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Evaluating the Development of the Crescent Visibility Criteria

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Abstract

Predicting the visibility of the lunar crescent drew great attention during the Middle Ages, especially among Muslims, because the dates of religious practices in Islam and calendars, in addition to being aware of the motion of the moon in its orbit, are based on the lunar month.

This paper aims to study the development of crescent visibility criteria from ancient times through the medieval period to the modern update. Some basics and astronomical terms related to crescent visibility are presented and then a comprehensive view of crescent visibility criteria is examined by surveying about 54 published research papers related to the issue of crescent visibility. Each research paper is summarized, classified, and the results evaluated. Also, the previous studies are divided into two parts, including the ancient and medieval ages, which depend on the visual observations of the naked eye. We found that predicting the crescent's visibility started at the end of the Islamic era. The second part shows that at the beginning of the twentieth century, engineering criteria were introduced into research for the prediction of crescent visibility, then physical engineering criteria, weather conditions, and observer location on the Earth's surface were introduced. Currently, modern technologies have been introduced, such as CCD cameras and image processing technology. The study concludes that there are no 100% accurate criteria that correspond to the truth and completely match the observations, as these criteria need improvements by increasing the number of observations and introducing modern technologies to the crescent's visibility prediction.

Keywords: Crescent visibility, Lunar, Sun-moon elongation

تقييم تطور معايير رؤية الهلال

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الخلاصة:

ان توقع ظهور الهلال كان له اهتمام كبير في العصور الوسطى وخاصة المسلمين لان مواعيد الشعائر الدينية في الإسلام والتقويم – تستند إلى الشهر القمري، بالإضافة إلى معرفة حركة القمر في مداره.

يهدف هذا البحث الى دراسة تطور معايير رؤية الهلال بدءاً من العصور القديمة مرورا بالوسطى حتى وقتنا الحالي. تم تقديم بعض المصطلحات الفلكية المتعلقة برؤية الهلال ثم البدء بعملية عرض شامل لمعايير رؤية الهلال من خلال مراجعة حوالي (54) دراسة وورقة بحثية منشورة تتعلق بموضوع رؤية الهلال. تم تلخيص كل ورقة بحثية وتصنيفها وتقييم النتائج حيث تم تقسيم الدراسات الى قسمين الاولى دراسات العصور القديمة والوسطى والتي تعتمد على الارصادات البصرية بالعين المجردة حيث وجدنا في نهاية العصر الاسلامي بداية التنبؤ برؤية

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الهلال. اما القسم الثاني وفي بداية القرن العشرين تم ادخال المعايير الهندسية في الدراسات والبحوث للتنبؤ برؤية الهلال بعدها ادخلت المعايير الهندسية الفيزيائية والظروف الجوية وموقع الراصد على سطح الارض وصولا الى وقتنا الحالي حيث تم إدخال تقنيات حديثة مثل كاميرات (CCD) وتقنية معالجة الصور، وخلصت هذه الدراسة بأنه لا توجد معايير دقيقة بنسبة 100% تتوافق مع الارصاد الحقيقية حيث تحتاج هذه المعايير الى تحسينات من خلال زيادة عدد الارصادات وادخال التقنيات الحديثة للتنبؤ برؤية الهلال.

1. Introduction

Due to the importance of the crescent's lunar visibility being related to many calendars adopted by many different Islamic and non-Islamic civilizations during the past ages as a sign of the beginnings of the lunar months, astronomers throughout the ages have been interested in determining the conditions necessary to see the crescent with the naked eye after sunset, this resulted in the appearance of "lunar visibility criteria". The crescent means the minimum limits or astronomical conditions that must be realized in order for the observer to be able to see the crescent after sunset in good weather conditions [1][2][3]. This paper provides a comprehensive overview of the criterion of lunar crescent's visibility from ancient times to the present day and the most common observable variables that have been employed to predict the crescent's visibility as defined below:

- Age of moon (Age): It is the time amid (the conjunction) of the crescent and the period of observation (the moment after sunset), and it is measured in (hours).
- Lag time of the moon (Lag) or (Makth time): The period between sunset and moonset on the local horizon, it is measured in minutes.
- Altitude of the moon: The moon's angular separation above the local horizon, expressed in degrees.
- Moon arc of light (ARCL): The sun's and moon's angular separation (elongation) after sunset and measured in degrees.
- Moon arc of vision (ARCV): The Moon's and the sun's angular difference in altitude at the moment of sunset measured in degrees.
- The moon relative azimuth (DAZ): The angle of azimuth separation difference between the moon and the sun at the moment about sunset measured in degrees.
- Width of the crescent (W): The thickness for the illuminated region of the lunar in arcminutes relative to the desk of the moon measured in degrees.

The three fundamental variables are shown in (Figure 1) below:[2][3].

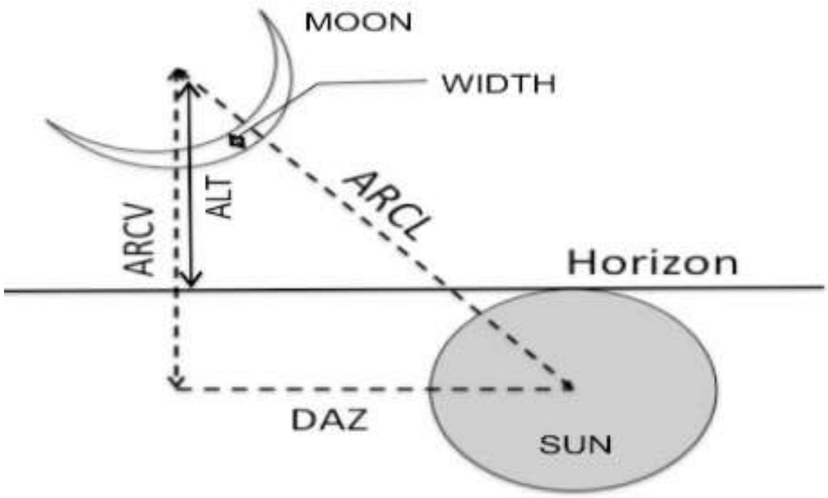

Figure 1: Basic geometric parameters for predicting the crescent's visibility [2][3]

2. Studies of the Ancient and Islamic Middle Ages

Based on extensive experience, Babylonian astronomers developed a straightforward criterion for the first sight of the lunar crescent that took into account the age of the moon and difference in time (Lag) between sunset and moonset. According to Ilyas [4] and Fatoohi [3] [5], the criterion used during the Babylonian age depended on observations and carried the requirement that the moon's age must be more than 24 hours from the conjunction of the moon and the sun to the local sunset time. The moon should be visible 48 minutes above the horizon after sunset.

Islamic astronomers began to create computational techniques in the ninth century including (the apparent angular separation of the sun and the moon (DAZ) and the interval between sunset and moonset, the lunar (Lag)). According to King [6], Muslim astronomers such as (Yaqub bin Tariq, AL farazi, AL Khwarizmi, and AL naziri) adhered to the Hindu rule (Lag). AL Khwarizmi's work on the sighting of the crescent (ninth century) and (Abdul Rahman AL Sufi, AL battani, AL biruni, Ghiyath AL din AL kashani) utilized the same amounts ARCL ≥ 12°.

Two centuries later, a renowned Egyptian astronomer named Ibn Yunus performed more thorough research on the visibility of the lunar crescent[6] and his theory on lunar visibility can be given ARCL ≥ 12° and Lag (48 minutes).

3. New criteria for the first observation of the lunar crescent

This astronomical issue was revived by Fotheringham's publication (1910) at the beginning of the 20th century after the huge interest of ancient and Islamic Middle Ages' astronomers in the appearance of the lunar crescent [7], and that criterion was not accurate because his study was statistical. After that, Fotheringham, Maunder, and Yallop [8], their observations and criteria are shown in Figure(2), set a geometric condition between the crescent, the sun, and the pace of observation, but this is not sufficient to solve the problem of the crescent's visibility. The comparison criteria between Fotheringham, Maunder, and Yallop are shown in Figure (2):

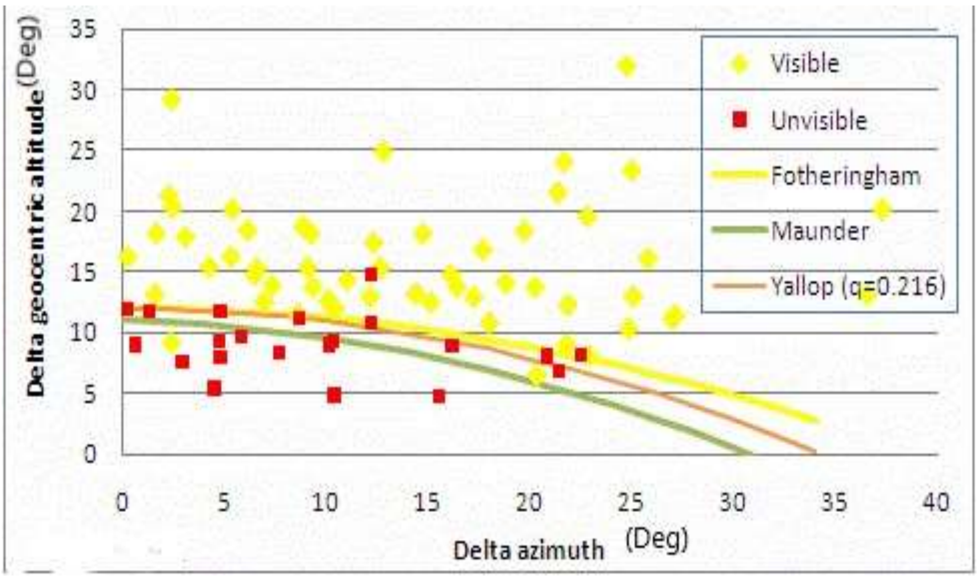

Figure 2: Observations used by Fotheringham, Maunder, and Yallop [7]

After that, Danjon, a French astronomer, reached a conclusion stating that the crescent is impossible to see with the naked eye when the moon elongation is less than 7 degrees, and he showed that the length of the crescent arc when the elongation is less than 7 degrees is zero, which he called the Danjon limit (ARCL) equation (1) [9][10]**.** The values above for the Danjon limit were later modified due to the entry of modern technology into observations.

$cos(ARCL) = cos(ARCV) * cos(DAZ)$ (1)[9]

After four decades, a new model based on an astrophysical view of the issue of crescent visibility was presented by Bruin [11]. This model depends on the ratio of the moon's and the sky's brightness as seen by an observer at a certain moment; that is, it takes into account (in an indirect and empirical way) the extent of the eye's perception and the phenomena of absorption and distortion that it performs. In the atmosphere, and by knowing the minimum value of the difference between the moon's brightness and the background sky that can be seen from the human eye, it is possible to predict the chance of the crescent's visibility or its impossibility, but this model still suffers from the limitation that it does not take into account the local visibility conditions that exist between one town and another. The Bruin model in Figure 3 shows that the behavior of ARCV decreased as crescent width increased.

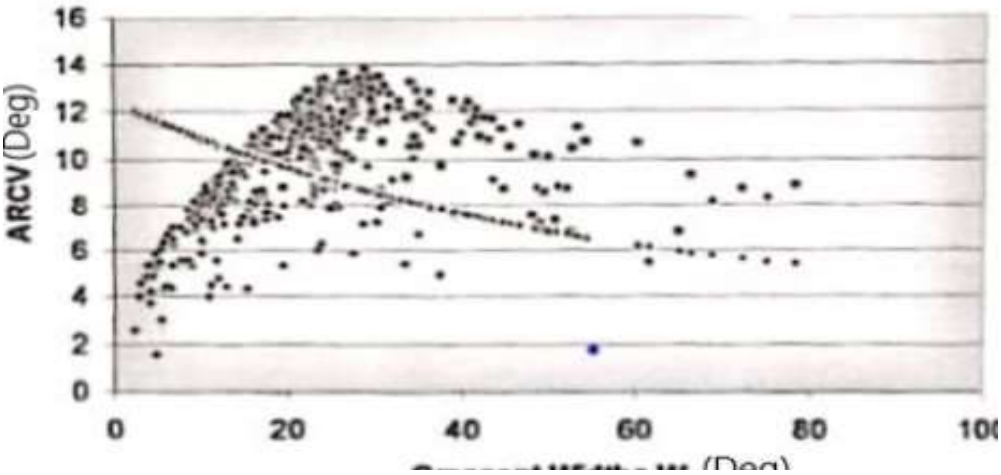

Figure 3: Bruin model [13]

During the 1980s, Ilyas research [12] [13] [4] [14]) [15] proposed a new model in which he tried to reconcile the geometric criteria (Maunder's model) and astrophysical criteria (Bruin model), where he addressed an issue from a general point of view with respect to the Earth and devised a criterion that determines the possibility of the first of the lunar month, a line for the lunar date (L.D.L). The two parameters (altitude and delta azimuth) criteria for (Fotheringham, Maunder, Yallop, Indian, Bruin, Caldweel, Schoch, and Neugebauer) are shown in Figure (4). Fotheringham and Maunder's curves are higher because their observations were old, while other curves are so close due to modern observations.

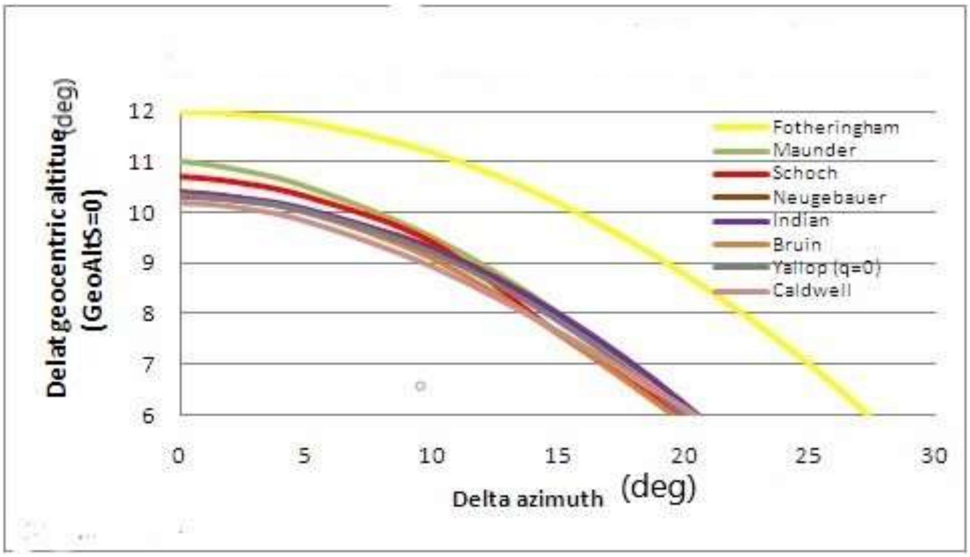

Figure 4: Numeric comparison of different criteria between (Fotheringham, Maunder, Yallop, Indian, Bruin, Caldweel, Schoch, and Neugebauer [13]

After that, Schaefer[16][17], Krisciunas and Schaefer [18], Schaefer, Ahmad and Doggett [19] Doggett and Schaefer [20] proposed a more ambitious model from Bruin and Ilyas, that incorporated atmospheric factors into the visibility conditions (temperature, humidity, and pollution), which change the probability of visibility, location factors (knowledge of latitude and longitude), altitude (relative to sea level), as well as astronomical data such as the sun's relative positions, the moon, and the Earth. Schaeffer made a qualitative transition by introducing the above factors that are very important in the probability of seeing the crescent and the need for further exploration of the physiology of human's vision capability.

In the nineties of the last century, Alnaimiy and Salih [21]set astronomical conditions for the visibility of the crescent that corresponded to the results of many observations, which are: the age of the crescent is ≥ 10 hours, its altitude above the horizon is ≥ 3 degrees, and the angular separation (elongation) from the sun is ≥ 5 degrees at sunset.

After that, among the criteria that are accurate is also the Yallop criterion[22], which he created based on correlations between the relative azimuth (DAZ), arc of vision (ARCV), and arc of light (ARCL). He examined three approaches from the 20th century: Maunder (1911), the Indian Astronomical Ephemeris (1996), and Bruin. From the database shown in the Tables (1,2,3) below, Yallop was able to conclude a new criterion in Figure (4) [17].

Table 1: The criterion of Maunder

DAZ	0°	5°	10°	15°	20°
ARCV	11°	10.5°	9.5°	8°	6°

Table 2: The Indian criterion

DAZ	0°	5°	10°	15°	20°
ARCV	10.4°	10 °	9.3°	8°	6.2°

Table 3: The criterion of Bruin

W	0.3'	0.5'	0.7'	1'	2'
ARCV	10°	8.4 °	7.5°	6.4°	4.7°

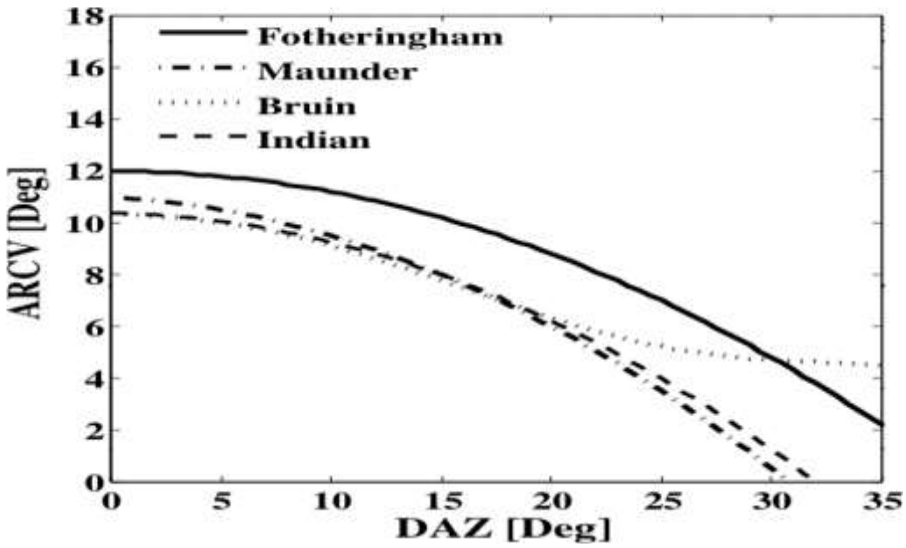

Figure 5: Comparing Bruin with Maunder and Indian criteria

From the figure above, we notice that the values of the Fotheringham criterion are high, and later with the increase in observations and technologies, we find that the values in the later criteria began to decline, reaching a value of (ARCV) that is less than 10 degrees. Yallop also calculates a quantity of focus for it (q), the q-values of which are defined by equation (2):
$q = ARCV - 11.871 - 6.3226 * W + 0.7319 * W^2 - 0.1018 * W^3/10$ [22] (2)
which is a measure of the visibility to see the crescent shown in Table (4)

Table 4: The q-test criteria of Yallop [22]

Criterion	Range	Remarks	Visibility Code
(A)	q > +0.216	Easily visible(ARCL≥12◦)	V
(B)	+216 ≥ q > −0.014	Visible under perfect conditions	V(V)
(C)	-0.014 ≥ q> -0.160	May need optical aid to crescent detection	V(F)
(D)	-0.160 ≥ q> -0.232	Will need optical aid to find crescent	I(V)
(E)	-0.232 ≥ q> -0.293	Not visible with optical aid (ARCL ≤ 8.5◦)	I(I)
(F)	-0.293 ≥ q	Not visible below Danjon limit, (ARCL ≤ 8◦)	I

After that, Salih [23] depended on astronomical conditions set by Alnaimiy and Salih [21] to draw the critical lunar date line (CLDL) for all geographical latitudes and they found that for all geographical latitudes the conditions of visibility were not appropriate with the same accuracy for all geographical latitudes. They were more accurate in the areas close to the equator, implying that there must be special visibility conditions for each location, based on the critical lunar date line (CLDL), which is adopted for the status of the global Hijri calendar.

Another accurate set of criteria are the South African Astronomical Observatory (SAAO) criteria [24][25]. They relate the altitude of the moon to the relative azimuth difference between the sun and the moon, as shown in Table (5):

Table 5: (SAAO) Criteria which relate both altitude and relative Azimuth [24] [25]

Relative Azimuth	It is not possible to see (even with a telescope) if the altitude of the crescent from the horizon is less than	Visibility with the naked eye is not possible if the altitude of the crescent from the horizon is less than
0°	6.3°	8.2°
5°	5.9°	7.8°
10°	4.9°	6.8°
15°	3.8°	5.7°
20°	2.6°	4.5°

In the last ten years, Salih [26] studied the relationship between the visibility of the crescent and the half-monthly synodical time (full moon date–date of start) and found that the crescent did not coincide with the period's midpoint.
We summarize the related papers in Table (6) below with each study.

Table 6: A summary of related papers and studies

References	Author	Summary
[27]	Kastner	He examined the near-sun objects in the twilight and took into account the twilight sky background, atmospheric extinction, and night glow.
[28]	Lestrade	He explained how to calculate a semi-analytical solution for relativistic perturbations of the point mass-moon and varying accelerations between the Earth and the moon.
[29]	McNally	He demonstrated that the lit crescent's outer terminator frequently deviated significantly from the predicted value of 180°.
[30]	Loewinger	He studied the Lag of the moonset and discovered that for a visible moon, the smallest Lag was 33 minutes, and the biggest Lag for an unseen moon was discovered to be 67 minutes.
[31]	Fatoohi	He explained that the crescent's visibility is not possible when (elongation) of moon (ARCL) is less than 7 degrees.
[32]	Jamaluddin	He documented the crescent's visibility during 1962-1997 in Indonesia and did a systematical analysis.
[33]	Guessoum	He used the 115 dates' group of data identical with the rituals of religion between (1963 to 2000) in Algeria and compared it with the dates of officials with the astronomical data and criteria of earliest visibility and found that the astronomical data and those dates were significantly irreconcilable.
[34]	Hoffman	Applying Yallop's criterion to the 539 observations made by several experienced observers in favorable weather, the moon is definitely visible if q is more than 0.43 and definitely not visible if q is less than -0.06 (q is Yallop's value for ease of visibility).
[35]	Hoffman	He used the most of two parameters of lunar visibility criteria: a limited latitude and the (LAG) time. They were only applicable and combined together into simple normal diagnostic parameter that predicted the

		crescent's visibility, whereas other variations cannot be predicted like temperature, humidity, pressure, and height above the see.
[36]	Xin	They discussed the different prediction criteria of crescent visibility and made sure that none of them was 100% accurate, then they tested the astronomical software MoonCalc, which is considered to be one of the most dependable options.
[37]	Sultan	He studied the length of the new crescent moon and his photometric model gave the same results as Schaefer's empirical model .
[38]	Sultan	He came to the conclusion that the site elevation and moon altitude are directly and inversely proportional to the " best time " for the initial visibility of the thin crescent moon.
[39]	Sultan	He utilized his photometric model to predict the lunar crescent's visibility by naked-eye and came to the conclusion that the minimum naked-eye visibility limit is 7.5° (±0.25°) and the lunar crescent's length is around 7.1° in state 180.
[40]	Qureshi	He compared the mathematical model for the criteria (Yallop, Bruin) under which the new moon crescent might be visible at a certain location on the Earth. The comparison of these models has been made and the models have been modified, where possible.
[41]	Hidayat	He provided an update on the development of an information system for moon-crescent astronomical observations using a tiny telescope equipped with a simple digital detector and connected to a server to provide information on moon-crescent observations.
[42]	Mostafa	He presented calculations indicating the possibility of two crescents in a single day.
[43]	Fouka	Using the functional link between the arc of vision (ARVC), arc of light (ARCL), and the relative of azimuth (DAZ), or between the arc of vision (ARVC) and the crescent width(W), he claimed that experimental procedures have been established.
[44]	Galal	Studied the relationship between the new moon crescent's helical rising and setting on the day of conjunction. The sun's declination determines how long the crescent will stay positive as a result of its helical rise and how much of a seasonal impact it will make.
[45]	Utama	He discussed crescent visibility predictions on telescopic–based optical observations and the angular magnification correction to predictions that contain many correction factors.
[46]	Fakhar	He presented a new method for the lunar crescent's visibility depending on the processing of lunar crescent image algorithms and used this method to assist having more precise criteria.
[47]	Özlem	Using the two reliable parameters (altitude and crescent width(W)), he developed a substitute criterion for the lunar crescent's naked-eye vision and made it feasible to assess the visibility for each Moon phase.
[48]	Ahmed	He presented a new criterion for the earliest crescent's visibility depending on the brightness variation between the new crescent and its background sky at sunset, the average sky brightness through sunset measured was 6.80 ±,1.13 magnitude/arc second.
[49]	Alrefay	Using 545 observations collected over a period of 27 years (1988-2015) at various locations in Saudi Arabia at latitudes ranging from 20° to 29° N, he presented a new criterion for the visibility of the lunar crescent using arc of vision (ARCV) and lunar-crescent width (W). This new model may be used to predict whether the lunar crescent will be seen with the unaided eye or with aided eye.
[50]	Salih	Using a modified version of Meeus's 1998 formula, which is used to compute the orbit's elements and also allows for the specification of a potential date for monitoring the crescent moon, he estimated the

		locations of the moon, its velocity, and its distance over a period of one hundred years.
[51]	Salih	He studied the synodic months and how they were related to the mean anomaly for the moon's orbit for (100 years). He also designed a software that accurately calculates the length of synodic months and the moon's coordinates at the time of the young moon. He discovered that the synodic month is at its shortest when the new moon occurs close to perigee (mean anomaly equal to 0°).
[52]	Raharto	He proposed a unique criterion for the Hijri lunar calendar, using an (ARCL) and one value in Indonesia, and can be used for the unification of the Islamic calendar ideally.
[53]	Zainon	He compared a new lunar criteria of the Islamic international calendar which notion from the international conference in 2016 in Istanbul with universal calendar notion from the (Jakarta 2017) conference, and discovered that both calendars were intended to have the same definition of being suggested as the unite calendar of the Islamic world in a single time system, even though they used different new lunar criteria and parameters application .
[54]	Ahmad	After 254 observations collected from Baitul Hilal in Malaysia, they concluded that the new criterion for crescent visibility is the moon's elongation (ARCL) at 7.28°, altitude at 3.33°, and (ARCV) at 3.74 at sunset.
[55]	Ahmed	He illustrated that using simple parameters like (the moon's age, difference altitude, sun-moon Lag, different azimuth, atmospheric condition, moon phase) can provide an accurate prediction for modeling this phenomenon based on astronomy. That approach equally addresses false positives and false negatives.
[56]	Zulkeflee	They suggested utilizing (Contrast Limited Adaptive Histogram Equalization) (CLAHE) to successfully improve the crescent moon's recent images' contrast. They concluded that this method can aid astronomers in detecting the moon's crescent appearance to announce Islamic calendar's first month, even when it is not visible to the human eye.

So, we studied the development of crescent's visibility criteria from ancient times through the medieval period to the modern update, and we gave a general, comprehensive and in-depth overview of the studies and research related to the topic to make it easier for students and researchers in the future to study the issue of crescent visibility and take what benefits them from related research.

4. Result and conclusion

We can classify the criterion into many categories based on: (A- visual observations from the ancient era), and (B- criterion based on theoretical calculations for future prediction such as the Islamic era), and (C- geometrical physical criteria that include local weather conditions). We took a survey of about 54 research papers related to the crescent's visibility criteria taken from the ancient ages to the present time. After these studies, there was a development in future prediction maps for the visibility of the crescent, for example (Yallop, Odeh, and SAAO). These maps can easily be found on websites.

This study concluded that the crescent's visibility requirement now faces the following challenges:

1. Must re-test the lunar crescent length, for a model of Danjon (1932) with additional data collection and verify the effects of the turbulence in the atmosphere on the width of the young crescent moon.
2. To check the effect of the local atmospheric conditions on the visibility characteristic of the lunar crescent.
3. It is important to identify the convenience of parameters and the crescent's visibility criteria and they may be used for regional calendar purposes.
4. The previous studies were divided into two parts: criteria for ancient and medieval ages which depended on human naked eye observation, and modern criteria that depends on the geometrical conditions, weather conditions, and the location of the observer.
5. Using modern technology, such as a CCD camera and digital image processing technology led to the detection of the lunar crescent even in the morning after the conjunction.
6. There are no 100% accurate criteria that correspond to the truth and completely match the observations but Yallop and (SSAO) reached acceptable criteria that are more accurate than other criteria according to their conformity with observations by a large percentage.
7. The border areas (edges) in the visibility prediction maps, separating the areas that are seen with the telescope and the areas that are difficult to see, are inaccurate and need more study in order to overlap the two areas together.

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