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Studying the Effect of the Instantaneous and Continuous Pollutants Sources on the Advection-diffusion Equation and Its Applications

Khaled S. M. Essa [1], Sawsan I. M. El Saied [1], A. A. Wheida [2], Mostafa El-Nazer [2]*

[1] *Mathematical and Theoretical Physics Department, Nuclear Research Centre, Egyptian Atomic Energy Authority, Cairo, Egypt*

[2] *Theoretical Physics Department, Physics Research Institute, National Research Centre, Cairo, Egypt*

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Abstract

This study investigates instantaneous and continuous sources as point, line, and area sources. Gaussian concentration in the case of the puff model with an instantaneous point source inhomogeneous longitudinal diffusion is investigated. The concentration is calculated using different dispersion parameters to get the proposed normalized concentration of the puff model at ground level around the centerline, which is compared with observed data by the Copenhagen experiment and previous work [1].

Also, the continuous point source is used to get the Gaussian plume model in three dimensions using dispersion parameters to compare with the observed concentration data measured by the Egyptian Atomic Energy Authority for Iodine-135 (I135) in an unstable condition.

Keywords: Instantaneous sources, Continuous sources, Advection equation, Diffusion equation, Gaussian model, Dispersion.

دراسة تأثير مصادر الملوثات اللحظية والمستمرة على معادلة التأفق والانتشار وتطبيقاتها

خالد صادق محمد عيسى[1], سوسن إبراهيم محمد [1], علي أحمد عبد الحميد وحيدة [2]، مصطفى محمود الناظر [2]*

[1] قسم الفيزياء الرياضية والنظرية، مركز البحوث النووية، هيئة الطاقة الذرية المصرية، القاهرة، مصر

[2] قسم الفيزياء النظرية ، معهد البحوث الفيزيقية، المركز القومي للبحوث، القاهرة، مصر

الخلاصة

بحثت هذه الدراسة في المصادر الفورية والمستمرة كمصادر نقطية وخطية ومنطقة. تم التحقيق من التركيز الغاوسي في حالة نموذج Puff مع مصدر نقطي لحظي الانتشار طولي غير متجانس. تم حساب التركيز باستخدام معلمات تشتت مختلفة للحصول على التركيز الطبيعي المقترح لنموذج puff على مستوى الأرض حول خط المنتصف والذي تتم مقارنته بالبيانات المرصودة في تجربة كوبنهاغن والعمل السابق [1].

أيضًا ، تم استخدام مصدر نقطي مستمر للحصول على نموذج عمود غاوسي في ثلاثة أبعاد باستخدام معاملات التشتت للمقارنة مع بيانات التركيز الملحوظة التي تم قياسها في هيئة الطاقة الذرية المصرية لليود 135 (I135) في حالة غير مستقرة.

* Email: mostafa.elnazer@gmail.com

1. Introduction

A suggested puff release scenario assumes that the sampling and release times are extremely short compared to the travel time starting from the source and ending at the receptor. Various versions of Gaussian models essentially differ in their techniques that are used to calculate the value of sigma as a function of the atmospheric stability and the downwind distance directly from the emission source [2]. One of the most popular Gaussian models is the AERMOD model [3], and among puff models, the CALPUFF model [4] has to be outlined. Among the non-Gaussian models, we only outline the puff model proposed by Van-Ulden [5].

The classic Gaussian-diffusion models are mostly used in affecting the impacts of finding and the proposed sources of air contaminants on local and urban air quality [6]. Homeliness, associated with the Gaussian analytical model, makes this approach particularly suitable for organizational usage in the mathematical modeling of air pollution. Indeed, such models are very useful in short-range forecasting. The horizontal and vertical dispersion parameters, respectively σy and σz that represent the turbulent parameterization key in this approach, once they contain the physical ingredients that describe the dispersion process and, consequently, express the spatial extent of the contaminant plume under the effect of the turbulent motion in the Planetary Boundary Layer (PBL) [7]. The solution was presented by Essa [8] for the advection-diffusion with variable vertical eddy-diffusivity and wind-speed parameters using the Hankel-transform to get the integrated cross-wind concentration.

This work introduced the instantaneous and continuous sources as (I) point, (II) line, and (III) area sources. Gaussian concentration in the case of the puff model with an instantaneous point source inhomogeneous longitudinal diffusion was investigated using different dispersion parameters to get the proposed normalized concentration of the puff model at ground level around the centerline, which was compared with observed data at the Copenhagen experiment and previous work from Lidiane [1].

Also, the Gaussian plume model in three dimensions using dispersion parameters from a continuous point source was used to compare with the observed concentration data, which was measured by the Egyptian Atomic Energy Authority for Iodine-135 (I135) in an unstable condition.

2. Abbreviations and Acronyms

I135 : Iodine-135
AERMOD : American Meteorological Society/Environmental Protection Agency Regulatory Model, used in Air Quality Dispersion Modeling
CALPUFF : California Puff Model
PBL : Planetary Boundary Layer
FB : Fraction-Bias
NMSE : Normalized Mean-Square-Error
COR : Correlation-Coefficient
FAC2 : Factor of Two

3. Methodology:

3.1. Advection-Diffusion Equation:

Consider a passive contaminant in an infinite, homogeneous medium that moves at a constant uniform velocity u in the x-direction, then the advection-diffusion equation is written as follows:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = K\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right) \quad (1)$$

Where, C is the pollutant concentration (g/m^3) or (Bq/m^3), u is the wind-speed (m/s), and K is the eddy-diffusivity (m^2/s).

This equation has solutions for different sources and boundary conditions [9]. The slighter condition of vanishing concentration at huge distances from the source is determined.

3.1.1. Instantaneous Point Source:

In the case of being at rest or moving at uniform velocity u, it is more suitable to see the expanding puff as a function of time in a reference frame moving with uniform velocity, taking u=0, and the diffusion equation (1) is simplified to:

$$\frac{\partial C}{\partial t} = K\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{2}$$

The solution of the diffusion equation (2) satisfying the boundary conditions and the integral mass continuity is as follows:

$$C(x, y, z, t) = \frac{Q_{ip}}{8(\pi K t)^{3/2}} exp\left(-\frac{x^2+y^2+z^2}{4Kt}\right) \tag{3}$$

Where Q_{ip} is an instantaneous point release rate, while x, y, and z are Cartesian coordinate systems.

The second moment of this distribution in σ^2 any direction equals:

$$\sigma^2 = 2Kt \tag{4}$$

This is a three-dimensional Gaussian distribution,

$$C(x, y, z, t) = \frac{Q_{ip}}{(2\pi)^{3/2}\ \sigma_x\sigma_y\sigma_z} exp\left(-\frac{x^2}{2\sigma_x^2}\right) exp\left(-\frac{y^2}{2\sigma_y^2}\right) exp\left(-\frac{z^2}{2\sigma_z^2}\right) \tag{5}$$

Where, $\sigma \propto t^{1/2}$, is the diffusion parameter that is used to measure the puff size.

For an instantaneous point source at (x', y', z') at time t', one can be obtained through a coordinate transformation as follows:

$$C(x, y, z, t) = \frac{Q_{ip}}{(2\pi)^{3/2}\ \sigma_x\sigma_y\sigma_z} exp\left(-\frac{(x-x_0)^2}{2\sigma_x^2}\right) exp\left(-\frac{(y-y_0)^2}{2\sigma_y^2}\right) exp\left(-\frac{(z-z_0)^2}{2\sigma_z^2}\right) \tag{6}$$

Where,

$$\sigma^2 = 2K(t - t') \tag{7}$$

3.1.2. Instantaneous Line Source:

The solution of the diffusion equation for an instantaneous line source of strength Q_{il} with dimension mass per unit length is obtained by integrated equation (5) concerning y from -∞ to ∞, where we get:

$$C(x, z, t) = \frac{Q_{il}}{2\pi\sigma^2} exp\left(-\frac{x^2+z^2}{2\sigma^2}\right) \tag{8}$$

Which is crosswind integrated concentration. Indeed, equation (8) is the solution of the diffusion equation in two-dimensions in the form:

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{9}$$

Where the mass continuity equals:

$$\iint_{-\infty}^{\infty} C dx dz = Q_{il} \tag{10}$$

3.1.3. Instantaneous Area Source:

We can obtain the solution of the diffusion equation for an area source of strength Q_{ia} in the form:

$$C(t,z) = \frac{Q_{ia}}{\sqrt{2\pi}\sigma} exp\left(-\frac{z^2}{2\sigma^2}\right) \tag{11}$$

It is the Gaussian equation in the z-direction. Equation (11) is in the direction normal to the source plane, and it is also the exact solution of the one-dimensional diffusion equation.

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial z^2}\right) \tag{12}$$

Which satisfies the mass conservation:

$$\int_{-\infty}^{\infty} C dz = Q_{ia} \tag{13}$$

We consider an instantaneous area source in the x-y plane, with an initial thickness "d" in the vertical and a uniform initial concentration "C_0" at t=0 of infinitesimal thickness "dz'", which, has a concentrated source strength of $Q_{ia}=C_0 dz'$. Integrating the elementary area source solution over a finite initial source is as follows:

$$C(z,t) = \frac{C_0}{2}\left[erf\left(\frac{0.5d+z}{\sqrt{2}\,\sigma}\right) + erf\left(\frac{0.5d-z}{\sqrt{2}\,\sigma}\right)\right] \tag{14}$$

Where the error function is defined as:

$$\mathrm{erf}(x) = \int_0^x exp(-x'^2)dx' \tag{15}$$

3.1.4. Continuous Point Source:

Considering a continuous point source has a fixed emission rate Q for a long enough time that has a steady-state diffusion equation for a point source in the uniform wind in the x-direction in the form:

$$u\frac{\partial C}{\partial x} = K\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{16}$$

Equation (16) is solved under the boundary condition:

$$C \to 0 \; as \; x, y, z \to \pm\infty \tag{17}$$

Then, the solution of the equation (16) will be in the form of:

$$C(x,y,z) = \frac{Q}{2\pi u\sigma^2} exp\left(-\frac{x-(x^2+y^2+z^2)}{2\sigma^2}\right) \tag{18}$$

After substituting equation (18) becomes:

$$C(x,y,z)=\frac{Q}{2\pi u\sigma_y\sigma_z}e^{\frac{-y^2}{2\sigma_y^2}}[e^{\frac{-(z-H)^2}{2\sigma_z^2}}+e^{\frac{-(z+H)^2}{2\sigma_z^2}}]e^{-\frac{\upsilon x}{u}} \quad (19)$$

Where; H is the effective height, $ree^{-\upsilon x/u}$ is the radioactive-decay of the isotope, $\upsilon=2.9\text{x}10^{-5}\ s^{-1,}$ and the diffusion parameter equals:

$$\sigma=\left(\frac{2Kx}{u}\right)^{1/2} \quad (20)$$

3.1.5. Continuous Cross-wind Line Source:

The approximate diffusion equation, where ignoring diffusion in the x-direction is in the form:

$$\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial z^2}\right) \quad (21)$$

The solution above is in the form:

$$C(x,z)=\frac{Q_{cl}}{\sqrt{2\pi}\,u\sigma}exp\left(-\frac{z^2}{2\sigma^2}\right) \quad (22)$$

3.1.6. Continuous Area Source:

The relevant diffusion equation is as follows:

$$\frac{\partial C}{\partial t}=D\left(\frac{\partial^2 C}{\partial z^2}\right) \quad (23)$$

Where; the mass conservation condition is:

$$\int_{-\infty}^{\infty}Cdz=Q_{ca}t \quad (24)$$

The solution of equation (23) for an area source is given by Sutton [9]:

$$C(z,t)=\frac{Q_{ca}}{D}\left[\left(\frac{Dt}{u}\right)^{1/2}exp\left(-\frac{z^2}{4Dt}\right)-\frac{z}{2}\left\{1-erf\left(\frac{z}{\sqrt{4Dt}}\right)\right\}\right] \quad (25)$$

4. The Experiment:

4.1. *Gaussian Instantaneous Puff Model:*

The final concentration field of equation (6) is given as a super-position of all puffs concentration distributions as follows:

$$\frac{C(x,y,z)}{Q}=\frac{\Delta t}{(2\pi)^{\frac{3}{2}}}\sum_{k=1}^{n}\frac{1}{\sigma_{xk}\sigma_{yk}\sigma_{zk}}\ exp\left(-\frac{(x_k-x_0)^2}{2\sigma_{xk}{}^2}-\frac{(y_k-y_0)^2}{2\sigma_{yk}{}^2}-\frac{(z_k-z_0)^2}{2\sigma_{zk}{}^2}\right) \quad (26)$$

Where; the $Q\Delta t$ is the source term, (x_k, y_k, z_k) is the position of the k^{th} puff, n is the number of puffs, and σ_{xk}, σ_{yk}, and σ_{zk} are a deviation of the Gaussian distribution inside the k^{th} puff in the x, y, z directions respectively, differing from the puff models, where they represent a sum of many calculated components. While each puff is defined as:

Where; $Q\Delta t$ is the source term, n is the number of puffs, (x_k, y_k, z_k) is the position of the k^{th} puff and σ_{xk}, σ_{yk} and σ_{zk} are a deviation of the Gaussian distribution inside the k^{th} puff in the x, y, and z directional respectively, with the difference that at the puff models, a sum of many components is calculated. While each puff is defined as:

$$x_0 = \bar{u}\Delta t; \quad y_0 = \bar{v}\Delta t; \quad and \quad z_0 = \bar{w}\Delta t$$

The puffs are emitted in time intervals Δt_1= 600s, and the calculation of the concentration of pollutants is made with a time resolution Δt_2= 60s.
The total concentration of a pollutant at a point in space is given by the sum of all puffs namely [10]:

$$\mathrm{C_T(x, y, z, t)} = \sum_{\mathrm{puffs}}^{\mathrm{total\ of\ puffs}} \Delta \mathrm{M_{puff}} \left\{ \int_{\mathrm{t}=0}^{\infty} \mathrm{c_{puff}(x, y, z, t) H(t - t_0)} \right\} \quad (27)$$

Where, H is the Heaviside-function, that H(t-t_0) =0, if (t-t_0) <0, and H(t-t_0) =1, if (t-t_0) ≥0, and:

$$\mathrm{c_{puff}(x, y, z, t) = c_1(x, t) c_2(y, t) c_3(z, t)} \quad (28)$$

Where; c_1, c_2, and c_3 presented in the Gaussian models are given as follows:

$$\mathrm{c_1} = \frac{1}{(\sqrt{2\pi})\sigma_x} \exp\left(-\frac{1}{2}\left(\frac{x - x_0}{\sigma_x}\right)^2\right) \quad (29)$$

$$\mathrm{c_2} = \frac{1}{(\sqrt{2\pi})\sigma_y} \exp\left(-\frac{1}{2}\left(\frac{y - y_0}{\sigma_y}\right)^2\right) \quad (30)$$

$$\mathrm{c_3} = \frac{1}{(\sqrt{2\pi})\sigma_z} \exp\left(-\frac{1}{2}\left(\frac{z - z_0}{\sigma_z}\right)^2\right) \quad (31)$$

The following generalized algebraic expression for the dispersion-parameters is taken from [1] as follows:

$$\sigma_x = z_i \sqrt{\frac{1.06 c_u \psi^{2/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_u^{-2/3} X^2}{1 + \frac{2\sqrt{1.06 c_u}}{\gamma} \left[\psi^{1/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_u^{2/3} X \right]}}$$

$$\sigma_y = z_i \sqrt{\frac{1.06 c_v \psi^{2/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_v^{-2/3} X^2}{1 + \frac{2\sqrt{1.06 c_v}}{\gamma} \left[\psi^{1/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_v^{2/3} X \right]}}$$

$$\sigma_z = z_i \sqrt{\frac{1.06 c_w \psi^{2/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_w^{-2/3} X^2}{1 + \frac{2\sqrt{1.06 c_w}}{\gamma} \left[\psi^{1/3} \left({}^{z}\!/_{z_i} \right)^{2/3} (f_m^*)_w^{2/3} X \right]}}$$

Where; α= x, y, z are the three components in horizontal, lateral, and vertical directions, and i = u, v, and w are the three components of the velocity in the three directions respectively. $X = \frac{x w_*}{U z_i}$ is a non-dimension distance, defined by the ratio of travel-time (x/U) to the convective-time scale (z_i/w_*), $c_i = \alpha_i \{0.5 \pm 0.05\} (2\pi k)^{-2/3}$ and $\alpha_i = 1.\frac{4}{3}.\frac{4}{3}$, for u, v, and w components respectively [11]. k=0.4 is von Karman constant. ψ=0.65 [12], $(f_m^*)_w$= $\left({}^{z}\!/_{z_i} \right)$, $\gamma = \frac{\sqrt{\pi}}{4}$ [1] and $(f_m^*)_v$=${}^{z}\!/_{1.5 z_i}$ [13].

4.2. *Gaussian Model from a Continuous Point Source:*

The effective height has form as follows:

$$H = h_s + \Delta h = h_s + \frac{3wD}{u} \tag{32}$$

Where, w is the pollutants exit velocity, and D is the diameter of the internal stack.
Also, the crosswind and vertical dispersion parameters for the convective condition are taken from Lidiane [1] in the form:

$$\frac{\sigma_y^2}{h^2} = \frac{0.66}{\pi^2}\int_0^\infty \frac{\sin^2\left(0.75\pi\Psi^{\frac{1}{3}}\, X\acute{n}\right)}{\acute{n}^2(1+\acute{n})^{\frac{5}{3}}}\,d\acute{n} \tag{33}$$

$$\frac{\sigma_z^2}{h^2} = \frac{0.98}{\pi^2}\int_0^\infty \frac{\sin^2\left(0.98\pi\Psi^{\frac{1}{3}}\, X\acute{n}\right)}{\acute{n}^2(1+\acute{n})^{\frac{5}{3}}}\,d\acute{n} \tag{34}$$

Where, $\acute{n} = \frac{1.5z}{u\,(f_m^*)_i}n$; $(f_m^*)_i$ is the reduced frequency of the convective spectral peak in the form $(f_m^*)_i = \frac{z}{h}$.

5. Results:

5.1. *Experimental Data (Puff Model):*

The used data were observed by Lidiane [1], and we have evaluated the performance of the algebraic/integral parameterization for x, y, and z dispersion parameters, by applying the Gaussian puff plume model to the Copenhagen-experimental tracer for hexafluoride SF6 concentration data set. For this, a comparison is done using the observed ground-level centerline normalized concentration with the source emission rate [14]. The resulting data were obtained from Essa [15]. The predicted-normalized puff model ground-level centerline concentration, the observed normalized concentration, and previous work are shown in Table (1) as follows:

Table 1: Shows the comparison between observed and predicted ground-level centerline concentration models under unstable conditions and downwind distance

Run	Distance (m)	Concentration /Q (10-7sm-3)		
		Observed	Previous work (2008)	Predicted
1	1900	10.5	5.34	6.67
1	3700	2.14	2.17	2.03
2	2100	9.85	7.67	6.85
2	4200	2.83	2.93	2.24
3	1900	16.33	13.74	15.33
3	3700	7.95	5.95	5.14
3	5400	3.76	3.72	6.67
4	4000	15.71	17.51	16.59
5	2100	12.11	20.94	14.73
5	4200	7.24	11.49	4.23
5	6100	4.75	7.52	3.44
6	2000	7.44	8.02	5.22
6	4200	3.37	3.24	4.44
6	5900	1.74	2.07	2.62
7	2000	9.48	5.55	5.15

7	4100	2.62	2.03	3.41
7	5300	1.15	1.44	2.56
8	1900	9.76	8.43	7.66
8	3600	2.64	4.06	3.55
8	5300	0.98	2.59	2.04
9	2100	8.52	6.86	4.52
9	4200	2.66	2.55	2.33
9	6000	1.98	1.53	0.98

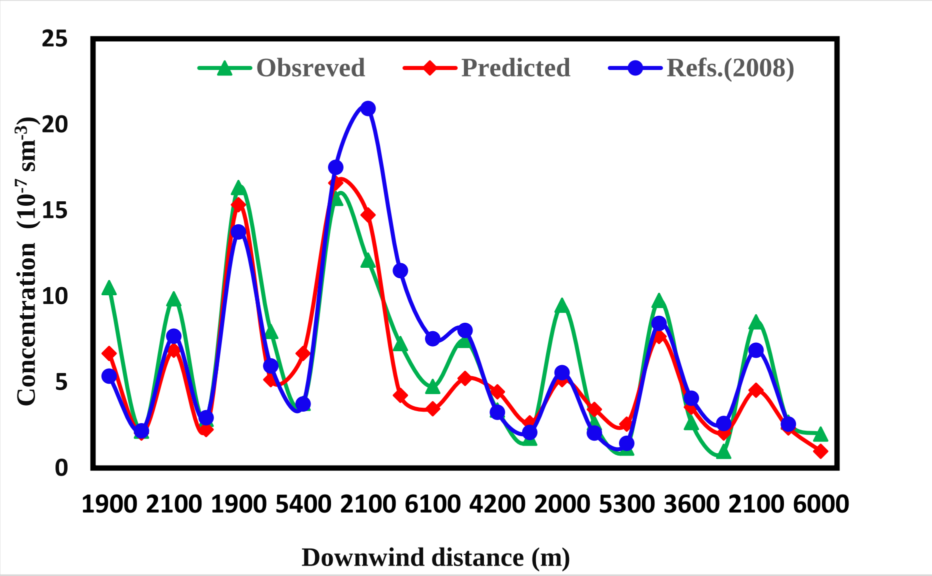

Figure 1: The variation of normalized observed and predicted puff model concentrations via downwind distance.

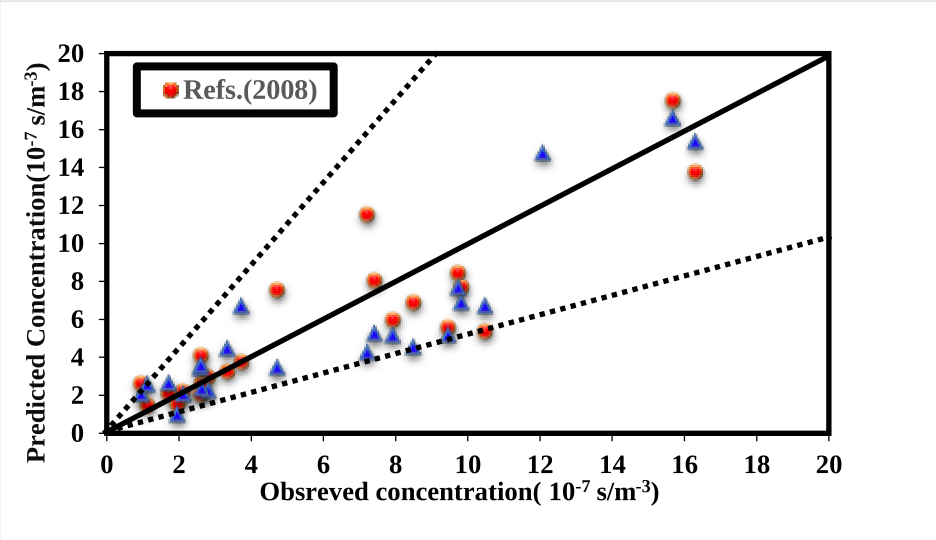

Figure 2: Observed and predicted normalized ground-level centerline concentration

The puff-predicted normalized ground-level centerline concentration is in good agreement with the observed normalized ground-level centerline concentration than the previous work [1], as shown in Figure (1). Also, the puff-predicted normalized ground-level centerline concentration and previous work [1], which are inside a factor of two with observed normalized ground-level centerline concentration, as shown in Figure (2)

5.1.1. Statistical Technique:

σ_p and σ_o are the standard deviations of the predicted (C_p = Cpred/Q), and the observed (Co = Cobs/Q) concentrations, respectively. The overbar indicates the average value. The perfect model must have the following performances:
NMSE = FB = 0 and COR= FAC2 = 1.0.

Table 2: Statistical evaluation of the present puff model against the Copenhagen experiment

Copenhagen experiment	NMSE	FB	COR	FAC2
Puff model	0.14	0.12	0.89	1.03
Previous work (Liadiane et al. 2008)	0.19	-0.01	0.84	1.1

The statistics reveal a better agreement between puff predicted with observed normalized ground-level centerline concentrations by the Copenhagen experiment in unstable conditions than plume previous concentrations [1], as shown in Table (2).

5.2. Experimental Data (Continuous Point Source):

The observed data of I^{135} isotope concentrations were obtained from the dispersion calculations as experiments conducted in unstable air samples collected around the Egyptian Atomic Energy Authority. The vertical height is 0.7 meters above ground from a stack height of 43 meters, for twenty-four hours of working, where the air samples were collected for half an hour at a height of 0.7 meters with a roughness length of 0.6 cm. The observed concentration of the I^{135} isotope and the meteorological data during the experiments were taken from Essa [16] and presented in Table (3).

The concentrations predicted by equations (19, 32, 33, 34) below the plume's centerline are also presented in Table (4). A comparison between predicted and observed concentrations of radioactive I135 via downwind distance in unstable conditions at Inshas is shown in Figure (3). Also, the relationship between observed and predicted concentration data is shown in Figure (4).

Table 3: Meteorological data of the nine convective-test runs at the Inshas site in March and May 2006.

Run number	Working hours of the source	Release rate (Bq)	Wind-speed (m s-1)	Wind-direction (deg)	W* (ms-1)	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	2.27	A	600.85	**5**
2	49	1050000	4	278.7	3.05	A	801.13	**10**
3	1.5	42857.14	6	190.2	1.61	B	973	**5**
4	22	471428.6	4	197.9	1.23	C	888	**5**
5	23	492857.1	4	181.5	0.958	A	921	**2**
6	24	514285.7	4	347.3	1.3	D	443	**8.0**
7	28	1007143	4	330.8	1.51	C	1271	**7.5**
8	48.7	1043571	4	187.6	1.64	C	1842	**7.5**
9	**48.25**	**1033929**	**4**	**141.7**	**2.1**	**A**	**1642**	**5.0**

Table 4: Predicted, observed, and Gaussian concentrations of the nine-runs experiments

Test	Downwind distance (m)	Observed conc. (Bq/m3)	Gaussian conc. Eqns(19,32,33,34) (Bq/m3)
1	100	0.025	0.039975
2	98	0.037	0.03302
3	136	0.091	0.082803
4	135	0.197	0.166257
5	106	0.272	0.274148
6	186	0.188	0.107066
7	165	0.447	0.216606
8	154	0.123	0.151414
9	106	0.032	0.044721

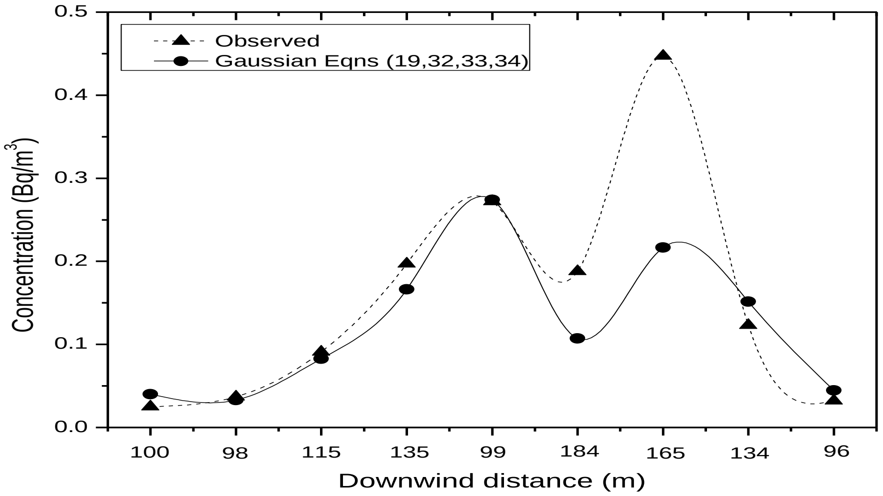

Figure 3: The relation between Gaussian and observed concentrations (Bq/m^3) via downwind distances.

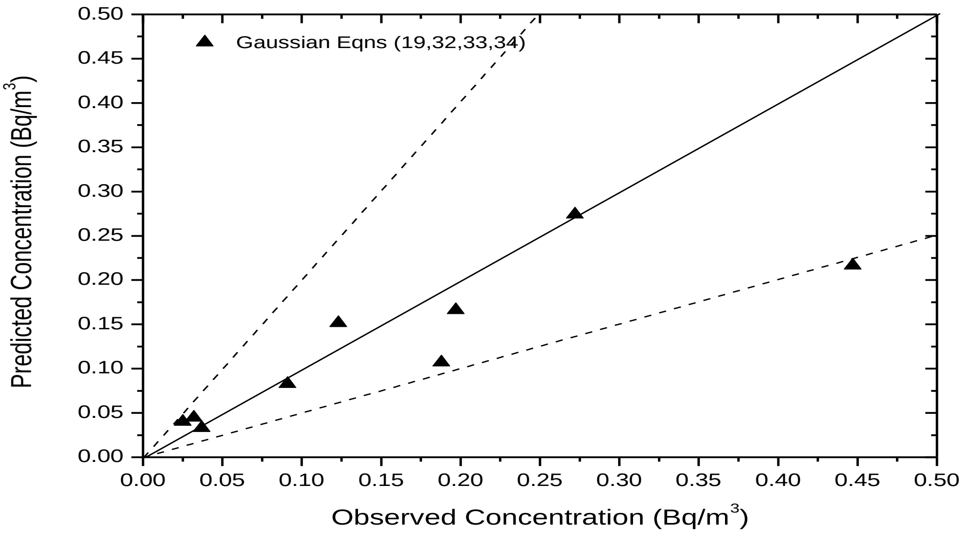

Figure 4: The relation between Gaussians with observed concentration.

From the two figures, we find that the Gaussian model (19, 32, 33, 34) is the best model, which gives a better result because of the strongest vertical dispersion. Also, the Gaussian lies a factor of two with the observed concentration.

5.2.1. Statistical Technique:

Comparing Gaussian predicted and observed concentrations are introduced by Hanna [17].

Table 5: The comparison between observed and Gaussian concentrations in an unstable-condition

	FB	NMSE	COR	FAC2
Gaussian Equations (19,32,33,34)	0.23	0.35	0.84	0.99

Where, FB is the Fraction-Bias, NMSE is the Normalized Mean-Square-Error, COR is the Correlation-Coefficient and FAC2 is the Factor of Two.

One can easily see from Table (5), that the statistical-technique shows that the proposed model is inside a factor of two with observed concentration data. Also, the statistics show that the Gaussian model (19,32,33,34) is in a good agreement with the observed concentration data for homogeneity. The Gaussian model achieved about 99% from observed concentration data.

6. Conclusions:

The puff predicted and previous data [1] lie inside a factor of two and the puff predicted data agrees with the observed concentration data, as the previous plume model [1]. The statistics' values reveal a good agreement between puff predicted with observed concentrations by the Copenhagen experiment in unstable conditions as the plume's previous work [1].

Also, we find that the Gaussian model (19, 32, 33, 34) gives the best result because of the strength of the vertical dispersion. The Gaussian model achieved about 99% from observed concentration data.

7. Statements on compliance with ethical standards and standards of research involving animals

"This article does not contain any studies involving animals performed by any of the authors."

8. Disclosure and conflict of interest

"Conflict of Interest: The authors declare that they have no conflicts of interest."

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