},*,\beta)$, then

$$S \cdot S = S^2 \in SB(\mathcal{S}).$$

And $S^2 \cdot S = S^3 \in SB(\mathcal{S})$. Similarly, $S^4, S^5, \dots \dots \in SB(\mathcal{S})$. Thus, the set $L = \bigcup_{n=1}^{\infty} S^n$ being the union of semi-bounded sets is semi bounded set.

Proposition 3.1.8: If $(\mathcal{S},\beta)$ is an S-bornological semigroup and H is a sub semigroup of $\mathcal{S}$. If H contains a non-empty semi- bounded set, then H is a semi-bounded set in $(\mathcal{S},\beta)$.

Proof: Let S be a non-empty semi bounded subset of H with $S \subset H$. For any $h \in H$ the set $l_h(S) = h\,.S$ is a semi-bounded set in $(\mathcal{S}, \beta)$ and is a subset of H. So, the sub semigroup $H = \bigcup_{h \in H}(h \cdot S)$ is a semi-bounded set in $(\mathcal{S}, \beta)$ as it is the union of semi-bounded sets.

3.2. S*-Bornological Semigroup

In this section, we construct new class which is called S*-bornological semigroup, to restrict the condition of bounded map for bornological semigroups.

Definition 3.2.1: A bornological semigroup $(\mathcal{S}, *, \beta)$ it is called ***S*-bornological semigroup*** if for every semi-bounded sets S_1 and S_2 contain $g_1, g_2 \in \mathcal{S}$, there is a bounded set B contains $g_1 * g_2$ in $\mathcal{S}$. Such that, $S_1 * S_2 \subset B$.
In discrete bornology on infinite set, any semi-bounded set is a bounded set.

Proposition 3.2.2: Every bornological semigroup $(\mathcal{S}, *, \beta)$ is an S*-bornological semigroup.
Proof: Let all bounded sets B_1, B_2 contains $g_1, g_2 \in \mathcal{S}$, respectively. Since $(\mathcal{S}, \beta)$ is a bornological semigroup. Then, there is a bounded set B contains g_2 in $\mathcal{S}$. Such that $B_1 * B_2 \subset B$ see [8]. Since any bounded set is a semi-bounded set.
Thus, by Definition 3.2.1. The proof is obvious.

Definition 3.2.3: A function f from bornological set (X, β) to (Y, β) is called an ***S*-bornological isomorphism*** if f it is bijective and f, f^{-1} are S*-bounded maps.

Example 3.2.4:
Let $X = \{1,2\}$ and $Y = \{a, b\}$ be two discrete bornological sets. And let $\beta_X = \{\emptyset, X, \{1\}, \{2\}\}$ and $\beta_Y = \{\emptyset, Y, \{a\}, \{b\}\}$. Define $f: (X, \beta_X) \to (Y, \beta_Y)$ by $f(B) = B'$, $\forall\, B \in \beta_X$, $B' \in \beta_Y$. Such that $f(X) = Y$, $f(\emptyset) = \emptyset$, $f(\{1\}) = \{a\}$ and $f(\{2\}) = \{b\}$. It is clear that f is bijective. As $f(\emptyset) = \emptyset$, $f(X) = Y$, $f(\{1\}) = \{a\}$ and $f(\{2\}) = \{b\}$. Then f is a bounded map. Since every bounded map is an S*-bounded map. So f is an S*-bounded map. Also, as $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$, $f^{-1}(\{a\}) = \{1\}$, and $f^{-1}(\{b\}) = \{2\}$. Furthermore, f^{-1} is an S*-bounded map. Thus, f is an S*-bornological isomorphism.

Theorem 3.2.5: If f is a map from an S*-bornological semigroup $(H, ., \beta)$ to a bornological semigroup $(L, \circ, \beta')$ such that f is a semigroup homomorphism and an S*- bornological isomorphism, then $(L, \circ, \beta')$ is also an S*-bornological semigroup.
Proof: Since S_1, S_2 are two semi bounded sets in $(L, \circ, \beta')$ contain $g_1, g_2 \in L$.
Let $x = f^{-1}(g_1)$ and $y = f^{-1}(g_2)$. Since f^{-1} is an S*-bounded map, then $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are bounded sets contain x and y , respectively. In addition, as every bounded set is a semi-bounded set, then $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are semi-bounded sets. Since H is an S*-bornological semigroup (By hypothesis). So, there exists a bounded set $f^{-1}(B)$ contains $x \cdot y$ where B is a bounded set contains $g_1 \circ g_2$ with $f^{-1}(S_1) \cdot f^{-1}(S_2) \subset f^{-1}(B)$. Since f is an S*-bounded map. Then $f(f^{-1}(S_1)) \cdot f^{-1}(S_2)) \subset B$. Thus, $S_1 \circ S_2 \subset B$.Then, there is a bounded set B which contains $g_1 \circ g_2$. Hence, $(L, \circ, \beta')$ is an S* -bornological semigroup.

Remark 3.2.6: Let $SB(\mathcal{S})$ be a class of all semi-bounded sets and $(\mathcal{S}, *, \beta)$ is an S*-bornological semigroup, if $S \in SB(\mathcal{S})$ and $B \in \beta$, then $S \cdot B$ and $B \cdot S$ both in $SB(\mathcal{S})$, where $S \cdot B = \{g \in \mathcal{S}\colon\ g = g_1 \cdot g_2 \colon g_1 \in S, g_2 \in B\}$.
In the result below we give sufficient condition for left or right S*-bornological semigroup to be an S*-bornological isomorphism.

Proposition 3.2.7: Let $(\mathcal{S},*,\beta)$ be an S*-bornological semigroup. Then each left translation $l_g\colon \mathcal{S} \rightarrow \mathcal{S}$ is an S*-bornological isomorphism.
Proof: As we know that left translations are bijective map. We must prove that for any fixed arbitrary $g \in \mathcal{S}$ the translation l_g is an S*-bounded map. Suppose that g, g_1 be two elements in $(\mathcal{S},\beta)$ and S_1, S_2be semi-bounded sets contain g, g_1. By Definition 3.2.1 there is a bounded set B containing $l_g(g_1) = g \cdot g_1$,
such that, $S_1 \cdot S_2 \subset B$. In particular, we have $g \cdot S_2 \subset B$. By Remark 3.2.6 the set S_2 is semi- bounded set contains g_1, as a result the last contains l_g which is an S* -bounded map at g. Since l_g is an S* -bounded on $\mathcal{S}$ and $g \in \mathcal{S}$ is an element in $\mathcal{S}$. Hence, ${l^{-1}}_g = (l_g)^{-1}$ which is also an S*-bounded map. Then each left translation is S*-bornological isomorphism.

3.3. S**- Bornological Semigroup

In this section we show a new class of bornological semigroup called an ***S**-bornological semigroup*** with respect to an S**-bounded map. The motivation for this study is to reduce the condition of boundedness for bornological semigroup and give the notion with respect to S**-bounded map.

Definition 3.3.1: A bornological semigroup$(\mathcal{S},*,\beta)$ it is called an ***S**-bornological semigroup*** if for every semi-bounded sets S_1, S_2 contain $g_1, g_2 \in \mathcal{S}$. There is a semi-bounded set S contains $g_1 \cdot g_2$ in $\mathcal{S}$ such that $S_1 \cdot S_2 \subset S$.
The following theorem develops an equivalent concept for an S**-bornological group.

Theorem 3.3.2: If $(\mathcal{S},*,\beta)$ is an S**-bornological semigroup, then the product map $f\colon \mathcal{S} \times \mathcal{S} \rightarrow \mathcal{S}$ is an S**-bounded map.
Proof: Assume that (g_1, g_2) be an element of $\mathcal{S} \times \mathcal{S}$ and both S_1, S_2 be a semi-bounded set contain g_1, g_2. Since $(\mathcal{S},\beta)$ is an S**-bornological semigroup and $S_1 \times S_2$ is semi-bounded set contains (g_1, g_2). Then there exists a semi-bounded set S contains $f(g_1, g_2) = g_1 * g_2$ in $\mathcal{S}$. Such that $S_1 * S_2 \subset S$. From Remark 3.2.6, S_2 is a semi-bounded set contains g_2. So, $f(S_1 \times S_2) \subset S$. That implies, f is an S**-bounded map at (g_1, g_2). Because, (g_1, g_2) is an arbitrary element of $\mathcal{S} \times \mathcal{S}$, then f is an S**-bounded map on $\mathcal{S} \times \mathcal{S}$.
We get the next definition instantly.

Definition 3.3.3: A bornological semigroup $(\mathcal{S},*,\beta)$ it is called an ***S**-bornological semigroup*** if $(\mathcal{S},*)$ is a semigroup, $(\mathcal{S},\beta)$is a bornological set, and the product map f : $(\mathcal{S},*,\beta) \times (\mathcal{S},*,\beta) \rightarrow (\mathcal{S},*,\beta)$ defied by $f(g_1, g_2) = g_1 \cdot g_2$, for each $g_1, g_2 \in \mathcal{S}$, is an S**-bounded map,
Moreover, since for each bounded map is an S**-bounded map, it follows that every bornological semigroup is an S**-bornological semigroup.

Proposition 3.3.4: Every S- bornological semigroup $(\mathcal{S},*,\beta)$ is an S**- bornological semigroup.
Proof: The proof is obvious by the definition of S- bornological semigroup and S**-bornological semigroup.

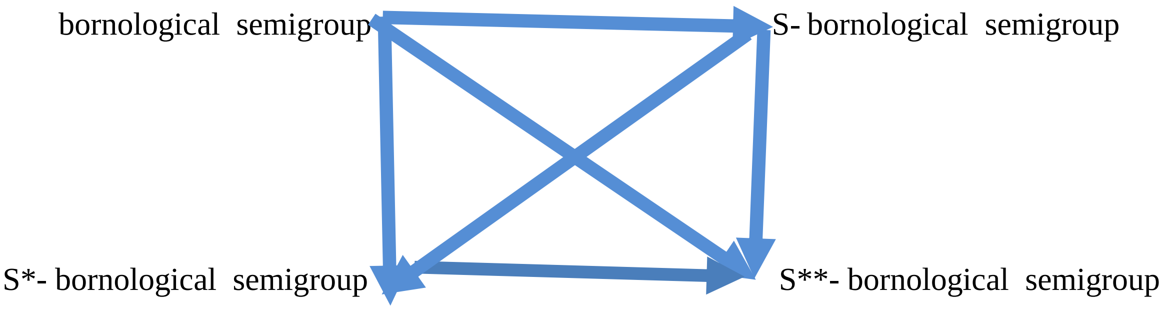
Proposition 3.3.5: Every bornological semigroup $(\mathcal{S},*,\beta)$ is an S**-bornological semigroup.
Proof: The proof similar to the proof of the Proposition 3.3.4.

Definition 3.3.6: A function f from bornological set (X,β) to (Y,β) is called an ***S**-bornological isomorphism*** if f is bijective and f, f^{-1} are S**-bounded maps.

Proposition 3.3.7: Every bornological isomorphism is an S*-bornological isomorphism and an S**-bornological isomorphism.
Proof: Since every bounded map is an S*-bounded map and an S**-bounded map. Then, the result it is clear.
By the next theorem we give the certain condition for any codomain of an S**- bornological semigroup to be S** -bornological semigroup.

Theorem 3.3.8: If f is a map from an S**-bornological semigroup $(H,*,\beta)$ to a bornological semigroup $(L,\circ,\beta')$ is a semigroup homomorphism and an S**- bornological isomorphism, then $(L,\circ,\beta')$ is also an S**-bornological semigroup.
Proof: Let S_1, S_2 be two semi-bounded sets contain $g_1, g_2 \in L$, respectively.
Let $x = f^{-1}(g_1)$ and $y = f^{-1}(g_2)$. Since f^{-1} is an S**-bounded map then $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are semi-bounded sets contain x and y , respectively. By hypothesis H is an S** -bornological semigroup. Then there is semi-bounded set $f^{-1}(S)$ contains $x \cdot y$ where S is semi- bounded set containing $g_1 \circ g_2$ with $f^{-1}(S_1) \cdot f^{-1}(S_2) \subset f^{-1}(S)$. Since f is an S** -bounded map. Then $f(f^{-1}(S_1)) \cdot f^{-1}(S_2)) \subset S$. Thus, $S_1 \circ S_2 \subset S$. Then, there is a semi- bounded set which contain $g_1 \circ g_2$. Thus, $(L,\circ,\beta')$ is an S**-bornological semigroup.
In result below we give sufficient condition for left or right S**-bornological semigroup to be an S**-bornological isomorphism.

Proposition 3.3.9: Let$(\mathcal{S},*,\beta)$ be an S**-bornological semigroup. Then every left translation $l_g: \mathcal{S} \to \mathcal{S}$ is an S**-bornological isomorphism.
Proof: The proof similar to the proof of Proposition 3.2.7.
The following diagram shows the relation between of the new classes of bornological group.



4. Conclusions

The main conclusion is that, if we want to restrict the condition of boundedness of semigroup operation, there is one way. By constructing new classes of bornological semigroup are called S-bornological semigroup, S*-bornological semigroup and S**-bornological semigroup. For this purpose, we used an S-bounded map and S*- bounded map, S**- bounded map of the semigroup operations instead of boundedness in the case of bornology.

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