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Influence of Rotation and Inclined Magnetic Field with Mixed Convective Heat and Mass Transfer in an Inclined Symmetric Channel on Peristaltic Flow with Slip Conditions

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Abstract

In this paper, we study the impact of the rotation of an inclined magnetic felid and inclined symmetric channel with slip condition on peristaltic transport using incompressible non-Newtonian fluid. Slip conditions for the concentration and heat transfer are considered. We use the conditions on the fluid, namely infinite wavelength and low - Reynolds number to simplify the governed equations that described - motion flow, energy and concentration. These equations of the problem are solved by the perturbation technique and restricted the number of Bingham to a small value to find the final expression of the stream function. The Bingham number, Brinkman number, Soret number, Dufour number, temperature, Hartman number and other parameters are tested. The effects of different values of these parameters are discussed and illustrated graphically through the set of figures. Numerical results are computed by using MATHEMATICA software.

Keywords: **I**nclined magnetic Field, Peristaltic Transport, Slip condition, Heat transfer, Concentration, Rotation.

تأثير الدوران والمجال المغناطيسي المائل مع حرارة الحمل المختلط ونقل الكتلة في قناة متناظرة مائلة على التدفق التمعجي مع ظروف الانزلاق

حاتم ناهي محيسن جويد

قسم الرياضيات, مكتب الوزير, وزارة التعليم العالي والبحث العلمي, بغداد, العراق

الخلاصة

في هذا البحث قمنا بدراسة تأثير الدوران لمجال مغناطيسي مائل وقناة متناظرة مائلة مع حالة انزلاق على النقل التمعجي باستخدام سائل غير نيوتوني غير قابل للضغط. وضع في نظر الاعتبار حالة الانزلاق للتركيز ونقل الحرارة. في ظل هذه الظروف ، يتم استخدام السوائل ذات الطول الموجي اللانهائي وعدد رينولدز المنخفض لتبسيط المعادلات المحكومة التي توصف تدفق الحركة والطاقة والتركيز. يتم حل معادلات المسألة هذه من خلال تقنية الاضطراب ، ويقيد عدد بينغهام ذو القيمة الصغيرة للعثور على التعبير النهائي لوظيفة الدفق. يتم اختبار رقم بينغهام ورقم برينكمان ورقم سورت ورقم دوفور ودرجة الحرارة ورقم هارتمان ومعلمات أخرى. تمت مناقشة تأثيرات القيم المختلفة لهذه المعلمات وتوضيحها بيانياً من خلال مجموعة الأرقام. تم حساب النتائج العددية باستخدام برنامج MATHEMATICA.

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1. Introduction

The peristaltic motion is defined, it's the repose and shrinkage in the fluid motion along the wall. Many researchers have recently taken attention and are interested in peristalsis benefits which due to its various applications in medicine and physiology (ymph, blood, etc.), biomedical (swallowing of food, urine flow from the kidney to bladder, circulation blood in small vessels, etc.), physics and engineering. In many applications, it is primarily used in the current material business, such as the polymer industry and applications in medicine, particularly in the polishing of artificial hearts, the existence of slip condition plays a significant role of peristaltic flow in comparison to the absence of slip. Many researchers studied the effect of slip conditions for Newtonian and Non-Newtonian fluids, in addition to inclined channel on the peristaltic flow. Mohaisen H.N. et al. [1], examined how the rotation affects the analysis of mixed convection heat transfer for the peristaltic transport of viscoplastic fluid in an asymmetric channel. Murada et al. [2], studied how MHD effects on mixed convectional heat and mass transfer for peristaltic transport of viscoplastic fluid. Mohaisen H.N. et al. [3], applied the rotation on mixed convection heat transfer for the peristaltic transport of Bingham plastic fluid with induced magnetic. Mohaisen H.N. et al. [4], investigated how the rotation and a magnetic field affected the analysis of mixed convection heat transfer for peristaltic transport. Adnan et al. [5], studied and discussed the influence of an inclined magnetic field on the peristaltic transport of incompressible Bingham plastic fluid in an inclined symmetric channel with heat transfer and mass transfer along with slip requirements for heat transfer and concentration. Ajaz et al. [6], examined how an inclined channel and magnetic field affect the peristaltic flow of a micropolar fluid. Y. Elmhed, et al. [7], studied the peristaltic flow of a micropolar fluid in a vertical symmetric channel with rotation and heat. The aim study of T. Hayat, et al. [8], is how the Hall current affects the peristaltic movement of conducting Eyring-Powell fluid in an inclined symmetric channel is the goal. The Joule heating effect is taken into account while modelling the energy equation. Saba et al. [9], focused on how the Rabinowitsch fluid model's peristaltic action behaves when the inclination and rotation are present at the same time in a symmetric inclined channel. Heat source/sink analysis and the analysis of heat transmission are taken into account. Kamal et al. [10], demonstrated the presence of peristaltic transport of a pseudoplastic nanofluid through a porous material in a two-dimensional inclined tapering asymmetric channel under convective circumstances of heat and mass transfer. T.Hayat, et al. [11], examined the impact of an inclined magnetic field on the peristaltic flow of an incompressible Williamson fluid with heat and mass transfer in an inclined conduit. The circumstances of convection are used for the transmission of heat and mass. T. Sh. Ahmed [12], by utilising the influence of non-slip boundary conditions, there is a propensity to demonstrate the peristaltic activity of magnetohydrodynamics flow of carreau fluid with heat transfer influence in an inclined tapering asymmetric channel through porous media. T. Sh. Alshareef [13], analysed non-Newtonian fluid waveform flow through a porous medium of a non-symmetric sloping canal while being affected by rotation and a magnetic force that has been applied through an inclined route. In B.B. Divya, et al. [14], the aim of the study is to describe the peristaltic behaviour of a fluid with changing viscosity, a Bingham fluid. The fluid is thought to move through a porous media and is exposed to a magnetic field that has a significant inclination. In Abdulla et al. [15], the research focuses on efficient heat transfer with associating inclined magnetic field activity in an asymmetric channel through a porous medium. In the research of Najma Saleem, et al. [16], they studied peristaltic flow slip boundary conditions of the second velocity by the Jeffery fluid in the inclined magnetic field of means of heat and mass transfer and simplified under lubrication technique for an asymmetric channel.

In this study, the impact of the rotation of an inclined magnetic field also inclined channel on the peristaltic transport with slip conditions of concentration and mass for the Bingham plastic. Throughout this study, we use conditions, namely infinite wavelength and low - Reynolds number to simplify the equations of the problem with slip conditions. The change of behaviour, when using different values of parameter are discussed and illustrated graphically through a set of figures.

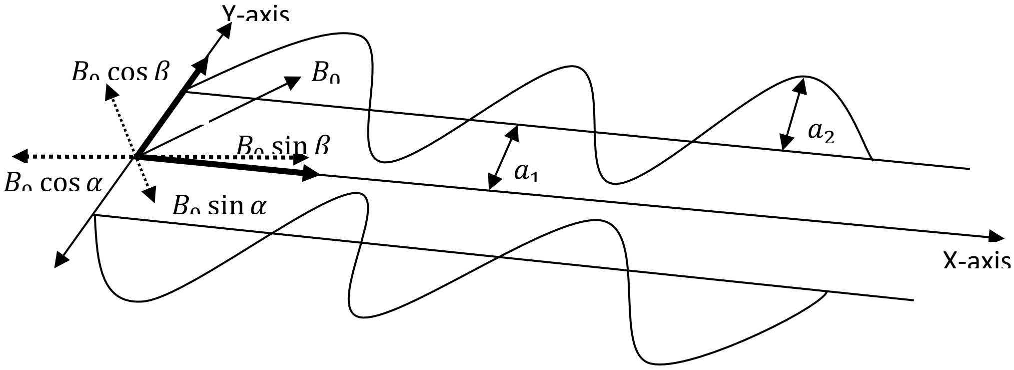
2. Constitutive Equation of the Problem

The peristaltic motion of a non-Newtonian of an incompressible fluid in two dimensional coordinate in a symmetric channel with width $(2a_1)$ is considered when both the channel and magnetic field are inclined at α, β angles. The x- axis is along the centreline of the channel, while y-axis is transverse to it, see Figure 1. The magnetic field $B = (B_0 sin\beta, B_0 cos\beta, 0)$is applied. The flow is created by sinusoidal wave trains reproduction with speed constant (c) along channel walls.

$\bar{H}_1(X,\bar{t}) = a_1 + a_2 sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})$ Upper wall (1)

$\bar{H}_2(X,\bar{t}) = -a_1 - a_2 sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})$ Lowe wall (2)

Where$\bar{H}_1, \bar{H}_2$ are the upper and lower wall respectively. The parameter a_2 is the amplitude wave, λ is the wavelength, t represents the time and c is the speed. The shell of the wall geometry is substance of the problem as follows:



3. Fundamental equations

The governed equations of the flow are by four coupled non-linear partial differential of continuity, momentum, energy and the concentrated equations, the system is written in two dimensional in laboratory frame as follows:

-The continuity equation is:

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad (3)$$

-The momentum equation on X-axis is given by

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) - \rho\Omega\left(\Omega\bar{U} + 2\frac{\partial \bar{V}}{\partial \bar{t}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} + \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} - \sigma {B_0}^2 \cos\beta\,(\bar{U}\cos\beta - \bar{V}\sin\beta) + \rho g \sin\alpha \quad (4)$$

- Momentum equation on Y-axis is

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) - \rho\Omega\left(\Omega\bar{V} - 2\frac{\partial \bar{U}}{\partial \bar{t}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} + \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} + \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} + \sigma {B_0}^2 \cos\beta\,(\bar{U}\cos\beta - \bar{V}\sin\beta) - \rho g \sin\alpha \quad (5)$$

Energy equation

$$\rho C_\rho\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{T}}{\partial \bar{Y}}\right) = K\left(\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2}\right) + \bar{S}_{\bar{X}\bar{X}}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{S}_{\bar{X}\bar{Y}}\left(\frac{\partial \bar{V}}{\partial \bar{X}} + \frac{\partial \bar{U}}{\partial \bar{Y}}\right) + \bar{S}_{\bar{Y}\bar{Y}}\frac{\partial \bar{V}}{\partial \bar{Y}} + \sigma {B_0}^2(\bar{U}^2 cos^2\beta + \bar{V}^2 sin^2\beta - 2\bar{U}\bar{V}\sin\beta\cos\beta) + \frac{DK_T}{C_S}\left(\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{Y}^2}\right) \quad (6)$$

- The concentrated equation is given by

$$\frac{\partial \bar{C}}{\partial \bar{t}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{Y}^2}\right) + \frac{DK_T}{T_m}\left(\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2}\right) \tag{7}$$

Where $C_S, K_T, T_m, D, \sigma, \rho, C_\rho, g$ represent the concentration susceptibility, the thermal diffusion ratio, the mean temperature, the coefficient of mass diffusivity, the electrical conductivity, the density of the fluid, specific heat and the acceleration due to gravity respectively. Ω is the rotation parameter and B_0 is the strength of the applied magnetics force. T is the temperature of material. The Bingham plastic fluid stress tensor is defined by [5] as follows:

$$\bar{S} = \left(\mu(\bar{Y}) + \frac{\tau_y}{\dot{\gamma}}\right)\bar{A}_1, for\ \tau > \tau_y \tag{8}$$

The Cauchy stress tensor is described by:

$$\bar{\tau} = -\bar{P}\bar{I} + \bar{S} \tag{9}$$

Where $\bar{P}$ is the pressure and $\bar{I}$ is the identity tensor and $\bar{A}_1$ is:

$$\bar{A}_1 = \nabla\bar{V} + (\nabla\bar{V})^T \tag{10}$$

With the boundary conditions:

$$\left.\begin{aligned}
&\bar{U} = 0 \quad at\ \bar{H}_1 \quad and\ \bar{H}_2\\
&K\frac{\partial \bar{T}}{\partial \bar{Y}} + w_1(T - T_0) = 0 \quad at \quad \bar{H}_1\\
&K\frac{\partial \bar{T}}{\partial \bar{Y}} + w_1(T_0 - T) = 0 \quad at \quad \bar{H}_2\\
&D\frac{\partial \bar{C}}{\partial \bar{Y}} + w_2(C - C_0) = 0 \quad at \quad \bar{H}_1\\
&D\frac{\partial \bar{C}}{\partial \bar{Y}} + w_2(C_0 - C) = 0 \quad at \quad \bar{H}_2
\end{aligned}\right\} \tag{11}$$

Where $w_1\ and\ w_2$ are the heat transfer coefficient, mass transfer coefficient to the mixed condition of heat transfer and mass transfer respectively. $C_0, and\ T_0$ are the concentration at the lower wall and the temperature at upper wall respectively.

Peristaltic motion in nature is an unsteady phenomenon, however, it can be assumed a steady by using the transformation from laboratory frame (fixed frame) $(\bar{X}, \bar{Y}, \bar{t}\,)$ to wave frame (move frame)$(\bar{x}, \bar{y})$. The relationship between coordinates, velocities and pressure in the laboratory frame is provided by the following transformations:

$$\bar{X} = \bar{x} + c\bar{t}, \bar{Y} = \bar{y}, \bar{U}(\bar{X}, \bar{Y}, \bar{t}\,) = \bar{u} + c, \bar{P}(\bar{X}, \bar{Y}, \bar{t}) = \bar{p}(\bar{x}, \bar{y}),$$
$$\bar{T}(\bar{X}, \bar{Y}, \bar{t}) = \bar{T}(\bar{x}, \bar{y}), \bar{C}(\bar{X}, \bar{Y}, \bar{t}) = \bar{C}(\bar{x}, \bar{y}), \bar{V}(\bar{X}, \bar{Y}, \bar{t}) = \bar{v}(\bar{x}, \bar{y}).$$
(12)

Where $\bar{u}, \bar{v}, \bar{p}$ are the velocity components and the pressure in the wave frame. The transform equations (1- 8) in wave frame with helping of equation (12) and normalize the resulting equations with dimensionless quantitie:

$$\left.\begin{aligned}
&\bar{x} = \frac{x\lambda}{2\pi}, \bar{y} = y\,a_1, \bar{u} = uc, \bar{v} = \delta vc, \bar{p} = \frac{p\lambda c\mu}{2\pi a_1}, \bar{H}_1 = h_1 a_1, \ \ \bar{H}_2 = h_2 a_1,\\
&\bar{t} = \frac{t\lambda}{2\pi c}, \delta = \frac{2\pi a_1}{\lambda}, R_e = \frac{\rho c a_1}{\mu}, \dot{\gamma} = \bar{\dot{\gamma}}\frac{\rho a_1{}^2}{\mu}, \bar{S} = \frac{S\mu c}{a_1}, M^2 = \frac{\sigma B_0{}^2 a_1{}^2}{\mu},\\
&P_r = \frac{\mu C_\rho}{K}, E_c = \frac{c^2}{T_0 C_\rho}, B_r = P_r E_c, \theta = \frac{T - T_0}{T_0}, B_n = \frac{\tau_y a_1}{\mu c}, \varphi = \frac{C - C_0}{C_1 - C_0},\\
&S_c = \frac{\upsilon}{D}, S_r = \frac{DK_T T_0}{\upsilon T_m (C_1 - C_0)}, F_r = \frac{c^2}{g a_1}, D_u = \frac{DK_T T_0 (C_1 - C_0)}{\mu C_\rho T_0}.
\end{aligned}\right\} \tag{13}$$

Where $\delta, R_e, P_r, E_c, B_r, \theta, B_n, \varphi, S_c, \upsilon, S_r, F_r, D_u, M$ are wave number, Reynolds number, Prandtl number, Eckert number, Brinkman number, temperature, Bingham number, concentration, Schmidt number, Kinematic viscosity, Soret number, Froude number, Dufour number and Hartman number respectively.

So, from the equations (1-8), we gets:

$$h_1 = 1+\in \sin x \quad (14)$$
$$h_2 = -1-\in \sin x \quad (15)$$
$$\delta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \quad (16)$$
$$\delta R_e\left((u+1)\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\rho {a_1}^2\Omega^2}{\mu_0}(u+1)-\delta R_e\frac{4\pi\Omega a_1}{\lambda}\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-$$
$$M^2\cos\beta\left((u+1)\cos\beta-v\delta\sin\beta\right)+\frac{R_e}{F_r}\sin\alpha \quad (17)$$
$$\delta^3 R_e\left((u+1)\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)-\delta^2\frac{\rho {a_1}^2\Omega^2}{\mu_0}v-\delta^2 R_e\Omega\frac{\partial(u+1)}{\partial t}=-\frac{\partial p}{\partial y}+\delta\frac{\partial S_{yy}}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+$$
$$M^2\delta\sin\beta\left((u+1)\cos\beta-v\delta\sin\beta\right)-\frac{R_e}{F_r}\delta\cos\alpha \quad (18)$$
$$\delta R_e P_r\left((u+1)\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+B_r\delta S_{xx}\frac{\partial u}{\partial x}+B_r\delta^2 S_{xy}\frac{\partial v}{\partial x}+B_r S_{yx}\frac{\partial u}{\partial y}+$$
$$B_r\delta S_{yy}\frac{\partial v}{\partial y}+M^2B_2\left((u+1)^2cos^2\beta+\delta^2v^2sin^2\beta+2\delta v\sin\beta\cos\beta\,(u+1)\right)+$$
$$D_uP_r\left(\delta^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right) \quad (19)$$
$$\delta R_e\left((u+1)\frac{\partial\varphi}{\partial x}+v\frac{\partial\varphi}{\partial y}\right)=\frac{1}{S_c}\left(\delta^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)+S_r\left(\delta^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right) \quad (20)$$
$$S_{xy}=\mu(y)\frac{\partial u}{\partial y}+B_n \quad (21)$$

Which $\in=\frac{a_2}{a_1}$ is called amplitude ratio.

If ψ(x, y, t) is the stream function, Then the velocity components in term of the streams function are $u=\psi_y \; and \; v=-\delta\psi_x$. The final boundary conditions:

$$\left.\begin{aligned}
&u=-1 \; at \; h_1 \; and \; h_2\\
&\frac{\partial\psi}{\partial x}=-1 \; at \; h_1 \; and \; h_2\\
&\frac{\partial\theta}{\partial y}+B*\theta=0 \; at \; h_1\\
&\frac{\partial\theta}{\partial y}-B*\theta=0 \; at \; h_2\\
&\frac{\partial\varphi}{\partial y}+A*\varphi=0 \; at \; h_1\\
&\frac{\partial\varphi}{\partial y}-A*\varphi=0 \; at \; h_2
\end{aligned}\right\} \quad (22)$$

Where $B=\frac{w_1a_1}{K} \; and \; A=\frac{w_2a_1}{D}$ represent the heat transfer Biot number and the mass transfer Biot number respectively. The continuity equation (16) satisfies identically, when applying $(\delta\ll 1)$, small Reynolds number and long wavelength, the equations from (17-20), gets:

$$\frac{\partial p}{\partial x}=\frac{\rho {a_1}^2\Omega^2}{\mu_0}\left(\psi_y+1\right)+\frac{\partial S_{xy}}{\partial y}-M^2(\cos\beta)^2\left(\psi_y+1\right)+\frac{R_e}{F_r}\sin\alpha \quad (23)$$
$$\frac{\partial p}{\partial y}=0 \quad (24)$$
$$\theta_{yy}+B_rS_{xy}\frac{\partial u}{\partial y}+M^2B_r\left(\left(\psi_y+1\right)^2cos^2\beta\right)+D_uP_r\varphi_{yy}=0 \quad (25)$$
$$\frac{1}{S_c}\varphi_{yy}+S_r\theta_{yy}=0 \quad (26)$$

$F=q-2=\int_{h_2}^{h_1}\frac{\partial p}{\partial y}dy=\psi(h_1)-\psi(h_2)$, where F and q are the dimensionless mean flow rate in fixed and wave frames, respectively.

4. The solution to the problem

The resulting system consists of highly nonlinear partial differential equations, due to the hardness of finding the exact solution, we do pressure elimination between (23) and (24) by derivation with regard to y, the result is as follows:

$$\left(L^2 - M^2(\cos\beta)^2) * \psi_{yy}\right) + \psi_{yyyy} = 0 \tag{27}$$

$$\theta_{yy} + B_r((\psi_{yy})^2 + B_n\psi_{yy}) + M^2B_r\left(\left(\psi_y + 1\right)^2 cos^2\beta\right) + D_uP_r\varphi_{yy} = 0 \tag{28}$$

$$\frac{1}{S_c}\varphi_{yy} + S_r\theta_{yy} = 0 \tag{29}$$

Where $L^2 = \frac{\rho {a_1}^2\Omega^2}{\mu_0}$.

Using the perturbation method and Mathematica software insert the following expressions:

$$\psi = \psi_0 + B_n\psi_1 + O(B_n)^2 \tag{30}$$

$$\theta = \theta_0 + B_n\theta_1 + O(B_n)^2 \tag{31}$$

$$\varphi = \varphi_0 + B_n\varphi_1 + O(B_n)^2 \tag{32}$$

Substituting equations 30, 31 and 32 into equations 27, 28 and 29 respectively, gets after using the power of B_n changing (0, 1) to obtain the following systems, namely the zero and first the order systems:

2.1 The zero-order system

When the $({B_n}^0)$, the system will be in the following form:

$$\frac{\partial^4\psi_0}{\partial y^4} + (L^2 - M^2(\cos\beta)^2)\frac{\partial^2\psi_0}{\partial y^2} = 0. \tag{33}$$

$$\frac{\partial^2\theta_0}{\partial y^2} + B_r\left((\frac{\partial^2\psi_0}{\partial y^2})^2 + B_n\frac{\partial^2\psi_0}{\partial y^2}\right) + M^2B_r\left(\left(\frac{\partial\psi_0}{\partial y} + 1\right)^2 cos^2\beta\right) + D_uP_r\frac{\partial^2\varphi_0}{\partial y^2} = 0. \tag{34}$$

$$\frac{1}{S_c}\frac{\partial^2\varphi_0}{\partial y^2} + S_r\frac{\partial^2\theta_0}{\partial y^2} = 0. \tag{35}$$

With the boundary conditions:

$$\psi_0 = \frac{q}{2}, \frac{\partial\psi_0}{\partial y} = -1, \frac{\partial\theta_0}{\partial y} + B\ \theta_0 = 0, \frac{\partial\varphi_0}{\partial y} + A\ \varphi_0 = 0\ at\ y = h_1$$

$$\psi_0 = -\frac{q}{2}, \frac{\partial\psi_0}{\partial y} = -1, \frac{\partial\theta_0}{\partial y} - B\ \theta_0 = 0, \frac{\partial\varphi_0}{\partial y} - A\ \varphi_0 = 0\ at\ y = h_2$$

The solution of equations (33-35) with the condition above, one can get:

$$\psi_0 = \text{t3} + y\text{t4} + \frac{2e^{-\frac{y\sqrt{-2L+M+M\text{Cos}[2\beta]}}{\sqrt{2}}}(e^{\sqrt{2}y\sqrt{-2L+M+M\text{Cos}[2\beta]}}\text{t1}+\text{t2})}{-2L+M+M\text{Cos}[2\beta]};$$

$$\theta_0 = \text{t5} + y\text{t6} - \frac{1}{2(-2L+M+M\text{Cos}[2\beta])^2(-1+D_uP_rS_cS_r)}(e^{\sqrt{2}y\sqrt{-2L+M+M\text{Cos}[2\beta]}}\text{t1}^2(2L - M - M\text{Cos}[2\beta] - 2M\text{Cos}^2[\beta]) + e^{-\sqrt{2}y\sqrt{-2L+M+M\text{Cos}[2\beta]}}\text{t2}^2(2L - M - M\text{Cos}[2\beta] - 2M\text{Cos}^2[\beta]) + y^2(-2L + M + M\text{Cos}[2\beta])(2\text{t1t2}(2L - M - M\text{Cos}[2\beta]) - M(M - 4\text{t1t2} + 2M\text{t4} + M\text{t4}^2 - 2L(1 + \text{t4})^2 + M(1 + \text{t4})^2\text{Cos}[2\beta])\text{Cos}^2[\beta]) + 4e^{-\frac{y\sqrt{-2L+M+M\text{Cos}[2\beta]}}{\sqrt{2}}}\text{t2}(2\sqrt{2}M(1 + \text{t4})\sqrt{-2L + M + M\text{Cos}[2\beta]}\text{Cos}^2[\beta] + (2L - M - M\text{Cos}[2\beta])B_n) - 4e^{\frac{y\sqrt{-2L+M+M\text{Cos}[2\beta]}}{\sqrt{2}}}\text{t1}(2\sqrt{2}M(1 + \text{t4})\sqrt{-2L + M + M\text{Cos}[2\beta]}\text{Cos}^2[\beta] + (-2L + M + M\text{Cos}[2\beta])B_n))B_r;$$

$$\varphi_0 = \mathrm{t7} + y\mathrm{t8} - \frac{1}{2(-2L+M+M\mathrm{Cos}[2\beta])^2(-1+D_uP_rS_cS_r)}(-e^{\sqrt{2}y\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}}\mathrm{t1}^2(2L - M - M\mathrm{Cos}[2\beta] - 2M\mathrm{Cos}^2[\beta]) - e^{-\sqrt{2}y\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}}\mathrm{t2}^2(2L - M - M\mathrm{Cos}[2\beta] - 2M\mathrm{Cos}^2[\beta]) - y^2(-2L+M+M\mathrm{Cos}[2\beta])(2\mathrm{t1t2}(2L-M-M\mathrm{Cos}[2\beta]) - M(M - 4\mathrm{t1t2} + 2M\mathrm{t4} + M\mathrm{t4}^2 - 2L(1+\mathrm{t4})^2 + M(1+\mathrm{t4})^2\mathrm{Cos}[2\beta])\mathrm{Cos}^2[\beta]) + 4e^{-\frac{y\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}}{\sqrt{2}}}\mathrm{t2}(-2\sqrt{2}M(1+\mathrm{t4})\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}\mathrm{Cos}^2[\beta] + (-2L+M + M\mathrm{Cos}[2\beta])B_n) + 4e^{\frac{y\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}}{\sqrt{2}}}\mathrm{t1}(2\sqrt{2}M(1+\mathrm{t4})\sqrt{-2L+M+M\mathrm{Cos}[2\beta]}\mathrm{Cos}^2[\beta] + (-2L+M+M\mathrm{Cos}[2\beta])B_n))B_rC_sS_r;$$

4.2 The first-order system

When the $({B_n}^1)$, the system will be in the following form:

$$\frac{\partial^4\psi_1}{\partial y^4} - y\ \frac{\partial^4\psi_0}{\partial y^4} - 2\frac{\partial^3\psi_0}{\partial y^3} + (L^2 - M^2(\cos\beta)^2)\frac{\partial^2\psi_0}{\partial y^2} = 0$$

$$\frac{\partial^2\theta_1}{\partial y^2} + 2B_r\ \frac{\partial^2\psi_1}{\partial y^2}\ \frac{\partial^2\psi_0}{\partial y^2} - y\ B_r\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2 + B_nB_r\frac{\partial^2\psi_1}{\partial y^2} + 2M^2B_r\left(\frac{\partial\psi_0}{\partial y}+1\right)^2 cos^2\beta\frac{\partial\psi_1}{\partial y} + D_uP_r\frac{\partial^2\varphi_1}{\partial y^2} = 0.$$

$$\frac{1}{S_c}\frac{\partial^2\varphi_1}{\partial y^2} + S_r\frac{\partial^2\theta_1}{\partial y^2} = 0.$$

With the boundary conditions:

$$\psi_1 = -\frac{q}{2}, \frac{\partial\psi_1}{\partial y} = 0, \frac{\partial\theta_1}{\partial y} + B\ \theta_1 = 0, \frac{\partial\varphi_1}{\partial y} + A\ \varphi_1 = 0\ at\ y = h_1$$

$$\psi_1 = -\frac{q}{2}, \frac{\partial\psi_1}{\partial y} = 0, \frac{\partial\theta_1}{\partial y} - B\ \theta_1 = 0, \frac{\partial\varphi_1}{\partial y} - A\ \varphi_1 = 0\ at\ y = h_2$$

Due to the long terms of $(\psi_1, \theta_1, \varphi_1)$, so we will not include any of them here.

5. Result and Discussions:

In this part, we will give the explanation in the next paragraphs, we also analyze and simplify the equations by setting the boundary conditions. Further, we will discuss the velocity, temperature, shear stress, pressure gradient, concentration and stream function and we study the effect of the parameters on these cases.

5.1. Velocity Profile

In this section, we discuss the change velocity behavior with different parameters given that x=0.3. Observed that the velocity in Figure 2 increases when an increasing values of the Bingham number, the flow rate and rotation. This is clearly seen in Figure 2 (A, B and F), with respect to the Bingham number it is increased after crossing from the center of the channel. Also, it is observed that the effect of rotation on velocity is continuously even crossing the center of the channel and it is nearly the wall of the channel. An increase in the values of (M, β, ϵ) implies that to decreasing the value of velocity decreases as it can be obviously seen in Figure 2 (C, D and E). Also with the increasing value to amplitude wave value the wave near the wall of the channel.

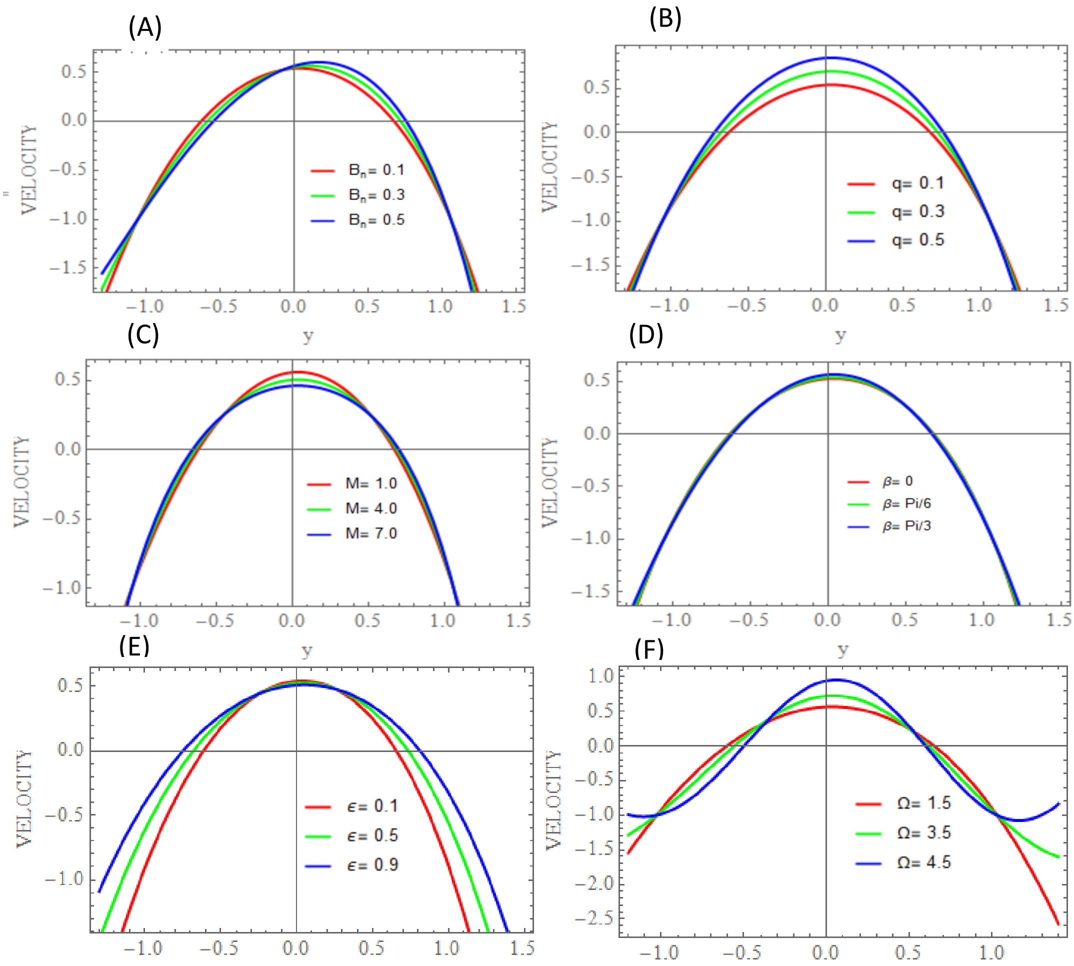
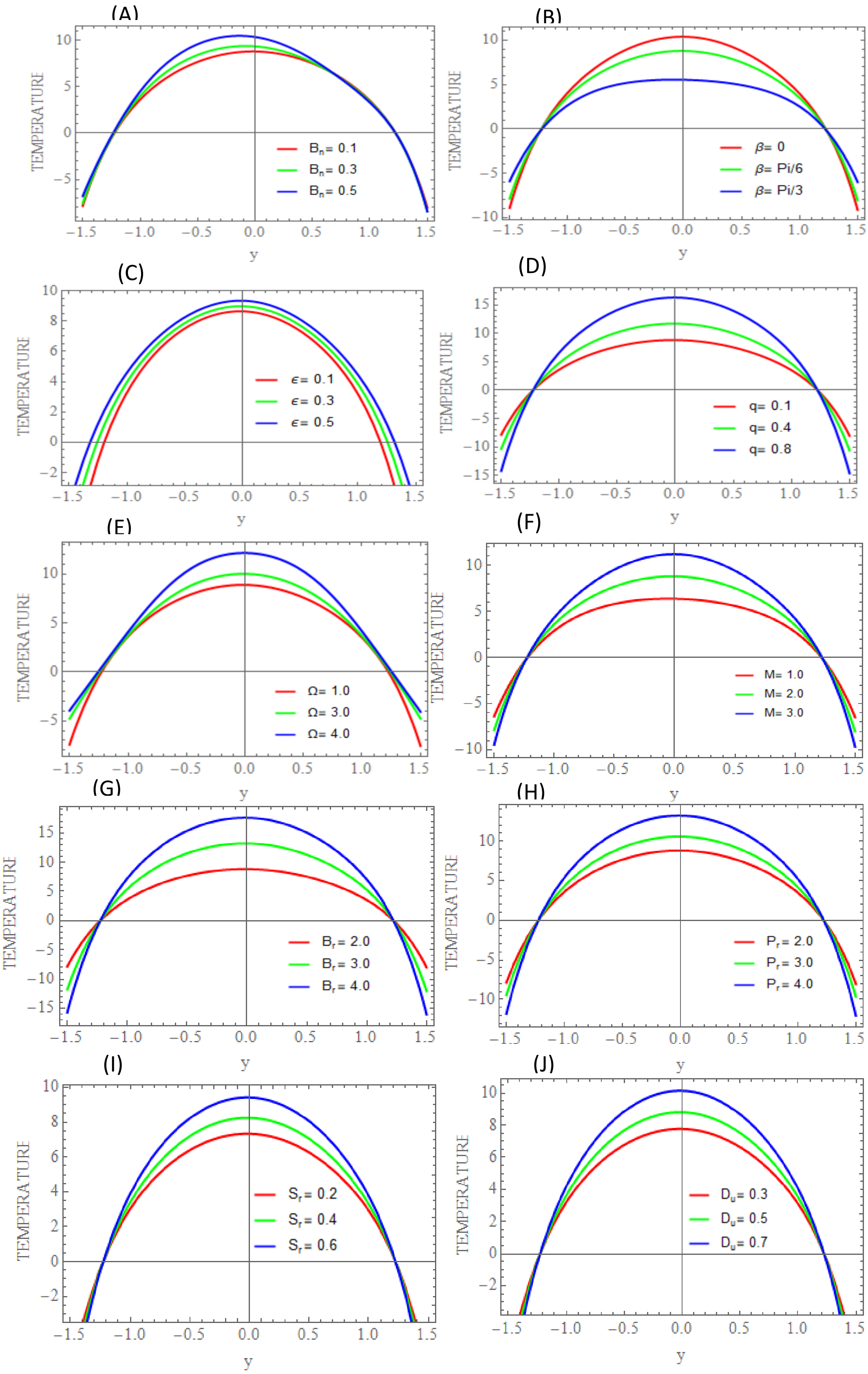


Figure 2: Velocity with different values of parameters $\{B_n = 0.1, \beta = \text{Pi}/6, M = 2.0, q = 0.1, \Omega = 0.2, \epsilon = 0.1\}$

5.2 Temperature Distribution

We illustrate the impact of different parameters on the temperature profile in Figure 3 (A-L) with x=0.3, it is observing that an increase of values of parameters $B_n, \in, q, \Omega, M, B_r, P_r, S_r, D_u$ *and* S_c implies that the value of temperature increases. Also, it is clearly in case of the Bingham number starts before arrives to the center of the channel, see Figure 3(A, C, D, E, F, G, H, I, J and K). While an increase in (β, B) leads to decrease in the temperature value. This is clearly in Figure 3 (B and L).



(A)
TEMPERATURE
y
B_n= 0.1
B_n= 0.3
B_n= 0.5
(B)
β= 0
β= Pi/6
β= Pi/3
(C)
ϵ= 0.1
ϵ= 0.3
ϵ= 0.5
(D)
q= 0.1
q= 0.4
q= 0.8
(E)
Ω= 1.0
Ω= 3.0
Ω= 4.0
(F)
M= 1.0
M= 2.0
M= 3.0
(G)
B_r= 2.0
B_r= 3.0
B_r= 4.0
(H)
P_r= 2.0
P_r= 3.0
P_r= 4.0
(I)
S_r= 0.2
S_r= 0.4
S_r= 0.6
(J)
D_u= 0.3
D_u= 0.5
D_u= 0.7

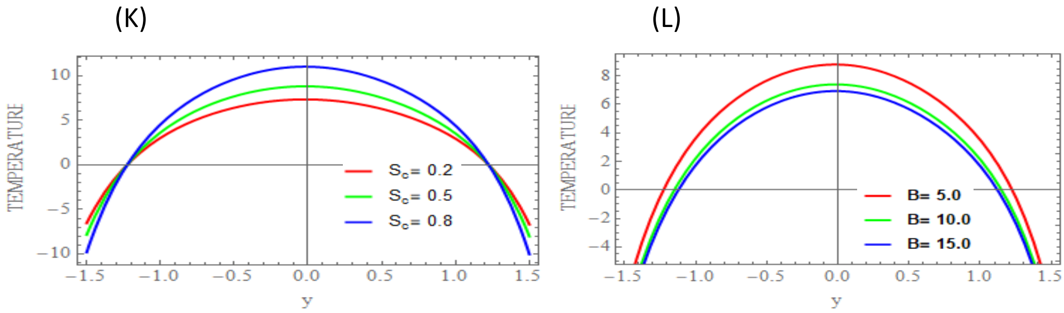


Figure 3: Temperature with different values of parameters $\{B_n = 0.1, \beta = \mathrm{Pi}/6, M = 2.0, q = 0.1, \Omega = 0.2, \epsilon = 0.1, B = 5, B_r = 2, P_r = 2, S_r = 0.5, D_u = 0.5, S_c = 0.5\}$

5.3 Gradient Pressure Distribution

In this section, we discussion how the behavior of gradient pressure changes with changing values of parameters with y=0.5, it is observed that increasing values of (B_n, q, M, F_r) leads to decrease in the value of gradient pressure see Figures 4 (A, B, E and I). Otherwise, it leads to increase in its value, see Figures 4(C, D, F, G and H).

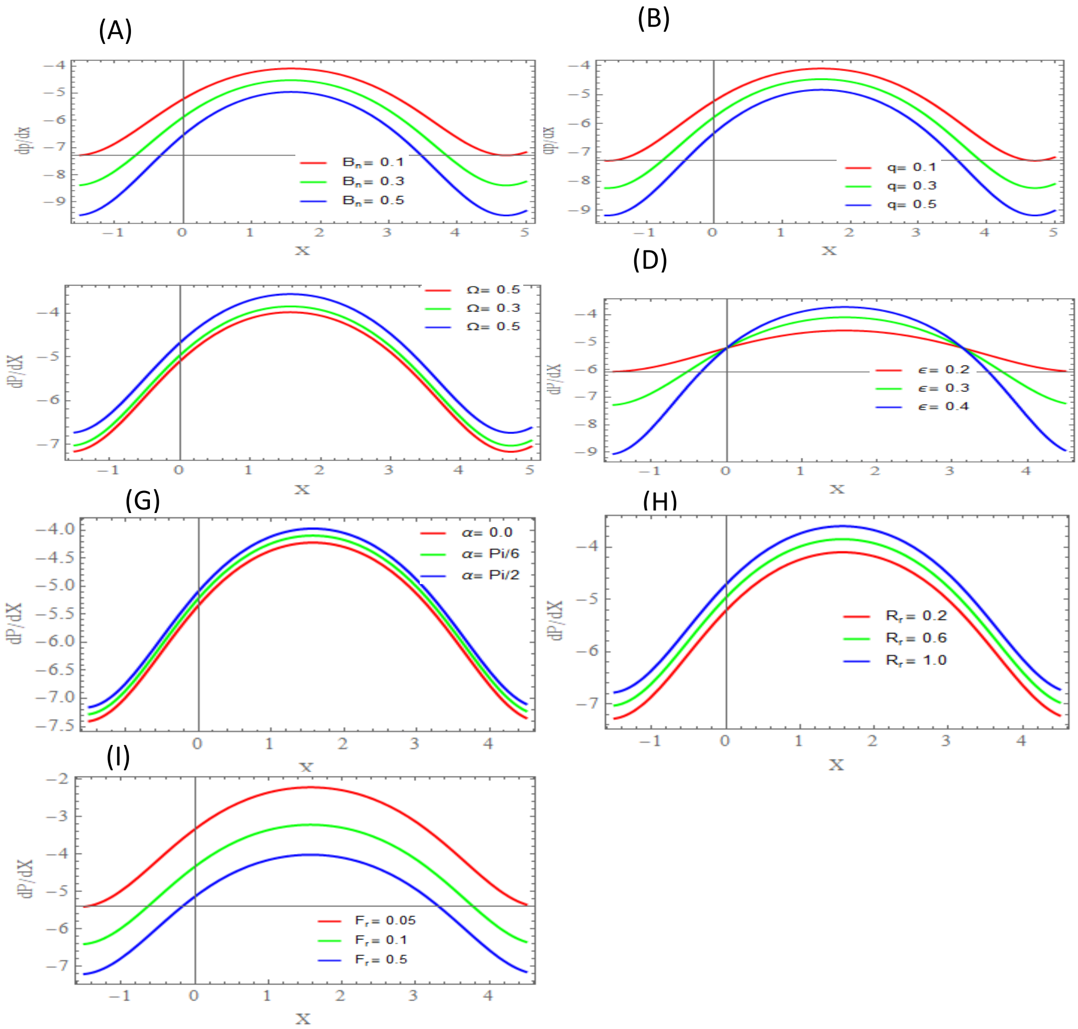


Figure 4: Gradient Pressure with different values of parameters $\{B_n = 0.1, \beta = \mathrm{Pi}/6, M = 2.0, q = 0.1, \Omega = 0.2, \epsilon = 0.1, \alpha = \mathrm{Pi}/6, R_r = 0.2, F_r = 0.8\}$

5.4. Shear Stress Profile

In this section, we discuss the influence of different parameters on the local shear stress with y=0.5. It is obvious that an increase in the value of $(B_n, \in, M)$ leads to increase in the shear stress, see Figure 5(A, D, E), while a decreasing in values of parameter (q, Ω, β) implies that the values of local shear stress increase, see Figure 5(B, C, F).

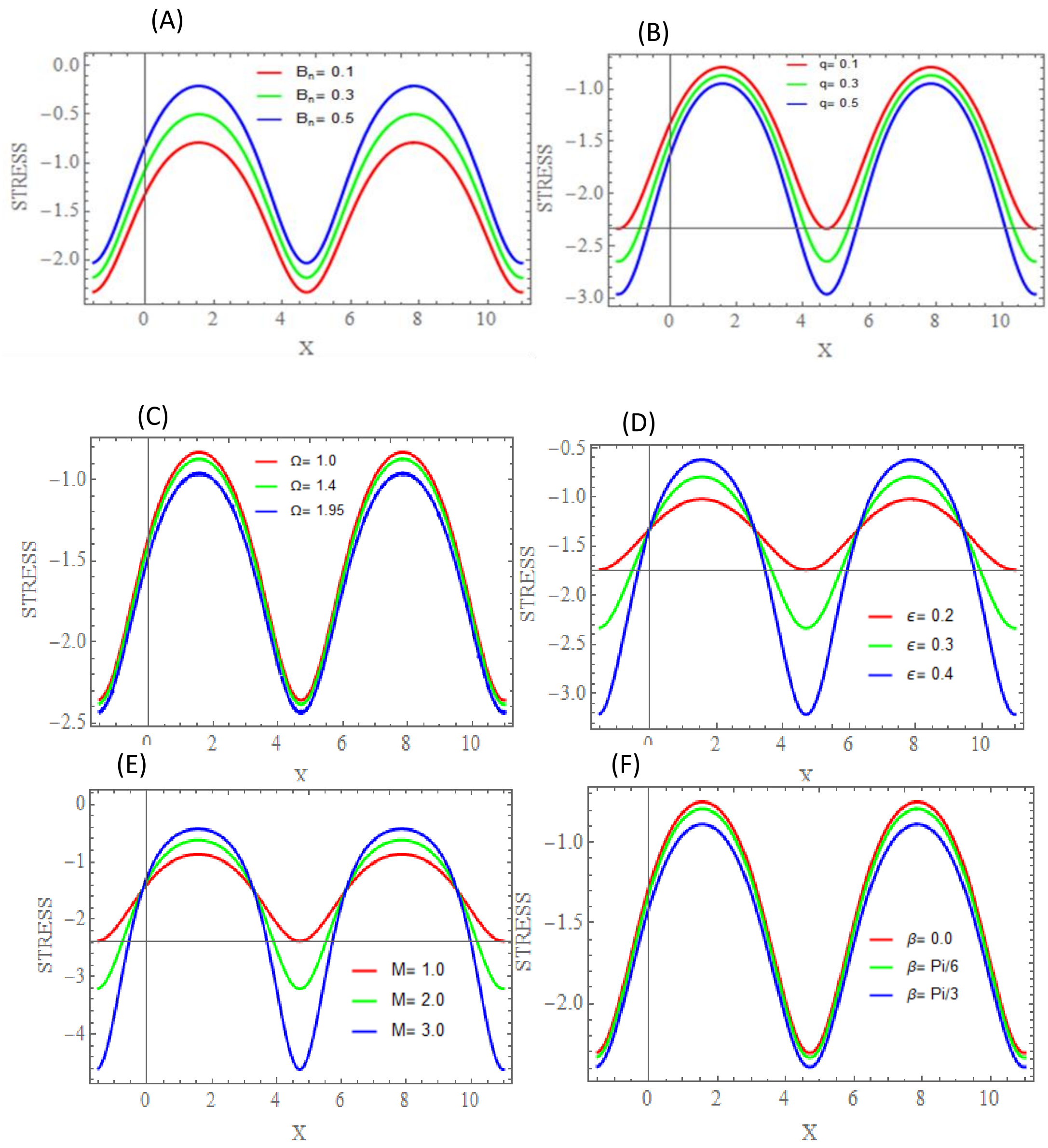
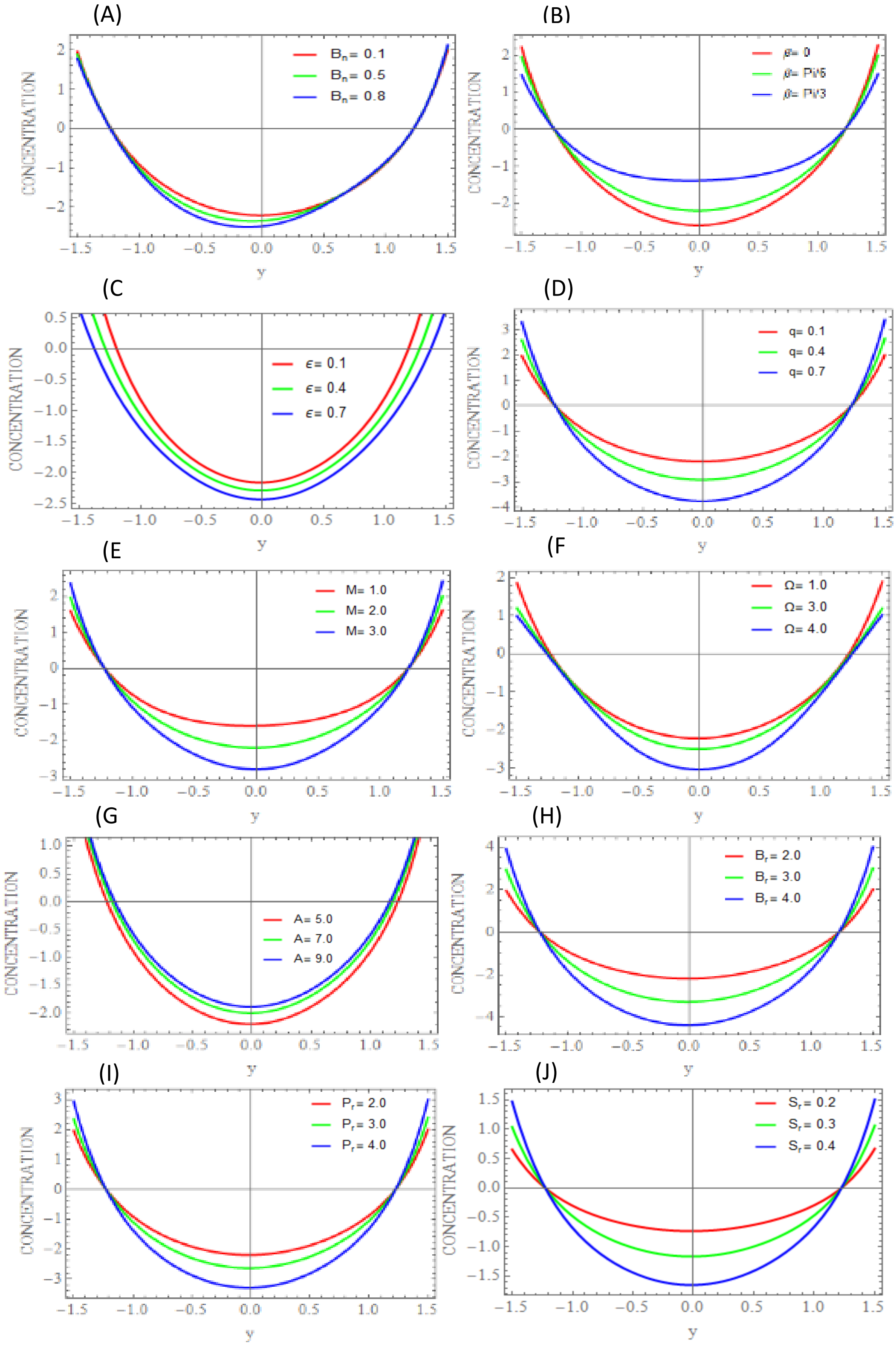


Figure 5: Shear Stress with different values of parameters $\{B_n = 0.1, \beta = \text{Pi}/6, M = 2.0, q = 0.1, \Omega = 0.2, \epsilon = 0.2\}$

5.5 Concentration Distribution

In this paragraph, we discuss how the concentration behaviour variation with different values of the parameter with x=0.3. The variation profile is obviously in Figure 6, it is observed that an increasing in the values of parameters $B_n, \epsilon, q, M, \Omega, B_r, P_r, S_r, D_u$ *and* S_c leads to decrease in the value of the concentration, see Figures 6(A, C, D, E, F, H, I, J, L and M) respectively, while a increasing in the values β *and* A leads to increase the value of the concentration, see Figure 6(B and G). We observed the Bingham started to lower before the centre of the channel.


(A)
B_n= 0.1
B_n= 0.5
B_n= 0.8
(B)
β= 0
β= Pi/6
β= Pi/3
(C
ϵ= 0.1
ϵ= 0.4
ϵ= 0.7
(D)
q= 0.1
q= 0.4
q= 0.7
(E
M= 1.0
M= 2.0
M= 3.0
(F
Ω= 1.0
Ω= 3.0
Ω= 4.0
(G)
A= 5.0
A= 7.0
A= 9.0
(H)
B_r= 2.0
B_r= 3.0
B_r= 4.0
(I)
P_r= 2.0
P_r= 3.0
P_r= 4.0
(J)
S_r= 0.2
S_r= 0.3
S_r= 0.4
CONCENTRATION
y

(L) (M)

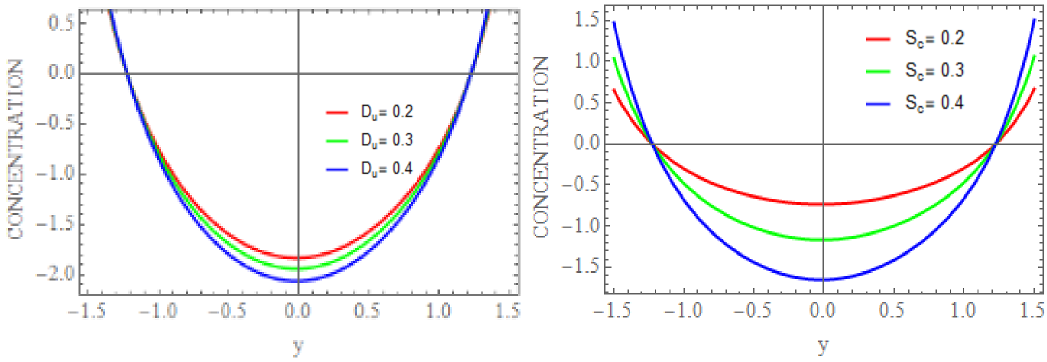
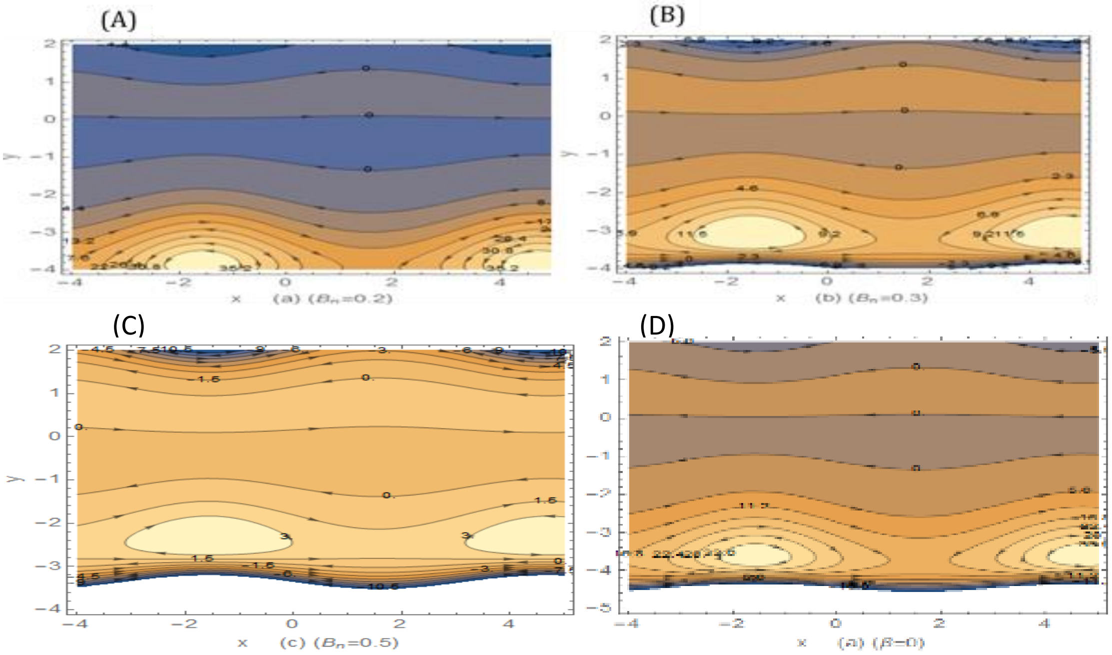
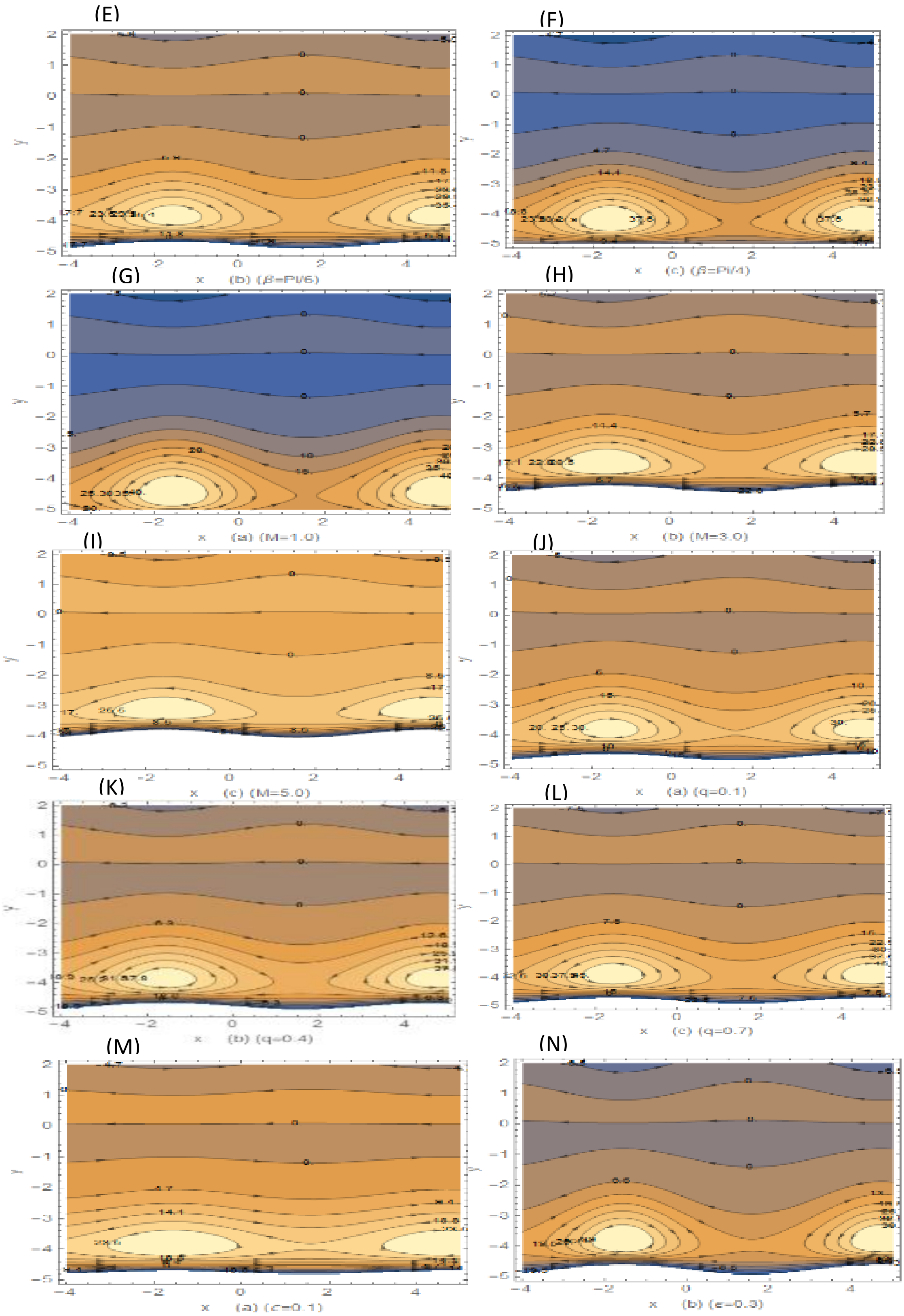


Figure 6: Concentration with different values of parameters $\{B_n = 0.1, \beta = \mathrm{Pi}/6, M = 2.0, q = 0.1, \Omega = 0.2, \epsilon = 0.1, B = 5, B_r = 2, P_r = 2, S_r = 0.5, D_u = 0.5, S_c = 0.5\}$

5.6. Stream Function

The phenomenon of trapping is an interesting subject in the peristaltic transport. The figuration of an inside diffusing bolus of fluid that crosses locked stream lines is called trapping and this trapped bolus is mobile ahead a crosses the peristaltic waves with the selfsame speed as the wave. Physically, this phenomenon appears in blood thrombus and locomotion the bolus of blood. If the value of B_n increases, we note that the number of bolus is decreasing and expanding toward of walls, see Figure 7(A, B and C), while an increase in the value of β leads to an increase in the number of bolus and nears to the centre, see Figure 7(D, E and F). An increase in the value of M leads to increase in the number of bolus and disappear upward, see Figure 7(G, H and I). In the case of increasing the value of q, it leads to the separating of boluses, and keeps away from the centre of the channel, see Figure 7(J, K and L). An increase in the value of ϵ implies that the bolus starts to separate from other and converge to each at the canal walls, see Figure 7(M, N and O), when an increasing in the value of Ω leads to decrease in the number of boluses and keeps away from each other toward of the canal of walls, see Figure 7(P, R and S).




(E)
(b) (β=Pi/6)
(F)
(c) (β=Pi/4)
(G)
(a) (M=1.0)
(H)
(b) (M=3.0)
(I)
(c) (M=5.0)
(J)
(a) (q=0.1)
(K)
(b) (q=0.4)
(L)
(c) (q=0.7)
(M)
(a) (ε=0.1)
(N)
(b) (ε=0.3)

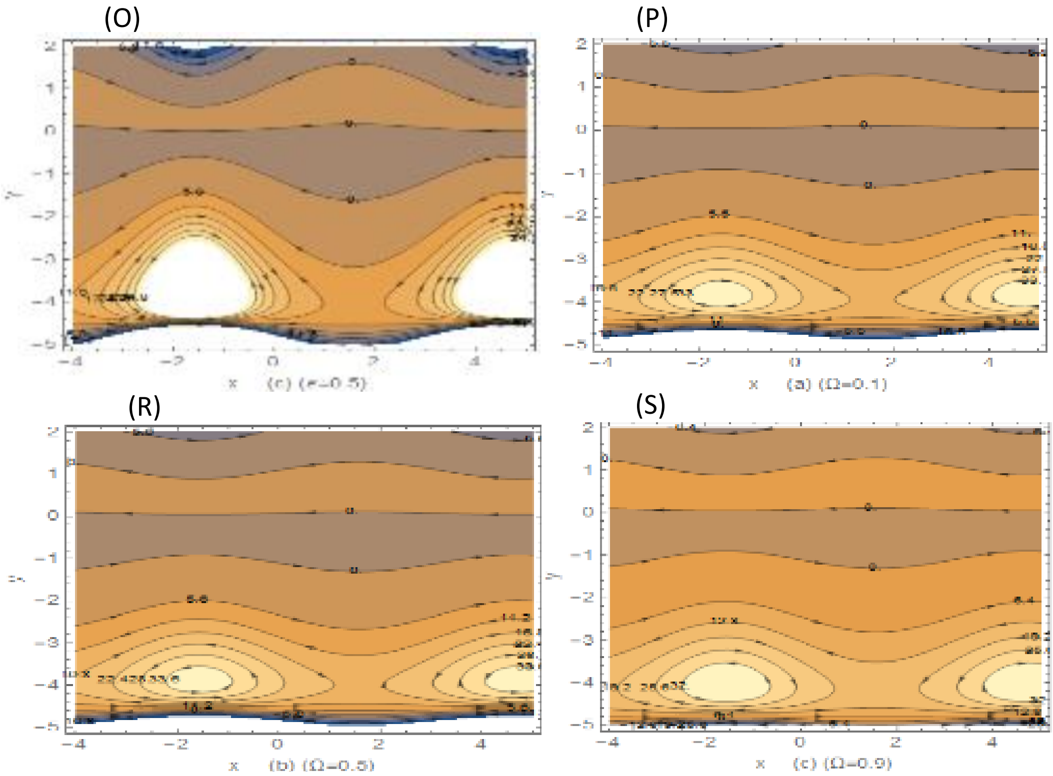


Figure 7: Stream Function variation with different values of parameters $\{B_n = 0.1, \beta = \text{Pi}/6, M = 2.0, q = 0.2, \Omega = 0.2, \epsilon = 0.2\}$

6. Conclusions

1. In general, the velocity distribution is a parabola, as well as, the velocity increase if the values of the Bingham number, flow rate and rotation are increasing, conversely an increase in the values of (M, β, ϵ) leads to a decreasing in the value of velocity.
2. Note that, the temperature profile is also parabola, when an increasing in the values of $B_n, \in, q, \Omega, M, B_r, P_r, S_r, D_u, and\ S_c$ implies that increases in the value of temperature. However, it decreases when an increase in the values of $\beta\ and\ B$.
3. The behavior of gradient pressure changes with changing the values of parameters, it is observed that when an increase in the values of $B_n, q, M, and\ F_r$ leads to a decrease in the value of gradient pressure. Otherwise, it leads to increasing in the value of the gradient pressure. .
4. The influence of different parameters on the local shear stress. It is obvious that increase in the value of $B_n, \in\ and\ M$ leads to an increase in the shear stress, see Figure 5 while a decrease in the values of parameters $q, \Omega\ and\ \beta$ implies that increases in the values of local shear stress.
5. In the case of concentration, it is observed that the increase in the values of parameters $B_n, \epsilon, q, M, \Omega, B_r, P_r, S_r, D_u\ and\ S_c$ lead to a decrease in the value of concentration, while it increases when a decrease in the values of $\beta\ and\ A$.
6. Stream function, the number and size of the bolus are increasing and decreasing which depend on the types of parameters when they are increasing or decreasing.
7. Physically, an application of this phenomenon appear in blood thrombus and locomotion the bolus of blood.

References

[1] Hatem Nahi Mohaisen, Ahmed M.Abedulhadi, “Effects of the Rotation on the Mixed Convection Heat Transfer Analysis for the Peristaltic Transport of Viscoplastic Fluid in Asymmetric Channel”, *Iraqi Journal of Science*, Vol. 63, No. 3, PP: 1240-1257, 2022, DOI: 10.24996/ijs.2022.63.3.29.

[2] Mohammed Ali Murada, Ahmed M. Abdulhadi, "Influence of MHD on mixed convective heat and mass transfer analysis for the peristaltic transport of viscoplastic fluid with porous medium in tapered channel", *Journal of Al-Qadisiyah for Computer Science and Mathematics* Vol.12 (4) 2, pp. Math. 79–90, 2020.

[3] [3] Hatem Nahi Mohaisen, Ahmed M.Abedulhadi, "Influence of the Induced Magnetic and Rotation on Mixed Convection Heat Transfer for the Peristaltic Transport of Bingham plastic Fluid in an Asymmetric Channel", *Iraqi Journal of Science,* Vol. 63, No. 4, pp: 1770-178, 2022, DOI:10.24996/ijs.2022.63.4.35.

[4] Hatem Nahi Mohaisen, Ahmed M.Abedulhadi, "Effects of The Rotation and A Magnetic Field onThe Mixed Convection Heat Transfer Analysis for The Peristaltic Transport of Viscoplastic Fluid through A Porous Medium in Asymmetric Channel", *2nd International Conference on Physics and Applied Sciences (ICPAS 2021), Journal of Physics: Conference Series 1963, 012165, IOP Publishing,* 2021,doi:10.1088/1742-6596/1963/1/012165.

[5] Farah Alaa Adnan, Ahmad M. Abdul Hadi, "Effect of an inclined magnetic field on peristaltic flow of Bingham plastic fluid in an inclined symmetric channel with slip conditions", *Iraqi Journal of Science,*Vol. 60, No.7, PP: 1551-1574, 2019, DOI: 10.24996/ijs.2019.60.7.16.

[6] Ajaz Ahmad Dar and K. Elangovan, "Influence of an Inclined Magnetic Field and Rotation on the Peristaltic Flow of a Micropolar Fluid in an Inclined Channel", *Hindawi Publishing Corporation New Journal of Science*، Vol.2016, Article ID 5717542, 2016, http://dx.doi.org/10.1155/2016/5717542.

[7] Y. Elmhed, A. Abd-Alla and M. Abdallah, "Effect of heat transfer and rotation on the peristaltic flow of a micropolar fluid in a vertical symmetric channel", *Sohag J. Sci.. 7,* No. 3, 117-122,2022, https://dx.doi.org/10.21608/sjsci.2022.148710 .1011.

[8] T. Hayat, Naseema Aslam, M. Rafiq and Fuad E. Alsaadi,"Hall and Joule heating effects on peristaltic flow of Powell-Eyring liquid in an inclined symmetric channel", *Results in Physics* (7) PP: 518–528, 2017 doi: http://dx.doi.org/10.1016/j. rinp.2017.01.008.

[9] Saba S. Hasen and Ahmed M. Abdulhadi, "Influence of a Rotating Frame on the Peristaltic Flow of a Rabinowitsch Fluid Model in an Inclined Channel", *Journal of Al-Qadisiyah for Computer Science and Mathematics* Vol.12 (1), pp. Math 21-33, 2020.

[10] Ali M. Kamal and Ahmed M. Abdulhadi, "Influence of an Inclined Magnetic Field on Peristaltic Transport of Pseudoplastic Nanofluid through a Porous Space in an Inclined Tapered Asymmetric Channel with Convective Conditions", *American Journal of Mechanics and Applications*، Vol. 3, Issue 5, Pages: 42-55, 2015.

[11] T.Hayat, Shahid, Bibi, M.Rafiq, A.Alsaedi and F.M.Abbasic, "Effect of an inclined magnetic field on peristaltic flow of Williamson fluid in an inclined channel with convective conditions", *Journal of Magnetism and Magnetic Materials,* 2015, http://dx.doi.org/10.1016/j.jmmm.2015.10.107.

[12] Tamara Sh. Ahmed, "Effect of Inclined Magnetic Field on Peristaltic Flow of Carreau Fluid through Porous Medium in an Inclined Tapered Asymmetric Channel", *Al-Mustansiriyah Journal of Science,* Vol. 29, Issue 3, 2018, DOI: http://doi.org/10.23851/mjs.v29i3.641.

[13] T Sh Alshareef, "Impress of rotation and an inclined MHD on waveform motion of the non-Newtonian fluid through porous canal", *Journal of Physics: Conference Series 1591,012061, IOP Publishing,* 2020, doi:10.1088/1742-6596/1591/1/012061.

[14] B.B. Divya, G. Manjunatha, C. Rajashekhar, H. Vaidya and K.V. Prasad, (2021), "Effects of Inclined Magnetic Field and Porous Medium onPeristaltic Flow of a Bingham Fluid with Heat Transfer ", *J. Appl. Comput. Mech.*, 7(4), PP: 1892–1906, 2021, DOI: 10.22055/JACM.2019.31060.1822.

[15] Shahad Ali Abdullaa and Liqaa Zeki Hummadya, "Inclined magnetic field and heat transfer of asymmetric and Porous Medium Channel on hyperbolic tangent peristaltic flow*", Int. J. Nonlinear Anal. Appl.* Vol.12, No. 2, PP: 2359-2372, 2021, http://dx.doi.org/10.22075/ijnaa.2021.5382.

[16] Najma Saleem, Safia Akram, Farkhanda Afzal, Emad H. Aly and Anwar Hussain, "Impact of Velocity Second Slip and Inclined Magnetic Field on Peristaltic Flow Coating with Jeffrey Fluid in Tapered Channel", *Coatings*, vol.10, No.30, 2020, doi:10.3390/coatings10010030.