Iraqi Journal of Science, 2024, Vol. 65, No. 10, pp: 5597-5609
DOI: 10.24996/ijs.2024.65.10.23

ISSN: 0067-2904

Influence of Forced Field on Blood Flow in an Overlapping Stenosed Arterial Section

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Received: 3/12/2022 Accepted: 27/8/2023 Published: 30/10/2024

Abstract

This article presents the theoretical analysis of the nonlinear behavior of the blood flow through an angled arterial segment with overlapping stenosis. A mathematically created time-variant stenosis emerges from the formation of arterial narrowing brought on by atheroma. An elastic cylindrical tube with a moving wall is used to represent the artery, and the Casson liquid is used to simulate blood that flows through it. The nonlinear elements that arise in the equations, govern blood flow are taken into account. The impact of the pulsatile pressure gradient that caused by the regular heartbeat on the flow process in the stenosed artery is demonstrated mathematically. By employing the boundary conditions, the present analytical technique can easily compute the velocity profiles, wall shear stress, and flow resistivity. To carry out a systematic quantitative study, the desired quantities are numerically computed. The results are graphically presented in the discussion section. They provide an overview of how the degree of stenosis and the malleability of the artery wall influence blood flow abnormalities. The application of the current model is adequately justified by many significant conclusions.

Keywords: Casson fluid, Forced field, Overlapping Stenosis, Flow Resistivity, Shear-stress.

1. Introduction

One of the diseases that affects the human cardiovascular system is the constriction of blood arteries brought on by improper tissue development. As a result, the flow of the blood may be decreased or obstructed, which might result in significant cardiovascular diseases. One of the most significant causes of death in the developed world today is the cardiovascular disease. The cardiovascular system, which is made up of the heart and blood arteries, is what allows blood to flow via an artery. Vascular conditions may significantly change how blood flows. Blood vessels and heart conditions, such as heart attacks and strokes, face serious health hazards today and are responsible for a significant portion of mortality. The properties of blood flow and vascular behavior are directly related to these diseases. These deaths are mostly due to stenosis. The term "stenosis" describes the narrowing of an artery as a result of the development of arteriosclerotic plaques or another kind of abnormal tissue growth. It has been suggested that deposits of fatty material, cholesterol, and cellular waste products on the arterial wall are the causes of stenosis, even if the exact causes remain unclear. When an artery develops stenosis, blood flow is decreased. The usual working of the circulatory system may be impaired by injuries sustained by the occurrence of stenosis in an artery. In particular,

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heart attacks might happen as a consequence of it. Blood flow restriction can injure the inner cells of the wall and hasten the onset of stenosis. Therefore, there is a link between the development of stenosis and blood flow in the artery since one affects the other. Young [1] was the first to study stenosis and looked at how it affected the steady flow of blood through a pipe. Models of the flow patterns in stenosed blood arteries were created by Azuma and Fukushima [2]. Vascular stenosis's impact on the steady flow was mentioned by MacDonald [3]. Then, several research are examined the flow characteristics of blood in a pipe with mild contraction using blood under various conditions such as the Newtonian or non-Newtonian fluids (see [4-12]).

Through the use of a mathematical model, Chakraborty and Mandal [13, 14] investigated the blood flow in overlaying stenosis with body acceleration. Two-layered non-Newtonian flow and overlapping stenosis's impact on the arterial flow have both been studied by Srivastava et al. [15, 16]. In the context of overlapping stenosis, the arterial flow was examined by Riahi et al.[17]. Mathematical modelling of irregular blood flow through elastic tapering arteries with overlapping stenosis was explored by Haghighi et al. [18]. Following that, other studies investigated the effects of overlapping stenosis in blood flow through varied artery geometry (see [19–24]).

Non-Newtonian blood flow has recently attracted the attention of scientists because it can be used to study blood flow via constricted arteries. Most investigations in the literature employ the Herschel-Bulkley, micropolar, Jeffrey, and Newtonian models. Due to the presence of the yield stress, this technique is unable to explain the physiological behavior of the circulation in feeding channels. The Casson model resembles the blood moving at low shear rates better than the Herschel-Bulkley fluid, despite the latter's yield stress constraint (see ScottBlair [25]). Recently, multiple researchers examined Casson fluid under various physiological conditions (see [26-29]).

It is well known that many pipes in physiological systems are inclined to the axis rather than horizontally. By Prasad and Radhakrishnamacharya [30], the blood flow via an artery with many stenoses and an uneven cross-section was investigated. An inclined elastic tube with a permeable wall and a creeping Casson liquid flow were studied by Gudekote and Choudhari [31]. According to Umadevi et al. [32], the copper nanoparticles and a magnetic force paired with an interwoven and restricted oblique artery were used to study the blood flow. Recently, several scientists examined the characteristics of blood flow within an artery in the presence of stenosis (See [33-46]).

The problem that is addressed in this study has potential in engineering and biomedical applications. Many researchers have worked on the stenosed artery, according to the literature review. However, there is no study has shown how the force field and angle of proclivity affect blood flow in an overlapping stenosed artery by treating blood as the Casson fluid. With the above inspiration, an attempt was made to investigate the effects of overlapping stenosis on an MHD Casson fluid with mild stenosis conditions. The research is carried out analytically. The impacts of numerous significant limitations are noticed through the graphs. The expressions for velocity, flow resistance, and wall shear stress are computed.

2. Mathematical Formulation and Solution

Consider the flowing of an incompressible Casson fluid via an inclined axisymmetric stenosed artery (pipe) of the uniform cross-section. The stenosis should be minor and evolve in an axially symmetric pattern. The surface's geometry is shown in Figure1.

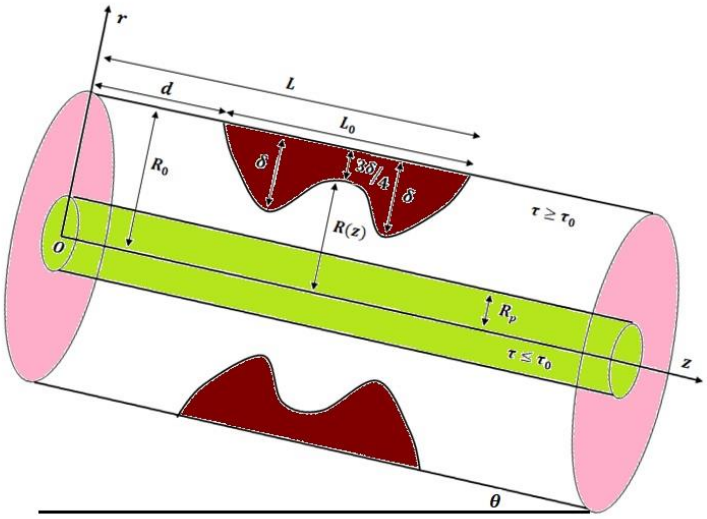

Figure 1: Design of overlying stenosis.

The formula involving the geometrical of the surface is (Chakraborty and Mandal [13], Srivastava et al. [15])

$$h = \frac{R(z)}{R_0} = \begin{cases} 1 - \frac{3\delta}{2R_0 L_0^4}[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4] & : d \leq z \leq d + L_0 \\ 1 & : \text{otherwise} \end{cases} \quad (1)$$

Here, the tube's radius at the stenotic area is $R(z)$, the radius of the artery's normal segment is $R_0(z)$, the start of the stenotic area lies at position d, and the length of the tube is L. L_0 measures how long the stenotic area and the proclivity angle is θ. As measured from the origin, δ is the maximum height of the stenosis at $z = d + L_0/6$, $z = d + 5L_0/6$, and $\frac{3\delta}{4}$ is the critical height of the stenosis at $z = d + L_0/2$.

2.1 Leading equations

In this study, blood was supposed to be a non-Newtonian fluid that was homogeneous and incompressible. The viscosity of blood can be described using a variety of non-Newtonian models, including the power-law model, micropolar. Herschel-Bulkley liquid schemes, and others. In this investigation, we used the Cassion model to describe the material property of the blood viscosity because it accurately represents the viscosity property of physiological blood in real life compared to other viscosity models (Pratumwal et al. [41]).

According to [1, 26, and 17], the important formulation of the existing circumstances is as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{r\,z}) = \rho g sin\theta - \frac{1}{\mu}\frac{\partial p}{\partial z} - M'\mu_0\frac{\partial H'}{\partial z}, \quad (2)$$

where,

$$\sqrt{\tau_{r\,z}} = \begin{cases} \sqrt{\mu}\sqrt{-\frac{\partial u}{\partial r}} + \sqrt{\tau_0} & : \tau \geq \tau_0 \\ 0 & : \tau \leq \tau_0. \end{cases} \quad (3)$$

Here, θ is the proclivity angle, τ_0 is the yield stress, μ represents the blood viscosity, H' is the forced field intensity, M' represents the magnetization, $\tau_{r\,z}$ is the shear stress, p is the

pressure, μ_0 is the magnetic permeability, g is the acceleration due to gravity, ρ is the density of the fluid, and $(z,\ r)$ is the correspondingly axial and radial coordinates.
When the forces in the plug region are considered, the result is:

$$2\pi r_0\, L\, \tau_0 = P\, \pi\, r_0^2\, L \ \ \therefore\ \tau_0 = \frac{P\, r_0}{2}, \quad \text{where} \quad P = \frac{\partial p}{\partial z}\ . \tag{4}$$

2.2 Borderline conditions (BC) and mathematical solution

In calculating the solutions to simulated physical problems, the boundary constraints are crucial. Because the blood particles stick to the inner surface of the arterial segment in question, the axial swiftness (u) of blood molecules on the surface, which corresponds to one-dimensional flow, may be assumed to be equal to the velocity of vascular wall material points in the same direction.
The quantitative description of the stenosed portion of this case is as follows:

$$u = 0 \ \ at \ \ r = h\ . \tag{5}$$

Suppose that there is no liquid shear rate along the axis of the artery section in the problem, the fluid stream speed gradient along the axis can be written as follows:

$$\tau_{rz} \ \text{is finite at} \quad r = \ 0. \tag{6}$$

About half of the patients who had mild to moderate stenosis at the start of the study experienced increasing valve calcification, resulting in hemodynamically severe aortic stenosis symptoms. The cusps (flaps) of the aortic valve may thicken and stiffen or they may fuse in moderate stenosis circumstances. The aortic valve aperture narrows as a result of this. Due to the constricted valve cannot fully open, the blood flow from the human heart to the aorta and the rest of the body is reduced or blocked (Ott [46]).

The speed of the liquid is determined by taking into account the constraint for reasonable stenosis and addressing (2) within the boundary constraints (5) and (6) as follows:

$$u = \left[\frac{4}{3} r_0^{\frac{1}{2}} \left(r^{\frac{3}{2}} - h^{\frac{3}{2}}\right) - \frac{1}{2}(r^2 - h^2) - r_0(r - h)\right] \frac{(P+f+M)}{2\mu}\ . \tag{7}$$

When $r = r_0$ is substituted in the previous formula, the push rapidity becomes:

$$u_p = \left[-\frac{1}{6} r_0^2\ - \frac{4}{3} r_0^{\frac{1}{2}}\, h^{\frac{3}{2}} + \frac{1}{2} h^2 + h\, r_0\right] \frac{(P+f+M)}{2\mu}\ , \tag{8}$$

where $\quad F = \dfrac{\mu\ u^{\frac{1}{2}}}{\rho\ g\ R_0^{\frac{3}{2}}}, f = \dfrac{sin\alpha}{F},\ M = \dfrac{\mu_0\, \bar{M} H_0}{\rho\ U_0^2}.$

There are several ways to observe the flow flux Q of the fluid:

$$Q = 2\left[\int_0^{r_0} u_p\, r\, dr \quad + \quad \int_{r_0}^{h} u\, r\, dr\ \right]. \tag{9}$$

Then,

$$Q = \left[-\frac{1}{168} r_0^4\ - \frac{2}{7} r_0^{\frac{1}{2}}\, h^{\frac{7}{2}} + \frac{1}{8} h^4 + \frac{1}{6} h^3 r_0\right] \frac{(P+f+M)}{\mu} \tag{10}$$

The following dimensionless amounts were employed:

$$\left.\begin{aligned} r' = \frac{r}{R_0}\ ,\ r_0' = \frac{r_0}{R_0},\ \delta' = \frac{\delta}{R_0},\ H = \frac{h}{R_0}, z' = \frac{z}{L}, L_0' = \frac{L_0}{L},\ u = \frac{u'}{U_0}, d_1' = \frac{d_1}{L},\\ d_2' = \frac{d_2}{L}\ ,\ \ Q' = \frac{Q}{U_0 R_0^2}\ ,\quad p' = \frac{p}{\frac{\mu\, U_0\, B}{R_0^2}},\ H' = \frac{\bar{M}}{H_0}\ \ r_0' = \frac{r_0}{R_0}, r' = \frac{r}{R_0}. \end{aligned}\right\} \tag{11}$$

Equation (10) results from Eq. (11) as follows:

$$Q = \left[-\frac{1}{168} r_0^4\ - \frac{2}{7} r_0^{\frac{1}{2}}\, H^{\frac{7}{2}} + \frac{1}{8} H^4 + \frac{1}{6} H^3 r_0\right](P + f + M) \tag{12}$$

Equation (12) can be written as follows:

$$\frac{\partial p}{\partial z} = M + f - \frac{Q}{\left[-\frac{1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} H^{\frac{7}{2}} + \frac{1}{8}H^4 + \frac{1}{6}H^3 r_0\right]} \ . \tag{13}$$

Equation (13) computes the pressure difference Δp lengthways of the whole distance of the pipe as follows:

$$\Delta p = \int_0^1 \frac{\partial p}{\partial z} dz \ = \int_0^1 \left\{ M + f - \frac{Q}{\left[-\frac{1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} H^{\frac{7}{2}} + \frac{1}{8}H^4 + \frac{1}{6}H^3 r_0\right]} \right\} dz \ . \tag{14}$$

Flow opposition is described by

$$\lambda = \frac{\Delta p}{Q} \quad . \tag{15}$$

Starting Eqs. (12,) (14) and (15), we may infer that

$$\frac{1}{Q}\int_0^1 \left\{ M + f - \frac{Q}{\left[-\frac{1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} H^{\frac{7}{2}} + \frac{1}{8}H^4 + \frac{1}{6}H^3 r_0\right]} \right\} dz = \lambda \ . \tag{16}$$

Due to the nonappearance of stricture ($H = 1$), the pressure reduction is described as follows:

$$\int_0^1 \left\{ M + f - \frac{Q}{\left[-\frac{1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} + \frac{1}{8} + \frac{1}{6}r_0\right]} \right\} dz = (\Delta p)_n \ . \tag{17}$$

Flow impedance is defined as follows when there is no stenosis:

$$\lambda_n = \frac{(\Delta p)_n}{Q} \tag{18}$$

The expression is given by Eqs. (12), (17), and (18) as follows:

$$\lambda_n = \frac{1}{Q}\int_0^1 \left\{ M + f - \frac{Q}{\left[-\frac{1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} + \frac{1}{8} + \frac{1}{6}r_0\right]} \right\} dz \ . \tag{19}$$

The normalized opposition of a stream is written by:

$$\frac{\lambda}{\lambda_n} = \overline{\lambda} \ . \tag{20}$$

Shear stress is applied to the channel's surface as a consequence of

$$\tau_w = -\mu \frac{\partial u}{\partial r}\Big|_{r=h} \quad . \tag{21}$$

Using Eq. (11) to Eq. (21), it turns into:

$$\tau_w' = \frac{\tau_w}{\left[\frac{\mu U}{R_0}\right]} \quad . \tag{22}$$

On the other hand, Eq. (22) is reduced to the following

$$\tau_w' = -\frac{\partial u'}{\partial r'} \ . \tag{23}$$

Using the dimensionless method, we modify Eqs (7) and (11) in Eq (23), and the outcome is then

$$M + f - \frac{Q}{2}\left\{ \frac{2r_0^{\frac{1}{2}} H^{\frac{1}{2}} - H - r_0}{\frac{1}{168}r_0^4 + \frac{2}{7}r_0^{\frac{1}{2}} H^{\frac{7}{2}} - \frac{1}{8}H^4 - \frac{1}{6}H^3 r_0} \right\} = \tau_w \ . \tag{24}$$

Equation (24) is used to determine the shear stress at the surface in the nonappearance of stenosis ($H = 1$) as follows:

$$M + f - \frac{Q}{2}\left\{ \frac{2r_0^{\frac{1}{2}} - 1 - r_0}{\frac{1}{168}r_0^4 + \frac{2}{7}r_0^{\frac{1}{2}} - \frac{1}{8} - \frac{1}{6}H^3 r_0} \right\} = (\tau_w)_n \ . \tag{25}$$

It is possible to calculate the normalized surface shear stress as follows:

$$\frac{\tau_w}{(\tau_w)_n} = \overline{\tau}_w \quad . \tag{26}$$

3. Results and Discussion

Resistance to the flow and wall shear stress are two important variables in the study of blood flow through an overlapping stenosed artery. Investigative results for liquid velocity (u), flow opposition $(\bar{\lambda})$, and wall shear-stress $(\bar{\tau}_w)$ are shown in Eqs. (7), (20), and (26), respectively. The several restrictions on the opposition to the flow, wall shear stress, and fluid velocity are numerically estimated with the help of the *MATHEMATICA* software. The results are then visualized using graphs. The parameter values were chosen through a careful reading of previously published studies in reputable journals, through which the appropriate parameter values were revealed. In addition, the values that were taken are the appropriate values in terms of linking them to realistic models to simulate the real model or to be close to it.

3.1 Opposition to the flow

Opposition to the flow is found to yield higher values for the arteries with higher stenosis heights, but the opposite is true for the arteries with lower stenosis heights. It is important to note the physical cause of these observations. The blocked liquid in the stenosis area rapidly transfers near the core flowing area. As a result, the liquid encounters a brief obstruction in the pre-stenotic region before it decreases in size in the post-stenotic region. The effects of the impedance on various limitations, including stenosis height (δ), are depicted in Figs. 2-4. When there is stenosis, it is observed that the radial distance (r) of the linked region increases and the flow impedance (δ) increases (see Figure 2). The impedance rises when the Casson liquid possesses non-Newtonian characteristics.

According to Figure 3, as the angle of bent (θ) increases, the flow impedance $(\bar{\lambda})$ rises for the height of the stenosis (δ). These findings show that the decreased lumen size of the slanted artery, which influences the flow, causes a significant change in the plug flow radius. In comparison to arteries that are not inclined, the plug flow radius is larger in sloped arteries. It supports Srivastava's claims [33]. Figure 4 shows that as forced field constraint (M) rises, the flow opposition $(\bar{\lambda})$ increases with the height of the stenosis (δ). The induced magnetic effect provides a resistive force when the force field is applied correctly to the liquid's surface, preventing the liquid from moving. These results are in line with earlier findings by [5], [12], [27], and they also support the results of experiments by Bureau et al. [39] and McMillan et al. [40] on fluid flow resistance.

3.2 shear-stress of the wall

It is crucial to understand wall shear stress to figure out what is going on with the tiny arteries and arterioles. The arteries are impacted by the pressure gradient and wall shear stress. These arteries grow exceedingly hard and lose their flexibility over time. When these damaged arteries are subjected to excessive blood pressure, the arterial wall ruptures. The effects of surface shear stress $(\bar{\tau}_w)$ on various restrictions with a height of stenosis are shown in Figs. 5-7. As r of the linked flowing area increases, it is demonstrated that surface shear stresses both decrease and increase in response to the elevation of stenosis (see Figure 5). According to Figs. 6- 7, the height of the stenosis (δ) causes an increase in the proclivity angle (θ) and forced field constraint (M), which causes an increase in wall shear stress. The numerical findings of Young [1], Chakraborty-Mandal [13], and Prasad-Radhakrishnamacharya [30] are in agreement with these findings. In the case of fluid wall shear stress, our results concur with those of Bureau et al. [39], and McMillan [40] experiments.

3.3 Fluid Velocity

Figures 8-12 show how different restrictions impact the velocity (u) of the fluid. As stenosis height (δ) rises, it is demonstrated that fluid velocity (u) drops (see Figure 8). It is observed that the velocity (u) of the fluid is high in the middle of the tube and decreases towards the wall, and the velocity is zero at the wall of the tube. It clears that the magnitude of velocity is greater in the normal artery as compared with the stenosed artery. It is explored that the velocity (u) of the fluid ascends with an angle of proclivity (θ) for the case of stenosis (Figs 9, 11). Magnetic forces are used for the transport, separation, positioning, and sorting of magnetic and non-magnetic objects. Many areas in microfluidic applications involve the manipulation of particles in a controllable manner. The effect of magnetic field constraint is observed through Figure 10. It is also experiential that the velocity(u) of the fluid rises with an increase in the forced field constraint (M) for the cases of stenosis (Figs. 10, 12). In Figs. 8–12, the fluid velocity is seen to be decreasing as the radial distance and stenosis height increase. These results concur with previously published outcomes of Young [1] and Chakraborty-Mandal [13].

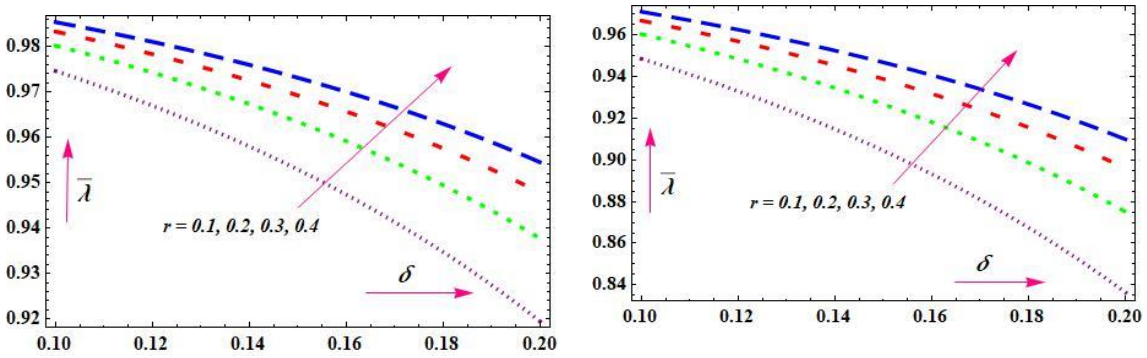

Figure 2: Design of $\bar{\lambda}$ for r through $\theta = {}^{\pi}/_{6}$, $Q = 0.1$, $d = 0.2$, $M = 2.0$, $L_0 = 0.4, F = 0.1$ (left) & $F = 0.3$ (right)

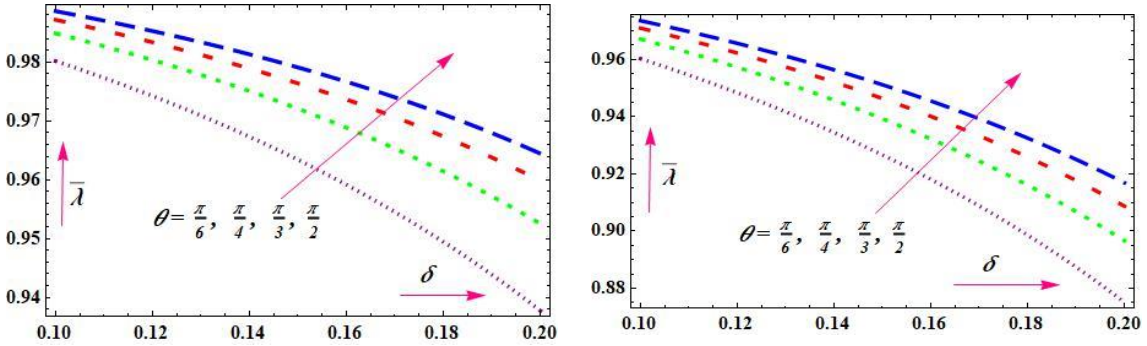

Figure 3: Design of $\bar{\lambda}$ for θ through $r = 0.2$, $Q = 0.1$, $d = 0.2$, $M = 2.0$, $L_0 = 0.4$, $F = 0.1$ (left) & $F = 0.3$ (right)

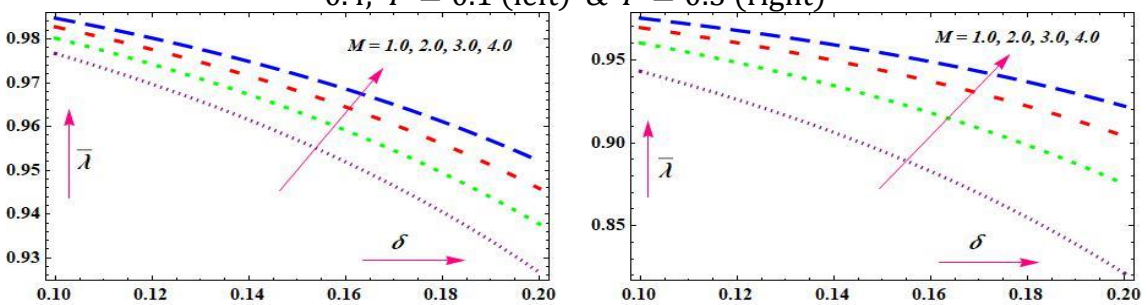

Figure 4: Design of $\bar{\lambda}$ for M through $r = 0.2$, $Q = 0.1, d = 0.2$, $\theta = {}^{\pi}/_{6}, L_0 =$

0.4, $F = 0.1$ (left) & $F = 0.3$ (right)

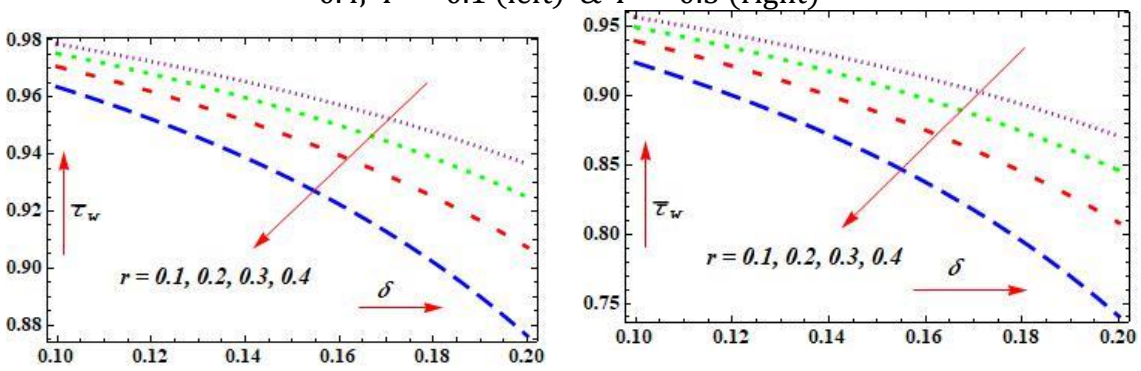

Figure 5: Design of $\bar{\tau}_w$ for r through M= 2.0, $Q = 0.1$, $d = 0.2$, $\theta = {}^{\pi}/_{6}$, $L_0 = 0.4$, $F = 0.1$ (left) & $F = 0.3$ (right)

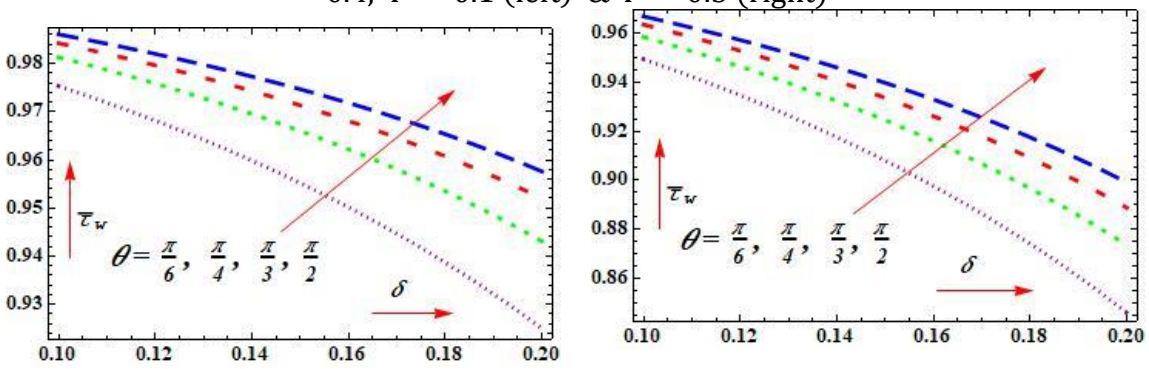

Figure 6: Design of $\bar{\tau}_w$ for θ through $M = 2.0$, $Q = 0.1$, $d = 0.2$, $r = 0.2$, $L_0 = 0.4$, $F = 0.1$ (left) & $F = 0.3$ (right)

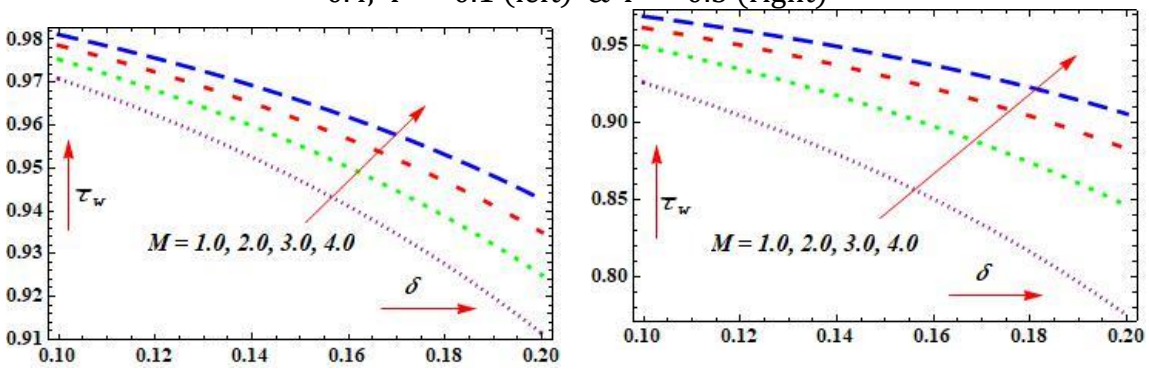

Figure 7: Design of $\bar{\tau}_w$ for M through $r = 0.2$, $Q = 0.1$, $d = 0.2$, $\theta = {}^{\pi}/_{6}$, $L_0 = 0.4$, $F = 0.1$ (left) & $F = 0.3$ (right)

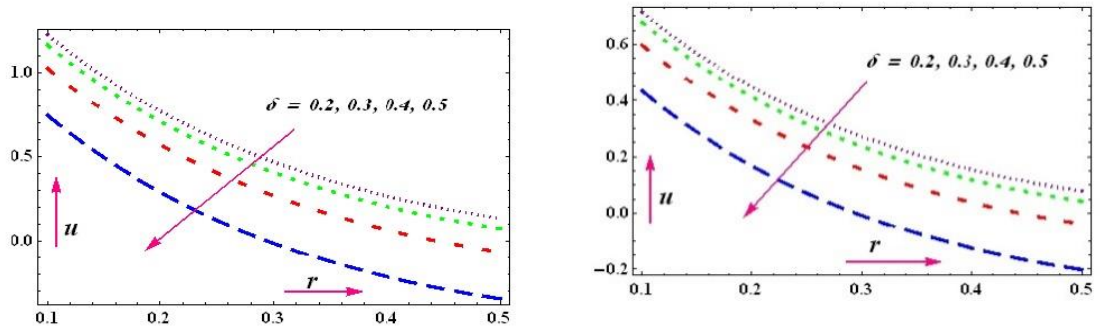

Figure 8: Design u & r for δ through $M = 2.0, z = 0.5, d = 0.2$, $\theta = {}^{\pi}/_{6}$, $L_0 = 0.4$, $F = 0.1$ (left) & $F = 0.3$ (right)

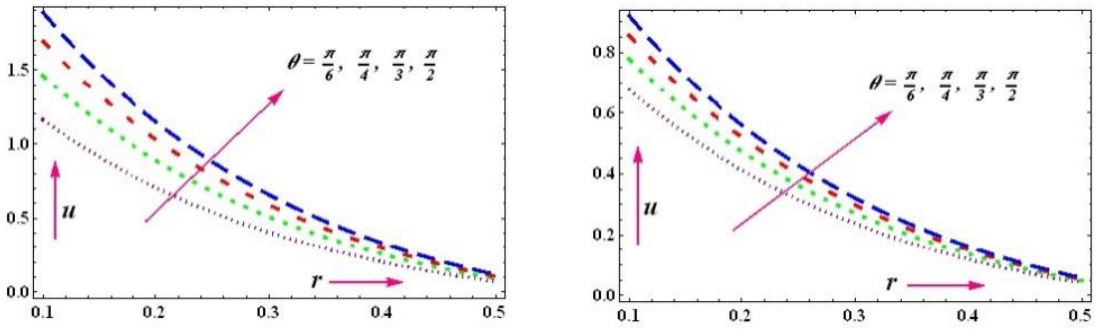

Figure 9: Design of u & r for θ through $\delta = 0.3, z = 0.5, M = 2.0,\ d = 0.2,\ L_0 = 0.4, F = 0.1$(left) & $F = 0.3$ (right)

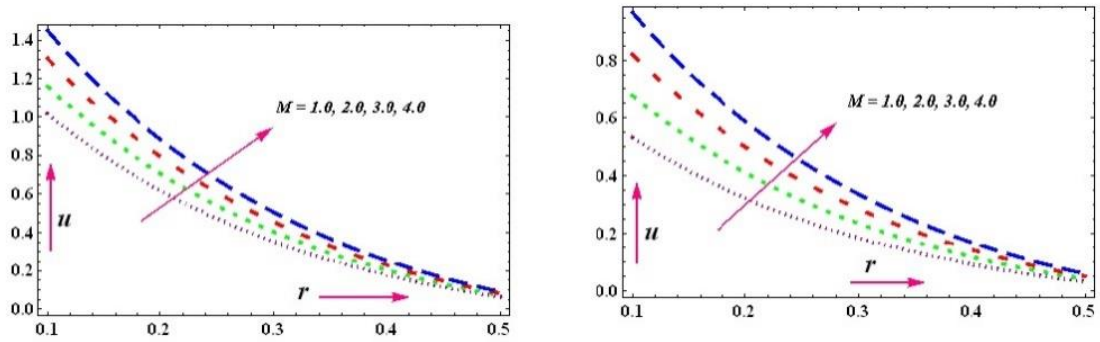

Figure 10: Design u & r for M through $\delta = 0.3, z = 0.5, d = 0.2,\ \theta = {}^{\pi}/_{6},\ L_0 = 0.4,\ F = 0.1$ (left) & $F = 0.3$ (right)

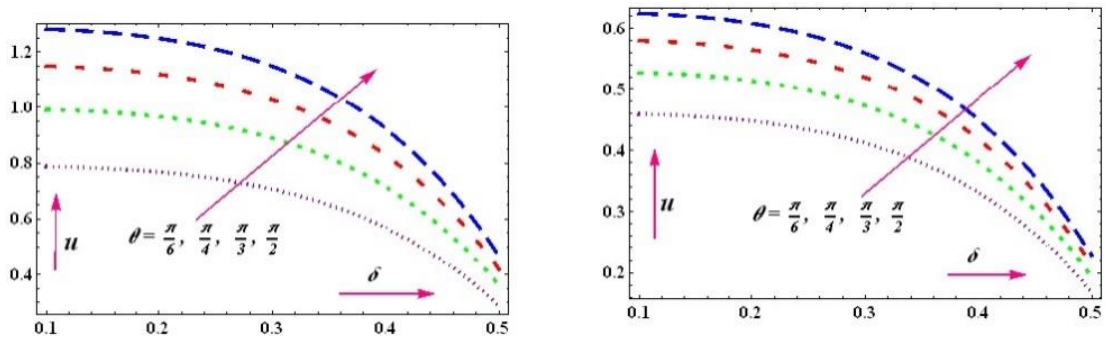

Figure 11: Design of u & δ for θ through $r = 0.2, z = 0.5, d = 0.2, \mathrm{M} = 2.0,\ L_0 = 0.4,\ F = 0.1$ (left) & $F = 0.3$ (right)

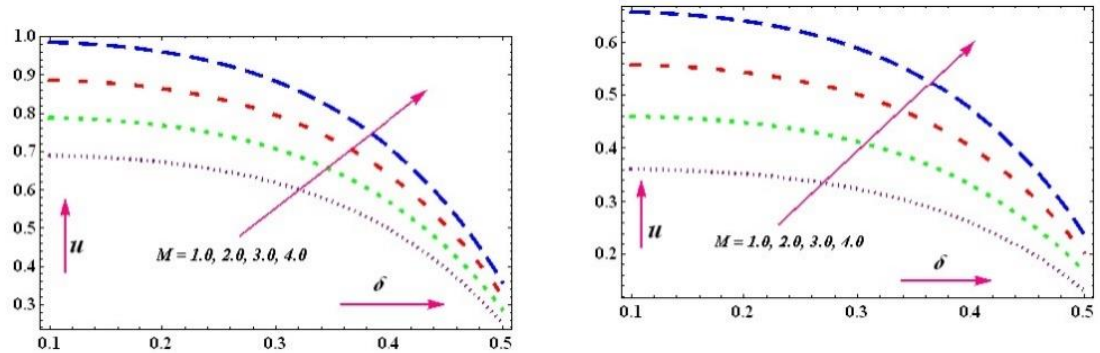

Figure 12: Design of u & δ for M through $r = 0.2, z = 0.5, d = 0.2,\ \theta = {}^{\pi}/_{6},\ L_0 = 0.4,\ F = 0.1$ (left) & $F = 0.3$ (right)

4. Conclusion

The mathematical model of Casson liquid in a steady, consistent tube with overlapping stenosis is examined. The results are represented graphically for varied radial distance, inclination angle, and stenosis height. The main conclusions are as follows:

- Surface shear stress falls and flowing resistivity rises as the radial length (r) of the flowing region increases.
- About the height of stenosis, the surface shear stress and the resistivity to flow increase together with an increase in the proclivity angle and forced field constraint.
- As the level of stenosis height increases, the velocity of the blood decreases.
- The fluid's velocity increases in response to an increase in proclivity angle and a force field constraint.
- For heights of stenosis, an increase in the radial length of the linked area results in a decrease in the fluid's velocity.

The above-mentioned results might be applied to improve blood vessel function. Drugs could be administered to people with aberrant blood vessel narrowing using this method. Moreover, the associated finding of the current physical model will act as a prototype for pharmaceutical and biological researchers engaged in research and development work. This mathematically based work may serve as a biomedical engineering prototype to treat vascular-related disorders.

Nomenclature

L:	The length of the tube (m).	$\bar{\lambda}$:	The resistance to the flow (kg/m^4s).
L_0:	The length of stenosis (m).	$\bar{\tau}_w$:	The wall shear stress (N/m^2).
d:	The length of non-stenotic segment (m).	$\tau_{r\,z}$:	The shear stress (N/m^2).
δ:	The maximum height of stenosis (m).	μ:	The blood viscosity $\left(\frac{kg}{ms}\right)$.
H':	The magnetic field intensity $\left(Tesla\ (T)\right)$.	τ_0:	The yield stress (N/m^2).
r:	The radial coordinate (m).	z:	The axial coordinate (m).
θ:	The angle of proclivity $(radian)$.	p:	The pressure across the region $\left(\frac{kg}{ms^2}\right)$.
$R(z)$:	The radius of the stenosed artery (m).	$R_0(z)$:	The radius of the normal artery (m).
M':	Tmhe agnetization $\left(\frac{A}{m}\right)$.	μ_0:	The magnetic permeability $\left(\frac{H}{m}\right)$.
g:	The acceleration due to gravity (m/s^2).	ρ:	The density (kg/m^3).
u:	The velocity of fluid $\left(\frac{m}{s}\right)$.	Q:	The volume flow rate $\left(\frac{kW}{m^2}\right)$.
M:	The forced field constraint $(Tesla\ (T))$	H:	The geometry of the artery wall.

Acknowledgments: The authors are grateful for the helpful comments from all reviewers which have led to a marked improvement in the manuscript.

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