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Numerical Comparison by using Some Criteria to Estimate the Two Parameters of the Exponentiated Exponential Distribution

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Abstract
The present paper shows a numerical comparison by using the number of the following criteria: mean squared, root mean Square, mean absolute, and relative mean squared error values between the methods, namely maximum likelihood and Bayesian estimators for the two parameters of the exponentiated exponential distribution. In estimating the maximum likelihood, The equations cannot be solved directly, Then Newton-Raphson method is used. Because of Bayes estimators under scale invariant squared and weighted composite linear-exponential loss functions the ratios cannot be simplified in a closed form. So, we use Lindley approximation. MATLAB program is used to display the results.

Keywords: Exponentiated Exponential Distribution, Maximum Likelihood, Bayes Estimation, Lindley's Approximation, Scale-invariant squared Loss Function.

مقارنة عددية باستخدام بعض المعايير لتقدير معلمتى التوزيع الأسي الاسي

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قسم الرياضيات, كلية العلوم, الجامعة المستنصرية, بغداد, العراق

الخلاصة
يوضح البحث الحالي مقارنة عددية باستخدام عدد من المعايير متوسط التربيعي، مربع متوسط الجذر، متوسط قيم الخطأ التربيعي المطلق والنسبي بين الطريقتين: أقصى احتمال، مقدرات بايزي لمعاملي التوزيع الأسي الأسي. لتقدير الاحتمالية القصوى، لا يمكن حل المعادلات مباشرة، تم استخدام طريقة نيوتن رافسون. إما مقدرات بايز تحت دوال الخسارة الخطية–الأسية المركبة ذات المقياس الثابت والمربّع الموزون، لا يمكن تبسيط النسب في شكل مغلق، لذلك نستخدم تقريب لندلي. البرنامج المستخدم لعرض النتائج هو ماتلاب.

1. Introduction

Exponentiated Exponential Distribution (EX2D) is generalization of exponential distribution by adding q as a shape parameter. It was proposed Kunda and Gupta [1]. The EX2D is also called the General Exponential Distribution [2]. Al-Sultany and Mohammed, [3] presented Bayesian estimation for the parameters and reliability function of a Perks distribution based on two different loss functions by using Lindley's approximation. Al-Sultany [4] discussed a constructing new exponentiated family distribution with reliability estimation.

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Let $u_1, u_2, \dots, u_n$ be a random sample of size n from EX2D then the cumulative distribution function is can then be obtained as: [5]:

$$F(u; p, q)_{\text{EX2D}} = (1 - e^{-qu})^{p}; u \geq 0; p, q > 0. \quad (1)$$

A probability density function of EX2D is obtained by:

$$f(u; p, q)_{\text{EX2D}} = \frac{\partial F(u; p, q)_{\text{EX2D}}}{\partial u}$$

$$f(u; p, q)_{\text{EX2D}} = \begin{cases} pq(1 - e^{-qu})^{p-1} \; e^{-qu} & ; u \geq 0; p, q > 0, \\ 0 & ; o.w. \end{cases} \quad (2)$$

where p is the shape parameter and q is the scale parameter.

The likelihood function (L-K) $\mathfrak{L}(p, q | \underline{u})_{\text{EX2D}}$ can then be obtained as:

$$\mathfrak{L}(p, q | \underline{u})_{\text{EX2D}} = p^n q^n \prod_{i=1}^{n} (1 - e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i} \quad (3)$$

The log – L-K is:

$$\iota\mathfrak{L} = \ln \mathfrak{L}(p, q | \underline{u})_{\text{EX2D}} = nlnp + nlnq + (p - 1) \sum_{i=1}^{n} \ln(1 - e^{-qu}) - q \sum_{i=1}^{n} u_i \quad (4)$$

2. Different methods of estimation the two parameters p and q

2.1 Maximum Likelihood Method (MLM)

Differentiating the log – L-K ,partially with respect to p and q, respectively.

$$\frac{\partial \iota\mathfrak{L}}{\partial p} = \frac{n}{p} + \sum_{i=1}^{n} ln(1 - e^{-qu_i}) = 0 . \quad (5)$$

$$\frac{\partial \iota\mathfrak{L}}{\partial q} = \frac{n}{q} + (p - 1) \sum_{i=1}^{n} \frac{u_i e^{-qu_i}}{1 - e^{-qu_i}} - \sum_{i=1}^{n} u_i = 0. \quad (6)$$

Equations (5) and (6) cannot be solved directly, so we use Newton-Raphson's $(n - r)$ method and the second partial derivatives are obtained as follows;

$$\frac{\partial^2 \iota\mathfrak{L}}{\partial p^2} = -np^{-2} \quad (7)$$

$$\frac{\partial^2 \iota\mathfrak{L}}{\partial q^2} = -nq^{-2} - (p - 1) \sum_{i=1}^{n} \frac{(u_i)^2 e^{-qu_i}}{(1 - e^{-qu_i})^2} \quad (8)$$

$$\frac{\partial^2 \iota\mathfrak{L}}{\partial p \partial q} = \frac{\partial^2 \iota\mathfrak{L}}{\partial q \partial p} = \sum_{i=1}^{n} \frac{u_i e^{-qu_i}}{1 - e^{-qu_i}} \quad (9)$$

Where $\hat{p}_{MLM}$ and $\hat{q}_{MLM}$ are the MLM of p and q of the EX2D by using $(n - r)$ method respectively.

2.2 Bayesian Method (BM)

The prior distributions of p and q of $EX2D$ are taken to be independent exponential $(\mathfrak{m})$ and exponential (v) respectively leads to a joint prior distribution of the form ,

$$\mathcal{T}(p, q) = \mathfrak{m} v e^{-(\mathfrak{m} p + v q)} . \quad (10)$$

A joint posterior density function using the two equations (10) and (3) ,can be obtained by the following expression,

$$\Psi(p, q | \underline{u}) = \frac{p^n q^n \prod_{i=1}^{n} (1 - e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i} e^{-(\mathfrak{m} p + v q)}}{\int_q \int_p p^n q^n \prod_{i=1}^{n} (1 - e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i} e^{-(\mathfrak{m} p + v q)} dp \, dq} \quad (11)$$

2.2.1 Bayes Estimators Under Scale Invariant Squared Loss Function using Lindley's Approximation Method (BM-SIS).

Bayes estimation of any function of the parameter $h(p, q)$ under scale invariant squared error (SIS) ,can be obtained by the following expression, [6]:

$$\hat{h}_{SIS}(p,q)=\frac{E\left(\frac{1}{h(p,q)}|\underline{u}\right)}{E\left(\frac{1}{(h(p,q))^2}|\underline{u}\right)}=\frac{A}{B}\ . \tag{12}$$

Where the numerator A is

$$A=\frac{\int_q\int_p\frac{1}{h(p,q)}\ p^nq^n\prod_{i=1}^n(1-e^{-qu_i})^{p-1}\ \prod_{i=1}^n e^{-qu_i}\ e^{-(mp+vq)}dp\ d}{\int_q\int_p\ p^nq^n\prod_{i=1}^n(1-e^{-qu_i})^{p-1}\ \prod_{i=1}^n e^{-qu_i}\ e^{-(mp+vq)}dp\ dq}\ . \tag{13}$$

And the denominator B is

$$B=\frac{\int_q\int_p\frac{1}{(h(p,q))^2}\ p^nq^n\prod_{i=1}^n(1-e^{-qu_i})^{p-1}\ \prod_{i=1}^n e^{-qu_i}\ e^{-(mp+vq)}dp\ d}{\int_q\int_p\ p^nq^n\prod_{i=1}^n(1-e^{-qu_i})^{p-1}\ \prod_{i=1}^n e^{-qu_i}\ e^{-(mp+vq)}dp\ dq}. \tag{14}$$

We note that the ratios A and B cannot be simplified in a closed form. So, we use Lindley Approximation [7].

If $h(p,q)=p$ then the numerator A ,can be obtained by the following expression,

$$A=E\left(\frac{1}{p}|\underline{u}\right)=\frac{1}{\hat{p}_{MLM}}+\left[\left(\frac{1}{\hat{p}_{MLM}{}^3}+\frac{m}{\hat{p}_{MLM}{}^2}\right)\hat{S}_{pp}+\frac{v}{\hat{p}_{MLM}{}^2}\hat{S}_{pq}\right]-\frac{1}{2\hat{p}^2}\left[\hat{S}_{pp}(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp})+\hat{S}_{qp}(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp}+\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq})\right]\ . \tag{15}$$

Where the denominator B, can be obtained by the following expression,

$$B=E\left(\frac{1}{p^2}|\underline{u}\right)=\frac{1}{\hat{p}_{MLM}{}^2}+\left[\left(\frac{3}{\hat{p}_{MLM}{}^4}+\frac{2m}{\hat{p}_{MLM}{}^3}\right)\hat{S}_{pp}+\frac{2v}{\hat{p}_{MLM}{}^3}\hat{S}_{pq}\right]-\frac{1}{\hat{p}_{MLM}{}^3}\left[\hat{S}_{pp}(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp})+\hat{S}_{qp}(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp}+\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq})\right]\ . \tag{16}$$

So, Bayes estimate of p under SISELF is:

$$\hat{p}^L_{SIS}=\frac{A}{B}=\frac{E\left(\frac{1}{p}|\underline{u}\right)}{E\left(\frac{1}{p^2}|\underline{u}\right)} \tag{17}$$

If $h(p,q)=q$ then the numerator A, can be obtained by the following expression,

$$A=E\left(\frac{1}{q}|\underline{u}\right)=\frac{1}{\hat{q}}+\frac{m}{\hat{q}^2}\ \hat{S}_{qp}+\left(\frac{1}{\hat{q}^3}+\frac{v}{\hat{q}^2}\right)\hat{S}_{qq}-\frac{1}{2\hat{q}^2}[\hat{S}_{pq}(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp})+\hat{S}_{qq}(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp}+\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq})]. \tag{18}$$

And the denominator B will be

$$B=E\left(\frac{1}{q^2}|\underline{u}\right)=\frac{1}{\hat{q}^2}+\frac{2m}{\hat{q}^3}\ \hat{S}_{qp}+\left(\frac{3}{\hat{q}^4}+\frac{2v}{\hat{q}^3}\right)\hat{S}_{qq}-\frac{1}{\hat{q}^3}[\hat{S}_{pq}(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp})+\hat{S}_{qq}(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq}+\iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp}+\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq})] \tag{19}$$

So, Bayes estimate of q under SISELF is:

$$\hat{q}^L_{SIS}=\frac{A}{B}=\frac{E\left(\frac{1}{q}|\underline{u}\right)}{E\left(\frac{1}{q^2}|\underline{u}\right)} \tag{20}$$

2.2.2 Bayes Estimators Under Weighted Composite Linear-Exponential Loss Function using Lindley's Approximation Method (BM-WCLE) [8]

Bayes estimation of any function of the parameter $h(p,q)$ under weighted composite linear-exponential loss function (WCLE) ,can be obtained by the following experssion [9]:

$$\hat{h}_{WCLE}(p,q)=\frac{1}{2a}\ln\left[\frac{E\left(e^{-(w-a)h(p,q)}|\underline{u}\right)}{E\left(e^{-(w+a)h(p,q)}|\underline{u}\right)}\right]=\frac{1}{2a}\ln\left[\frac{C}{D}\right]. \tag{21}$$

Where, $a>0,\ w$ is the weighted function

$$C = \frac{\int_{q}\int_{p} e^{-(w-a)h(p,q)}\, p^n q^n \prod_{i=1}^{n}(1-e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i}\, e^{-(mp+vq)} dp\, d}{\int_{q}\int_{p} p^n q^n \prod_{i=1}^{n}(1-e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i}\, e^{-(mp+vq)} dp\, dq} \tag{22}$$

and

$$D = \frac{\int_{q}\int_{p} e^{-(w+a)h(p,q)}\, p^n q^n \prod_{i=1}^{n}(1-e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i}\, e^{-(mp+vq)} dp\, d}{\int_{q}\int_{p} p^n q^n \prod_{i=1}^{n}(1-e^{-qu_i})^{p-1} \prod_{i=1}^{n} e^{-qu_i}\, e^{-(mp+vq)} dp\, dq}. \tag{23}$$

Using the Lindley approximation, we find C and D

If $h(p,q) = p$

Then

$$C = -(w-a)e^{-(w-a)\hat{p}} + \frac{1}{2}\Big[\big((w-a)^2 e^{-(w-a)\hat{p}} - 2m(w-a)e^{-(w-a)\hat{p}}\big)\hat{S}_{pp} - 2v(w-a)e^{-(w-a)\hat{p}}\hat{S}_{pq}\Big] + \frac{1}{2}\Big[\big(-(w-a)e^{-(w-a)\hat{p}}\hat{S}_{pp}\big)\big(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp}\big) + \big(-(w-a)e^{-(w-a)\hat{p}}\hat{S}_{qp}\big)\big(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp} + \iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq}\big)\Big]. \tag{24}$$

And

$$D = -(w+a)e^{-(w+a)\hat{p}} + \frac{1}{2}\Big[\big((w+a)^2 e^{-(w+a)\hat{p}} - 2m(w+a)e^{-(w+a)\hat{p}}\big)\hat{S}_{pp} - 2v(w+a)e^{-(w+a)\hat{p}}\hat{S}_{pq}\Big] + \frac{1}{2}\Big[\big(-(w+a)e^{-(w+a)\hat{p}}\hat{S}_{pp}\big)\big(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp}\big) + \big(-(w-a)e^{-(w-a)\hat{p}}\hat{S}_{qp}\big)\big(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp} + \iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq}\big)\Big]. \tag{25}$$

So, Bayes estimate of p under WCLE is:

$$\hat{p}^{L}_{WCLE} = \frac{1}{2a}\ln\left[\frac{C}{D}\right] = \frac{1}{2a}\ln\left[\frac{E\big(e^{-(w-a)p}|\underline{u}\big)}{E\big(e^{-(w+a)p}|\underline{u}\big)}\right] \tag{26}$$

If $h(p,q) = q$

Then

$$C = -(w-a)e^{-(w-a)\hat{q}} + \frac{1}{2}\Big[\big(-2m(w-a)e^{-(w-a)\hat{q}}\big)\hat{S}_{qp} + \big((w-a)^2 e^{-(w-a)\hat{q}} - 2v(w-a)e^{-(w-a)\hat{q}}\big)\hat{S}_{qq}\Big] + \frac{1}{2}\Big[\big(-(w-a)e^{-(w-a)\hat{q}}\hat{S}_{pq}\big)\big(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp}\big) + \big(-(w-a)e^{-(w-a)\hat{q}}\hat{S}_{qq}\big)\big(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp} + \iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq}\big)\Big]. \tag{27}$$

And Lindley approximation for D is

$$D = -(w+a)e^{-(w+a)\hat{q}} + \frac{1}{2}\Big[\big(-2m(w+a)e^{-(w+a)\hat{q}}\big)\hat{S}_{qp} + \big((w+a)^2 e^{-(w+a)\hat{q}} - 2v(w+a)e^{-(w+a)\hat{q}}\big)\hat{S}_{qq}\Big] + \frac{1}{2}\Big[\big(-(w+a)e^{-(w+a)\hat{q}}\hat{S}_{pq}\big)\big(\iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{ppp}\hat{S}_{pp}\big) + \big(-(w+a)e^{-(w+a)\hat{q}}\hat{S}_{qq}\big)\big(\iota\hat{\mathfrak{L}}_{qqq}\hat{S}_{qq} + \iota\hat{\mathfrak{L}}_{qpq}\hat{S}_{qp} + \iota\hat{\mathfrak{L}}_{pqq}\hat{S}_{pq}\big)\Big]. \tag{28}$$

Note that the symbol S used in Lindley approximation is defined as follows,

$$S = \begin{bmatrix} -\frac{\partial^2 \iota\mathfrak{L}}{\partial p^2} & -\frac{\partial^2 \iota\mathfrak{L}}{\partial p \partial q} \\ -\frac{\partial^2 \iota\mathfrak{L}}{\partial p \partial q} & -\frac{\partial^2 \iota\mathfrak{L}}{\partial q^2} \end{bmatrix}^{-1} = \begin{bmatrix} S_{pp} & S_{pq} \\ S_{qp} & S_{qq} \end{bmatrix} \tag{29}$$

and

$$\iota\hat{\mathfrak{L}}_{ppp} = \frac{\partial^3 \iota\mathfrak{L}(p,q|\underline{u})}{\partial p^3}\Big|_{\substack{p=\hat{p}\\ q=\hat{q}}} = 2np^{-3}. \tag{30}$$

$$\widehat{\iota\mathfrak{L}}_{qqq} = \frac{\partial^3 \iota\mathfrak{L}(p,q|\underline{u})}{\partial q^3}\Big|_{\substack{p=\hat{p}\\ q=\hat{q}}} = 2n\, q^{-3} + (p-1)\sum_{i=1}^{n} \frac{(u_i)^3 e^{-qu_i}(1+e^{-qu_i})}{(1-e^{-qu_i})^3} \quad . \tag{31}$$

$$\widehat{\iota\mathfrak{L}}_{qpq} = \frac{\partial^3 \iota\mathfrak{L}(p,q|\underline{u})}{\partial q\,\partial p\,\partial q}\Big|_{\substack{p=\hat{p}\\ q=\hat{q}}} = \widehat{\iota\mathfrak{L}}_{pqq} = \frac{\partial^3 \iota\mathfrak{L}(p,q|\underline{u})}{\partial p\,\partial q\,\partial q}\Big|_{\substack{p=\hat{p}\\ q=\hat{q}}} = -\sum_{i=1}^{n} \frac{(u_i)^2 e^{-qu_i}}{(1-e^{-qu_i})^2} \quad . \tag{32}$$

3. Simulation Study

In this section ,simulation has been conducted based on Monte carlo method to compare the Maximum Likelihood and Bayesian estimators method numerically under scale invariant squared and weighted composite linear-exponential loss functions for the scale and shape parameters of the Exponentiated Exponential Distribution by using four criteria Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values, which are written in the following forms, respectively [10].

$$MSE1(p) = \sum_{I=1}^{R} \frac{(\hat{p}_I - p)^2}{R} \quad , \tag{31}$$
$$MSE2(q) = \sum_{I=1}^{R} \frac{(\hat{q}_I - q)^2}{R} \quad , \tag{32}$$
$$ROMSE1(p) = \sqrt{MSE1(p)} \quad , \tag{33}$$
$$ROMSE2(q) = \sqrt{MSE2(q)} \quad , \tag{34}$$
$$mMSE1(p) = \sum_{I=1}^{R} \frac{|\hat{p}_I - p|}{R} \quad , \tag{35}$$
$$mMSE2(q) = \sum_{I=1}^{R} \frac{|\hat{q}_I - q|}{R} \quad , \tag{36}$$
$$rMSE1(p) = \frac{MSE1(p)}{p} \quad , \tag{37}$$
$$rMSE2(q) = \frac{MSE2(q)}{q} \tag{38}$$

Where

$\hat{p}_I$: is the estimate of p $at\ the\ I^{th} replicate$.

$\hat{q}_I$: is the estimate of q $at\ the\ I^{th} replicate$.

R : is the number of sample repetitions.

The default used values are shown below:

Sample sizes (n)	:	n=20 for small size, n=40,80 for medium size and n=160 for large size.
Cases p, q	:	0.4 and 1, 1 and 1, 1 and 0.8, 0.4 and 0.8.
non-informative Prior ($\mathfrak{m} = v$)	:	0.00001
informative Prior ($\mathfrak{m} = v$)	:	2
weighted function (w)	:	0.5
Number of iterations (R)	:	200

The simulation program has been written by using MATLAB [11].

Table 1: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the scale parameter q of the EX2D for different sample sizes with case p=0.4, $q = 1$.

n	criteria	$\hat{q}_{MLM}$	Bayes Estimates for q with non-informative priors (0.00001)				Bayes Estimates for q with informative priors (2)			
			$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a = 0.1$	$\hat{q}^L_{WCLE}$ $a = 0.5$	$\hat{q}^L_{WCLE}$ $a = 2$	$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a = 0.1$	$\hat{q}^L_{WCLE}$ $a = 0.5$	$\hat{q}^L_{WCLE}$ $a = 2$
20	MSE	0.02779	3.51228	0.19367	0.19368	0.29351	3.31555	0.19890	0.19891	0.29475
	RMSE	0.01390	1.75614	0.09684	0.09684	0.14676	1.65777	0.09945	0.09945	0.14737
	mMSE	0.13403	0.39251	0.30525	0.30525	0.30527	0.39072	0.30981	0.30981	0.30986
	rMSE	0.02779	3.51228	0.19367	0.19368	0.29351	3.31555	0.19890	0.19891	0.29475
40	MSE	0.02199	0.19243	0.12678	0.18679	0.19372	0.19770	0.12832	0.18783	0.19903
	RMSE	0.01099	0.09622	0.06339	0.09340	0.09686	0.09885	0.06416	0.09391	0.09951
	mMSE	0.11967	0.30509	0.27245	0.29188	0.29303	0.30970	0.27455	0.29389	0.29506
	rMSE	0.02199	0.19243	0.12678	0.18679	0.19372	0.19770	0.12832	0.18783	0.19903
80	MSE	0.01115	0.06521	0.06521	0.06521	0.06521	0.06534	0.06534	0.06534	0.06534
	RMSE	0.00557	0.03261	0.03261	0.03261	0.03261	0.03267	0.03267	0.03267	0.03267
	mMSE	0.08233	0.20358	0.20357	0.20357	0.20357	0.20386	0.20386	0.20386	0.20386
	rMSE	0.01115	0.06521	0.06521	0.06521	0.06521	0.06534	0.06534	0.06534	0.06534
160	MSE	0.00604	0.03670	0.03669	0.03669	0.03669	0.03671	0.03670	0.03670	0.03670
	RMSE	0.00302	0.01835	0.01835	0.01835	0.01835	0.01835	0.01835	0.01835	0.01835
	mMSE	0.06307	0.14836	0.14834	0.14834	0.14834	0.14840	0.14838	0.14838	0.14838
	rMSE	0.00604	0.03670	0.03669	0.03669	0.03669	0.03671	0.03670	0.03670	0.03670

Table 2: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the scale parameter q of the EX2D for different sample Sizes with case p=1, $q = 1$.

n	criteria	$\hat{q}_{MLM}$	Bayes Estimates for q with non-informative priors (0.00001)				Bayes Estimates for q with informative priors (2)			
			$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a = 0.1$	$\hat{q}^L_{WCLE}$ $a = 0.5$	$\hat{q}^L_{WCLE}$ $a = 2$	$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a = 0.1$	$\hat{q}^L_{WCLE}$ $a = 0.5$	$\hat{q}^L_{WCLE}$ $a = 2$
20	MSE	0.06275	0.58710	0.58663	0.58693	0.59479	0.96047	0.86385	0.76077	0.59072
	RMSE	0.03138	0.29355	0.29331	0.29347	0.29739	0.48024	0.43192	0.38039	0.29536
	mMSE	0.19903	0.41148	0.40800	0.40804	0.40906	0.46393	0.44292	0.44409	0.42581
	rMSE	0.06275	0.58710	0.58663	0.58693	0.59479	0.96047	0.86385	0.76077	0.59072
40	MSE	0.02604	0.10648	0.10640	0.10640	0.10640	0.10746	0.10737	0.10737	0.10737
	RMSE	0.01302	0.05324	0.05320	0.05320	0.05320	0.05373	0.05368	0.05368	0.05369
	mMSE	0.12982	0.26420	0.26399	0.26399	0.26398	0.26579	0.26556	0.26556	0.26557
	rMSE	0.02604	0.10648	0.10640	0.10640	0.10640	0.10746	0.10737	0.10737	0.10737
80	MSE	0.01274	0.06061	0.06060	0.06060	0.06060	0.06068	0.06067	0.06067	0.06067
	RMSE	0.00637	0.03031	0.03030	0.03030	0.03030	0.03034	0.03033	0.03033	0.03033
	mMSE	0.09123	0.19903	0.19900	0.19900	0.19900	0.19916	0.19912	0.19912	0.19912
	rMSE	0.01274	0.06061	0.06060	0.06060	0.06060	0.06068	0.06067	0.06067	0.06067
160	MSE	0.00545	0.03698	0.03697	0.03697	0.03697	0.03699	0.03698	0.03698	0.03698
	RMSE	0.00272	0.01849	0.01848	0.01848	0.01848	0.01849	0.01849	0.01849	0.01849
	mMSE	0.05809	0.14700	0.14697	0.14697	0.14697	0.14703	0.14700	0.14700	0.14700
	rMSE	0.00545	0.03698	0.03697	0.03697	0.03697	0.03699	0.03698	0.03698	0.03698

Table 3: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the scale parameter q of the EX2D for different sample Sizes with case $p=1$, $q = 0.8$.

N	criteria	$\hat{q}_{MLM}$	Bayes Estimates for q *with* non-informative priors (0.00001)				Bayes Estimates for q with informative priors (2)			
			$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a=0.1$	$\hat{q}^L_{WCLE}$ $a=0.5$	$\hat{q}^L_{WCLE}$ $a=2$	$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a=0.1$	$\hat{q}^L_{WCLE}$ $a=0.5$	$\hat{q}^L_{WCLE}$ $a=2$
20	MSE	0.30302	0.38502	0.38018	0.36718	0.36537	0.47707	0.45810	0.44586	0.36537
	RMSE	0.15151	0.19251	0.19009	0.18359	0.18269	0.23853	0.22905	0.22293	0.18269
	mMSE	0.47417	0.53553	0.53399	0.54879	0.53241	0.58711	0.58067	0.59574	0.53241
	rMSE	0.37878	0.48127	0.47522	0.45897	0.45671	0.59633	0.57263	0.55732	0.45671
40	MSE	0.23344	0.25268	0.25152	0.25152	0.25151	0.26169	0.26039	0.26039	0.25151
	RMSE	0.11672	0.12634	0.12576	0.12576	0.12576	0.13085	0.13020	0.13020	0.12576
	mMSE	0.44924	0.45620	0.45530	0.45529	0.45529	0.46375	0.46275	0.46275	0.45529
	rMSE	0.29181	0.31585	0.31440	0.31440	0.31439	0.32711	0.32549	0.32549	0.31439
80	MSE	0.22024	0.22890	0.22864	0.22864	0.22864	0.23074	0.23047	0.23047	0.22864
	RMSE	0.11012	0.11445	0.11432	0.11432	0.11432	0.11537	0.11523	0.11523	0.11432
	mMSE	0.44464	0.44535	0.44514	0.44514	0.44514	0.44705	0.44683	0.44683	0.44514
	rMSE	0.27530	0.28612	0.28580	0.28580	0.28580	0.28842	0.28808	0.28808	0.28580
160	MSE	0.21110	0.20417	0.20413	0.20413	0.20413	0.20458	0.20453	0.20453	0.20413
	RMSE	0.10555	0.10209	0.10206	0.10206	0.10206	0.10229	0.10227	0.10227	0.10206
	mMSE	0.44452	0.43189	0.43186	0.43186	0.43186	0.43231	0.43227	0.43227	0.43186
	rMSE	0.26388	0.25521	0.25516	0.25516	0.25516	0.25572	0.25567	0.25567	0.25516

Table 4: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the scale parameter q of the EX2D for different sample Sizes with case p=0.4, $q = 0.8$.

n	criteria	$\hat{q}_{MLM}$	Bayes Estimates for q *with* non-informative priors (0.00001)				Bayes Estimates for q with informative priors (2)			
			$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a=0.1$	$\hat{q}^L_{WCLE}$ $a=0.5$	$\hat{q}^L_{WCLE}$ $a=2$	$\hat{q}^L_{SIS}$	$\hat{q}^L_{WCLE}$ $a=0.1$	$\hat{q}^L_{WCLE}$ $a=0.5$	$\hat{q}^L_{WCLE}$ $a=2$
20	MSE	0.31786	0.39472	0.38212	0.38218	0.38356	0.48883	0.46253	0.46366	0.38356
	RMSE	0.15893	0.19736	0.19106	0.19109	0.19178	0.24441	0.23126	0.23183	0.19178
	mMSE	0.47902	0.54680	0.53919	0.53921	0.53968	0.59785	0.58572	0.58604	0.53968
	rMSE	0.39732	0.49341	0.47765	0.47773	0.47945	0.61104	0.57816	0.57957	0.47945
40	MSE	0.24410	0.26022	0.27118	0.26190	0.26287	0.26968	0.27118	0.27122	0.26287
	RMSE	0.12205	0.13011	0.13559	0.13095	0.13143	0.13484	0.13559	0.13561	0.13143
	mMSE	0.44887	0.45648	0.46463	0.45713	0.45756	0.46408	0.46463	0.46465	0.45756
	rMSE	0.30513	0.32527	0.33897	0.32738	0.32858	0.33710	0.33897	0.33903	0.32858
80	MSE	0.21598	0.21443	0.21423	0.21423	0.21423	0.21618	0.21598	0.21598	0.21423
	RMSE	0.10799	0.10722	0.10712	0.10712	0.10712	0.10809	0.10799	0.10799	0.10712
	mMSE	0.44280	0.42770	0.42753	0.42753	0.42753	0.42934	0.42918	0.42918	0.42753
	rMSE	0.26998	0.26804	0.26779	0.26779	0.26779	0.27023	0.26997	0.26997	0.26779
160	MSE	0.20346	0.18998	0.18994	0.18994	0.18994	0.19035	0.19031	0.19031	0.18994
	RMSE	0.10173	0.09499	0.09497	0.09497	0.09497	0.09518	0.09515	0.09515	0.09497
	mMSE	0.44026	0.41096	0.41093	0.41093	0.41093	0.41134	0.41131	0.41131	0.41093
	rMSE	0.25432	0.23748	0.23742	0.23742	0.23742	0.23794	0.23789	0.23789	0.23742

Table 5: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the shape parameter p of the EX2D for different sample Sizes with case p=0.4, $q = 1$.

N	criteria	$\hat{p}_{MLM}$	Bayes Estimates for p *with* non-informative priors (0.00001)				Bayes Estimates for p with informative priors (2)			
			$\hat{p}^{L}_{SIS}$	$\hat{p}^{L}_{WCLE}$ $a = 0.1$	$\hat{p}^{L}_{WCLE}$ $a = 0.5$	$\hat{p}^{L}_{WCLE}$ $a = 2$	$\hat{p}^{L}_{SIS}$	$\hat{p}^{L}_{WCLE}$ $a = 0.1$	$\hat{p}^{L}_{WCLE}$ $a = 0.5$	$\hat{p}^{L}_{WCLE}$ $a = 2$
20	MSE	0.39839	0.46174	1.02716	0.43849	0.42687	0.45520	1.01637	0.44386	0.43799
	RMSE	0.19920	0.23087	0.51358	0.21925	0.21343	0.22760	0.50818	0.22193	0.21900
	mMSE	0.60039	0.61098	0.66615	0.62804	0.60883	0.61498	0.67314	0.63314	0.60875
	rMSE	0.99095	1.15436	2.56791	1.09624	1.06717	1.13801	2.54092	1.10964	1.09498
40	MSE	0.37648	0.41916	0.80914	0.34471	0.41317	0.42448	0.80891	0.35574	0.41831
	RMSE	0.18824	0.20958	0.40457	0.17236	0.20658	0.21224	0.40445	0.17787	0.20916
	mMSE	0.60109	0.58474	0.63693	0.62492	0.59155	0.58930	0.64046	0.63190	0.59539
	rMSE	0.94120	1.04789	2.02285	0.86178	1.03291	1.06120	2.02227	0.88935	1.04579
80	MSE	0.36884	0.36007	0.62933	0.30252	0.35369	0.36114	0.54227	0.30278	0.35477
	RMSE	0.18442	0.18003	0.31467	0.15126	0.17685	0.18057	0.27113	0.15139	0.17738
	mMSE	0.59807	0.55320	0.59703	0.58376	0.55672	0.55416	0.58103	0.58471	0.55767
	rMSE	0.92209	0.90016	1.57333	0.75630	0.88423	0.90285	1.35567	0.75694	0.88692
160	MSE	0.36735	0.34561	0.54207	0.29035	0.34271	0.34587	0.36114	0.29143	0.34297
	RMSE	0.18368	0.17281	0.27104	0.14517	0.17136	0.17293	0.18057	0.14572	0.17148
	mMSE	0.59541	0.55135	0.58081	0.56619	0.55390	0.55158	0.55416	0.56642	0.55413
	rMSE	0.91838	0.86404	1.35519	0.72587	0.85678	0.86467	0.90285	0.72858	0.85742

Table 6: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the shape parameter p of the EX2D for different sample Sizes with case p=1, $q = 1$.

n	criteria	$\hat{p}_{MLM}$	Bayes Estimates for p *with* non-informative priors (0.00001)				Bayes Estimates for p with informative priors (2)			
			$\hat{p}^{L}_{SIS}$	$\hat{p}^{L}_{WCLE}$ $a = 0.1$	$\hat{p}^{L}_{WCLE}$ $a = 0.5$	$\hat{p}^{L}_{WCLE}$ $a = 2$	$\hat{p}^{L}_{SIS}$	$\hat{p}^{L}_{WCLE}$ $a = 0.1$	$\hat{p}^{L}_{WCLE}$ $a = 0.5$	$\hat{p}^{L}_{WCLE}$ $a = 2$
20	MSE	0.06275	0.14024	0.28183	0.10181	0.14083	0.38719	0.49712	0.09013	0.13842
	RMSE	0.03138	0.07012	0.14092	0.05091	0.07042	0.19360	0.24856	0.04307	0.06921
	mMSE	0.19903	0.29555	0.31798	0.30906	0.29836	0.33233	0.35458	0.33255	0.30932
	rMSE	0.06275	0.14024	0.28183	0.10181	0.14083	0.38719	0.49712	0.09013	0.13842
40	MSE	0.02604	0.08692	0.08545	0.08542	0.08472	0.08642	0.08558	0.08555	0.06719
	RMSE	0.01302	0.04346	0.04273	0.04271	0.04236	0.04321	0.04279	0.04277	0.03359
	mMSE	0.12982	0.23299	0.23226	0.23223	0.23125	0.23267	0.23292	0.23289	0.19511
	rMSE	0.02604	0.08692	0.08545	0.08542	0.08472	0.08642	0.08558	0.08555	0.66719
80	MSE	0.01274	0.06705	0.06727	0.06727	0.06727	0.06696	0.06719	0.06719	0.04176
	RMSE	0.00637	0.03353	0.03363	0.03363	0.03363	0.03348	0.03359	0.03359	0.02088
	mMSE	0.09123	0.19493	0.19507	0.19507	0.19507	0.19497	0.19511	0.19511	0.14793
	rMSE	0.01274	0.06705	0.06727	0.06727	0.06727	0.06696	0.06719	0.06719	0.04176
160	MSE	0.00545	0.04180	0.04180	0.04180	0.04180	0.04177	0.04176	0.04176	0.08479
	RMSE	0.00272	0.02090	0.02090	0.02090	0.02090	0.02088	0.02088	0.02088	0.04239
	mMSE	mMSE	0.05809	0.14799	0.14796	0.14796	0.14796	0.14796	0.14793	0.23174
	rMSE	rMSE	0.00545	0.04180	0.04180	0.04180	0.04180	0.04177	0.04176	0.08479

Table 7: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the shape parameter p of the EX2D for different sample Sizes with case p=1, $q = 0.8$.

N	criteria	$\hat{p}_{MLM}$	Bayes Estimates for p *with* non-informative priors (0.00001)				Bayes Estimates for p with informative priors (2)			
			$\hat{p}^L_{SIS}$	$\hat{p}^L_{WCLE}$ $a = 0.1$	$\hat{p}^L_{WCLE}$ $a = 0.5$	$\hat{p}^L_{WCLE}$ $a = 2$	$\hat{p}^L_{SIS}$	$\hat{p}^L_{WCLE}$ $a = 0.1$	$\hat{p}^L_{WCLE}$ $a = 0.5$	$\hat{p}^L_{WCLE}$ $a = 2$
20	MSE	0.15799	0.22606	0.83658	1.99876	0.19447	0.24372	0.82638	2.04366	0.22575
	RMSE	0.07899	0.11303	0.41829	0.99938	0.09724	0.12186	0.41319	1.02183	0.11287
	mMSE	0.31102	0.39086	0.45382	0.51826	0.39050	0.41297	0.47799	0.54679	0.41690
	rMSE	0.15799	0.22606	0.83658	1.99876	0.19447	0.24372	0.82638	2.04366	0.22575
40	MSE	0.09583	0.17583	0.41663	0.20631	0.16051	0.17626	0.41685	0.20650	0.16176
	RMSE	0.04792	0.08791	0.20831	0.10316	0.08025	0.08813	0.20842	0.10325	0.08088
	mMSE	0.25847	0.37219	0.40814	0.42320	0.37616	0.37294	0.40888	0.42391	0.37692
	rMSE	0.09583	0.17583	0.41663	0.20631	0.16051	0.17626	0.41685	0.20650	0.16176
80	MSE	0.08244	0.16712	0.29001	0.16151	0.16032	0.16861	0.29232	0.16159	0.16077
	RMSE	0.04122	0.08356	0.14500	0.08075	0.08016	0.08431	0.14616	0.08079	0.08038
	mMSE	0.25025	0.36515	0.37764	0.37072	0.36230	0.36531	0.38096	0.37406	0.36570
	rMSE	0.08244	0.16712	0.29001	0.16151	0.16032	0.16861	0.29232	0.16159	0.16077
160	MSE	0.15799	0.22606	0.83658	1.99876	0.19447	0.24372	0.82638	2.04366	0.16059
	RMSE	0.07899	0.11303	0.41829	0.99938	0.09724	0.12186	0.41319	1.02183	0.08029
	mMSE	0.31102	0.39086	0.45382	0.51826	0.39050	0.41297	0.47799	0.54679	0.36100
	rMSE	0.15799	0.22606	0.83658	1.99876	0.19447	0.24372	0.82638	2.04366	0.16059

Table 8: Mean squared, Root Mean Square**,** mean absolute and Relative mean squared error values for the shape parameter p of the EX2D for different sample Sizes with case p=0.4, $q =$ 0.8.

n	criteria	$\hat{p}_{MLM}$	Bayes Estimates for p *with* non-informative priors (0.00001)				Bayes Estimates for p with informative priors (2)			
			$\hat{p}^L_{SIS}$	$\hat{p}^L_{WCLE}$ $a = 0.1$	$\hat{p}^L_{WCLE}$ $a = 0.5$	$\hat{p}^L_{WCLE}$ $a = 2$	$\hat{p}^L_{SIS}$	$\hat{p}^L_{WCLE}$ $a = 0.1$	$\hat{p}^L_{WCLE}$ $a = 0.5$	$\hat{p}^L_{WCLE}$ $a = 2$
20	MSE	0.85458	0.82817	1.13227	0.68795	0.81184	0.90426	1.20109	0.76143	0.88662
	RMSE	0.42729	0.41409	0.56613	0.34398	0.40592	0.45213	0.60054	0.38072	0.44331
	mMSE	0.87090	0.82242	0.88993	0.88461	0.82976	0.85817	0.92370	0.91896	0.86461
	rMSE	2.13646	2.07044	2.83067	1.71988	2.02960	2.26064	3.00271	1.90358	2.21655
40	MSE	0.76320	0.66884	0.72179	0.63815	0.65992	0.67994	0.72179	0.64924	0.67079
	RMSE	0.38160	0.33442	0.36089	0.31908	0.32996	0.33997	0.36089	0.32462	0.33540
	mMSE	0.84887	0.73450	0.75394	0.75349	0.73678	0.74102	0.75394	0.75940	0.74257
	rMSE	1.90800	1.67210	1.80447	1.59538	1.64981	1.69985	1.80447	1.62309	1.67698
80	MSE	0.73023	0.57667	0.69787	0.52647	0.57219	0.57884	0.70044	0.52866	0.57438
	RMSE	0.36511	0.28833	0.34893	0.26324	0.28610	0.28942	0.35022	0.26433	0.28719
	mMSE	0.84280	0.67585	0.69621	0.68839	0.67622	0.67707	0.69751	0.68865	0.67749
	rMSE	1.82556	1.44167	1.74467	1.31618	1.43048	1.44710	1.75109	1.32164	1.43596
160	MSE	0.71566	0.54289	0.64328	0.49702	0.53736	0.54333	0.64376	0.49750	0.53784
	RMSE	0.35783	0.27145	0.32164	0.24851	0.26868	0.27167	0.32188	0.24875	0.26892
	mMSE	0.84026	0.65337	0.68115	0.68730	0.65843	0.65336	0.68142	0.68865	0.65870
	rMSE	1.78916	1.35723	1.60820	1.24255	1.34339	1.35833	1.60939	1.24376	1.34459

4. Conclusions

From Tables (1-8), the following conclusions are obtained:

1. From Tables (1) and (2), The maximum likelihood Method (MLM) for estimate the scale parameter q give the best performance in comparison with other Bayes estimates (with non-informative priors ($\mathfrak{m} = v$ =0.00001)) and with informative priors ($\mathfrak{m} = v$ =2) with all criteria and for all sample sizes.
2. From Tables (3) and (4), Bayes estimators under weighted composite linear-exponential loss function using Lindley's approximation method (BM-WCLE) for estimate the scale parameter q (with non-informative priors ($\mathfrak{m} = v$ =0.00001)) give the best performance in comparison with other estimates with all criteria for large sample size.
3. From Tables (6) and (7), MLM give the best performance in comparison with other Bayes estimates for estimating the shape parameter p (with non-informative priors ($\mathfrak{m} = v$ =0.00001)) and with informative priors ($\mathfrak{m} = v$ =2) with all criteria and for all sample sizes.
4. From Tables (5) and (8), Bayes estimators under weighted composite linear-exponential loss function using Lindley's approximation method (BM-WCLE) for estimate the shape parameter p (with non-informative priors ($\mathfrak{m} = v$ =0.00001)) and a=0.5 give the best performance in comparison with other estimates with all criteria except mean absolute error values, for medium and large sample sizes.

5. Recommendations

- Using the Performance BM-WCLE for estimate the scale parameter q with non-informative priors for large sample size.
- Using the Performance BM-WCLE for estimate the shape parameter p with non-informative priors and a=0.5 for large sample size with all criteria except mean absolute mMSE, for medium and large sample sizes.

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