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المعلمات.

Effects of Rotation and Inclined Magnetic Field on Walters' B Fluid in a Porous Medium using perturbation method or technique.

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Abstract

The peristaltic flow of Walter's B fluid through an asymmetric channel is analyzed. The constitutive equation for the balance of mass, momentum, trapped phenomena, and velocity distribution is modeled. Consideration of low Reynolds number and long wavelength assumption are used in the flow analysis to get an exact solution by using the perturbation method. The impact of impotent parameters on stream function and axial velocity has been gained. The set of figures is graphically analyzed of these parameters.

Keywords: Peristaltic flow, Walter's B, Inclined magnetic felid, Rotation, and Porous medium.

طريقة الاضطراب لحل مسألة تأثير الدوران والحقل المغناطيسي المائل على سائل والتر ب في وسط مسامى زينب على جعفر 1^* . لقاء زكى حمادى². مزعل حمد ذاوى³ ¹ قسم الرياضيات, كلية العلوم للحاسوب والرياضيات, جامعة تكريت, تكريت, العراق ²قسم الراضيات, كلية العلوم,جامعة بغداد,بغداد,العراق ³قسم الرباضيات, كلية العلوم للحاسوب والرباضيات, جامعة تكربت, تكربت, العراق الخلاصة تتناول الدراسة تحليل التدفق التمعجي لسائل والترب في أنبوب غير متماثل، كما تتم نمذجة المعادلة التركيبية لموازنة الكتلة والزخم والظواهر المحصورة وتوزيع السرعة. تم اعتماد طول موجى طويل مع افتراض عدد رينولد وإطئ عند تحليل الندفق ولإيجاد الحل الدقيق بتطبيق طريقة الاضطراب. وقد تم بالفعل إيجاد أنْر المعلمات الضعيفة لوظيفة التدفق والسرعة المحورية، كما تعطى الدراسة تحليلا بيانيا لمجموعة أعداد هذه

1. Introduction

Peristaltic pumping is known as a particular type of pumping when it can be easily transported a variety of complex rheological fluids from one place to another place. This pumping precept is called peristaltic. Peristalsis is what makes the small intestine move and it's crucial for digestion to work properly since it combines food with digestive juices, brings food to the surface of the intestine where nutrients may be absorbed, and moves food down the

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digestive tract. Instances of that include the extraction of crude oil from petroleum products, food mixing, and flow of blood which are studied by the references [1-3].

Non-Newtonian fluids are deemed superior to Newtonian fluids for a variety of engineering and biological applications. Both viscous and elastic peristaltic forces are present in a viscoelastic fluid which make it a non-Newtonian fluid [4].

The magneto hydrodynamic MHD phenomenon can be characterized by an interaction between the hydrodynamic and boundary layer and the electromagnetic field. The studies of the boundary layer flow of viscous and non-Newtonian fluids have received much attention because of their extensive application in the startegy metallurgy and chemical engineering. Such investigation of magneto hydrodynamic (MHD) flows are very important industrially and have application in different areas such as petroleum products and metallurgical processes. Now it is well known that in a technological application the non-Newtonian fluids are more applied than Newtonian fluids [5].

For the aforementioned cases, Walter's B fluid model is the most appropriate due to its confined viscosity at small shear rates with a short memory coefficient. Although industrial uses received a lot of interest [6-12] in Walter's B fluid, its focus was placed on a physiological perspective exclusively. In [13], the authors studied the peristaltic transport of Walter's B fluid in inclined ant symmetric channels and porous mediums. In [14], the authors applied the principle of nonlinear peristalsis transport to the movement of a porous medium along an inclinable planar channel. In [14], the authors examined the peristaltic flow of blood in a non-uniform channel under a magnetic field. The study of heat transfer and inclined magnetic field of the asymmetric channel on hyperbolic tangent peristaltic flow with the porous medium is discussed in [15]. The impact of the rotation on the mixed convection heat transfer analysis for the peristaltic transport of viscoelastic fluid in asymmetric channels is investigated in [16].

The purpose of this research is to investigate the effect of an angled magnetic field on the peristalsis flow of non-Newtonian Walter's B fluid. That fills in an asymmetric channel which is an incompressible fluid. Parameters including rotation, density, wave amplitude, and channel taper are varied. Also, different parameter values are studied by changing the axial velocity and stream function.

2. Mathematical Models of Problem

Consider the peristaltic transport of non-Newtonian Walter's B fluid of two dimensional in an asymmetric channel considering width d1+d2 and electrically conduction fluid through a porous medium. As a result, the symmetric channel has variable wave amplitudes, phase angles, and channel widths. The flow is caused because infinite sinusoidal wave trains reproduce with constant velocity c along the channel walls. Here is the modeling of the wall geometries

$$\overline{H}_{1}(\overline{x} \cdot \overline{t}) = d_{1} + a_{1} \sin\left[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t})\right], \tag{1}$$

$$\overline{H}_{2}(\overline{x} \cdot \overline{t}) = -d_{2} - a_{2} \sin\left[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t}) + \Phi\right], \tag{2}$$

where
$$d_1$$
 and d_2 are the width of the channel, a_1 and a_2 are the amplitudes of the waves, λ is the wavelength, c is the wave speed, Φ is the phase difference various in the range ($0 \le \Phi \le \pi$). Choose the two dimensions system, \overline{X} is along the centerline channel, while \overline{Y} is transverse to it. It shows that at $\Phi=0$, waves in the symmetric channel are shown to be out of phase, whereas when $\Phi=\pi$ the waves are in phase. Further, a_1 , a_2 , d_1 , d_2 and Φ satisfy the condition.
 $a_1^2 + a_2^2 + 2a_1a_2\cos(\Phi) \le (d_1 + d_2)^2$. (3)



Figure 1: physical illustration of the issue

3. Constitutive Equation

The constitutive equations (the continuity equation, momentum equation) for incompressible fluid (Walter's B) are illustrated in a representation of the laboratory framework $(\overline{x}, \overline{y})$ as follows:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 .$$
(4)

On the x-axis

$$\rho\left(\frac{\partial}{\partial\overline{t}} + \overline{u}\frac{\partial}{\partial\overline{x}} + \overline{v}\frac{\partial}{\partial\overline{y}}\right)\overline{u} - \rho\Omega\left(\Omega\overline{u} + 2\frac{\partial\overline{v}}{\partial\overline{t}}\right) = -\frac{\partial\overline{p}}{\partial\overline{x}} + \frac{\partial}{\partial\overline{x}}\overline{\tau}_{\overline{x}\overline{x}} + \frac{\partial}{\partial\overline{y}}\overline{\tau}_{\overline{x}\overline{y}} - \sigma B_0^2\cos\beta^*$$
(5)
$$(\overline{u}\cos\beta^* - \overline{v}\sin\beta^*) - \mu\frac{\overline{u}}{\overline{k}} \quad .$$

On the v-axis

$$\rho(\frac{\partial}{\partial \overline{t}} + \overline{u}\frac{\partial}{\partial \overline{x}} + \overline{v}\frac{\partial}{\partial \overline{y}})\overline{v} - \rho\Omega(\Omega\overline{v} - 2\frac{\partial\overline{u}}{\partial\overline{t}}) = -\frac{\partial\overline{p}}{\partial\overline{y}} + \frac{\partial}{\partial\overline{x}}\overline{\tau}_{\overline{x}\overline{y}} + \frac{\partial}{\partial\overline{y}}\overline{\tau}_{\overline{y}\overline{y}} + (6)$$

$$\sigma B_0^2 \sin\beta^* (\overline{u}\cos\beta^* - \overline{v}\sin\beta^*) - \mu\frac{\overline{v}}{\overline{k}} \quad .$$

Where the $\overline{\tau_{xx}}$, $\overline{\tau_{xy}}$ and $\overline{\tau_{yy}}$ are the components of the tensor of additional stress. The fluid density is ρ axial velocity, \overline{u} and \overline{v} are the transverse velocity, \overline{y} is the transverse coordinate, \overline{p} represents the pressure, μ is the viscosity, \overline{K} is the permeability parameter, B_0 is the constant magnetic field and σ denotes the electrical conductivity. In nature, the motion peristaltic is unsteady. The steady is assumed by using transformation from laboratory frame (\overline{x} , \overline{y}) to wave frame (\overline{X} , \overline{Y}) which is defined as follows:

$$\overline{X} = \overline{x} - c\overline{t}, \overline{Y} = \overline{y}, \ \overline{U} = \overline{u} - c, \overline{V} = \overline{v}, \overline{P}(\overline{X}, \overline{Y}) = \overline{P}(\overline{x}, \overline{y}, \overline{t})$$
(7)

The components of velocity are shown by $\overline{U}, \overline{V}$ and \overline{P} of the pressure in the wave frame. To perform the dimensional less analysis, we arrange the following dimensional less quantities as follows:

$$\bar{x} = \lambda x, \ \bar{y} = d_1 y, \ \bar{u} = c u, \bar{v} = c v \delta, \overline{H}_1 = h_1 d_1, \overline{H}_2 = h_2 d_1, \bar{p} = \frac{p \mu c \lambda}{d_1^2}$$
(8)

$$\overline{k} = kd_1^2, Re = rac{
ho cd_1}{\mu}, \delta = rac{d_1}{\lambda}, Ha = d_1 \sqrt{rac{\sigma}{\mu}} B_0, \overline{\tau} = rac{\tau\mu c}{d_1},$$

$$K = \frac{k_0 c}{\mu d_1}, \bar{t} = \frac{t\lambda}{c}.$$

Where *Ha*, *Re*, δ , and K are the Hartman number, the Reynold number, the wave number, and the viscoelastic parameter, respectively. Therefore, Eq (8), Eqs. (4) to (6) take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 . aga{9}$$

$$\operatorname{Re}\delta\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \frac{\rho\Omega^2 d_1^2}{\mu}u - \frac{2\delta^2}{c\mu}\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} + \delta\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} -$$

$$Ha^2\cos\beta^*(u\cos\beta^* - v\delta\sin\beta^*) - \frac{u}{x}$$
(10)

$$\operatorname{Re}\delta^{3}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\rho\Omega^{2}d_{1}^{2}}{\mu}v\delta^{2} + \frac{2\delta^{2}}{c\mu}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial y} + \delta^{2}\frac{\partial \tau_{xy}}{\partial x} + \delta\frac{\partial \tau_{yy}}{\partial y} + Ha^{2}\delta\sin\beta^{*}(u\cos\beta^{*} - v\delta\sin\beta^{*}) - \delta\frac{v}{k}$$

$$(11)$$

The components of the extra stress tenser of Walter's B are listed below

$$\bar{S} = 2\mu r - 2K_0\omega.$$

$$\omega = \frac{\partial r}{\partial t} + V.\nabla r - r\nabla V - (\nabla V)^T r$$

$$\tau_{xx} = 2\delta u_x - 2\delta^2 K[uu_{xx} + vu_{xy} - v_x u_y - \delta^2 v_x^2 - 2u_x^2].$$
(12)

$$\tau_{xy} = (u_y + \delta^2 v_x) - \delta K [u u_{yx} + \delta^2 u v_{xx} + v u_{yy} + \delta^2 v v_{xy} - 3 u_y u_x$$
(13)
$$-3\delta^2 v_x v_y - v_y u_y - \delta^2 u_x v_x]$$

$$\tau_{yy} = 2\delta v_y - K \Big[2\delta^2 u v_{yx} + 2\delta^2 v v_{yy} - 2u_y^2 - 2\delta^2 u_y v_x - 4\delta^2 v_y^2 \Big] .$$
(14)

The stream function is linked to the velocity components.
$$\psi$$
 due to the relations
 $U = \frac{\partial \psi}{\partial y}, V = -\frac{\partial \psi}{\partial x}$. (15)
Substituted Eq.(7) and Eq.(15) in Eqs.(10)-(14), respectively.

$$\operatorname{Re}\delta\left(\psi_{y}\frac{\partial\psi_{y}}{\partial x}-\psi_{x}\frac{\partial\psi_{y}}{\partial y}\right)-\frac{\rho\Omega^{2}d_{1}^{2}}{\mu}\psi_{y}+\frac{2\delta^{2}}{c\mu}\frac{\partial\psi_{x}}{\partial t}=-\frac{\partial p}{\partial x}+\delta\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}-Ha^{2}\cos\beta^{*}$$

$$\left((\psi_{y}+1)\cos\beta^{*}+\delta\psi_{x}\sin\beta^{*}\right)-\frac{1}{\mu}\psi_{y}.$$
(16)

$$\operatorname{Re}\delta^{3}\left(\psi_{y}\frac{\partial\psi_{x}}{\partial x}-\psi_{x}\frac{\partial\psi_{x}}{\partial y}\right)+\frac{\rho\Omega^{2}d_{1}^{2}\delta^{2}}{\mu}\psi_{x}-\frac{2\delta^{2}}{c\mu}\frac{\partial\psi_{y}}{\partial t}=-\frac{\partial p}{\partial y}+\delta^{2}\frac{\partial\tau_{xy}}{\partial x}+\delta\frac{\partial\tau_{yy}}{\partial y}+$$

$$\delta Ha^{2}\sin\beta^{*}\left((\psi_{y}+1)\cos\beta^{*}+\delta\psi_{x}\sin\beta^{*}\right)+\frac{\delta}{\mu}\psi_{x}.$$
(17)

$$\tau_{xx} = (2\delta\psi_{xy}) - 2\delta^2 K (\psi_y \psi_{xxy} - \psi_x \psi_{xyy} - 2\psi_{xy}^2 + \psi_{xx} \psi_{yy} + \delta^2 \psi_{xx}^2).$$
(18)
$$\tau_{xy} = (\psi_{yy} - \delta^2 \psi_{xx}) - \delta K (\psi_y \psi_{xyy} - \delta^2 \psi_y \psi_{xxx} - \psi_x \psi_{yyy})$$
(19)

$$+\delta^{2}\psi_{x}\psi_{xxy} - 2\psi_{xy}\psi_{yy} - 2\delta^{2}\psi_{xx}\psi_{xy}).$$

$$\tau_{yy} = -2\delta\psi_{xy} - K(-2\delta^{2}\psi_{y}\psi_{xxy} + 2\delta^{2}\psi_{x}\psi_{xyy} - 2\psi_{yy}^{2} + 2\delta^{2}\psi_{xx}\psi_{yy} - 4\delta^{2}\psi_{xy}^{2}).$$
 (20)

Elimination of pressure between Eqs.(16) and (17), we have

$$\operatorname{Re}\delta(\psi_{y}\frac{\partial}{\partial x}-\psi_{x}\frac{\partial}{\partial y})\nabla^{2}\psi + (\frac{\rho\Omega^{2}d_{1}^{2}\delta^{2}}{\mu}+\frac{2\delta^{2}}{c\mu}\frac{\partial}{\partial t})(\psi_{x}-\psi_{y}) = \delta[\frac{\partial^{2}}{\partial x\partial y}(\tau_{xx}-\tau_{yy})] + [\frac{\partial^{2}}{\partial y^{2}} - (21)$$

$$\delta^{2}\frac{\partial^{2}}{\partial x^{2}}]\tau_{xy} - Ha^{2}\cos^{2}\beta^{*}\frac{\partial^{2}\psi}{\partial y^{2}} + \delta^{2}Ha^{2}\sin^{2}\beta^{*}\frac{\partial^{2}\psi}{\partial x^{2}} - (21)$$

$$-\delta Ha^{2}\cos\beta^{*}\sin\beta^{*}\psi_{xy} - \frac{1}{k}(\psi_{yy} + \delta\psi_{xx}).$$
Where

$$\nabla^{2} = \delta^{2}\frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}}$$
The wave frames by the dimensionless boundary conditions are

$$\psi = \frac{F}{2}, \frac{\partial\psi}{\partial y} = -1 \text{ at } y = h_{I}.$$
(22)

$$\psi = \frac{-F}{2}, \frac{\partial\psi}{\partial y} = -1 \text{ at } y = h_{2}.$$
(23)

Without dimensions time mean flow rate F in the wave frame. In the laboratory frame without dimensions time mean flow rate Q1 is related to F by the expression:

Q1=F + 1 + d.(24) The h₁(x) and h₂(x) are the dimensionless forms h₁(x) = 1+a sin x, h₂(x) = -d-b sin (x + Φ), (25) where a ,b, Φ and d satisfy: $a^2 + b^2 + 2ab cos(\Phi) \le (1 + d)^2.$ (26)

4. The solution to the problem

Eq. (21) is exceedingly non-linear and convoluted. All arbitrary parameters of Eq. (21) are impossible to put in closed form solution. Therefore, we use the perturbation technique to find the solution. For the perturbation solution, we expand.

$$\begin{split} \psi &= \psi_0 + K\psi_1 + o(K^2), \\ F &= F_0 + KF_1 + o(K^2) , \\ p &= p_0 + Kp_1 + o(K^2), \\ U &= U_0 + KU_1 + o(K^2). \end{split}$$
(27)

We replace the expression (27) in Eq.s (16-21) with boundary conditions.

Since $\delta \ll 1$, Eq.s (22-23). Because the higher order terms including the powers of(δ)are smaller, then the convergent solution is almost non-existent. We gain the following by paralleling the coefficients of powers of (δ).

4.1 System of zero order

We can negligible the terms of order (δ), so we obtain from Eq.(21)

$$\frac{\partial^2 \tau_{0xy}}{\partial y^2} - m^2 \psi_{0yy} = 0 \quad . \tag{28}$$

$$m^{2} = Ha^{2} \cos^{2} \beta^{*} + \frac{1}{k}.$$
From Eq. (16), we get
$$(29)$$

$$\frac{\partial p_0}{\partial x} = \frac{\rho \Omega^{2d_1^2}}{\mu} \psi_{0y} + \frac{\partial}{\partial y} \tau_{0xy} - m^2 (\psi_{0y} - 1) \quad .$$

$$0 = \frac{\rho \Omega^{2d_{1}}}{\mu} \psi_{0yy} + \psi_{0yyyy} - m^{2} \psi_{0yy}.$$

$$0 = \psi_{0yyyy} - \psi_{0yy}(\xi) , \qquad (31)$$

(30)

when From Eq.(17), we get $\frac{\partial p}{\partial y} = 0.$ (32) Such that

$$\tau_{0xy} = \psi_{0yy} , \tau_{0xx} = 0, \tau_{0yy} = 2K\psi_{0yy}^2 .$$
(33)

$$\psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1 \quad \text{at } y = h_I. \tag{34}$$

$$\psi_0 = -\frac{F_0}{2}, \frac{\partial \psi_0}{\partial y} = -1 \text{ at } y = h_2.$$
 (35)

4.2 System of the first order

We substitute Eq. (27) in Eqs. (16)-(21) and negligible higher order of δ , $o(\delta^2)$, we obtain $\operatorname{Re}(\psi_{0y}\psi_{0yyx} - \psi_{0x}\psi_{0yyy}) - \frac{\rho\Omega^2 d_1^2}{\mu}\psi_{1yy} = \frac{\partial^2}{\partial x \partial y}\tau_{0yy} + \frac{\partial^2}{\partial y^2}\tau_{1xy} -$ (36) $Ha^2 \cos^2\beta^*\psi_{1yy} - Ha^2 \cos\beta^* \sin\beta^*\psi_{0xy} - \frac{1}{k}(\psi_{0yy} - \psi_{1yy}).$

And

$$\frac{\partial p_1}{\partial x} = \frac{\partial}{\partial y} \tau_{1xy} - m^2 (\psi_{1y} - 1) + \psi_{0x} \sin\beta^* - \operatorname{Re}(\psi_{0y}\psi_{0yx} - 1) + \psi_{0x} \sin\beta^* - \operatorname{Re}(\psi$$

$$\psi_{0x}\psi_{0yy}) + \frac{1}{\mu}\psi_{1y}.$$

$$\tau_{1xy} = \psi_{1yy} - K(\psi_{0y}\psi_{0xyy} - \psi_{0x}\psi_{0yyy} - 2\psi_{0xy}\psi_{0yy}) \quad .$$
(38)

$$\tau_{1xx} = 2\psi_{0xy} \quad . \tag{39}$$

$$\tau_{1yy} = -2\psi_{0xy} + 2K\psi_{1yy}^2 \,. \tag{40}$$

$$\psi_1 = \frac{1}{2}, \frac{3\psi_1}{3y} = -1$$
 at $y = h_1$. (41)

$$\psi_1 = -\frac{F_1}{2} \quad , \frac{\partial \psi_1}{\partial y} = -1 \text{ at } y = h_2.$$

$$\tag{42}$$

5. Discussion of the findings

This section contains two subsections. The trapping is discussed in the first subsection. While, in the second subsection, the software MATHEMATIC is used to represent the velocity distribution graphically.

5.1 Trapping Section

Trapping is a fascinating peristaltic motion phenomenon. A closed stream line forms a bolus of fluid that entirely circulates inside itself. We put the stream function profile to the test by modifying the values of parameters *K*, *Ha*. ϕ , Ω , *Q*, and β^*). The profile of stream lines behavior decreases when different parameters are changed. Notes that the changing of the stream line and appearing the bolus for every parameter are clearly shown in Figures (2-7).

5.2 Velocity Section

The velocity distribution is represented using various properties. The change in velocity for different values of parameters $K, Ha, \phi, \Omega, Q, \text{and }\beta^*$) is explained in Figures(8-13). In Figures(8,10), the axial velocity is the lowest at the channel's middle according to our observations with increasing the parameters Ω and Ha, whereas the axial velocity increases along the channel wall's boundary. As the fluid parameter K is increased, the axial velocity in the centre of the channel increases, while it decreases near the wall of the channel which is explained in Figure (9). Figures(12-13) show that there is no effect on the axial velocity with increase in the value of parameters Q and β^* .







6. Conclusion

-The study tackles the effects of rotation and inclined magnetic field on Walters' B fluid under the effects of rotation in an asymmetric channel with a porous medium. Special attention was placed in this inquiry to study trapping phenomena and velocity distribution using a simple analytical solution.

-In the case of stream function, the trapped bolus size increases with increasing the values of parameters (*K*, *Ha*, ϕ , Ω , *Q*, and β^*).

- The axial velocity decreases in the core part of the channel as both parameters Ω and Ha increase, whereas the axial velocity increases along the channel wall's perimeter.

- As the fluid parameter K is raised, the axial velocity increases in the channel's center and decreases in its proximity to the wall.

- The axial velocity increases the wall of the channel with increasing the parameter ϕ , whereas decreases in the central region.

- No effect on the axial velocity with increase in the values of parameters Q and β^* is seen

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