



ISSN: 0067-2904

## On Soft Strongly $b^*$ – Separation Axioms

Saif Z. Hameed\*, Abdelaziz E. Radwan, Essam El-Seidy

Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

Received: 15/1/2023 Accepted: 11/6/2023 Published: 30/6/2024

### Abstract

In this paper, we study some new types of soft separation axioms called soft strongly  $b^*$  – separation axioms. We show that, the properties of soft strongly  $b^* - T_i$  space ( $i=0, 1,2$ ) are soft topological properties under the bijection, soft irresolute and soft continuous mapping. Furthermore, the property of being soft strongly  $b^*$  – regular and soft strongly  $b^*$  – normal are soft topological properties under bijection, soft continuous functions. Moreover, their relationships with existing spaces are studied.

**Keywords:** soft strongly  $b^*$  –closed set, soft strongly  $b^*$  –open set, soft strongly  $b^* - T_i$  space, soft strongly  $b^*$  – regular, soft strongly  $b^*$  – normal.

### على الناعمة بقوة $b^*$ – بديهيات الفصل

سيف زهير حميد\* ، عبد العزيز علي رضوان ، عصام الصعيدي

قسم الرياضيات، كلية العلوم، جامعة عين شمس، القاهرة، مصر

### الخلاصة

في هذا البحث، ندرس بعض الأنواع الجديدة من بديهيات الفصل الناعمة المسماة الناعمة بقوة  $b^*$  – بديهيات الفصل. نوضح أن خصائص الفضاء  $b^* - T_i$  الناعمة بقوة ( $i=0, 1,2$ ) هي خصائص توبولوجية ناعمة تحت التقابل، غير الثابتة الناعمة وتطبيق المستمرة الناعمة. علاوة على ذلك، فإن خاصية كونها ناعمة بقوة  $b^*$  – منتظمة وناعمة بقوة  $b^*$  – طبيعية هي خصائص توبولوجية ناعمة تحت التقابل والدالة المستمرة الناعمة. علاوة على ذلك، يتم دراسة علاقاتهم مع الفضاءات الموجودة.

## 1. Introduction and Preliminaries

In 1999, Molodtsov [1], instigated the concept of soft set as a new mathematical tool to deal with uncertainties problems in different fields of science. In [2] and [3] defined soft generalized closed and open sets and soft continuous mappings in soft closure in soft topological spaces. I. Arockiarani and A. Arokialancy [4] defined soft  $\beta$  –open sets and continued to study weak forms of soft open sets in soft topological space.

Later, Akdag and Ozkan [5, 6] defined soft  $\alpha$ -open and introduced soft  $\alpha$ -separation axioms. In [7-10] the soft and Simply  $b$ -open are studied, A. Poongothai, R. Parimelazhagan defined  $sb^*$ -separation axioms [11]. Hameed, S. Z., Hussein, A. K [12] defined the soft  $b_c$  –open set. The soft  $b^*$  – closed are studied by Hameed, Saif Z., Fayza Ibrahim, and Essam El-Seidy [13]. The soft  $b^*$  –continuous and soft strongly  $b^*$  –closed sets and soft

\*Email: ssaifzuhair@gmail.com

strongly  $\mathbf{b}^*$  –continuous functions are studied by Hameed, Saif Z., Abdelaziz E. Radwan, and Essam El-Seidy [14] and [15]. In this paper, we define soft  $\mathbf{b}^*$  –separation axioms. The properties and relationships of the concept are discussed in detail. With help of counterexamples, we show the non-coincidence of these various types of separation axiom.

**Definition 1.1:** [1] Let  $Z$  be an initial universe set and  $E$  be a set of parameters. Let  $P(Z)$  denote the power set of  $Z$ , and  $A$  be a non-empty subset of  $E$ . A pair  $(S, A)$  is called a soft set over  $Z$ . Where  $S$  is a mapping given by  $S: A \rightarrow P(Z)$ . In what follows we denote by  $SS(Z, A)$  the family of all soft sets over  $Z$ .

**Definition 1.2:** [16] The soft set  $(S, A) \in SS(Z, A)$ , where  $S(c) = \emptyset$ , for every  $c \in A$  is called  $A$ -null soft set of  $SS(Z, A)$  and denoted by  $\tilde{\emptyset}$ . The soft set  $(S, A) \in SS(Z, A)$ , where  $S(c) = Z$ , for every  $c \in A$  is called the  $A$ -absolute soft set of  $SS(Z, A)$  and denoted by  $\tilde{Z}$ .

**Definition 1.3:** [16] For two sets  $(P, A), (S, B) \in SS(Z, A)$ , we say that  $(P, A)$  is a soft subset of  $(S, B)$  denoted by  $(P, A) \subseteq (S, B)$ , if

- (1)  $A \subseteq B$ .
- (2)  $P(e) \subseteq S(e), \forall e \in A$ .

In this case,  $(P, A)$  is said to be a soft superset of  $(S, B)$ , if  $(S, B)$  is a soft subset of  $(P, A)$ ,  $(S, B) \supseteq (P, A)$ .

**Definition 1.4:** [17] Let  $(P, A)$  be a soft set over  $Z$  and  $z \in Z$ . We say that  $z \in (P, A)$  read as  $z$  belongs to the soft set  $(P, A)$  whenever  $z \in P(e)$  for all  $e \in A$ . The soft set  $(P, A)$  over  $Z$  such that  $P(e) = \{z\} \forall e \in A$  is called singleton soft point and denoted by  $z_A$  or  $(z, A)$ .

**Definition 1.5:** [17] Let  $T$  be a collection of soft sets over  $Z$ , then  $T$  is said to be soft topological space on  $Z$  if

- (1)  $\tilde{\emptyset}$  and  $\tilde{Z}$  belong to  $T$ .
- (2) The union of any subcollection of soft sets of  $T$  belongs to  $T$ .
- (3) The intersection of any two soft sets in  $T$  belongs to  $T$ .

It is denoted by  $(Z, T, A)$  and briefly  $Z$ .

**Definition 1.6:** [17] Let  $(Z, T, A)$  be a soft space over  $Z$ , then the members of  $T$  are said to be soft open sets in  $T$ .

**Definition 1.7:** [17] Let  $(Z, T, A)$  be a soft space over  $Z$ . A soft set  $(P, A)$  over  $Z$  is said to be a soft closed set in  $Z$ , if its relative complement  $(P, A)'$  belongs to  $T$ .

**Definition 1.8:** [18] Let  $(Z, T, A)$  be a soft topological space and  $(P, A) \in SS(Z, A)$ . Then

- (1) The soft closure of  $(P, A)$  is the soft set  $cl(P, A) = \cap \{(L, A) : (L, A) \in T^c, (P, A) \subseteq (L, A)\}$ .
- (2) The soft interior of  $(P, A)$  is the soft set  $int(P, A) = \cup \{(H, A) : (H, A) \in T, (H, A) \subseteq (P, A)\}$ .

**Definition 1.9:** [4, 5, 7, 19] A soft set  $(S, A)$  of a soft topological space  $(Z, T, A)$  is said to be

- (1) soft  $\alpha$ - open if  $(S, A) \subset int(cl(int((S, A))))$ .
- (2) soft preopen if  $(S, A) \subset int(cl((S, A)))$ .
- (3) soft semi - open if  $(S, A) \subset cl(int((S, A)))$ .
- (4) soft  $\beta$ -open if  $(S, A) \subset cl(int(cl((S, A))))$ .

(5) soft  $\mathbf{b}$  –open if  $(S, A) \subset \text{int}(\text{cl}((S, A))) \cup \text{cl}(\text{int}((S, A)))$ .

**Definition 1.10:** [20] A soft set  $(P, A)$  is called soft  $\omega$  –closed if  $\text{cl}(P, A) \subseteq (S, A)$  whenever  $(P, A) \subseteq (S, A)$  and  $(S, A)$  is soft semi-open set in  $Z$ . The relative complement of  $(P, A)$  is called soft  $\omega$  –open in  $Z$ .

**Definition 1.11:** [6] A soft topological space  $(Z, T, A)$  is said to be:

- (1) Soft  $\alpha$ - $T_0$  space if for each pair of distinct points in  $Z$ , there is an  $s\alpha$  –open set containing one of the points but not the other.
- (2) Soft  $\alpha$ - $T_1$  space if for each pair of distinct points  $z$  and  $c$  of  $Z$ , there exists  $s\alpha$  –open sets  $(P, A)$  and  $(S, A)$  containing  $z$  and  $c$  respectively such that  $c \notin (P, A)$  and  $z \notin (S, A)$ .
- (3) Soft  $\alpha$ - $T_2$  space if for each pair of distinct points  $z$  and  $c$  of  $Z$ , there exist disjoint  $s\alpha$  –open sets  $(P, A)$  and  $(S, A)$  containing  $z$  and  $c$  respectively.

**Definition 1.12:** [9] Let  $(Z, T, A)$  be a soft topological space and  $z, c \in Z$  such that  $z \neq c$ . Then,  $(Z, T, A)$  is called a soft  $\mathbf{b}$ - $T_0$  space if there exist soft  $\mathbf{b}$ -open sets  $(P, A)$  and  $(S, A)$  such that either  $z \in (P, A)$  and  $c \notin (P, A)$  or  $c \in (S, A)$  and  $z \notin (S, A)$ .

**Definition 1.13:** [9] Let  $(Z, T, A)$  be a soft topological space and  $z, c \in Z$  such that  $z \neq c$ . Then,  $(Z, T, A)$  is called soft  $\mathbf{b}$ - $T_1$  space if there exist soft  $\mathbf{b}$ -open sets  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$  and  $c \notin (P, A)$  and  $c \in (S, A)$  and  $z \notin (S, A)$ .

**Definition 1.14:** [9] Let  $(Z, T, A)$  be a soft topological space and  $z, c \in Z$  such that  $z \neq c$ . Then  $(Z, T, A)$  is called a soft  $\mathbf{b}$  –Hausdorff space or soft  $\mathbf{b}$ - $T_2$  space if there exist soft  $\mathbf{b}$ -open  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$ ,  $c \in (S, A)$  and  $(P, A) \cap (S, A) = \emptyset$

**Definition 1.15:** [15] A soft set  $(P, A)$  of a soft topological space  $(Z, T, A)$  is called a soft strongly  $\mathbf{b}^*$  –closed (briefly  $sSb^*$  –closed) if  $\text{cl}(\text{int}(P, A)) \subseteq (S, A)$ , whenever  $(P, A) \subset (S, A)$  and  $(S, A)$  is soft  $\mathbf{b}$  –open. The complement of a soft strongly  $\mathbf{b}^*$  –closed set is called soft strongly  $\mathbf{b}^*$  –open set. The family of all soft strongly  $\mathbf{b}^*$  –open sets denoted by  $sSb^*OS(Z)$ .

**Theorem 1.16:** [15] The following statements are true in general.

- (i) Every soft open is soft strongly  $\mathbf{b}^*$  –open.
- (ii) Every soft  $\alpha$  –open is soft strongly  $\mathbf{b}^*$  –open.
- (iii) Every soft strongly  $\mathbf{b}^*$  –open set is soft  $\mathbf{b}$  –open.
- (iv) Every soft  $\omega$  –open is soft strongly  $\mathbf{b}^*$  –open.

**Definition 1.17:** [15] Let  $(Z, T, A)$  be a soft topological space. A subset  $(S, A) \subseteq Z$  is called a  $sSb^*$  –neighbourhood (briefly  $sSb^*$  –nbd) of a point  $z \in Z$  if there exists an  $sSb^*$  –open set  $(P, A)$  such that  $z \in (P, A) \subseteq (S, A)$ .

**Definition 1.18:** [15] Let  $(O, A)$  be a soft subset of  $Z$ . Then  $sSb^*\text{int}(O, A) = \cup \{(H, A) : (H, A) \text{ is a soft strongly } \mathbf{b}^* \text{ – open set and } (H, A) \subset (O, A)\}$ .

**Definition 1.19:** [15] Let  $(H, A)$  be a soft subset of a soft space  $Z$ . Then the soft strongly  $\mathbf{b}^*$  –closure of  $(H, A)$  is defined as the intersection of all soft strongly  $\mathbf{b}^*$  –closed set containing  $(H, A)$ , that is  $sSb^*\text{cl}(H, A) = \cap \{(P, A) : (P, A) \text{ is a soft strongly } \mathbf{b}^* \text{ – closed set and } (H, A) \subset (P, A)\}$ .

**Definition 1.20:** [15] A soft mapping  $f: Z \rightarrow D$ , from soft topological space  $(Z, T, A)$  into soft topological space  $(D, \Omega, K)$ , is said to be soft strongly  $\mathfrak{b}^*$ -continuous (briefly  $sS\mathfrak{b}^*$ -continuous) if the inverse image of every soft open set in  $D$  is a soft strongly  $\mathfrak{b}^*$ -open set in  $Z$ .

**Definition 1.21:** [15] A soft mapping  $f: Z \rightarrow D$  is said to be soft strongly  $\mathfrak{b}^*$ -irresolute (briefly  $sS\mathfrak{b}^*$ -irresolute) if the inverse image of every soft  $\mathfrak{b}^*$ -closed set in  $D$  is a soft strongly  $\mathfrak{b}^*$ -closed set in  $Z$ .

## 2. Soft strongly $\mathfrak{b}^*$ -separation axioms

In this section, we introduce the concept of some soft separation axioms such as the soft strongly  $\mathfrak{b}^* - T_0$ , soft strongly  $\mathfrak{b}^* - T_1$ , soft strongly  $\mathfrak{b}^* - T_2$ , soft strongly  $\mathfrak{b}^* - T_3$  and soft strongly  $\mathfrak{b}^* - T_4$  axioms. Moreover, we give some of their properties and the relations between these concepts.

**Definition 2.1:** A soft topological space  $Z$  is said to be soft strongly  $\mathfrak{b}^* - T_0$  space if for every pair of distinct points  $z$  and  $c$  of  $Z$ , there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(S, A)$  such that  $z \in (S, A)$  and  $c \notin (S, A)$  or  $c \in (S, A)$  and  $z \notin (S, A)$ .

**Corollary 2.2:** (i) Every soft strongly  $\mathfrak{b}^* - T_0$  space is soft  $\mathfrak{b} - T_0$ -space.

(ii) Every soft  $\alpha - T_0$  space is soft strongly  $\mathfrak{b}^* - T_0$  space.

(iii) Every soft  $\omega - T_0$  space is soft strongly  $\mathfrak{b}^* - T_0$  space.

(iv) Every soft  $T_0$ -space is soft strongly  $\mathfrak{b}^* - T_0$  space.

**Proof.** (i) Suppose that  $Z$  be a soft strongly  $\mathfrak{b}^* - T_0$  space. Let  $z$  and  $c$  be any two distinct points in  $Z$ . Then there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(L, A)$  such that  $z \in (L, A)$  and  $c \notin (L, A)$  or  $c \in (L, A)$  and  $z \notin (L, A)$ . By Theorem 1.14 (iii),  $(L, A)$  is a soft  $\mathfrak{b}$ -open set such that  $z \in (L, A)$  and  $c \notin (L, A)$  or  $c \in (L, A)$  and  $z \notin (L, A)$ . Thus  $Z$  is soft  $\mathfrak{b} - T_0$  space.

(ii) Let  $Z$  be a soft  $\alpha - T_0$  space. Let  $z$  and  $c$  be any two distinct points in  $Z$ . Then there exists a  $s\alpha$ -open set  $(S, A)$  such that  $z \in (S, A)$  and  $c \notin (S, A)$  or  $c \in (S, A)$  and  $z \notin (S, A)$ . By Theorem 1.14 (ii),  $(S, A)$  is a soft strongly  $\mathfrak{b}^*$ -open set such that  $z \in (S, A)$  and  $c \notin (S, A)$  or  $c \in (S, A)$  and  $z \notin (S, A)$ . Thus  $Z$  is soft strongly  $\mathfrak{b}^* - T_0$  space.

(iii) and (iv) The proof is similar to the proof of (ii).

**Theorem 2.3:** Every soft subspace of a soft strongly  $\mathfrak{b}^* - T_0$  space is soft strongly  $\mathfrak{b}^* - T_0$  space.

**Proof.** Let  $D$  be a soft subspace of a soft strongly  $\mathfrak{b}^* - T_0$  space. Let  $z, c$  be two distinct points of  $D$ , as  $D \subseteq Z$ . Then there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(P, A)$  such that  $z \in (P, A)$  and  $c \notin (P, A)$  since  $Z$  is soft  $\mathfrak{b}^* - T_0$  space. Then  $(P, A) \cap D$  is soft strongly  $\mathfrak{b}^*$ -open set in  $D$  which contains  $z$  but does not contain  $c$ . Hence  $D$  is soft strongly  $\mathfrak{b}^* - T_0$  space.

**Definition 2.4:** A soft topological space  $Z$  is said to be soft strongly  $\mathfrak{b}^* - T_1$  space if for every pair of distinct points  $z$  and  $c$  in  $Z$ , there exists a soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(L, A)$  such that  $z \in (P, A)$  and  $c \notin (P, A)$ ,  $c \in (L, A)$  and  $z \notin (L, A)$ .

**Corollary 2.5:** Every soft strongly  $\mathfrak{b}^* - T_1$  space is soft strongly  $\mathfrak{b}^* - T_0$  space.

**Proof.** Suppose that  $Z$  be a soft strongly  $\mathfrak{b}^* - T_1$  space. Let  $z$  and  $c$  be any two distinct points in  $Z$ . Then there exists soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(L, A)$  such that  $z \in (P, A)$  and  $c \notin (P, A)$ , and  $c \in (L, A)$  and  $z \notin (L, A)$ . Obviously then we have  $z \in (P, A)$  and  $c \notin (P, A)$ , or  $c \in (L, A)$  and  $z \notin (L, A)$ . Hence  $Z$  is soft strongly  $\mathfrak{b}^* - T_0$  space.

The following example shows that the converse of the above corollary need not be true in general.

**Example 2.6:** Let  $Z = \{J_1, J_2, J_3, J_4\}$ ,  $A = \{e_1, e_2\}$  and  $T = \{\Phi, X, (P_1, A), (P_2, A), (P_3, A)\}$  where

$$(P_1, A) = \{(e_1, \{J_1, J_2, J_3\}), (e_2, \{J_1, J_2, J_3\})\},$$

$$(P_2, A) = \{(e_1, \{J_2, J_3\}), (e_2, \{J_2, J_3\})\},$$

$$(P_3, A) = \{(e_1, \{J_2\}), (e_2, \{J_2\})\}.$$

Then  $(Z, T, A)$  is a soft topological space over  $Z$ . Also,  $(Z, T, A)$  is a soft strongly  $\mathfrak{b}^* - T_0$  space over  $Z$  which is not a soft strongly  $\mathfrak{b}^* - T_1$  space because  $J_2, J_3 \in Z$  but there do not exist soft strongly  $\mathfrak{b}^* -$ open sets  $(P, A)$  and  $(S, A)$ , such that  $J_2 \in (P, A)$  and  $J_3 \notin (P, A)$ , and  $J_3 \in (S, A)$  and  $J_2 \notin (S, A)$ .

**Corollary 2.7:** Every soft strongly  $\mathfrak{b}^* - T_1$  space is soft  $\mathfrak{b} - T_1$  space.

**Proof.** Suppose  $Z$  is a soft  $\mathfrak{b}^* - T_1$  space. Let  $z$  and  $c$  be two distinct points in  $Z$ . Then there exists soft strongly  $\mathfrak{b}^* -$ open sets  $(P, A)$  and  $(L, A)$  such that  $z \in (P, A)$  and  $c \notin (P, A)$  and  $c \in (L, A)$  and  $z \notin (L, A)$ . By Theorem 1.14 (iii),  $(P, A)$  and  $(L, A)$  are soft  $\mathfrak{b} -$ open such that  $z \in (P, A)$  and  $c \notin (P, A)$  and  $c \in (L, A)$  and  $z \notin (L, A)$ . Thus  $Z$  is soft  $\mathfrak{b} - T_1$  space.

The following example shows that the converse of the Corollary 2.7 need not be true in general.

**Example 2.8:** Let  $Z = \{c, b\}$ ,  $A = \{e_1, e_2\}$  and  $T = \{\tilde{Z}, \tilde{\emptyset}, (P_1, A), (P_2, A), (P_3, A)\}$

$(P_1, A), (P_2, A), (P_3, A)$  are soft sets over  $Z$  defined as follows:

$$(P_1, A) = \{(e_1, Z), (e_2, \{b\})\}$$

$$(P_2, A) = \{(e_1, \{c\}), (e_2, Z)\}$$

$$(P_3, A) = \{(e_1, \{c\}), (e_2, \{b\})\}.$$

Then,  $T$  defines a soft topology on  $Z$ . Also,  $(Z, T, A)$  is soft  $\mathfrak{b} - T_1$  space, but it is not a soft strongly  $\mathfrak{b}^* - T_1$  space.

**Theorem 2.9:** Every soft subspace of a soft strongly  $\mathfrak{b}^* - T_1$  space is soft strongly  $\mathfrak{b}^* - T_1$  space.

**Proof.** It is same as the proof of Theorem 2.3.

**Theorem 2.10:** Every soft  $\omega - T_1$  space is soft strongly  $\mathfrak{b}^* - T_1$  space.

**Proof.** Suppose that  $Z$  is a soft  $\omega - T_1$  space. Let  $z$  and  $c$  be two distinct points in  $Z$ . Then there exists soft  $\omega -$ open sets  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$  but  $c \notin (P, A)$  and  $c \in (S, A)$  but  $z \notin (S, A)$ . By Theorem 1.14 (iv),  $(P, A)$  and  $(S, A)$  are soft strongly  $\mathfrak{b}^* -$ open such that  $z \in (P, A)$  but  $c \notin (P, A)$  and  $c \in (S, A)$  but  $z \notin (S, A)$ . Thus  $Z$  is soft strongly  $\mathfrak{b}^* - T_1$  space.

The following example shows that the converse of the above Theorem need not be true in general.

**Example 2.11:** Let  $Z = \{c, b, c\}$ ,  $A = \{e_1, e_2\}$  and  $T = \{\tilde{Z}, \tilde{\emptyset}, (P, A)\}$

$(P, A)$  are soft sets over  $Z$  defined as follows:

$$(P, A) = \{(e_1, \{c, b\}), (e_2, \{c, b\})\}$$

Then,  $T$  defines a soft topological space on  $Z$ . Also,  $(Z, T, A)$  is soft strongly  $\mathfrak{b}^* - T_1$  space, but it is not a soft  $\omega - T_1$  space.

**Theorem 2.12:** A soft topological space  $(Z, T, A)$  is soft strongly  $\mathfrak{b}^* - T_1$  if and only if for every  $z \in Z$ ,  $sS\mathfrak{b}^*cl\{z\} = \{z\}$ .

**Proof.** Let  $(Z, T, A)$  is a soft  $\mathfrak{b}^* - T_1$  and  $z \in Z$ . Then for each  $z \neq c$ , there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(P, A)$  such that  $z \in (P, A)$  but  $c \notin (P, A)$ . This implies that  $c \notin sS\mathfrak{b}^*cl\{z\}$ , for every  $c \in Z$  and  $z \neq c$ , this  $\{z\} = sS\mathfrak{b}^*cl\{z\}$ . Conversely, suppose  $sS\mathfrak{b}^*cl\{z\} = \{z\}$  for every  $z \in Z$ . Let  $z, c$  be two disjoint points in  $Z$ , then  $z \notin \{c\} = sS\mathfrak{b}^*cl\{c\}$  implies there exists a soft strongly  $\mathfrak{b}^*$ -closed set  $(L, A)$  such that  $c \in (L, A)$ ,  $z \notin (L, A)$  implies  $(L, A)^c$  is a soft strongly  $\mathfrak{b}^*$ -open set such that  $z \in (L, A)^c$  but  $c \notin (L, A)^c$ . Also,  $c \notin \{z\} = sS\mathfrak{b}^*cl\{z\}$  implies there exists a soft strongly  $\mathfrak{b}^*$ -closed set  $(H, A)$  such that  $z \in (H, A)$ ,  $c \notin (H, A)$  which implies that  $(H, A)^c$  is a soft strongly  $\mathfrak{b}^*$ -open set such that  $c \in (H, A)^c$  but  $z \notin (H, A)^c$ . Then  $(Z, T, A)$  is soft strongly  $\mathfrak{b}^* - T_1$  space.

**Definition 2.13:** A soft topological space  $Z$  is said to be soft strongly  $\mathfrak{b}^* - T_2$  space if for every pair of distinct points  $z$  and  $c$  in  $Z$ , there are disjoint soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$  in  $Z$  containing  $z$  and  $c$ , respectively.

**Corollary 2.14:** (i) Every soft  $\alpha - T_2$  space is soft strongly  $\mathfrak{b}^* - T_2$  space.

(ii) Every soft strongly  $\mathfrak{b}^* - T_2$  space is soft  $\mathfrak{b} - T_2$  space.

(iii) Every soft  $\omega - T_2$  space is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Proof.** Omitted.

**Theorem 2.15:** Every soft strongly  $\mathfrak{b}^* - T_2$  space is soft strongly  $\mathfrak{b}^* - T_1$  space.

**Proof.** Suppose that  $Z$  be a soft strongly  $\mathfrak{b}^* - T_2$  space. Let  $z$  and  $c$  be any two distinct points in  $Z$ . Then there exists two disjoint soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(L, A)$  such that  $z \in (P, A)$  and  $c \in (L, A)$ . Since  $(P, A)$  and  $(L, A)$  are disjoint,  $z \in (P, A)$  and  $c \notin (P, A)$  and  $c \in (L, A)$  and  $z \notin (L, A)$ . Hence  $Z$  is soft strongly  $\mathfrak{b}^* - T_1$  space.

The following example shows that the converse of the Theorem 2.15 need not be true in general.

**Example 2.16:** Let  $A$  any set of parameters and  $T$  is the soft co-finite topological space on the set of integers numbers  $\mathbb{Z}$ . The soft subset of  $(\mathbb{Z}, T_{cof}, A)$  is soft strongly  $\mathfrak{b}^*$ -open since it is soft open set. Then for each  $z \neq c \in \mathbb{Z}$ , we have  $\mathbb{Z} \setminus \{c\}$  and  $\mathbb{Z} \setminus \{z\}$  are soft strongly  $\mathfrak{b}^*$ -open sets such that  $z \in \mathbb{Z} \setminus \{c\}$  and  $c \notin \mathbb{Z} \setminus \{c\}$ ; and  $c \in \mathbb{Z} \setminus \{z\}$  and  $z \notin \mathbb{Z} \setminus \{z\}$ . Therefore,  $(\mathbb{Z}, T_{cof}, A)$  is soft strongly  $\mathfrak{b}^* - T_1$  space. However, there do not exist two disjoint soft strongly  $\mathfrak{b}^*$ -open sets except for the  $\tilde{\emptyset}$  and  $\tilde{Z}$  soft sets. Hence,  $(\mathbb{Z}, T_{cof}, A)$  is not soft strongly  $\mathfrak{b}^* - T_2$  space.

**Theorem 2.17:** For a soft topological space  $(Z, T, A)$ , the following are equivalent:

(i)  $Z$  is a soft strongly  $\mathfrak{b}^* - T_2$  space;

(ii) Let  $z \in Z$ . then for each  $z \neq c$  there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(P, A)$  such that  $z \in (P, A)$  and  $c \notin sS\mathfrak{b}^*cl(P, A)$ ;

(iii) For each  $z \in Z, \cap \{sS\mathfrak{b}^*cl(P, A) : (P, A) \in sS\mathfrak{b}^*OS(Z) \text{ and } z \in (P, A)\} = \{z\}$ .

**Proof.**

(i)  $\Rightarrow$  (ii) Let  $Z$  is soft strongly  $\mathfrak{b}^* - T_2$  space. Then for each  $z \neq c$  there exists disjoint soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$  and  $c \in (S, A)$ . Since  $(S, A)$  is soft strongly  $\mathfrak{b}^*$ -open, then  $(S, A)^c$  is soft strongly  $\mathfrak{b}^*$ -closed and  $(P, A) \subseteq (S, A)^c$ . This implies that  $sS\mathfrak{b}^*cl(P, A) \subseteq (S, A)^c$ . Since  $c \notin (S, A)^c$  and  $c \notin sS\mathfrak{b}^*cl(P, A)$ .

(ii)  $\implies$  (iii) If  $z \neq c$  there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(H, A)$  such that  $z \in (H, A)$  and  $c \notin sS\mathfrak{b}^*cl(P, A)$ . Therefore,  $c \notin \cap \{sS\mathfrak{b}^*cl(H, A): (H, A) \in sS\mathfrak{b}^*OS(Z) \text{ and } z \in (H, A)\}$ . Therefore,  $\cap \{sS\mathfrak{b}^*cl(H, A): (H, A) \in sS\mathfrak{b}^*OS(Z) \text{ and } z \in (H, A)\} = \{z\}$ .

(iii)  $\implies$  (i) Let  $z \neq c$  then  $c \notin \cap \{sS\mathfrak{b}^*cl(P, A): (P, A) \in sS\mathfrak{b}^*OS(Z) \text{ and } z \in (P, A)\}$ . This implies that there exists a soft strongly  $\mathfrak{b}^*$ -open set  $(P, A)$  such that  $z \in (P, A)$  and  $c \notin sS\mathfrak{b}^*cl(P, A)$ . Let  $(S, A) = (sS\mathfrak{b}^*cl(P, A))^c$ . Then  $(S, A)$  is soft strongly  $\mathfrak{b}^*$ -open and  $c \in (S, A)$ . Now  $(P, A) \cap (S, A) = (P, A) \cap (sS\mathfrak{b}^*cl(P, A))^c \subseteq (P, A) \cap (P, A)^c = \emptyset$ . Therefore,  $Z$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Definition 2.18:** Let  $(Z, T, A)$  be a soft topological space,  $(H, A)$  be a soft strongly  $\mathfrak{b}^*$ -closed set in  $Z$  and  $z \in Z$  such that  $z \notin (H, A)$ . If there exist soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$ ,  $(H, A) \subseteq (S, A)$  and  $(P, A) \cap (S, A) = \emptyset$ . Then  $(Z, T, A)$  is called a soft strongly  $\mathfrak{b}^*$ -regular space.

The following example shows that every soft strongly  $\mathfrak{b}^*$ -regular does not have to be soft strongly  $\mathfrak{b}^* - T_1$  space.

**Example 2.19:** Let  $Z = \{J_1, J_2, J_3\}$ ,  $A = \{e_1, e_2\}$  and  $\tau = \{\emptyset, Z, (P_1, A), (P_2, A)\}$  where  $(P_1, A) = \{(e_1, \{J_1\}), (e_2, \{J_1\})\}$ ,  $(P_2, A) = \{(e_1, \{J_2, J_3\}), (e_2, \{J_2, J_3\})\}$ . Then  $(Z, \tau, A)$  is a soft topological space over  $Z$ . Also,  $(Z, T, A)$  is a soft strongly  $\mathfrak{b}^*$ -regular space over  $Z$  which is not a soft strongly  $\mathfrak{b}^* - T_1$  space because  $J_1, J_2 \in Z$  but there do not exist soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$ , such that  $J_1 \in (P, A)$  and  $J_2 \notin (P, A)$ , and  $J_2 \in (S, A)$  and  $J_1 \notin (S, A)$ .

**Definition 2.20:** Let  $(Z, T, A)$  be a soft topological space over  $Z$ . Then  $(Z, T, A)$  is said to be soft strongly  $\mathfrak{b}^* - T_3$  space if it is soft strongly  $\mathfrak{b}^*$ -regular space and soft strongly  $\mathfrak{b}^* - T_1$  space.

**Theorem 2.21:** Every soft strongly  $\mathfrak{b}^* - T_3$  space is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Proof.** Suppose that  $(Z, T, A)$  be a soft strongly  $\mathfrak{b}^* - T_3$  space. Let  $z, c \in Z$  where  $z \neq c$ . Since  $Z$  is a soft strongly  $\mathfrak{b}^* - T_3$  space, hence  $\{c\}$  is soft strongly  $\mathfrak{b}^*$ -closed and  $z \notin \{c\}$ . Also,  $Z$  is soft strongly  $\mathfrak{b}^*$ -regular. Hence there exists a soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$  such that  $z \in (P, A)$ ,  $\{c\} \subseteq (S, A)$  and  $(P, A) \cap (S, A) = \emptyset$ . Thus  $z \in (P, A)$ ,  $c \in \{c\} \subseteq (S, A)$  and  $(P, A) \cap (S, A) = \emptyset$ . Therefore,  $(Z, T, A)$  be a soft strongly  $\mathfrak{b}^* - T_2$  space.

**Definition 2.22:** Let  $(Z, T, A)$  be a soft topological space over  $Z$ ,  $(P, A)$  and  $(S, A)$  be a soft strongly  $\mathfrak{b}^*$ -closed sets over  $Z$  such that  $(P, A) \cap (S, A) = \emptyset$ . If there exist soft strongly  $\mathfrak{b}^*$ -open sets  $(P_1, A)$  and  $(P_2, A)$  such that  $(P, A) \subset (P_1, A)$ ,  $(S, A) \subset (P_2, A)$  and  $(P_1, A) \cap (P_2, A) = \emptyset$ , then  $(Z, T, A)$  is called a soft strongly  $\mathfrak{b}^*$ -normal space.

The following example show that every soft strongly  $\mathfrak{b}^*$ -normal space does not have to be both soft strongly  $\mathfrak{b}^*$ -normal and soft strongly  $\mathfrak{b}^* - T_1$  space.

**Example 2.23:** Let  $Z = \{J_1, J_2, J_3\}$ ,  $A = \{e_1, e_2\}$  and  $T = \{\emptyset, Z, (P_1, A), (P_2, A), (P_3, A)\}$  where

$$(P_1, A) = \{(e_1, \{J_1\}), (e_2, \{J_1\})\},$$

$$(P_2, A) = \{(e_1, \{J_2\}), (e_2, \{J_2\})\},$$

$$(P_3, A) = \{(e_1, \{J_1, J_2\}), (e_2, \{J_1, J_2\})\}.$$

Then  $(Z, \mathcal{T}, A)$  is a soft topological space over  $Z$ . Moreover,  $(Z, \mathcal{T}, A)$  is a soft strongly  $\mathfrak{b}^*$ -normal space over  $Z$ , but neither soft strongly  $\mathfrak{b}^*$ -regular nor soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

**Definition 2.24:** Let  $(Z, \mathcal{T}, A)$  be a soft topological space over  $Z$ . Then  $(Z, \mathcal{T}, A)$  is said to be a soft strongly  $\mathfrak{b}^*$ - $T_4$  space if it is soft strongly  $\mathfrak{b}^*$ -normal and soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

**Example 2.25:** Let  $Z = \{J_1, J_2\}$ ,  $A = \{e_1, e_2\}$  and  $\mathcal{T} = \{\emptyset, Z, (P_1, A), (P_2, A)\}$  where  $(P_1, A) = \{(e_1, \{J_1\}), (e_2, \{J_1\})\}$ ,  
 $(P_2, A) = \{(e_1, \{J_2\}), (e_2, \{J_2\})\}$ .

Then  $(Z, \mathcal{T}, A)$  is a soft topological space over  $Z$ . Moreover,  $(Z, \mathcal{T}, A)$  is a soft strongly  $\mathfrak{b}^*$ - $T_4$ -space over  $Z$ .

**Theorem 2.26:** Every soft strongly  $\mathfrak{b}^*$ - $T_4$  space is soft strongly  $\mathfrak{b}^*$ - $T_3$  space.

**Proof.** Suppose that  $Z$  be a soft  $\mathfrak{b}^*$ - $T_4$  space. Since  $Z$  is also the soft strongly  $\mathfrak{b}^*$ - $T_1$  space, only soft strongly  $\mathfrak{b}^*$ -regular is enough to show that space. Let  $(H, A)$  be a soft strongly  $\mathfrak{b}^*$ -closed set in  $Z$  and  $z \notin (H, A)$ . Since  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space,  $\{z\}$  is soft strongly  $\mathfrak{b}^*$ -closed. Then  $(H, A) \cap \{z\} = \emptyset$ , and since  $Z$  is soft strongly  $\mathfrak{b}^*$ -regular, there exist soft strongly  $\mathfrak{b}^*$ -open sets  $(P_1, A)$  and  $(P_2, A)$  in  $Z$  such that  $\{z\} \subset (P_1, A)$  and  $(H, A) \subset (P_2, A)$  and  $(P_1, A) \cap (P_2, A) = \emptyset$ . Thus  $Z$  is soft strongly  $\mathfrak{b}^*$ -regular, and so soft strongly  $\mathfrak{b}^*$ - $T_3$  space.

**Theorem 2.27:** Let  $f: Z \rightarrow D$  be a soft strongly  $\mathfrak{b}^*$ -irresolute, injective map, if  $D$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space. Then  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

**Proof.** Assume that  $D$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space. Let  $z, c \in D$  such that  $z \neq c$ . Then there exists a pair of soft strongly  $\mathfrak{b}^*$ -open sets  $(P, A), (H, A)$  in  $D$  such that  $f(z) \in (P, A), f(c) \in (H, A)$  and  $f(z) \notin (H, A), f(c) \notin (P, A)$ . Then  $z \in f^{-1}((P, A)), c \notin f^{-1}((P, A))$  and  $c \in f^{-1}((H, A)), z \notin f^{-1}((H, A))$ . Since  $f$  is soft strongly  $\mathfrak{b}^*$ -irresolute,  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

**Theorem 2.28:** Let  $f: Z \rightarrow D$  be bijective. Then we have the following:

- (i) If  $f$  is soft strongly  $\mathfrak{b}^*$ -continuous and  $D$  is soft  $T_1$  space. Then  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space.
- (ii) If  $f$  is soft strongly  $\mathfrak{b}^*$ -open and  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space. Then  $D$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

**Proof.** Let  $f: Z \rightarrow D$  be bijective.

(i) Suppose  $f: Z \rightarrow D$  is soft strongly  $\mathfrak{b}^*$ -continuous and  $D$  is soft  $T_1$  space. Let  $z_1, z_2 \in Z$  with  $z_1 \neq z_2$ . Since  $f$  is bijective,  $y_1 = f(z_1) \neq f(z_2) = y_2$  for some  $y_1, y_2 \in D$ . Since  $D$  is soft  $T_1$  space, there exist soft open sets  $(P, A)$  and  $(S, A)$  such that  $y_1 \in (P, A)$  but  $y_2 \notin (P, A)$  and  $y_2 \in (S, A)$  but  $y_1 \notin (S, A)$ . Since  $f$  is bijective,  $z_1 = f^{-1}(y_1) \in f^{-1}((P, A))$  but  $z_2 = f^{-1}(y_2) \notin f^{-1}((P, A))$  and  $z_2 = f^{-1}(y_2) \in f^{-1}((S, A))$  but  $z_1 = f^{-1}(y_1) \notin f^{-1}((S, A))$ . Since  $f$  is soft strongly  $\mathfrak{b}^*$ -continuous,  $f^{-1}((P, A))$  and  $f^{-1}((S, A))$  are soft strongly  $\mathfrak{b}^*$ -open sets in  $Z$ . it follows that  $Z$  is a soft strongly  $\mathfrak{b}^*$ - $T_1$  space.

(ii) Suppose  $f$  is a soft strongly  $\mathfrak{b}^*$ -open and  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space. Let  $y_1 \neq y_2 \in D$ . Since  $f$  is bijective, there exist  $z_1, z_2$  in  $Z$ , such that  $f(z_1) = y_1$  and  $f(z_2) = y_2$  with  $z_1 \neq z_2$ . Since  $Z$  is soft strongly  $\mathfrak{b}^*$ - $T_1$  space, there exist soft  $\mathfrak{b}^*$ -open sets  $(P, A)$  and  $(S, A)$  such that  $z_1 \in (P, A)$  but  $z_2 \notin (P, A)$  and  $z_2 \in (S, A)$  but  $z_1 \notin (S, A)$ . Since  $f$  is soft strongly  $\mathfrak{b}^*$ -open.  $f((P, A))$  and  $f((S, A))$  are soft strongly  $\mathfrak{b}^*$ -open in  $D$  such that,



$y_1 = f(z_1) \in f((P, A))$  and  $y_2 = f(z_2) \in f((S, A))$ . Again since  $f$  is bijective,  $y_2 = f(z_2) \notin f((P, A))$  and  $y_1 = f(z_1) \notin f((S, A))$ . Thus  $D$  is a soft strongly  $\mathfrak{b}^* - T_1$  space.

**Theorem 2.29:** Let  $f: Z \rightarrow D$  be bijective. Then we have the following:

- (i) If  $f$  is soft strongly  $\mathfrak{b}^* -$ open and  $Z$  is soft  $T_2$  space. Then  $D$  is soft strongly  $\mathfrak{b}^* - T_2$  space.
- (ii) If  $f$  is soft strongly  $\mathfrak{b}^* -$ continuous and  $D$  is soft  $T_2$  space. Then  $Z$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Proof.** Let  $f: Z \rightarrow D$  be bijection.

(i) Suppose  $f$  is soft strongly  $\mathfrak{b}^* -$ open and  $Z$  is soft  $T_2$  space. Let  $c_1 \neq c_2 \in D$ . Since  $f$  is a bijection, there exist  $z_1, z_2$  in  $Z$ , such that  $f(z_1) = c_1$  and  $f(z_2) = c_2$  with  $z_1 \neq z_2$ . Since  $Z$  is soft  $T_2$  space, there exist disjoint soft open sets  $(H, A)$  and  $(M, A)$  in  $Z$  such that  $z_1 \in (H, A)$  and  $z_2 \in (M, A)$ . Since  $f$  is soft strongly  $\mathfrak{b}^* -$ open.  $f((H, A))$  and  $f((M, A))$  are soft strongly  $\mathfrak{b}^* -$ open in  $D$  such that  $c_1 = f(z_1) \in f((H, A))$  and  $c_2 = f(z_2) \in f((M, A))$ . Again since  $f$  is bijection,  $f((H, A))$  and  $f((M, A))$  are disjoint in  $D$ ,  $D$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

(ii) Suppose  $f: Z \rightarrow D$  is soft strongly  $\mathfrak{b}^* -$ continuous and  $D$  is soft  $T_2$  space. Let  $z_1, z_2 \in Z$  with  $z_1 \neq z_2$ . Let  $c_1 = f(z_1)$  and  $c_2 = f(z_2)$ . Since  $f$  is one -one,  $c_1 \neq c_2 \in D$ . Since  $D$  is soft  $T_2$  space, there exist disjoint soft open sets  $(H, A)$  and  $(M, A)$  containing  $c_1$  and  $c_2$  respectively. Since  $f$  is soft strongly  $\mathfrak{b}^* -$ continuous bijective,  $f^{-1}((H, A))$  and  $f^{-1}((M, A))$  are disjoint soft strongly  $\mathfrak{b}^* -$ open sets containing  $z_1$  and  $z_2$  respectively. Thus  $Z$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Theorem 2.30:** Let  $f: Z \rightarrow D$  be bijective, soft strongly  $\mathfrak{b}^* -$ irresolute map and  $Z$  is soft strongly  $\mathfrak{b}^* - T_2$  space. Then  $(Z, T_2, A)$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Proof.** Suppose  $f: (Z, T_1, A) \rightarrow (D, T_2, A)$  is bijective. And  $f$  is soft strongly  $\mathfrak{b}^* -$ irresolute, and  $(D, T_2, A)$  is soft strongly  $\mathfrak{b}^* - T_2$  space. Let  $z_1, z_2 \in Z$  with  $z_1 \neq z_2$ . Since  $f$  is bijective,  $c_1 = f(z_1) \neq f(z_2) = c_2$  for some  $c_1, c_2 \in D$ . Since  $(D, T_2, A)$  is soft strongly  $\mathfrak{b}^* - T_2$  space, there exist disjoint soft strongly  $\mathfrak{b}^* -$ open sets  $(H, A)$  and  $(L, A)$  such that  $c_1 \in (H, A)$  and  $c_2 \in (L, A)$ . Again since  $f$  is bijective,  $z_1 = f^{-1}(c_1) \in f^{-1}((H, A))$  but  $z_2 = f^{-1}(c_2) \notin f^{-1}((L, A))$ . Since  $f$  is soft strongly  $\mathfrak{b}^* -$ irresolute,  $f^{-1}((H, A))$  and  $f^{-1}((L, A))$  are soft strongly  $\mathfrak{b}^* -$ open sets in  $(Z, T_1, A)$ . Also,  $f$  is bijective,  $(H, A) \cap (L, A) = \emptyset$  implies that  $f^{-1}((H, A)) \cap f^{-1}((L, A)) = f^{-1}((H, A) \cap (L, A)) = f^{-1}(\emptyset) = \emptyset$ . It follows that  $(Z, T_2, A)$  is soft strongly  $\mathfrak{b}^* - T_2$  space.

**Theorem 2.31:** If  $f: Z \rightarrow D$  is soft continuous, soft strongly  $\mathfrak{b}^* -$ closed map from soft normal space  $Z$  onto a space  $D$ , then  $D$  is a soft normal.

**Proof.** Let  $(S, A)$  and  $(M, A)$  be a disjoint soft closed set of  $D$ . Then  $f^{-1}(S, A), f^{-1}(M, A)$  are disjoint soft closed sets of  $Z$ . since  $Z$  is normal, there are disjoint soft open sets  $(N, A), (P, A)$  in  $Z$  such that  $f^{-1}(S, A) \subset (N, A)$  and  $f^{-1}(M, A) \subset (P, A)$ . Since  $f$  is soft strongly  $\mathfrak{b}^* -$ closed, there are soft open sets  $(G, A), (W, A)$  in  $D$  such that  $(S, A) \subset (G, A), (P, A) \subset (W, A)$  and  $f^{-1}(G, A) \subset (N, A)$  and  $f^{-1}(W, A) \subset (P, A)$ . Since  $(N, A), (P, A)$  are disjoint,  $int(G, A), int(W, A)$  are disjoint soft open sets. Since  $(G, A)$  is soft strongly  $\mathfrak{b}^* -$ open,  $(S, A)$  is soft closed and  $(S, A) \subset (G, A)$ .  $(S, A) \subset cl(int((G, A)))$ . Similarly,  $(M, A) \subset cl(int((W, A)))$ . Hence,  $D$  is a soft normal.

**Theorem 2.32:** If  $f: Z \rightarrow D$  is a soft open, soft strongly  $\mathfrak{b}^* -$ closed surjection map, soft continuous, where  $Z$  is soft regular then  $D$  is a soft regular.

**Proof.** Let  $(H, A)$  be a soft open set of  $D$  and  $c \in (H, A)$ . Since  $f$  is surjection there exists a point  $z \in Z$  such that  $f(z) = c$ . Since  $Z$  is soft regular and  $f$  is soft continuous, there is a soft open set  $(S, A)$  in  $Z$  such that  $z \in (S, A) \subset cl(S, A) \subset f^{-1}(H, A)$ . Here  $c \in f(S, A) \subset f(cl(S, A)) \subset (H, A)$ . Since  $f$  is a soft strongly  $\mathfrak{b}^*$ -closed set contained in the soft open set  $(H, A)$ . By hypothesis,  $cl(f(cl(S, A))) = f(cl(S, A))$  and  $cl(f((S, A))) = cl(f(cl(S, A)))$ . Therefore,  $c \in f((S, A)) \subset cl(f((S, A))) \subset (H, A)$  and  $f((S, A))$  is a soft open, since  $f$  is a soft open. Hence,  $D$  is a soft regular.

### 3. Conclusions

The separation axioms are various conditions that are sometimes imposed upon topological spaces which can be described in terms of the various types of separated sets. In the paper, we introduce the notion of new class of separation axiom is called soft strongly  $\mathfrak{b}^*$ -separation axiom. We study the properties of the soft strongly  $\mathfrak{b}^*$ -regular and soft strongly  $\mathfrak{b}^*$ -normal spaces. Also, some of the properties and relationships with other types of soft separation axiom are studied.

### References

- [1] D. Molodtsov, "Soft set theory-first results," *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, 1999.
- [2] K. Kannan, "Soft generalized closed sets in soft topological spaces," *Journal of Theoretical and Appl. Inform. Technology*, vol. 37, no. 1, p. 17 – 21, 2012.
- [3] Ekram, S. T., & Majeed and R. N., "Soft Continuous Mappings in Soft Closure Spaces," *Iraqi Journal of Science*, vol. 62, no. 8, p. 2676–2684, 2021.
- [4] I. Arockiarani and A. Arokialancy, "Generalized soft  $g\beta$ -closed sets and soft  $gs\beta$ -closed sets in soft topological spaces," *International Journal of Mathematical Archive*, vol. 4, no. 2, pp. 1-7, 2013.
- [5] M. Akdag and A. Ozkan, "Soft  $\alpha$ -open sets and soft  $\alpha$ -continuous functions," *Abstr. Anal. Appl.*, pp. 1-7, 2014.
- [6] M. Akdag and A. Ozkan, "On Soft  $\alpha$ -separation axioms," *journal of advanced studies in topology*, vol. 5, no. 4, pp. 16-27, 2014.
- [7] M. Akdag, A. Ozkan and A., "Soft  $b$ -open sets and soft  $b$ -continuous functions," *Math Sci*, vol. 8, no. 127, pp. 1-9, 2014.
- [8] Das, S., & Tripathy and B. C., "Neutrosophic Simply  $b$ -Open Set in Neutrosophic Topological Spaces," *Iraqi Journal of Science*, vol. 62, no. 12, p. 4830–4838, 2021.
- [9] El-Sheikh, S.A., Hosny, R.A., Abd-e-Latif and A.M., "Characterization of  $b$  soft separation Axioms in soft topological spaces," *Information Sciences Letters*, vol. 4, no. 3, pp. 125-133, 2015.
- [10] Majeed, R. N., & El-Sheikh and S. A., "Separation Axioms in Topological Ordered Spaces Via  $b$ -open Sets," *Iraqi Journal of Science*, vol. 62, no. 8, p. 2685–2693, 2021.
- [11] A. Poongothai and R. Parimelazhagan, " $s\mathfrak{b}^*$ -Separation axioms," *Int. Journal of Math. and soft computing*, vol. 5, no. 2, p. 155 – 164, 2015.
- [12] Hameed, S. Z. and A. K. & Hussein, "On Soft  $\mathfrak{b}c$ -open Sets in Soft Topological Spaces," *Iraqi Journal of Science*, pp. 238-242, 2020.
- [13] Hameed, S. Z., F. A. Ibrahim and Essam A. El-Seidy, "On soft  $\mathfrak{b}^*$ -closed sets in soft topological space," *International Journal of Nonlinear Analysis and Applications*, vol. 12, no. 1, pp. 1235-1242, 2021.
- [14] Hameed, S. Z., A. E. Radwan and Essam A. El-Seidy, "On soft  $\mathfrak{b}^*$ -continuous functions in soft topological spaces," *Measurement: Sensors*, vol. 27, pp. 1-5, 2023.
- [15] Hameed, Saif Z., A. E. Radwan and Essam A. El-Seidy, "On soft strongly  $\mathfrak{b}^*$ -closed sets and soft

- strongly  $\mathfrak{L}^*$  – continuous functions in soft topological spaces," *Under the Publication*, 2023.
- [16] Maji, P.K., Biswas, R., Roy and A.R., "Soft set theory," *Comput. Math. Appl.*, vol. 45, p. 555–562, 2003.
- [17] Shabir, M., Naz and M., "On soft topological spaces," *Comput. Math. Appl.*, vol. 61, p. 1786–1799, 2011.
- [18] I. Zorlutuna, M. Akdag, W. K. Min and a. S. Atmaca, "Remarks on soft topological spaces," *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 171-185, 2012.
- [19] B. Chen, "Soft Semi-open sets and related properties in soft topological spaces," *Applied Mathematics and Information Sciences*, vol. 7, no. 1, pp. 287-294, 2013.
- [20] N. R. Paul, "Remarks on soft  $\omega$  – closed sets in soft topological spaces," *Boletim da Sociedade Paranaense de Matemática*, vol. 33, no. 1, pp. 183-192, 2015.