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Certain Strong and Weak Types of Paracompact Maps

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Abstract
In this work, the concept of the paracompact map has been generalized to other new types of maps, in addition to the fact that the paracompact map has been linked to these new maps. Accordingly, we will separate these new types of maps into two classes, the first class is called a strong form which implies a paracompact map under certain conditions. While the other class is called a weaker form of a paracompact map, whereas the paracompact map implies them. Finally, the composition operations of paracompact maps are studied.

Keywords: Paracompact map, Compact map, Metacompact map, countably paracompact map, Pa-closed space, countably metacompact map, S-paracompact map.

أنواع معينة ضعيفة وقوية من الدوال فوق المتراصة

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الخلاصة
في هذا العمل تم تعميم مفهوم الدالة فوق المتراصة لأنواع جديدة من الدوال بالإضافة الى ربط الدالة فوق المتراصة مع تلك الدوال وبناءاً على ذلك تم تصنيف تلك الدوال الى صنفين الصنف الاول تسمى صيغة قوية بالنسبة للدالة فوق المتراصة وفقاً لشروط معينة على فضاءات تلك الدوال والصنف الثاني يسمى صيغة ضعيفة بالنسبة للدالة فوق المتراصة في ظل شروط معينة أيضاً. وأخيراً، تم دراسة التركيب للأنواع المختلفة للدالة فوق المتراصة.

1. Introduction

The motivation of compactness into topology was beginning to generalize the properties of the bounded and closed subset $\mathbb{R}^n$. In 1944, Dieudonné [1] introduced a wider class of compact spaces, namely paracompact spaces. In 1951, Dowker [2] had given generalization of paracompact spaces by introducing the class of countably paracompact spaces. Through the use

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of α-open, pre-open, semi-open, regular open, and β-open sets, new generalizations of paracompact spaces were given. Nearly paracompact space was defined by Singal and Arya [3] using the regular open. In 2006, Al-Zoubi [4] introduced the notion of S -paracompact space using a semi-open set. Demir and Ozbakir [5] defined in 2013 β -paracompact spaces, by replacing the open cover with a β-open cover in the definition of paracompact space. On the other hand, there are maps known as parallel advanced spaces. In 1947, Halfar [6] introduced the concept of a compact map in a metric space. Garg and Goel [7] in 1993 initiated a countably compact map and then Buhagiar [8] in 1997 introduced the notion of a paracompact map. After that, in 2003 a countably paracompact map was defined by AL-Zoubi and Hdeib [9]. In this paper, we introduce two classes of paracompact maps, namely, strong, and weak, and we investigate their compositions. A space $\mathbb{W}$ means a topological space ($\mathbb{W}$,τ), by a map $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$, we mean continuous surjection map $\mathcal{L}$ of a space $\mathbb{W}$ into a space $\mathbb{M}$.

2. Preliminaries

This section includes several basic definitions and theorems on paracompactness that are essential in the paper are reviewed.

Definitions 2.1:

1. A Hausdorff space $\mathbb{W}$ is known as a paracompact provided any open cover of it includes a locally finite open refinement [1].
2. Let $\mathbb{W}$ and $\mathbb{M}$ be two spaces. A map $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is known as compact providing the pre-image of any compact set in $\mathbb{M}$ is compact in $\mathbb{W}$ [6].
3. A Hausdorff space $\mathbb{W}$ is known as extremally disconnected provided the closure of each open set in $\mathbb{W}$ is open [10].
4. A Hausdorff space $\mathbb{W}$ is known as completely extremally disconnected provided it is extremally disconnected and $\bar{A} \cap \bar{B} = \emptyset$ for any $A, B \subseteq \mathbb{W}$, [11].
5. A space $\mathbb{W}$ is known as countably paracompact (sometimes called binormal) provided each open countable covering includes a locally finite open refinement, [2].
6. A space $\mathbb{W}$is known as S-closed (res. Countably S-closed) provided for each semi-open (res., countable semi-open) cover of $\mathbb{W}$ includes a finite subfamily the closures of whose members cover $\mathbb{W}$, [12].
7. A space $\mathbb{W}$ is known as an S-paracompact provided any open cover of it includes a locally finite semi-open refinement, [4].
8. A space $\mathbb{W}$ is metacompact provided per open cover of $\mathbb{W}$includes an open point finite refinement, [13].
9. A space $\mathbb{W}$ is countably metacompact provided each countable open cover of $\mathbb{W}$includes a point finite open refinement, [14].
10. A space $\mathbb{W}$ is known as β-paracompact provided per open cover of $\mathbb{W}$includes a β-locally finite β-open refinement, [5].
11. A space $\mathbb{W}$is known as fully T_4 provided per open cover of $\mathbb{W}$includes a star refinement, [10].
12. A Hausdorff space $\mathbb{W}$ is known as fully normal provided per open cover includes star open refinement, [15].
13. A space $\mathbb{W}$ is known as submaximal provided any dense subset of W is open in W, [16].

Theorem 2.2: [10]

1. Each paracompact space is countably paracompact.
2. A space $\mathbb{W}$is compact whenever it is countably compact and paracompact.
3. Each countably paracompact and Lindelöf space is paracompact.
4. All countably paracompact (or binormal) space is normal.

5. Any countably paracompact space is countably metacompact.
6. Each metacompact space is countably metacompact.
7. Any paracompact space is metacompact.
8. All metacompact countably compact space is compact.
9. Each fully T_4 and Hausdorff space is paracompact.
10. A fully T_4 and T_1-space is fully normal.
11. any fully normal space is fully T_4.
12. Paracompactness of space implies countably paracompactness.
13. Any compact space is a countably compact space
14. Each compact space is Lindelöf.
15. Each countably compact space is countably paracompact.

Theorem 2.3:
1. The compactness of space implies paracompactness, [2].
2. Each closed subspace of compact (res., paracompact, countably compact, countably paracompact) space is compact (res., paracompact, countably compact, countably paracompact), [2].
3. Each extremally disconnected S-paracompact T_2-space is paracompact, [4].
4. Each paracompact space is S-paracompact, [4].
5. Each S-paracompact space is β-paracompact, [5].
6. Each paracompact space is a β-paracompact, [5].
7. Permit $\mathbb{W}$ to be an extremally disconnected submaximal space. If $\mathbb{W}$ is a β-paracompact space, then $\mathbb{W}$ is a paracompact space, [5].
8. Each normal metacompact space is countably paracompact space, [17].
9. Each Lindelöf countably metacompact space is metacompact space, [18].
10. Each fully normal and T_1- space is paracompact, [19].
11. A Hausdorff paracompact space is fully normal, [20].
12. Each proper map of $\mathbb{W}$ onto $\mathbb{M}$ is paracompact, [21].
13. Let the map $\mathcal{L}$ of $\mathbb{W}$ onto $\mathbb{M}$ be open and proper, then the image for any paracompact set in $\mathbb{W}$ is paracompact set in $\mathbb{M}$, [21].
14. A closed subset of a Lindelöf (res., normal) space, is a Lindelöf (resp., normal) subspace, [22].
15. Let $\mathbb{W}$ be a Hausdorff space. $\mathbb{W}$ is hereditarily extremally disconnected if and only if it is completely extremally disconnected, [23].
16. A continuous image of a compact space is compact, [24].

3. Strong forms of paracompact maps

This section aimed to introduce new types that are stronger than a paracompact map under certain conditions. Initially, we introduce the definition of a paracompact map as follows:

Definition 3.1: A surjective continuous map $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is known as a paracompact map if the inverse image for any paracompact set in $\mathbb{M}$ is a paracompact set in $\mathbb{W}$.

Example 3.2: Any map of a metrizable space into any space is a paracompact map.

Definition 3.3: A space $\mathbb{W}$ is known as Pa-closed provided each paracompact subset of $\mathbb{W}$ is closed.

Example 3.4: $(\mathbb{Z}, \tau_D)$ is a Pa-closed space.

Definition 3.5: A map $\mathcal{L}: \mathbb{W} \to \mathbb{M}$is known as countably paracompact providing the pre-image of any closed and countably paracompact set in $\mathbb{M}$ is countably paracompact in $\mathbb{W}$.

Theorem 3.6: Each countably paracompact map of a Lindelöf space onto a Pa-closed space is a paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a countably paracompact map such that $\mathbb{W}$ is a Lindelöf space and $\mathbb{M}$ is a Pa-closed space. Assume that $\mathbb{K}$ is a paracompact set in $\mathbb{M}$. So, $\mathbb{K}$ is a countably paracompact subset of $\mathbb{M}$ by Theorem 2.2 part (1). Since $\mathbb{M}$ is a Pa-closed, then $\mathbb{K}$ is a closed subset of $\mathbb{M}$. Thus, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact set in $\mathbb{W}$. In addition,$\mathcal{L}^{-1}(\mathbb{K})$ is closed in $\mathbb{W}$. Now, Theorem 2.3 part (14) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a Lindelöf subspace of $\mathbb{W}$.Then, Theorem 2.2 part (3) implies that $\mathcal{L}^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$. Therefore, $\mathcal{L}$ is a paracompact map.

The countable compactness and Pa-closedness are required for establishing the paracompactness for all the compact maps, which can be proven by the same argument of Theorem 3.6.

Theorem 3.7: Each compact map onto a countably compact Pa-closed space is paracompact.

Definition 3.8: A map $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is known as a countably compact map providing the pre-image of any closed and countably compact set in $\mathbb{M}$ is countably compact in $\mathbb{W}$.

Theorem 3.9: Each countably compact map of a Lindelöf space onto Pa-closed and compact space is a paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a countably compact map. Assume that $\mathbb{K}$ is a paracompact set in $\mathbb{M}$. Since $\mathbb{M}$ is a Pa-closed space, then $\mathbb{K}$ is a closed subset of $\mathbb{M}$andso, $\mathbb{K}$ is a compact subspace of $\mathbb{M}$ owing to Theorem 2.3 part (2). Thus, $\mathbb{K}$ is countably compact by Theorem 2.2 part (13). Now,$\mathcal{L}^{-1}(\mathbb{K})$ is a countably compact set in $\mathbb{W}$ because $\mathcal{L}$ is a countably compact map and so, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact subspace of $\mathbb{W}$. Since $\mathbb{W}$ is Lindelöf space, then $\mathcal{L}^{-1}(\mathbb{K})$ is a Lindelöf which implies that $\mathcal{L}^{-1}(\mathbb{K})$is paracompact set in $\mathbb{W}$ due to Theorem 2.2 part (3). Hence, $\mathcal{L}$ is a paracompact map.

Definition 3.10: A map $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is known as a meta compact providing the pre-image of any closed and metacompact set in $\mathbb{M}$ is metacompact in $\mathbb{W}$.

Theorem 3.11: Each metacompact map of a Lindelöf and normal space onto a Pa-closed space is a paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a metacompact map such that $\mathbb{W}$ is a Lindelöf and normal space and $\mathbb{M}$ is a Pa-closed space. Suppose that $\mathbb{K}$ is a paracompact set in $\mathbb{M}$. Then Theorem 2.2 part (7) implies that $\mathbb{K}$ is a metacompact subset of $\mathbb{M}$. Since $\mathbb{M}$ is a Pa-closed space, then $\mathbb{K}$ is a closed subset of $\mathbb{M}$, thus $\mathcal{L}^{-1}(\mathbb{K})$ is a metacompact set in $\mathbb{W}$ due to $\mathcal{L}$ is a metacompact map. So, $\mathcal{L}^{-1}(\mathbb{K})$ is a normal subspace in $\mathbb{W}$ owing to Theorem 2.3 part (14). As a result, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact subspace of $\mathbb{W}$ due to Theorem 2.3 part (8) and$\mathcal{L}^{-1}(\mathbb{K})$is a Lindelöf subspace of $\mathbb{W}$ by Theorem 2.3 part (14). Now, Theorem 2.2 part (3)asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$. Hence, $\mathcal{L}$ is a paracompact map.
Next, the countably paracompact map and the metacompact map are intimately linked.

Theorem 3.12: Each countably paracompact map of a Lindelöf space onto a normal space is a metacompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a countably paracompact map, where $\mathbb{W}$ is a Lindelöf space and $\mathbb{M}$ is a normal space. Assume that $\mathbb{K}$ is a closed and metacompact set in $\mathbb{M}$. So, $\mathbb{K}$ is a normal

subspace of 𝕄 by Theorem 2.3 part (14). Then, Theorem 2.3 part (8) implies that 𝕂 is a countably paracompact subset of 𝕄. Thus, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact set in 𝕎 due to $\mathcal{L}$ is a countably paracompact map. Hence, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably metacompact set in 𝕎 due to Theorem 2.2 part (5). By Theorem 2.3 part (14) $\mathcal{L}^{-1}(\mathbb{K})$ is a Lindelöf set in 𝕎. Theorem 2.3 part (9) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a metacompact set in 𝕎. Hence, $\mathcal{L}$ is a metacompact map.

Definition 3.13: A map $\mathcal{L}$: 𝕎→𝕄 is known as a countably metacompact providing the pre-image of any closed and countably metacompact set in 𝕄 is countably metacompact in 𝕎.

Theorem 3.14 Each countably metacompact map of a Lindelöf space is a metacompact map.
Proof: Let $\mathcal{L}$: 𝕎→𝕄 be a countably metacompact map where𝕎 is a Lindelöf space. Suppose that 𝕂 is a closed and metacompact set in 𝕄. So, 𝕂 is a countably metacompact subspace of 𝕄 by Theorem 2.2 part (6). Thus, $\mathcal{L}^{-1}(K)$ is a countably metacompact set in 𝕎 due to $\mathcal{L}$being a countably metacompact map. Indeed, $\mathcal{L}^{-1}(\mathbb{K})$ is closed in 𝕎 by the continuity of $\mathcal{L}$. Theorem 2.3 part (14) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a Lindelöf subspace of 𝕎 due to $\mathcal{L}^{-1}(\mathbb{K})$ is closed in 𝕎 . As a result, Theorem 2.3 part (9) implies that $\mathcal{L}^{-1}(\mathbb{K})$ is a metacompact set in 𝕎. Hence, $\mathcal{L}$ is a metacompact map.

Corollary 3.15: Each countably metacompact map of a Lindelöf and normal space onto a Pa-closed space is a paracompact map.
Proof: The prove is straight forward by using Theorem 3.11 and Theorem 3.14.

Definition 3.16: A map $\mathcal{L}$: 𝕎→𝕄is known as an S-paracompact providing the pre-image of any closed and S-paracompact set in 𝕄 is S-paracompact in 𝕎.

Theorem 3.17: Each S-paracompact map of a Hausdorff completely extremally disconnected space is a paracompact map.
Proof: Let $\mathcal{L}$: 𝕎→𝕄 be an S-paracompact map such that 𝕎 is a completely extremally disconnected space. Assume that 𝕂 is a paracompact set in 𝕄. Theorem 2.3 part (4) implies that 𝕂 is an S-paracompact set in 𝕄. Since $\mathcal{L}$ is an S-paracompact map, then $\mathcal{L}^{-1}(\mathbb{K})$ is anS-paracompact set in 𝕎. But 𝕎 is a Hausdorff, completely extremally disconnected space, thus Theorem 2.3 part (15) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is extremely disconnected set in 𝕎. As a result, $\mathcal{L}^{-1}(\mathbb{K})$ is an S-paracompact and extremally disconnected set in 𝕎, which implies that $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact in 𝕎 by Theorem 1.2.2. Hence, $\mathcal{L}$ is a paracompact map.

Definition 3.18: A map $\mathcal{L}$: 𝕎→𝕄 is known as a β-paracompact providing the pre-image of any closed and β-paracompact set in 𝕄 is β-paracompact in 𝕎.

Theorem 3.19: Each β-paracompact map of a Hausdorff, completely extremally disconnected and submaximal space is a paracompact map.
Proof: Let $\mathcal{L}$: 𝕎→𝕄 be a β-paracompact map such that 𝕎 is a completely extremally disconnected and submaximal space. Assume that 𝕂 is a paracompact set in 𝕄. So, K is β-paracompact by Theorem 2.3 part (6). Then, $\mathcal{L}^{-1}(\mathbb{K})$ is a β-paracompact set in 𝕎 because $\mathcal{L}$is a β-paracompact map. Added to, $\mathcal{L}^{-1}(\mathbb{K})$ is an extremally disconnected subspace of 𝕎 by Theorem 2.3 part (15) and it is submaximal of 𝕎. $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact set in 𝕎 by Theorem 2.3 part (7). Hence, $\mathcal{L}$ is a paracompact map.

Theorem 3.20: Each β-paracompact map of a Hausdorff, completely extremally disconnected and submaximal space is an S-paracompact map.

Proof: Let $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ be a β-paracompact map such that $\mathbb{W}$ is a Hausdorff completely extremally disconnected space. Assume that $\mathbb{K}$ is a closed S-paracompact set in $\mathbb{M}$. Thus, $\mathbb{K}$ is a β-paracompact subset of $\mathbb{M}$ owing to Theorem 2.3 part (5). So, $\mathcal{L}^{-1}(\mathbb{K})$ is β-paracompact set in $\mathbb{W}$ because $\mathcal{L}$ is a β-paracompact map. Added to, $\mathcal{L}^{-1}(\mathbb{K})$ is an extremally disconnected subspace of $\mathbb{W}$ by Theorem 2.3 part (15) and it is submaximal. Theorem 2.3 part (7). implies that $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact. Therefore, $\mathcal{L}^{-1}(\mathbb{K})$is S-paracompact in $\mathbb{W}$ owing to Theorem 2.3 part (4). Hence, $\mathcal{L}$ is an S-paracompact map.

Definition 3.21: A map $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ is known as a fully normal map providing the pre-image of any fully normal set in $\mathbb{M}$ is fully normal in $\mathbb{W}$.

Theorem 3.22: Each fully normal map of a T_1-space onto a Hausdorff space is a paracompact map.
Proof: Let $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ be a fully normal map such that $\mathbb{W}$ is a T_1- space and $\mathbb{M}$ is a Hausdorff space. Assume that $\mathbb{K}$ is a paracompact set in $\mathbb{M}$. Thus, $\mathbb{K}$ is a Hausdorff subspace of $\mathbb{M}$ because $\mathbb{M}$ is a Hausdorff space and so, $\mathbb{K}$ is a fully normal subset of $\mathbb{M}$ by Theorem 2.3 part (11). Now, $\mathcal{L}^{-1}(\mathbb{K})$ is fully normal in $\mathbb{W}$ due to $\mathcal{L}$being fully normal. Hence, Theorem 2.3 part (10) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$. Hence, $\mathcal{L}$ is a paracompact map.

Definition 3.23: A map $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ is known as a fully T_4 providing the pre-image of any fully T_4 set in $\mathbb{M}$ is fully T_4 in $\mathbb{W}$.

Theorem 3.24: Each fully T_4 map of a T_1- space is a fully normal map.
Proof: Let $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ be a fully T_4 map such that $\mathbb{W}$ is a T_1- space. Assume that $\mathbb{K}$ is a fully normal set in $\mathbb{M}$. So, $\mathbb{K}$ is a fully T_4 set in $\mathbb{M}$ by Theorem 2.2 part (11). thus, $\mathcal{L}^{-1}(\mathbb{K})$ is fully T_4 in $\mathbb{W}$ due to $\mathcal{L}$ being a fully T_4 map. Since $\mathbb{W}$ is a T_1- space, then$\mathcal{L}^{-1}(\mathbb{K})$ is fully normal by Theorem 2.2 part (10). Hence, $\mathcal{L}$ is a fully normal map.

Corollary 3.25: Each fully T_4map of a T_1- space onto a Hausdorff space is a paracompact map.

4. Weaker forms of compact maps

The principal purpose of this section is to reveal more new and weaker definitions of maps by using the concept of paracompactness.

Theorem 4.1: Each paracompact map of a countably compact space onto a Hausdorff space is a compact map.
Proof: Let $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ be a paracompact map. Assume that $\mathbb{K}$ is a compact set in $\mathbb{M}$. Then, $\mathbb{K}$ is closed in $\mathbb{M}$, because $\mathbb{M}$ of is a Hausdorff space. Therefore, $\mathbb{K}$ is also paracompact due to Theorem 2.3 part (1). Consequently, $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact set in $\mathbb{W}$ because $\mathcal{L}$ is paracompact mapping. Indeed, $\mathcal{L}^{-1}(\mathbb{K})$ is a closed in $\mathbb{W}$ by continuity of $\mathcal{L}$. Since $\mathbb{W}$ is a countably compact space, so $\mathcal{L}^{-1}(\mathbb{K})$ is countably compact subspace owing to Theorem 2.3 part (2). Thus, $\mathcal{L}^{-1}(\mathbb{K})$ is compact in $\mathbb{M}$ by Theorem 2.2 part (2). Hence, $\mathcal{L}$ is a compact map.

Theorem 4.2: Each paracompact map onto a normal and Lindelöf space is a metacompact map.
Proof: Let $\mathcal{L}:\mathbb{W}\to\mathbb{M}$ be a paracompact map where $\mathbb{M}$ is a normal and Lindelöf space. Assume that $\mathbb{K}$ is a closed metacompact set in $\mathbb{M}$. Since $\mathbb{M}$ is normal, so $\mathbb{K}$ is a countably paracompact set due to Theorem 2.3 part (8), also we have $\mathbb{M}$ is Lindelöf, then $\mathbb{K}$ is Lindelöf by Theorem 2.3 part (14). So, $\mathbb{K}$ is a paracompact set in $\mathbb{W}$ from Theorem 2.2 part (3). Thus, $\mathcal{L}^{-1}(\mathbb{K})$ is a

paracompact in$\mathbb{W}$ due to $\mathcal{L}$ is a paracompact map. Now, by Theorem 2.2 part (7), $\mathcal{L}^{-1}(\mathbb{K})$ is a metacompact in $\mathbb{W}$. Hence, $\mathcal{L}$ is a metacompact map.

Theorem 4.3: Each metacompact map onto a Lindelöf space is a countably metacompact map.
Proof: The prove is straight forward by using Theorem 2.3 part (9) and Theorem 2.2 part (6).

Theorem 4.4: Each paracompact map onto a normal and Lindelöf space is a countably metacompact map.
Proof: The prove is straight forward by using Theorem 4.2 and Theorem 4.3.

Theorem 4.5: Each paracompact map onto a Lindelöf space is a countably paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a paracompact map where $\mathbb{M}$ is a Lindelöf space. Assume that $\mathbb{K}$ is a closed countably paracompact subset of $\mathbb{M}$. Since $\mathbb{M}$ is a Lindelöf space and $\mathbb{K}$ is closed,$\mathbb{K}$ is a Lindelöf subspace of $\mathbb{M}$ due to Theorem 2.3 part (14). Then, $\mathbb{K}$ is a paracompact subspace of $\mathbb{M}$ due to Theorem 2.2 part (3). Consequently, $\mathcal{L}^{-1}(\mathbb{K})$ is a paracompact subset of $\mathbb{W}$ due to $\mathcal{L}$ is a paracompact map. Hence, $\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact as a result of Theorem 2.2 part (1). Hence, $\mathcal{L}$ is a countably paracompact map.

Theorem 4.6: Each compact map of a Lindelöf space onto a countably compact, normal, and a Lindelöf space is a metacompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a compact map where $\mathbb{W}$ is a Lindelöf space and $\mathbb{M}$ is a countably metacompact, normal, and Lindelöf space. Assume that $\mathbb{K}$ is a closed metacompact subset of $\mathbb{M}$. Then, $\mathbb{K}$ is normal due to $\mathbb{M}$ is a normal space. Theorem 2.3 part (8) emphasizes that $\mathbb{K}$ is a countably paracompact subspace of $\mathbb{M}$. Since $\mathbb{M}$ is Lindelöf, thus $\mathbb{K}$ is a paracompact set in $\mathbb{M}$ by Theorem 2.2 part (3). Since $\mathbb{M}$ is a countably compact space, then $\mathbb{K}$ is countably compact by Theorem 2.3 part (2). Then, Theorem 2.3 part (5)implies that $\mathbb{K}$ is compact. Therefore, $\mathcal{L}^{-1}(\mathbb{K})$ is a compact subset of $\mathbb{W}$ due to $\mathcal{L}$ is a compact map. Theorem 2.2 part (13)insists that $\mathcal{L}^{-1}(\mathbb{K})$ is a countably compact set in $\mathbb{W}$ and by Theorem 2.2 part (15)$\mathcal{L}^{-1}(\mathbb{K})$ is a countably paracompact set in $\mathbb{W}$, therefore $\mathcal{L}^{-1}(\mathbb{K})$ is a countably metacompact due to Theorem 2.3 part (14). But we have $\mathbb{W}$ isLindelöf therefore, $\mathcal{L}^{-1}(\mathbb{K})$ is a Lindelöf and countably metacompact. Theorem 2.3 part (9) asserts that $\mathcal{L}^{-1}(\mathbb{K})$ is a metacompact. Hence, $\mathcal{L}$ is a metacompact map.
Directly**,** Theorem 4.6 and Theorem 4.3 lead us the next result.

Theorem 4.7: Each compact map of a Lindelöf space onto a countably metacompact, normal, and Lindelöf space is a countably metacompact map.
Proof: The prove is straight forward by using Theorem 4.6 and Theorem 4.3.

Theorem 4.8: Each paracompact map onto a Hausdorff and completely extremally disconnected space is an S-paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ be a paracompact map where $\mathbb{M}$ is a Hausdorff and completely extremally disconnected space. To show that $\mathcal{L}$ is an S-paracompact map. Assume that $\mathbb{K}$ is a closed S-paracompact subspace of $\mathbb{M}$. Theorem 2.3 part (15) asserts that $\mathbb{K}$ is an extremally disconnected subspace of $\mathbb{M}$, also$\mathbb{K}$ is a Hausdorff subspace of $\mathbb{M}$thus,$\mathbb{K}$ is a paracompact subspace of $\mathbb{M}$ due to Theorem2.3 part (3). Now, $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact set in $\mathbb{W}$ due to $\mathcal{L}$beinga paracompact map. Theorem 2.3 part (4)implies that $\mathcal{L}^{-1}(\mathbb{K})$is an S-paracompact subspace of $\mathbb{M}$. Hence, $\mathcal{L}$ is an S-paracompact.

Theorem 4.9: Each paracompact map onto a Hausdorff, completely extremally disconnected, and submaximal space is a β-paracompact map.

Proof: Let $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ be a paracompact map where $\mathbb{M}$ is a completely extremally disconnected and submaximal space. Suppose that $\mathbb{K}$ is a closed and β-paracompact set in $\mathbb{M}$. Since $\mathbb{K}$ is an extremally disconnected and submaximal subspace of $\mathbb{W}$ by Theorem 2.3 part (15), then $\mathbb{K}$ is a paracompact subspace of $\mathbb{W}$ by Theorem 2.3 part (7). Thus, $\mathcal{L}^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$ because $\mathcal{L}$ is a paracompact map. Consequently, $\mathcal{L}^{-1}(\mathbb{K})$ is β-paracompact set in $\mathbb{W}$ for Theorem 2.3 part (6). Hence, $\mathcal{L}$ is a paracompact map.

Theorem 4.10: Each paracompact map of a T_2-space onto is T_1-space is fully normal.
Proof: Let $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ be a paracompact map where $\mathbb{W}$ is a T_2-space and $\mathbb{M}$ is T_1-space. Suppose that $\mathbb{K}$ is a fully normal set in $\mathbb{M}$. Then, $\mathbb{K}$ is a paracompact set in $\mathbb{M}$ by Theorem 2.3 part (10). This implies $\mathcal{L}^{-1}(\mathbb{K})$ is paracompact in $\mathbb{W}$ owing to $\mathcal{L}$ is a paracompact map which follows $\mathcal{L}^{-1}(\mathbb{K})$ is fully normal because of Theorem 2.3 part (11). Hence, is a fully normal map.
Consequently, by similar arguments as in Theorem 4.10, the following results are recognized:

Theorem 4.11: Each fully normal map onto a T_1-space is fully T_4.

Corollary 4.12: Each paracompact map of a T_2-space onto a T_1-space is fully T_4.

Next, Figure 1 illustrates the relationships between certain types of strong paracompact maps under certain conditions as follows:

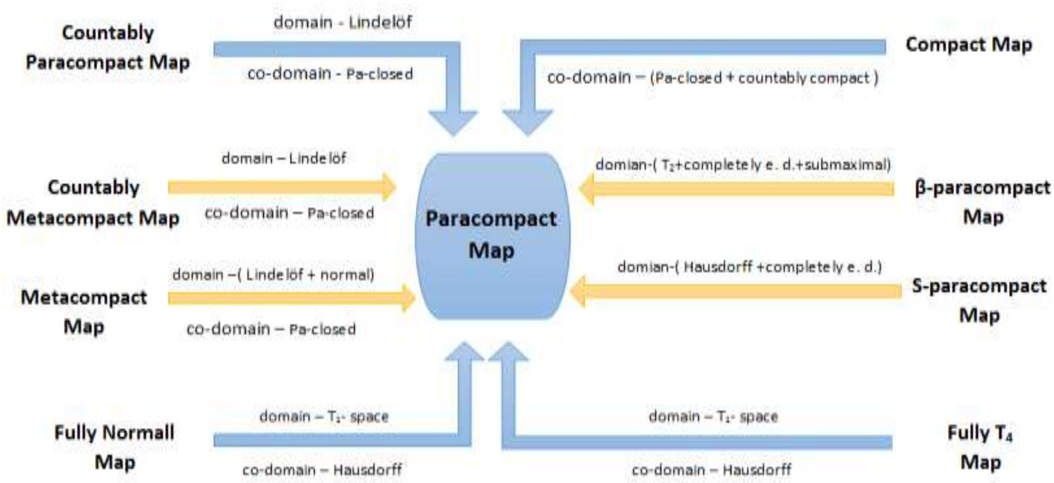

Figure 1. Relationships between Certain Types of Strong Paracompact Maps

The following figure shows the relationships between certain types of weaker paracompact maps under certain conditions.

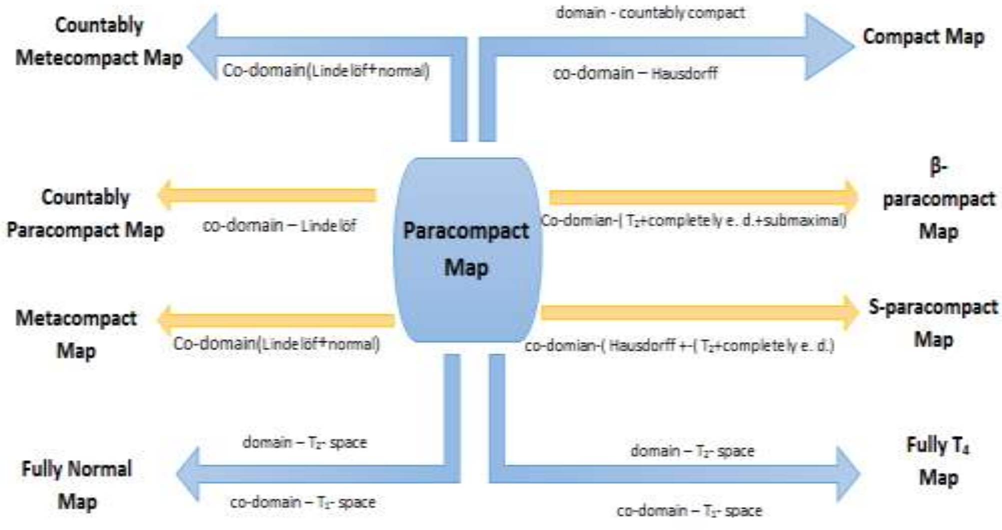

Figure 2. Relationships between Certain Types of weaker Paracompact Maps

5. Composition of certain types of paracompact maps

In this section, we investigate the composition of the strong and weaker forms of paracompact maps in various cases.

Theorem 5.1: Let $\mathbb{W}$ be a Pa-closed and compact space and let $\mathbb{M}$ be any space. Then, the continuous image of any paracompact set in $\mathbb{W}$ is paracompact in $\mathbb{M}$.
Proof: Let $\mathcal{L}\colon \mathbb{W} \to \mathbb{M}$ be a continuous map. Suppose that $\mathbb{K}$ is paracompact in $\mathbb{W}$. Since $\mathbb{W}$ is a Pa-closed space, then $\mathbb{K}$ is a closed set in $\mathbb{W}$, therefore, $\mathbb{K}$ is compact by Theorem 2.3 part (2). Now, $\mathcal{L}(\mathbb{K})$ is a compact set in $\mathbb{M}$ due to $\mathcal{L}$ being a continuous map. From Theorem 2.3 part (1), $\mathcal{L}(\mathbb{K})$ is a paracompact subspace of $\mathbb{M}$.

Corollary 5.2: Let $\mathbb{W}$ be a compact space and let $\mathbb{M}$be any space. Then, the continuous image of any closed set in $\mathbb{W}$ is paracompact in $\mathbb{M}$.

Theorem 5.3 The composition of paracompact maps is also a paracompact map.
Proof: Let $\mathcal{L}\colon \mathbb{W} \to \mathbb{M}$ and $\mathcal{J}\colon \mathbb{M} \to \mathbb{E}$ be two paracompact maps. To show that $\mathcal{J} \circ \mathcal{L}$ is also a paracompact map. Assume that$\mathbb{K}$ is a paracompact set in $\mathbb{E}$, For demonstrating that $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$. We have $\mathcal{J}^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{M}$ since $\mathcal{J}$ is a paracompact map. Thus, $\mathcal{L}^{-1}\left(\mathcal{J}^{-1}(\mathbb{K})\right)$ is a paracompact set in $\mathbb{W}$ due to, $\mathcal{L}$being a paracompact map, but $\mathcal{L}^{-1}\left(\mathcal{J}^{-1}(\mathbb{K})\right) = (\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$. So,$(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a paracompact set in $\mathbb{W}$. Hence, $\mathcal{J} \circ \mathcal{L}$ is a paracompact map.
As a direct consequence of employing similar arguments as in Theorem 5.3, the following results are recognized:

Theorem 5.4: Let $\mathbb{M}$ be a Pa-closed compact. If $\mathcal{J} \circ \mathcal{L}\colon \mathbb{W} \to \mathbb{E}$ is a paracompact map and $\mathcal{J}\colon \mathbb{M} \to \mathbb{E}$ is a continuous injective map, then $\mathcal{L}\colon \mathbb{W} \to \mathbb{M}$ is a paracompact map.

Theorem 5.5: Let $\mathbb{W}$ be a Pa-closed and compact space. If $\mathcal{J} \circ \mathcal{L}\colon \mathbb{W} \to \mathbb{E}$ is a paracompact map and $\mathcal{L}\colon \mathbb{W} \to \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}\colon \mathbb{M} \to \mathbb{E}$ is a paracompact map.

Theorem 5.6: The composition of countably paracompact maps is also a countably paracompact map.
Theorem 5.7: Let $\mathbb{W}$ be a Lindelöf, $\mathbb{M}$ is a Pa-closed compact and $\mathbb{E}$ is Pa-closed. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is a countably paracompact map and $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is a paracompact map.
Proof: Let $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is a countably paracompact map and $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is a continuous injective map where $\mathbb{M}$ is a Pa-closed compact. Assume that $\mathbb{K}$ is a paracompact set in $\mathbb{M}$. Then, $\mathcal{J}(\mathbb{K})$ is the paracompact subspace of $\mathbb{E}$ due to Theorem 5.1. Since $\mathbb{W}$ is a Lindelöf and $\mathbb{E}$ is Pa-closed, thus $\mathcal{J} \circ \mathcal{L}$ is a paracompact map by Theorem 3.6. Now, $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathcal{J}(\mathbb{K})) = \mathcal{L}^{-1}(\mathcal{J}^{-1}(\mathcal{J}(\mathbb{K}))) = \mathcal{L}^{-1}(\mathbb{K})$ is a paracompact subspace of $\mathbb{W}$ due to $\mathcal{J} \circ \mathcal{L}$ is paracompact. Hence, $\mathcal{L}$ is a paracompact map.

Theorem 5.8: Let $\mathbb{W}$ be a Pa-closed and compact space and $\mathbb{E}$ be a Lindelöf space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is a countably paracompact map.
Proof: Let $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is a continuous surjective map where $\mathbb{W}$ be a Pa-closed and compact space. Suppose that $\mathbb{K}$ is a closed countably paracompact set in $\mathbb{E}$. Since $\mathbb{E}$ is a Lindelöf space, then $\mathcal{J} \circ \mathcal{L}$is a countably paracompact subspace of $\mathbb{E}$ owing to Theorem 4.5 which follows $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a closed countably paracompact set in $\mathbb{W}$. But $\mathbb{W}$ is a compact space, thus $\mathbb{W}$ is a Lindelöf by Theorem 2.2 part (14) which implies $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is Lindelöf and so,$(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is paracompact. Because $\mathcal{L}$ a surjective continuous map then, $\mathcal{L}(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K}) = \mathcal{L}(\mathcal{L}^{-1}((\mathcal{J}^{-1}(\mathbb{K})))) = \mathcal{J}^{-1}(\mathbb{K})$ is paracompact in $\mathbb{M}$ by Theorem 6.1 therefore, $\mathcal{J}^{-1}(\mathbb{K})$ is countably paracompact by Theorem 1.2.24. Hence, $\mathcal{L}$ is a countably paracompact map.

Theorem 5.9: The composition of S-paracompact maps is also an S-paracompact map.
Proof: Let $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ and $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ be two S-paracompact maps. To show that $\mathcal{J} \circ \mathcal{L}$ is also an S-paracompact map. Assume that$\mathbb{K}$ is a closed and S-paracompact set in $\mathbb{E}$, For demonstrating that$(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a closed and S-paracompact set in $\mathbb{W}$. Since $\mathcal{J}^{-1}(\mathbb{K})$ is a closed and S-paracompact set in $\mathbb{M}$ owing to $\mathcal{J}$ being an S-paracompact map. Thus, $\mathcal{L}^{-1}(\mathcal{J}^{-1}(\mathbb{K}))$ is an S-paracompact set in $\mathbb{W}$ because $\mathcal{L}$ is an S-paracompact map, but $\mathcal{L}^{-1}(\mathcal{J}^{-1}(\mathbb{K})) = (\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$. So, $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a S-paracompact set in $\mathbb{W}$. Hence, $\mathcal{J} \circ \mathcal{L}$ is S-paracompact.

Theorem 5.10: Let $\mathbb{W}$ be a Hausdorff completely externally disconnected and $\mathbb{M}$ is a Pa-closed compact. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is an S-paracompact map and $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is a paracompact map.
Proof: Let $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is an S-paracompact map and $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is a continuous injective map where $\mathbb{M}$ is a Pa-closed compact. Assume that $\mathbb{K}$is a paracompact set in $\mathbb{M}$. Then, $\mathcal{J}(\mathbb{K})$ is a paracompact subspace of $\mathbb{E}$ due to Theorem 5.1. Since $\mathbb{W}$ is a Hausdorff and completely extremally disconnected space, thus $\mathcal{J} \circ \mathcal{L}$ is a paracompact map by Theorem 3.17. Now, $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathcal{J}(\mathbb{K})) = \mathcal{L}^{-1}(\mathcal{J}^{-1}(\mathcal{J}(\mathbb{K}))) = \mathcal{L}^{-1}(\mathbb{K})$ is a paracompact subspace of $\mathbb{W}$ due to $\mathcal{J} \circ \mathcal{L}$ is paracompact. Hence, $\mathcal{L}$ is a paracompact map.
Next, under certain conditions, the paracompact map is explored as an S-paracompact map, which can be satisfied by the same method as Theorem 5.10.

Theorem 5.11: Let $\mathbb{W}$ be a Pa-closed compact space, and $\mathbb{E}$ is a completely externally disconnected and Hausdorff space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \to \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \to \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \to \mathbb{E}$ is an S-paracompact map.

Proof: Let $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ be a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map. Suppose that $\mathbb{K}$ is a closed S-paracompact set in $\mathbb{E}$. Since $\mathbb{E}$ is a completely e.d. and Hausdorff space, then $\mathbb{K}$ is a Hausdorff externally disconnected subspace owing to Theorem 2.3 part (15) which follows $\mathbb{K}$ is a paracompact set in $\mathbb{E}$ by Theorem 2.3 part (4). Thus $(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K})$ is a closed paracompact set in $\mathbb{W}$. But $\mathbb{W}$ is a Pa-closed compact space, thus $\mathcal{L}(\mathcal{J} \circ \mathcal{L})^{-1}(\mathbb{K}) = \mathcal{L}(\mathcal{L}^{-1}(\mathcal{J}^{-1}(\mathbb{K}))) = \mathcal{J}^{-1}(\mathbb{K})$ is a paracompact subset of $\mathbb{M}$ by Theorem 5.1. Therefore, $\mathcal{J}^{-1}(\mathbb{K})$ is an S-paracompact set due to Theorem 2.3 part (4). Hence, $\mathcal{J}$ is an S-paracompact map.
As a direct consequence of using similar arguments as in Theorem 5.11, the following results are established:

Theorem 5.12: The composition of β-paracompact maps is also a β-paracompact map.

Theorem 5.13: Let $\mathbb{W}$ be a Hausdorff completely extremally disconnected and submaximal and $\mathbb{M}$ is a Pa-closed compact. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a β-paracompact map and $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a paracompact map.
Proof: By Theorem 5.1 and Theorem 2.3 part (6).

Theorem 5.14: Let $\mathbb{W}$ be a Pa-closed compact space, and $\mathbb{E}$ is a completely externally disconnected sub-maximal space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a β -paracompact map.
Proof: The prove is clear by using Theorem 5.1 and Theorem 2.3 part (7).

Theorem 5.15: The composition of metacompact maps is also a metacompact map.

Theorem 5.16: Let $\mathbb{W}$ be a Lindelöf and normal space, $\mathbb{M}$ be a Pa-closed compact, and $\mathbb{E}$ is Pa-closed space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a metacompact map and $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a paracompact map.
Proof: By Theorem 5.1 and Theorem 3.11.

Theorem 5.17: Let $\mathbb{W}$ be a countably compact space and $\mathbb{E}$ be a Lindelöf and normal space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a metacompact map.
Proof: The prove is clear by using Theorem 4.2 and Theorem 2.3 part (1) and Theorem 2.2 part (8).

Theorem 5.18: The composition of countably metacompact maps is also a countably metacompact map.

Theorem 5.19: Let $\mathbb{W}$ be a Lindelöf space, $\mathbb{M}$ be a Pa-closed compact, and $\mathbb{E}$ be a Pa-closed space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a countably metacompact map and $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a paracompact map.
Proof: The prove is clear by using Theorem 5.1 and Corollary 3.15.

Theorem 5.20: Let $\mathbb{W}$ be a countably compact space and $\mathbb{E}$ be a Lindelöf and normal space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a countably metacompact map.
Proof: The prove is clear by using Theorem 2.3 part (16) and Theorem 2.3 part (9) and Theorem 2.3 part (8).
Theorem 5.21: The composition of fully T_4 maps is also a fully T_4 map.

Theorem 5.22: Let $\mathbb{W}$ be a T_1-space and $\mathbb{M}$ is a Pa-closed compact space and $\mathbb{E}$ is a Hausdorff space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a fully T_4 map and $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a paracompact map.
Proof: The prove is clear by using Theorem 5.1 and Corollary 3.25.

Theorem 5.23: Let $\mathbb{W}$ be a T_2-space and countably compact and $\mathbb{E}$ is a T_1-space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a fully T_4 map.
Proof: The prove is clear by using Corollary 4.12 and Theorem 2.2 part (9) and Theorem 2.3 part (16).

Theorem 5.24: The composition of fully normal maps is also a fully normal map.

Theorem 5.25: Let $\mathbb{W}$ be a T_1-space and $\mathbb{M}$ is a Pa-closed compact space and $\mathbb{E}$ is a Hausdorff space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a fully normal map and $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a continuous injective map, then $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a paracompact map.
Proof: The prove is clear by using Theorem 5.1 and Theorem 3.22.

Theorem 5.26: Let $\mathbb{W}$ be a T_2-space and countably compact and $\mathbb{M}$ is a T_2-space and $\mathbb{E}$ is a T_1-space. If $\mathcal{J} \circ \mathcal{L}: \mathbb{W} \rightarrow \mathbb{E}$ is a paracompact map and $\mathcal{L}: \mathbb{W} \rightarrow \mathbb{M}$ is a continuous surjective map, then $\mathcal{J}: \mathbb{M} \rightarrow \mathbb{E}$ is a fully normal map.
Proof: The prove is clear by using Theorem 5.1 and Theorem 2.3 part (11) and Theorem 2.3 part (16).

6. Conclusions

To recapitulate, several types of maps are introduced by using the concept of paracompactness. These maps are classified based on their relations with the paracompact map into two forms, namely strong and weaker forms. As well as the links between these maps are investigated and satisfied under certain conditions. In addition, a new space is initiated which is utilized in the relations between the maps.

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