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Instability Analysis Study of the Jeffrey Nanofluid Flow through a Brinkman-Darcy Porous Medium

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Abstract

The analysis of thermal instability in a Brinkman-Darcy Jeffrey nanofluid flow through a porous medium is studied in this paper. The nanoparticles are immersed in the Jeffrey fluid so that the thermal conductivity of the system is maintained and high medium porosity is to be undertaken. Under the impact of the Jeffrey, nanoparticles and Brinkman-Darcy parameters, the momentum-balance equations of fluid flow are mutated. The dispersion relation for the Rayleigh number is derived by employing the normal mode analysis method and linear stability theory in terms of different parameters affecting the stability of the system. It is noticed that the Darcy-Brinkman number advances the convection while the Jeffrey parameter postpones the convection in a stationary mode. To verify the results numerically, graphs have been plotted by using Origin 6.1 software. Further, for the top-heavy nanoparticles distribution, oscillatory convection does not exist.

Keywords: Thermal convection, Rayleigh number, Jeffrey Model, porous medium, nanofluid.

1. Introduction

The instability of a non-Newtonian fluid has many applications in real-life problems as well as in various areas of modern technology and industry, viz. plastic production, polymer industry, paper and textile dyeing, food industry, geophysics, chemical and biological industry, etc. [1-9]. Motor oils, printing inks, egg white, wallpaper paste, toothpaste, soap solution, sauce, and biological fluids such as blood are some examples of non-Newtonian fluids. The Jeffrey fluid model [10] is one such kind of non-Newtonian fluid. He investigated some problems of an incompressible fluid that is heated from below, and now it is shown to be the best fluid model to describe the behaviour of physiological and industrial fluids [11-14].

Studying porous media has many applications in groundwater hydrology, Earth's molten core, and many others. Sandstone, limestone, human lungs, bile ducts and gall bladder with stones in the vessels are some examples of natural porous media. A simple Darcy model was used to initiate studies in a porous media. Later, the Darcy model was extended to the Brinkman-Darcy model due to its high porosity and was used in various industries for the

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production of highly porous materials such as paints, lubricants, metal foams, lightweight plastics, etc. Lapwood [15] examined the instability fluid in a porous medium while the instability of the boundary layer problem was investigated by Wooding [16]. Different authors [17-22] studied the effect of porosity on the onset of convection in the porous medium.

During the last few years, the studies of non-Newtonian nanofluids become an important topic of research due to their high thermal conductivity and various industrial applications. Sheu [23-24] studied the problem of a porous medium layer saturated by a nanofluid and found that oscillatory convection is possible for both bottom/top heavy nanoparticles distributions. Chand and Rana [25] studied the problem of thermal instability of Rivlin-Ericksen nanofluid and found that the Rivlin-Ericksen nanofluid behaves like an ordinary fluid in the case of stationary convection while Yadav [26] studied the same problem by taking Kuvshiniski fluid as base fluid. Natural convection in non-Newtonian nanofluids has been analysed by different authors [27-32] on the basis of Buongiorno's [33] model. They observed that nanofluids are the best coolants and find various applications in engineering industries, energy-saving industries and bio-medical industries, etc., viz, nanoparticle suspension develops medical applications such as in the treatment of hyperthermia.

In the present paper, the thermal instability of Jeffrey nanofluid layer permeated with high porosity medium is examined under the assumption that the nanoparticles are spherical in shape and the size of nanoparticles is quite smaller than the pore size of the matrix.

2. Mathematical model

Here, we consider the Brinkman-Darcy Jeffrey nanofluid flow through a porous layer having thickened *d* as shown in Figure 1

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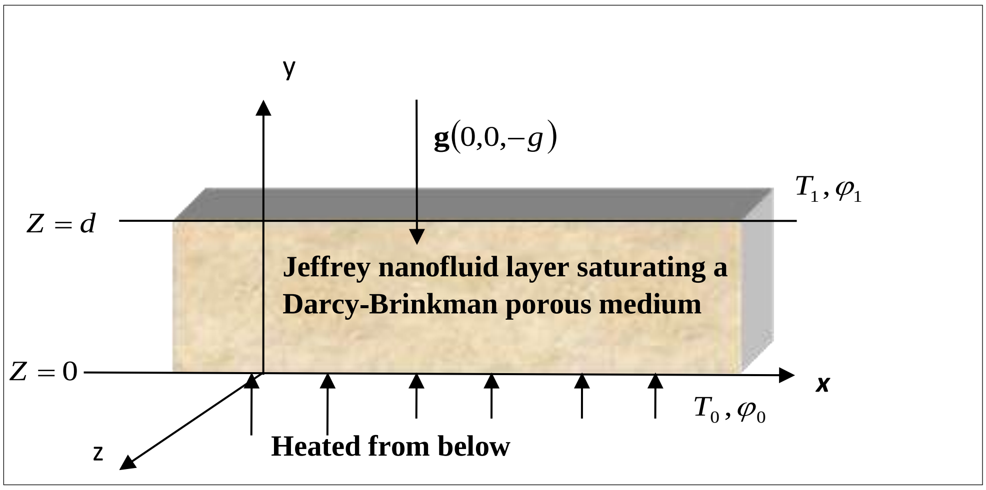

Figure 1: Physical sketch of the problem

2.1 Governing equations

The governing equations of Brinkman-Darcy Jeffrey nanofluid flow through a porous layer after applying Boussinesq approximation are given as follows:

The equation of mass-balance is:

$$\nabla.\mathbf{q_D} = 0, \quad (1)$$

Where, ∇ is the Laplacian operator and $\mathbf{q_D}$ is the flow velocity of the fluid.

Equations of modified momentum-balance [10-15] are:

$$\frac{\rho_f}{\varepsilon}\left(\frac{\partial \mathbf{q_D}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q_D}.\nabla)\mathbf{q_D}\right)=-\nabla p-\frac{\mu}{k_1(1+\lambda_3)}\mathbf{q_D}+\tilde{\mu}\nabla^2\mathbf{q_D}+\left[\phi\rho_p+(1-\phi)\rho_f\left\{1-\alpha(T-T_1)\right\}\right]\mathbf{g}, \qquad (2)$$

Where ρ_f is the fluid density, ρ_p is the density of nanoparticles, p is the fluid pressure, T is the fluid temperature, μ is the fluid viscosity, $\tilde{\mu}$ is the effective viscosity of the fluid, k_1 is the medium permeability, ε is the porosity, ϕ is the volumetric fraction of nanoparticles, medium permeability and λ_3 is the Jeffrey parameter (accounting for viscoelasticity).

The equation of momentum-balance equation of nanoparticles [22-29] is:

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{q_D}.\nabla}{\varepsilon}\right]\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_0}\nabla^2 T. \qquad (3)$$

Where, D_B is the Brownian diffusion coefficient and D_T is the Thermophoretic diffusion coefficient, they are given as follows:

$$D_B=\frac{k_B T}{3\pi\mu\, d_{np}}, \quad D_T=\frac{\mu}{\rho_f}\frac{0.26k_{nf}}{(2k_{bf}+k_{np})}\phi,$$

Where k_B is the Boltzmann's constant, d_{np} is the diameter of the nanoparticle, k_{bf} and k_{np} are the thermal conductivities of the base fluid and nanoparticles, respectively.

The equation of energy-balance [25-33] is:

$$(\rho_f c)_{bm}\left\{\frac{\partial T}{\partial t}+\mathbf{q_D}.\nabla T\right\}=k_m\nabla^2 T+\varepsilon(\rho_f c)_p\left\{D_B\nabla\phi.\nabla T+\frac{D_T}{T_0}\nabla T.\nabla T\right\}. \qquad (4)$$

Where $(\rho_f c)_{bm}$ is the effective capacity, $(\rho_f c)_{bf}$ is the heat capacity of nanofluid.

Using the following non-dimensional variables

$$(x',y',z')d=(x,y,z),(u',v',w')\kappa_{bm}=(u,v,w)d, t'\sigma d^2=t\kappa_{bm}, p'\mu\kappa_{bm}=pk_1,$$

$$(\phi_1-\phi_0)\phi'=\phi-\phi_0,(T_0-T_1)T'=T-T_1.$$

In non-dimensional form, equations (1-4) reduce to (deleting the (') for simplicity)

$$\nabla.q_D=0, \qquad (5)$$

$$\frac{1}{Va}\frac{\partial \mathbf{q_D}}{\partial t}=-\nabla p-\frac{1}{(1+\lambda_3)}\mathbf{q_D}+\tilde{D}a\nabla^2\mathbf{q}-Rm\hat{e}_z+RaT\hat{e}_z-Rn\phi\hat{e}_z, \qquad (6)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}q_{\mathbf{D}}.\nabla\varphi=\frac{1}{Le}\nabla^2\varphi+\frac{N_A}{Le}\nabla^2 T, \qquad (7)$$

$$\frac{\partial T}{\partial t}+q_{\mathbf{D}}\cdot\nabla T=\nabla^2 T+\frac{N_B}{Le}\nabla\phi\cdot\nabla T+\frac{N_A N_B}{Le}\nabla T\cdot\nabla T. \qquad (8)$$

Here, the thermal diffusivity of the base fluid $(\kappa_{bm})=\frac{k_m}{(\rho_{nf}c_{np})_{bf}}$, the thermal capacity ratio $(\sigma)=\frac{(\rho_{nf}c_{np})_{bm}}{(\rho_{nf}c_{np})_{bf}}$, the Prandtl number $\Pr=\frac{\mu}{\rho_{nf}\kappa_{bm}}$, the Darcy number $Da=\frac{k_1}{d^2}$, the Brinkman-Darcy number $\tilde{D}a=\frac{\tilde{\mu}k_1}{\mu d^2}$, the Vadasz number $Va=\frac{\varepsilon\Pr}{Da}$, the Rayleigh number $Ra=\frac{\rho_{nf}g\alpha dk(T_0-T_1)}{\mu_{bf}\kappa_{bm}}$, the nanoparticles Rayleigh number $Rn=\frac{(\rho_{np}-\rho_{bf})(\varphi_1-\varphi_0)gk_1 d}{\mu\kappa_{bm}}$, the modified particle density increment $N_B = \frac{\in \rho_{np} c_{np}}{(\rho c)_{bf}}(\varphi_1 - \varphi_0)$, the Lewis number $Le = \frac{\kappa_{bm}}{D_B}$, the modified diffusivity ratio $N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\varphi_1 - \varphi_0)}$, the basic density Rayleigh number and $Rm = \frac{\{\rho_{np}\varphi_0 + \rho_{bf}(1-\varphi_0)\} g k_1 d}{\mu \kappa_{bm}}$.

The appropriate boundary conditions are

$$w = 0, T = T_0, \varphi = \varphi_0 \text{ at z = 0 and } w = 0, T = T_1, \varphi = \varphi_1 \text{ at z = 1.} \tag{9}$$

2.2 Basic solutions

The basic state is supposed to be motionless, thus

$$u = 0 = v = w, \mathrm{p} = \mathrm{p}(z), T = T_b(z), \phi = \varphi_b(z). \tag{10}$$

Using equation (10), equations (5-8) reduce to

$$0 = -\frac{dp_b}{dz} - Rm + RaT_b + R_n \varphi_b. \tag{11}$$

$$\frac{d^2 T_b}{dz^2} + N_B\left(\frac{d\phi_b}{dz}\right)\left(\frac{dT_b}{dz}\right) + (Le)^{-1} N_A N_B \left(\frac{dT_b}{dz}\right)^2 = 0, \tag{12}$$

$$\frac{d^2 \phi_b}{dz^2} + N_A (Le)^{-1} N_B \frac{d^2 T_b}{dz^2} = 0. \tag{13}$$

Using the boundary conditions (9) in the above equations and retaining only the first-order terms, we obtain the solution as

$$T_b = 1 - z \text{ and } \varphi_b = z. \tag{14}$$

Which verifies the results that are obtained by [25-26].

2.3 Perturbation equations

Suppose that the quantities are perturbed from the equilibrium position as

$$q_D = q^*, p = p_b + p^*, \phi = \phi_b + \phi^*, T = T_b + T^*, \varepsilon = \varepsilon_b + \varepsilon^* \text{ with } T_b = 1 - z, \ \varphi_b = z. \tag{15}$$

Where * denotes the perturbations in the physical quantities from the position of equilibrium. Equations (5-8) can be written after using equation (15) in the form of the non-dimensional perturbed equations as

$$\nabla . q_D{}^* = 0, \tag{16}$$

$$\frac{1}{Va}\frac{\partial q_D}{\partial t} = -\nabla p - \frac{1}{(1+\lambda_3)} q_D + \tilde{D}a \nabla^2 q_D + RaT\hat{e}_z - Rn\varphi\hat{e}_z, \tag{17}$$

$$\frac{1}{\sigma}\frac{\partial \varphi^*}{\partial t} + \frac{q_D{}^*}{\varepsilon} = \frac{1}{Le}\nabla^2 \varphi^* + \frac{N_A}{Le}\nabla^2 T^*, \tag{18}$$

$$\frac{\partial}{\partial t} T^* - q_D{}^* = \nabla^2 T^* + (Le)^{-1} N_B (\nabla T^* - \nabla \varphi^*) - 2 N_A N_B (Le)^{-1} \nabla T^*, \tag{19}$$

and the boundary conditions are:

$$w^* = 0, T^* = T_0^*, \varphi = \varphi_0^* \text{ at z = 0and } w^* = 0, T^* = T_1^*, \varphi = \varphi_1^* \text{ at z = 0 .} \tag{20}$$

Applying $\nabla \times \nabla = grad\, div - \nabla^2$ in equation (17), we obtain

$$\frac{1}{Va}\frac{\partial(\nabla^2 w)}{\partial t} + \frac{1}{(1+\lambda_3)}\nabla^2 w - \nabla^4 w \tilde{D}a = Ra\nabla_H^2 T - Rn\nabla_H^2 \phi. \tag{21}$$

Where, $\nabla_H{}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a horizontal Laplacian operator.

3. Linear stability analysis and normal modes

Suppose the perturbations quantities w^*, ϕ^* and T^* are of the form

$$w^*, \phi^*, T^*(x, y, z, t) = \left[W(z), \Phi(z), \Theta(z),\right] \exp(irx + isy + \omega t). \tag{22}$$

Equations (16) – (19) and (21) together reduce to the ordinary linear differential equations after using equation (22) as follows:

$$\left(\frac{\omega}{Va} + \frac{1}{1+\lambda_3} - \tilde{D}a\left(D^2 - k^2\right)\right)\left(D^2 - k^2\right)W + k^2 Ra\Theta - k^2 Rn\Phi = 0, \tag{23}$$

$$\frac{1}{\varepsilon}W - N_A\left(Le\right)^{-1}\left(D^2 - k^2\right)\Theta - \left\{\frac{1}{Le}\left(D^2 - k^2\right) - \frac{\omega}{\sigma}\right\}\Phi = 0, \tag{24}$$

$$W + \left\{\frac{N_B}{Le}D + \left(D^2 - k^2\right) - 2\frac{N_A N_B}{Le} - \omega\right\}\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{25}$$

Where, $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{dz}}$ and $k^2 = r^2 + s^2$ is the dimensionless resultant wave number.

For free-free boundaries, conditions (20) take the form

W=0, D^2W=0, Θ=0, Φ=0 at z = 0, and

W=0, D^2W=0, Θ=0, Φ=0 at z = 1. (26)

To solve equations (22-25), we now suppose that

$$\mathrm{W} = W_0 \sin\pi z,, \ \Theta = \Theta_0 \sin\pi z, \ \phi = \Phi_0 \sin\pi z \tag{27}$$

Which satisfies boundary conditions (26).

Using the solutions (27) in equations (23-25), the dispersion relation is given by

$$Ra = \frac{\pi^2 + p^2}{p^2}\left(\frac{1}{1+\lambda_3} + \frac{\omega}{V_a} + \tilde{D}a\left(p^2 + \pi^2\right)\right)\left(\pi^2 + k^2 + \omega\right) - \frac{\varepsilon N_A\left(\pi^2 + k^2\right) + Le\left(\pi^2 + k^2 + \omega\right)}{\left(\pi^2 + k^2\right)\sigma + \omega Le}\frac{\sigma}{\varepsilon}Rn \tag{28}$$

For a steady state, the real part of ω is zero. Therefore, we put $\omega = i\omega_i$ in equation (28), we obtain

$$Ra = \Delta_0 + iw_i\Delta_1, \tag{29}$$

Where,

$$\Delta_0 = \frac{\left(\pi^2 + k^2\right)}{p^2}\left[\frac{\pi^2 + k^2}{1+\lambda_3} + \tilde{D}a\left(\pi^2 + k^2\right)^2 - \frac{\omega_i^2}{Va}\right] - \frac{\left(\pi^2 + k^2\right)^2 \sigma\left(\varepsilon N_A + Le\right) + Le^2\omega_i^2}{\left(\pi^2 + k^2\right)^2 \sigma^2 + \omega_i^2 Le^2}\frac{\sigma}{\varepsilon}Rn, \tag{30}$$

And,

$$\Delta_1 = \frac{\left(\pi^2 + p^2\right)}{k^2}\left[\frac{1}{1+\lambda_3} + \tilde{D}a\left(\pi^2 + k^2\right) + \frac{\left(\pi^2 + k^2\right)}{Va}\right] - \frac{Le\sigma - \left(\varepsilon N_A + Le\right)}{\left(\pi^2 + k^2\right)^2 \sigma^2 + \omega_i^2 Le^2}\frac{\left(\pi^2 + k^2\right)\sigma}{\varepsilon}Rn. \tag{31}$$

4. The stationary convection

For the stationary convection (ω = 0). Equation (28) yields

$$Ra_S = \frac{\left(k^2 + \pi^2\right)^2}{k^2}\left[\frac{1}{1+\lambda_3} + \tilde{D}a\left(k^2 + \pi^2\right)\right] - \left(N_A + \frac{Le}{\varepsilon}\right)Rn. \tag{32}$$

When $\tilde{D}a \to 0$, equation (32) becomes:

$$Ra_S = \frac{1}{1+\lambda_3}\frac{\left(k^2+\pi^2\right)^2}{k^2} - \frac{N_A\varepsilon + Le}{\varepsilon}Rn\,. \qquad (33)$$

Which verifies the result obtained by [33].

When both $\tilde{D}a \to 0$ and $\lambda_3 \to 0$, the equation (32) reduces to

$$Ra_S = \frac{\left(k^2+\pi^2\right)^2}{k^2} - \frac{N_A\varepsilon + Le}{\varepsilon}Rn\,, \qquad (34)$$

When $\tilde{D}a \to 0$ and $\lambda_3 \to 0$, $Rn \to 0$ *and* $N_A \to 0$, equation (32) becomes:

$$Ra_S = \frac{\left(k^2+\pi^2\right)^2}{a^2}\,. \qquad (35)$$

Equations (34) and (35) are identical to the standard result in a Newtonian nanofluid.

By definition, *Rn* has a negative value which implies N_A also has a negative value for heavy nanoparticles $\left(i.e., \rho_{np} > \rho_{bf}\right)$.

In the following discussion, values of *Rn* and N_A are taken to be negative if not specified (i. e., bottom-heavy).

We now study the nature of $\frac{\partial Ra_S}{\partial \lambda_3}, \frac{\partial Ra_S}{\partial \tilde{D}a}, \frac{\partial Ra_S}{\partial Le}, \frac{\partial Ra_S}{\partial N_A}, \frac{\partial Ra_S}{\partial Rn}$ *and* $\frac{\partial Ra_S}{\partial \varepsilon}$ analytically.

Differentiating equation (30) partially w. r. t. λ_3, Rn, Le, N_A, N_B, Pr, we obtain

$$\frac{\partial Ra_S}{\partial \lambda_3} = -\frac{1}{\left(1+\lambda_3\right)^2}\frac{\left(\pi^2+k^2\right)^2}{k^2} < 0, \text{ for all } \lambda_3 \in \Re \text{ and } Rn \in \Re \qquad (36)$$

$$\frac{\partial Ra_S}{\partial \tilde{D}a} = \frac{\left(\pi^2+k^2\right)^3}{k^2}\tilde{D}a > 0, \text{ for all } Rn \in \Re \qquad (37)$$

$$\frac{\partial Ra_S}{\partial Le} = -\frac{Rn}{\varepsilon} > 0, \text{if } Rn < 0, \qquad (38)$$

$$\frac{\partial Ra_S}{\partial N_A} = -Rn > 0, \text{ if } Rn < 0, \qquad (39)$$

$$\frac{\partial Ra_S}{\partial Rn} = -\left(N_A + \frac{Le}{\varepsilon}\right) < 0, \text{ for all } Rn \in \Re \qquad (40)$$

$$\frac{\partial Ra_S}{\partial \varepsilon} = \frac{LeRn}{\varepsilon^2} < 0, \text{ if } Rn < 0. \qquad (41)$$

Equations (36) and (40) show the destabilizing influence of the Jeffrey parameter and nanoparticles Rayleigh number implying thereby postpones the stationary convection for both bottom/top-heavy nanoparticles distributions as attained by [31-33]. From equation (37), we notice that the Brinkman-Darcy number $\left(\tilde{D}a\right)$advances the stationary convection for both bottom/top-heavy nanoparticles distribution which accord the result that are analysed by [21-23, 25-29].

Equations (38) and (39) show that the Lewis number (*Le*) and modified diffusivity ratio (N_A) have stabilizing effect on the system for bottom-heavy nanoparticles distribution. From (41), it is noticed that medium porosity has a destabilizing effect on the system.

5. Result and Discussion

We now examine the results obtained earlier graphically by giving some numerical values to the parameters.

Figures (2-4) show that the Rayleigh number decreases with an increase in the value of the Jeffrey parameter, the nanoparticles Rayleigh number and the medium porosity which imply that these parameters have a destabilizing effect on the stationary convection. Thus, the Jeffrey parameter, the nanoparticles Rayleigh number and the medium porosity postpone the stationary convection which is in good agreement with the results derived by Sheu [23-24], Chand et al. [27] and Rana and Gautam [32]. It has been observed from Figures (5-7) that the stationary Rayleigh number increases with an increase in the value of the Lewis number, the modified diffusivity ratio, and the Brinkman-Darcy number respectively which imply that these parameters stabilise the stationary convection which are in good agreement with the results derived by Sheu [23-24] Yadav et al. [26] and Chand et al [27].

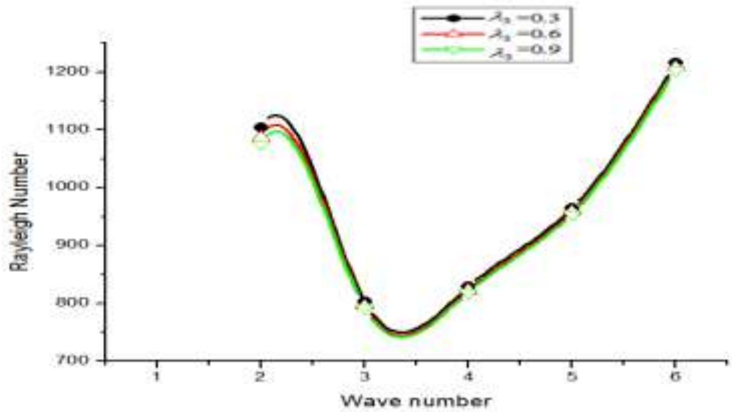

Figure 2: Variation with respect to λ_3.

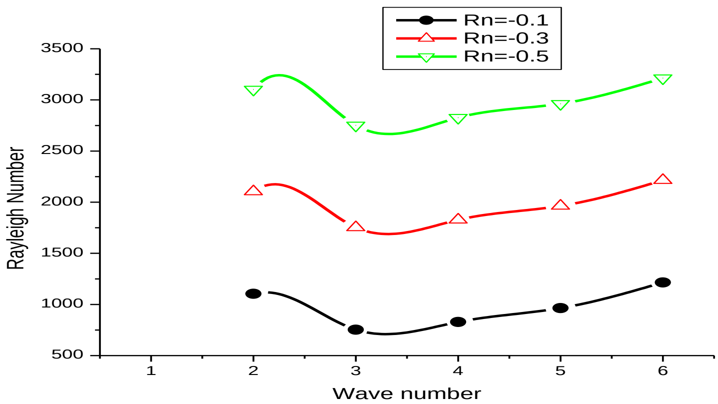

Figure 3: Variation with respect to Rn .

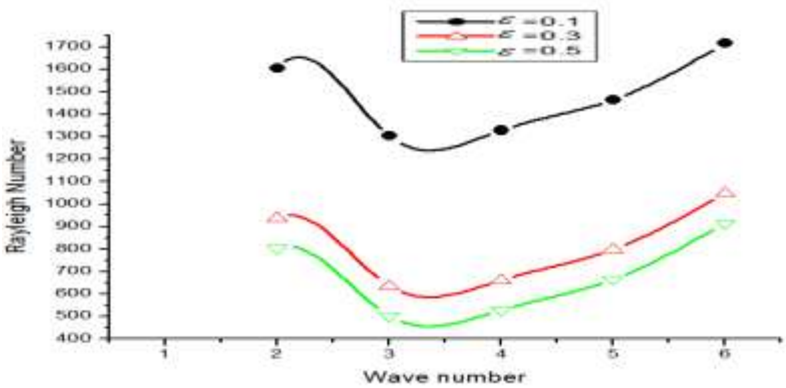

Figure 4: Variation with respect to ε. .

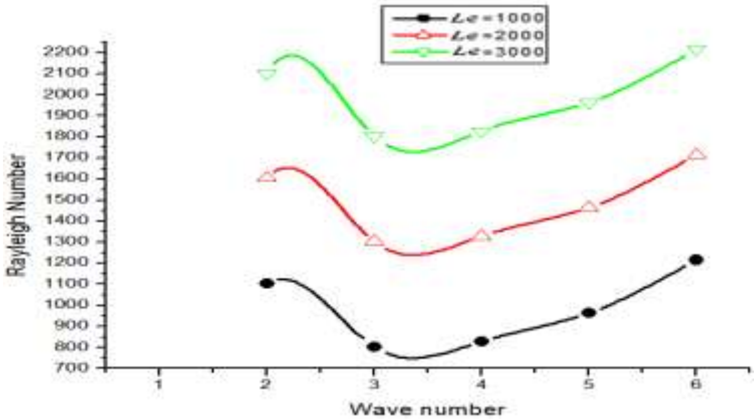

Figure 5: Variations with respect to Le.

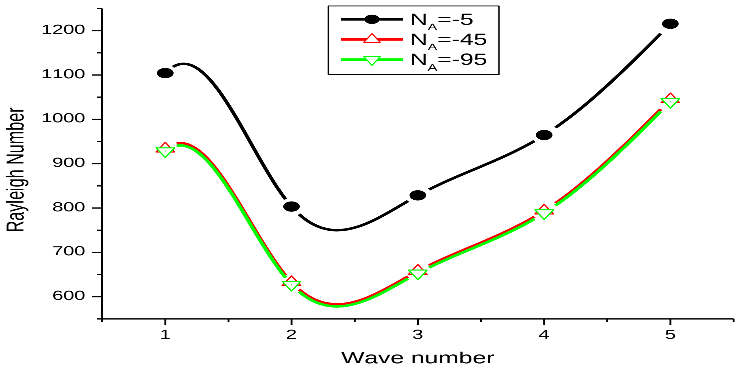

Figure 6: Variations with respect to N_A.

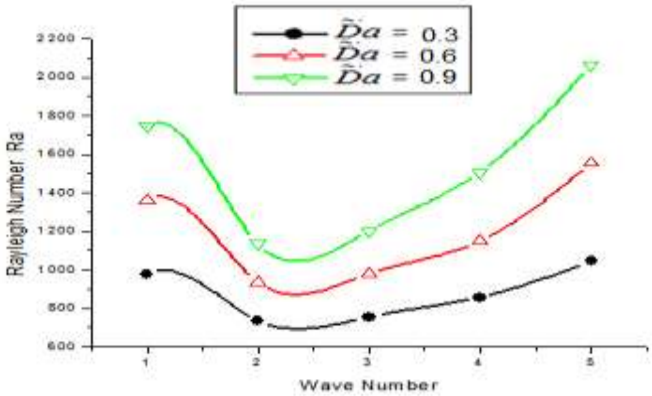

Figure 7: Variations with respect to $\tilde{D}a$.

6. The oscillatory convection

In oscillatory convection ($\omega_i \neq 0$), i.e., $\Delta_2 = 0$, which gives

$$\omega_i^2 = \frac{Le\sigma - (\varepsilon N_A + Le)}{\frac{1}{1+\lambda_3} + \tilde{D}a(\pi^2+k^2) + \frac{(\pi^2+k^2)}{Va}} \sigma \left(\frac{k^2}{\varepsilon Le^2}\right) Rn - \frac{(\pi^2+k^2)^2 \sigma^2}{Le^2}. \quad (42)$$

Equation (42) gives the frequency of oscillatory modes. If there is no positive value of the frequency of oscillatory modes ω_i^2, then equation (28) yields

$$Ra_{OSC}=\frac{\left(\pi^2+k^2\right)}{k^2}\left[\frac{1}{1+\lambda_3}+\tilde{D}a\left(\pi^2+k^2\right)+\frac{\left(\pi^2+k^2\right)}{Va}\right]-\frac{Le\sigma-\left(\varepsilon N_A+Le\right)}{+\left(\pi^2+k^2\right)^2\sigma^2+\omega_i^2 Le^2}\frac{\left(\pi^2+k^2\right)\sigma}{\varepsilon}Rn, \quad (43)$$

Where, ω_i^2 is given in equation (42).

If $Rn<0$ and $Le>\frac{\varepsilon N_A}{\sigma-1}$, then ω_i^2 is negative, hence oscillatory convection does not exist.

Thus, $Rn<0$ and $Le>\frac{\varepsilon N_A}{\sigma-1}$, are the sufficient conditions for the non-existence of oscillatory convection, the infraction of which does not certainly show the occurrence of oscillatory convection.

7. Conclusions

The thermal instability in a layer of Jeffrey nanofluid has been investigated analytically for free-free boundaries. It is found that the Jeffrey parameter, the medium porosity and the nanoparticles Rayleigh number destabilize the stationary convection, i. e., delay convection while the Lewis number, the Brinkman-Darcy number and the modified diffusivity ratio stabilize the physical system, i. e., promote convection. The sufficient conditions for the non-existence of oscillatory convection are obtained as $Rn < 0$ and $Le>\frac{\varepsilon N_A}{\sigma-1}$.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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