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Multimodal Medical Image Fusion Enhancement Based on Wavelet Transform

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Abstract

Image fusion is the process of combining multiple images to obtain one image with complementary information. Hence the importance of multimodal medical image fusion, which aims to obtain all the important information in one image to improve diagnosis in the medical field. The wavelet transform is a mathematical tool for multi-resolution analyses where the images are decomposed into a series of subbands. Wavelet transforms have a good representation of the original images. The averaging fusion rule and shift variance in the discrete wavelet transform, on the other hand, make the image that comes out of fusion under the wavelet transform blurry or of low quality. In this paper, two novel methods have been presented to improve the quality of the multimodal medical image fusion-based wavelet transform. The efficiency of the proposed methods has been measured using three metrics, all of which prove the efficiency of the proposed methods compared to the standard wavelet methods.

Keywords: Image fusion; multimodal; wavelet transform; discrete wavelet transform

تحسين اندماج الصور الطبية متعددة الانماط باستعمال تحويل الموجات

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الخلاصة

دمج الصور هي عملية دمج صور متعددة بغرض الحصول على صورة بمعلومات تكميلية. ومن هنا تأتي أهمية دمج الصورة الطبية متعددة الوسائط والتي تهدف إلى الحصول على جميع المعلومات المهمة في صورة واحدة بهدف تحسين التشخيص في المجال الطبي. تحويل الموجات هو أداة رياضية لتحليل متعدد الدقة حيث تتحلل الصور إلى سلسلة من النطاقات الفرعية ويتم تمثيل الصورة الأصلية بواسطة النطاقات الفرعية المنخفضة ويعتبر تمثيل جيد بالمقارنة مع التحويلات الأخرى. ومع ذلك، فإن الصورة الناتجة من عملية الاندماج باستعمال التحويل الموجي تعاني من جودة منخفضة أو ضبابية بسبب قاعدة الانصهار المتوسطة وخاصة تباين التحويل في تحويل الموجات المنفصل. في هذا البحث، تم تقديم طريقتين جديدتين تهدفان إلى تحسين جودة الصور الطبية المتعددة الوسائط. وتم قياس كفاءة أداء الطرق المقترحة باستعمال ثلاثة مقاييس أثبتت جميعها كفاءة الطرق المقترحة مقارنة بالطرق المعيارية التحويل الموجي المنفصل والتحويل الموجي الثابت.

1. Introduction

Computerized tomography (CT) is a medical imaging technique used to obtain details about internal images of the body or the brain. CT has been used in human medicine since 1972. Although CT is inferior to MRI in soft tissue imaging due to its low-resolution contrast, it is the preferred method for imaging bones [1]. Magnetic resonance imaging (MRI) is a common method used in medical imaging to clarify pathological changes in living tissues. MRI was invented in 1970; it mainly uses the properties of the hydrogen nucleus, and from a physical point of view, it depends on the magnetic field. MRI provides greater contrast between different body tissues than the image produced by a CT scan [2]. Multimodal image fusion is the process of fusing images from different modalities taken by different sensors, such as CT and MRI, visible and infrared, or panchromatic and multispectral satellite images. These images are merged to get images of improved quality for the purpose of producing a suitable image for the diagnosis while enhancing the important information acquired from different sensors. So, fusion can store the efforts of processing different images. Another advantage of fusion is the reduction of storage capacity for different scans [3]. There are three levels in the image fusion process: pixel level, feature level, and decision level. The pixel level is a low level that is used to analyze and integrate data from different sources directly without the need for a prior estimation process or recognition of the original information. For instance, when performing the merging process at the pixel level using the mathematical model that calculates the average values of the input images, the pixel strength of the merged image is obtained by merging the average values of the two images. Pixel-based fusion provides the best possible results. There are two domains of merging images: the fusion-based spatial domain and the fusion-based transform domain [4]. The resultant image from fusion in the spatial domain has spatial distortions. Consequently, transform fusion methods have been used to outweigh the negatives of spatial fusion [5]. The discrete wavelet transform plays a vital role in image merging as it reduces structural distortions among various other transformations. Shift variance, poor directional selectivity, and the absence of phase information are the disadvantages of the discrete wavelet transform [6].

The rest of the paper has been structured as follows: Section 2 presented the literature review and the aim of the study. Section 3 presented the methodology of the paper, which contains the theoretical side and performance metrics used to evaluate the proposed method. Section 4 offers experimental results and discussion, followed by Section 5, which presents the conclusion.

2. Related Work

There are numerous image fusion enhancement methods that have been presented. In [7], the authors showed a new way to improve the fusion of multimodal medical images like computed tomography (CT) and magnetic resonance imaging (MRI). They did this by using stationary wavelet transform (SWT)-based image fusion to get coefficients while using the laws of texture energy measures in the stationary wavelet transform to deal with low contrast and high computational complexity. They also used morphology processes to clean up the fusion process. The author in [8] presented a fusion method based on the Dual Tree-Complex Wavelet Transform (DT-CWT) and focus filter for fusing many types of images. In this method, images are enhanced by a focus filter before the fusion process. Focus filters consist of two filters, which are the wiener algorithm and sharpening filters. By using inverse DT-CWT, the coefficients are combined using the averaging and maximum select fusion rules. In [9], the authors showed how the empirical wavelet transform (EWT) and local energy maxima (LEM) can be used to improve the fusion of medical images like MRI and PET. In this method, the input images are analyzed by EWT to obtain the low-frequency coefficients and the high-frequency coefficients. Then, the merging method based on the Local Maximum

Energy (LEM) was used to combine the parameters from the two images, and the Inverse Empirical Wavelet Transform (IEWT) was used to reconstruct the final image from the combined coefficients. To measure the performance of the proposed method, four scales were used, including the mutual information scale. The authors in [10] presented a method for multimodal image fusion, such as visible images, infrared images, CT, and MRI, using stationary wavelet transform-based image fusion and principal component analysis (PCA). The results of the research show that the PCA method has better performance for those input images that have different contrast and brightness levels. The authors in [11] have presented the application of deep learning techniques to multi-modal medical image fusion for CT and MRI images.

Researchers have presented several methods for multimodal CT-MRI image fusion, and these methods have achieved good results. However, the resulting images contain blur, and image fusion is a renewable field that needs to be developed continuously and incrementally to overcome the increasing challenges. In this paper, a new method for improving multimodal CT-MRI image fusion using the restoration algorithm Lucy Richardson (LR) with image fusion based on the stationary wavelet transform (SWT) and a convolution process using the kernel with image fusion based on the discrete wavelet transform (DWT) is shown.

3. Subjects and Method

3.1 Wavelet Transform

The wavelet transform is a mathematical tool for processing signals that was made by Jean Morlet and Alex Grossmann in 1981. It lets you look at the local properties of complex signals with non-stationary parts on different time scales [14]. The wavelet method depends on small waves called wavelets, which are oscillatory functions of finite duration and have an average value equal to zero with limited energy [15]. Wavelets can be described in two functions: the wavelet function, defined as the mother wave and denoted by the symbol $\psi(x)$ and the scaling function, defined as the father wave and denoted by the symbol $\phi(x)$ [15]. The wavelet transform allows localized time-frequency, where time and frequency information can be obtained using waves, unlike other transforms that may lose time information [16]. The mother wavelet can be expressed as follows [17]:

$$\psi_{k,p} = \frac{1}{\sqrt{k}} \psi\left(\frac{t-k}{p}\right) \quad (1)$$

where k represents scale, p represents position, and t indicates time.

3.2 Discrete Wavelet Transform

Discrete Wavelet Transform (DWT) or Mallat algorithm It is an implementation of the wavelet transform using dyadic scales and locations. DWT handles a two-dimensional signal. DWT analyzes the signal into an orthogonal group of waves, which is the main difference from the Continuous Wavelet Transform (CWT). In DWT, the mother wavelet is scaled by ($a = 2j$) and positioned by ($b=k2j$) [17]. DWT can be obtained using the expression mentioned in [18]:

$$C(l, m) = \sum_{n=0}^{N-1} x[n] \psi_{l,m}(n) \quad (2)$$

Where, $x[n]$ is the signal, $\psi_{l,m}(n)$ is the mother wavelet, which can be expressed by [18]:

$$\psi_{l,m}(x) = 2^{-\frac{l}{2}} \psi(2^{-l}x - m) \quad (3)$$

In DWT, signal decomposition can be done using a filter bank approach or a downsampling system approach. In the filter bank approach, the input signal is passed

through low filters $L[m]$ and high filters $H[m]$ and then downsampled by a factor of two to obtain approximate and detailed coefficients [19]. Figure 1 illustrates the process of signal analysis in DWT.

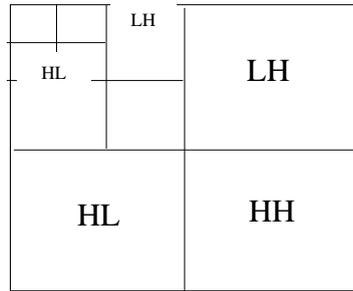


Figure 1: Image decomposition by discrete wavelet transform for 3 levels

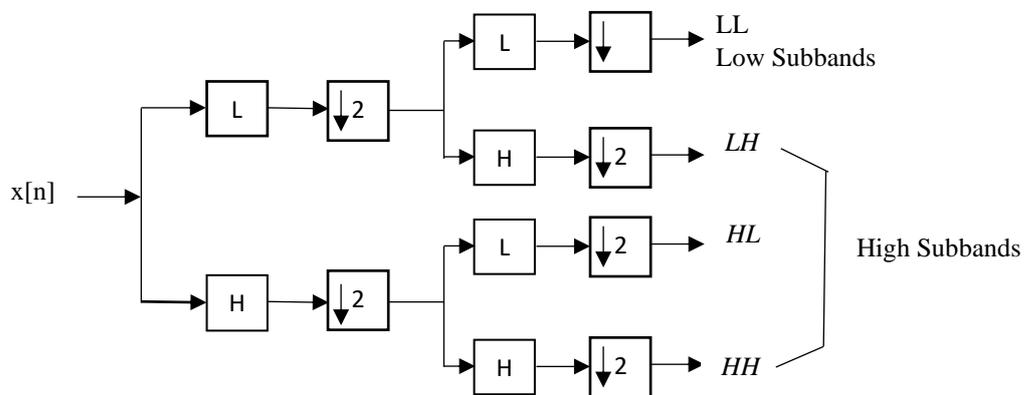


Figure 2: Discrete wavelet decomposition for a 2D signal Decomposition results in four subbands (LL, LH, HL, and HH) representing low subband approximation information and three high subbands (horizontal, vertical, and diagonal information, respectively).

3.3 Stationary wavelet transform

The discrete wavelet transform (DWT) suffers from shift variance due to the decimation process, which divides the signals in half at every level. Stationary wavelet transform (SWT) is the same as DWT in signal decomposition but with no decimation process by including zeros between the filter parameters [20]. Therefore, SWT is unlike DWT, which provides translation invariance. SWT is redundant, so some detail information may be retained in levels of transformation [21]. Figure 3 illustrates the process of SWT decomposition for a 2D signal.

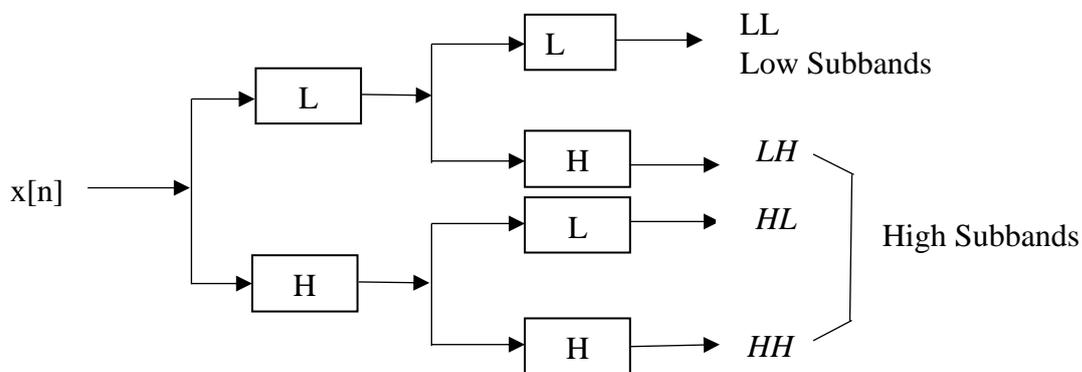


Figure 3: The process of 2-D SWT signal decomposition

Discrete wavelet transform-based image fusion can be implemented in three main stages:
 Step 1: Convert the input images to the transform domain by using one of the forms of the wavelet transform.
 Step 2: the resultant subbands are fused by the fusion rules.
 Step 3: image obtained by reconstructing the wavelet transform.

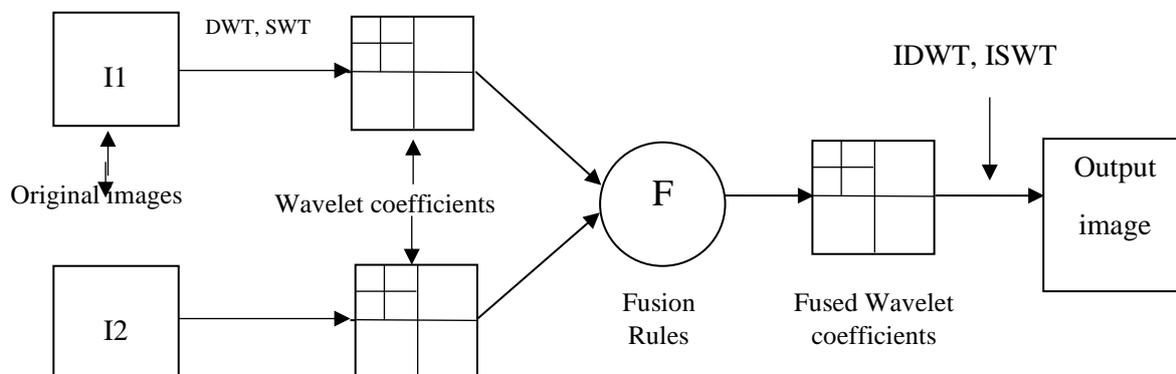


Figure 4: The process of image fusion based on DWT and SWT

3.4 Lucy Richardson algorithm

The Lucy-Richardson algorithm (LR) is a non-linear iterative algorithm commonly used to restore images from degradation, designed by Lucy and Richardson. Initially, deconvolution is performed using a blurred image and a point spread function (PSF). Image recovery in the LR algorithm can be achieved by doing many iterations of deconvolution to converge to the final result [22]. The LR algorithm originates from the maximum-likelihood formulation of the image using Poisson statistics, and the mathematical formula for the LR algorithm is as mentioned in [23]:

$$\hat{f}_{k+1}(x, y) = \hat{f}_k(x, y) \left[h(-x, -y) * \frac{g(x, y)}{h(x, y) * \hat{f}_k(x, y)} \right] \tag{4}$$

where, k represents the iterative number, $\hat{f}_k(x, y)$ represents the restore image in the previous step, $\hat{f}_{k+1}(x, y)$ represents the restore image in the current step.

3.5 Conventional Spatial linear filtering

The linear spatial filtering mechanism simply consists of moving the filter mask from pixel to pixel between the filter parameters and the corresponding image elements [24].

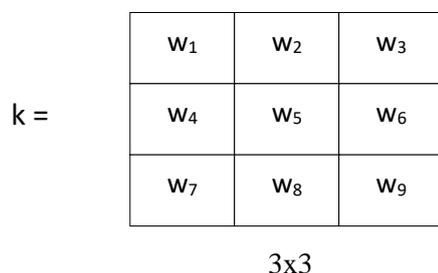


Figure 5: Spatial filter mask (3x3)

In general, a linear filtering of an image of size M x N with a filter mask of size m x n is as in the following mathematical expression, as pointed out in [24]:

$$R = k(-1,-1)f(x-1,y-1) + k(-1,0)f(x-1,y) + k(-1,1)f(x-1,y+1) + \dots + k(0,0)f(x,y) + \dots + k(1,0)f(x+1,y) + k(1,1)f(x+1,y+1) \tag{5}$$

The response R, filtering linearity with a filter mask (3x3) at the point (x,y) in the image, can be expressed mathematically as mentioned in [25]:

$$R=k_1 f_1+k_2 f_2+k_3 f_3+\dots+k f_{mn}=\sum_{i=1}^{mn} k_i f_i \tag{6}$$

where k represents the mask coefficients, f represents the values of the gray image levels corresponding to the mask coefficients, and mn represents the total number of mask coefficients. It is expressed in the general form as follows [25]:

$$g(x,y)=\sum_{i,j} k(i,j)f(x-i,y-j)$$

$$g = k * f \tag{7}$$

where g represents the result image, k represents the mask, and f represents the original image.

Linear spatial filtering is often called the convolution process, and the filter mask is also referred to as a convolution mask.

3.6 Proposed algorithm

For the purpose of enhancing the quality of the medical image on CT-MRI using image fusion based on the wavelet transform, two methods have been presented. The first method is using the stationary wavelet transform-based wavelet transform with the LR algorithm as a preprocessing step to pre-process the image before the image fusion process. The second way to get a high-quality image is to use a convolution process with a discrete wavelet transform inside the fusion process at the approximate fused coefficients. Figure -6 illustrates the first method (DWT-KE), and Figure -7 illustrates the second method (SWT-LR).

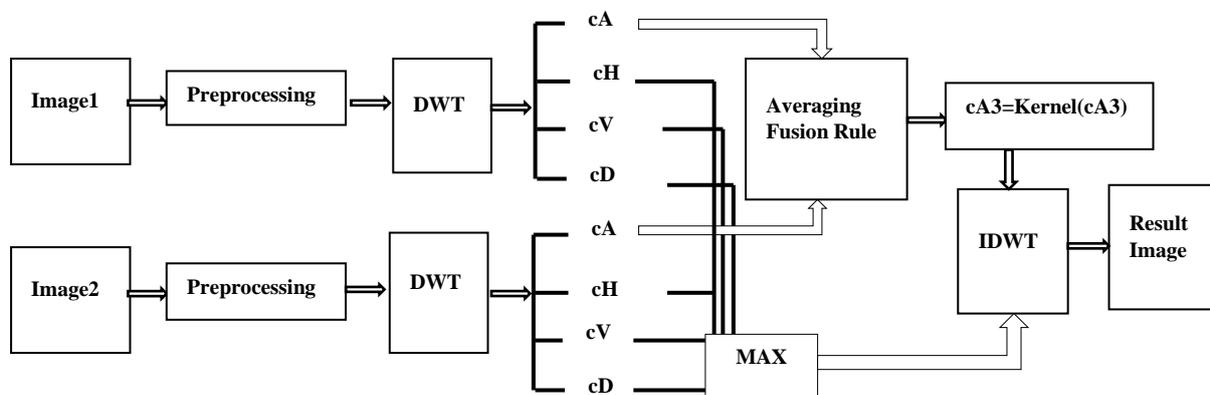


Figure 6: Structure of the First Proposed Method DWT-KE

The following are the steps of the presented algorithm for image fusion using DWT and convolution using a kernel:

- Step 1:** Insert the images, let's say image 1 and image 2.
- Step 2:** Preprocess the image by converting it to grayscale.
- Step 3:** Decompose the images with DWT to obtain the high- and low-frequency subbands.
 $[LL1, LH1, HL1, HH1] = \text{dwt2}(\text{image1}, \text{db2});$
 $[LL2, LH2, HL2, HH2] = \text{dwt2}(\text{image2}, \text{db2});$
- Step 4:** Integrate the approximate coefficients (LL) using the average value merging method and merge the detailed coefficients (LH), (HL), and (HH) using the maximum value selection.
 $LL3(i, j) = (LL1(i, j) + LL2(i, j))/2;$
 $LH3(i, j) = \max(LH1(i, j), LH2(i, j));$
 $HL3(i, j) = \max(HL1(i, j), HL2(i, j));$
 $HH3(i, j) = \max(HH1(i, j), HH2(i, j));$
- Step 5:** process the LL3 with convolution mask.
 $LL3 = \text{kernel}(LL3);$
- Step 6:** Inverse DWT has been used to obtain the final enhanced image.
 $\text{Fused image} = \text{idwt2}(LL3, LH3, HL3, HH3, 'db2');$

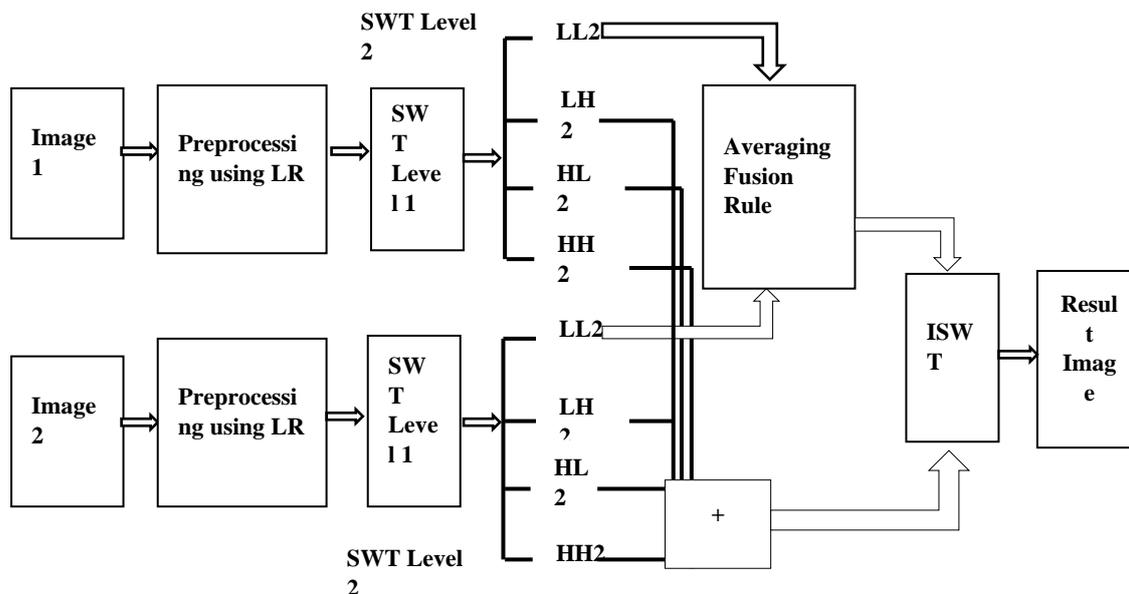


Figure 7: Structure of the Second Proposed Method (SWT-LR)

The following are the steps of the presented algorithm for image fusion using SWT and the Lucy Richardson algorithm:

Step 1: Insert the images, let's say image 1 and image 2.

Step 2: preprocessing images (CT, MRI) by Lucy Richardson algorithm.

Step 3: Image decomposition to obtain LL1, LH1, HL1, and HH1 coefficients using SWT in Level 1.

Step 4: Decompose the approximation coefficient LL1 from level 1 to obtain the LL2, LH2, HL2, and HH2 coefficients in level 2.

Step 5: The averaging fusion rule has been used for fusing the LL2 approximation coefficients from the two images in level 2 and using the addition process for fusing the three detailed coefficients LH2, HL2, and HH2 in level 2.

Step 6: fuse LL1 coefficients from the two images in level 1 by using the averaging fusion rule and fuse LH, HL, and HH in level 1 by using the addition process.

Step 7: The inverse process of SWT has been used to reconstruct the LL2, LH2, HL2, and HH2 coefficients in level 2.

Step 8: The inverse process of SWT has been used for reconstructing LL1, LH1, HL1, and HH1 coefficients in level 1 to obtain the final enhanced image.

3.7 Efficiency Evaluation Metrics

The following are some of the metrics utilized to evaluate the performance of the proposed image fusion method and traditional image fusion methods:

1- Contrast Focus Measure

The contrast meter is used to measure the focus of the image, where sharp images have high contrast. The contrast meter can be obtained by the following equation, as mentioned in [26]:

$$CF(x,y)=\sum_{m=x}\sum_{n=y}|I(x,y)-I(m,n)| \quad (8)$$

where, $I(x,y)$ represents the value of the pixel.

2- Histogram Entropy

A measure of evaluating the quantity of information included in a gray-scale image Elevated entropy in the image denotes an increase in image information. Entropy can be defined as pointed out in [26]:

$$HE = -\sum_{k=0}^{K-1} P_i \log P_i \quad (9)$$

Where, K is representing the whole grey level, $P_i, (i=1, 2, \dots, K-1)$ is the probability distribution.

3- Sum of Wavelet Coefficients

The sum of the wavelet coefficients counts the focus degree of the improved image by using the sum of the coefficients of the wavelet. Mathematically, it can be defined as [26]:

$$SW=\sum_{(i,j)\in\Omega_D} \text{abs}(W_{LH1}(i,j))+\text{abs}(W_{HL1}(i,j))+\text{abs}(W_{HH1}) \quad (10)$$

Where, Ω_D is the corresponding neighborhood ($\Omega(i,j)$) of the enhanced image elements $f(i,j)$.

4. Experimental results and Discussion

In this paper, two ways to improve CT-MRI image fusion are suggested. The first uses the complementary stationary wavelet transform (SWT) and the Lucy Richardson algorithm. The second uses the discrete wavelet transform with a kernel-based convolution process. The results obtained showed the effectiveness of the proposed methods. Traditional methods such as DWT, SWT, and PCA image fusion have been used for comparison of the results. Focus operators, such as the contrast focus measure (CF), the histogram entropy (HE), and the sum of coefficients of the wavelet (SW), were used to figure out how well the experiment worked. The proposed method has been accomplished on three pairs of multimodal images, namely (Pair1, Pair2, and Pair3), in the form of three CT images and three MRI images in the size of 256 x 256, as illustrated in Figure 8. The datasets have been collected from the MathWorks

website. This study is performed using the MATLAB programming language version 9.0.2 under Windows 10 Pro.

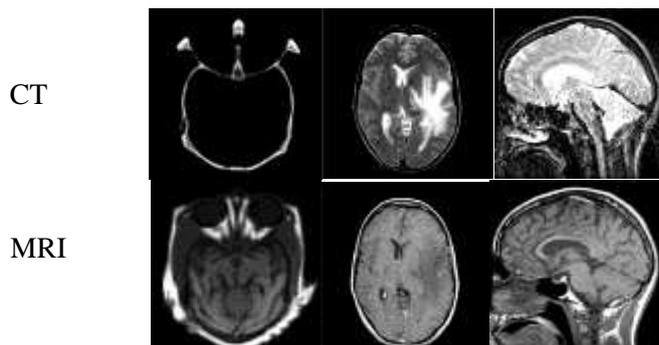


Figure 8: Images in the form of CT and MRI

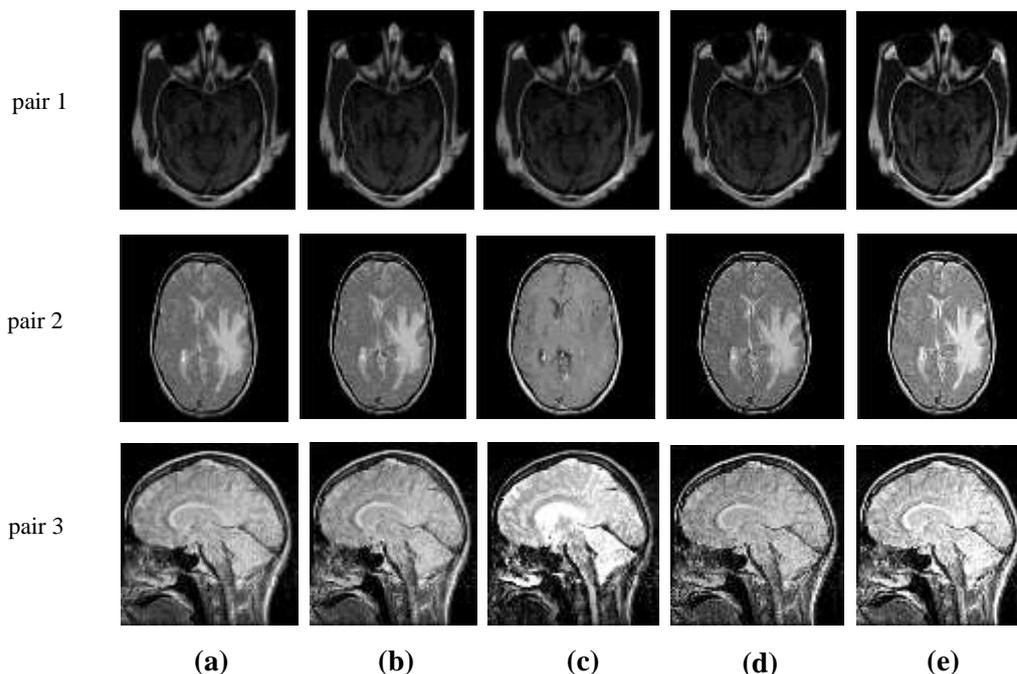


Figure 9: The resultant images are as follows: (a) resultant image from fusion-based DWT; (b) resultant image from fusion-based SWT; (c) resultant image from fusion-based PCA; (d) resultant image from fusion-based proposed method SWT-LR; (e) image from fusion-based DWT-Ke.

Table 1: Results for the DWT, SWT, PCA, and Proposed Methods SWT-LR, DWT-KE for Image Pair 1

Method/Metrics	CN	EN	SW
DWT	17.4065	6.0186	1.8604
SWT	17.6832	6.0701	1.6198
PCA	15.7670	5.9817	1.1661
SWT-LR	22.3195	6.1667	2.4522
DWT-KE	27.1346	6.4170	2.8441

DWT: Discrete Wavelet Transform; SWT: Stationary Wavelet Transform; PCA: Principal Component Analysis; SWT-LR: First proposed method; DWT-KE: Second proposed method

Table 2: Results for the DWT, SWT, PCA, and Proposed Methods SWT-LR, DWT-KE for Image Pair 2

Method/Metrics	CN	EN	SW
DWT	23.4387	4.4728	3.5687
SWT	24.4948	4.5095	3.6494
PCA	24.3639	4.3997	3.0223
SWT-LR	32.6505	4.5099	5.7893
DWT-KE	34.7840	4.5892	5.3766

DWT: Discrete Wavelet Transform; SWT: Stationary Wavelet Transform; PCA: Principal Component Analysis; SWT-LR: First Proposed Method; DWT-KE: Second Proposed Method

Table 3: Results for the DWT, SWT, PCA, and Proposed Methods SWT-LR, DWT-KE for Image Pair 3

Method/Metrics	CN	EN	SW
DWT	55.2934	7.5671	9.4718
SWT	58.1550	7.5766	9.5615
PCA	63.5599	7.6252	9.0438
SWT-LR	78.8002	7.5630	14.0611
DWT-KE	79.3218	7.7030	13.6327

DWT: Discrete Wavelet Transform; SWT: Stationary Wavelet Transform; PCA: Principal Component Analysis; SWT-LR: First Proposed Method; DWT-KE: Second Proposed Method Through the experiment results, the proposed fusion methods had better results than the traditional methods of DWT, SWT, and the PCA method. For instance, from Table 1, for image pair 1, the value of histogram entropy for DWT is 6.0186, for SWT is 6.0701, and for PCA is 5.9817, while for the first proposed method SWT-LR is 6.1667 and for the second method DWT-KE is 6.4170. And from table 2, for image pair 2, the value of histogram entropy for DWT is 4.4728, for SWT is 4.5095, and for PCA is 4.3997, while for the first proposed method SWT-LR is 4.5099 and for the second method DWT-KE is 4.5892. The values obtained proved the effectiveness of the proposed methods.

5. Conclusion

The objective of this paper is to enhance CT-MRI image fusion based on the wavelet transform. Although wavelet transform-based image fusion has good representation for the images, the resulting image has low contrast and blur. This will affect the quality of the image information. The proposed methods create high-quality merged images by using convolutional processing in a DWT-based image fusion process and the Lucy Richardson algorithm as a preprocessing step in a SWT-based image fusion process. Images used in the experiment are gathered from different modalities, such as CT and MRI images, and each of them provides different information about the human body. Three performance metrics were used to judge the experiment, and the results showed that the proposed fusion methods worked better than the traditional DWT, SWT, and PCA methods.

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