Hussein and Yaseen *Iraqi Journal of Science, 2023, Vol. 64, No. 7, pp: 3670-3678*
DOI: 10.24996/ijs.2023.64.7.42

ISSN: 0067-2904

Estimation of the Standard Atmospheric Earth Model Parameters at 86 km Altitude

Hussein Omran Hussein, Waleed Ibrahim Yaseen*
Department of Astronomy and Space, College of Science, University of Baghdad, Baghdad-Iraq

---

Received: 5/10/2022 Accepted: 15/1/2023 Published: 30/7/2023

Abstract

Utilizing the Turbo C programming language, the atmospheric earth model is created from sea level to 86 km. This model has been used to determine atmospheric Earth parameters in this study. Analytical derivations of these parameters are made using the balancing forces theory and the hydrostatic equation. The effects of altitude on density, pressure, temperature, gravitational acceleration, sound speed, scale height, and molecular weight are examined. The mass of the atmosphere is equal to about 50% between sea level and 5.5 km. g is equal to 9.65 m/s^2 at 50 km altitude, which is 9% lower than 9.8 m/s^2 at sea level. However, at 86 km altitude, g is close to 9.51 m/s^2, which is close to 15% smaller than 9.8 m/s^2.

These results have been compared with an international standard atmosphere. The presumed atmosphere model differs significantly from the actual atmosphere because weather fluctuations are not taken into consideration in this model.

Keywords: Atmospheric Earth, Density, Pressure, Temperature, Gravity acceleration, Turbo C language,

تقدير معلمات نموذج الغلاف الجوي الأرضي القياسي على ارتفاع 86 كم

حسين عمران حسين , وليد ابراهيم ياسين*
قسم الفلك والفضاء, كلية العلوم, جامعة بغداد, بغداد, العراق

الخلاصة

باستخدام لغة برمجة Turbo C ، يتم إنشاء نموذج الغلاف الجوي للأرض من مستوى سطح البحر إلى 86 كم. تم استخدام هذا النموذج لتحديد بارامترات الغلاف الجوي للأرض في هذه الدراسة. يتم إجراء الاشتقاقات التحليلية لهذه المعلمات باستخدام نظرية قوى التوازن والمعادلة الهيدروستاتيكية. يتم فحص تأثيرات الارتفاع على الكثافة والضغط ودرجة الحرارة وتسارع الجاذبية وسرعة الصوت وارتفاع المقياس والوزن الجزيئي. كتلة الغلاف الجوي تساوي حوالي 50٪ بين مستوى سطح البحر و 5.5 كم. g يساوي 9.65 م / ث 2 على ارتفاع 50 كم ، وهو أقل بنسبة 9٪ من 9.8 م / ث 2 عند مستوى سطح البحر. ومع ذلك ، عند ارتفاع 86 كم ، تكون g قريبة من 9.51 م / ث 2 ، وهو ما يقرب من 15٪ أصغر من 9.8 م / ث 2. تمت مقارنة هذه النتائج مع الغلاف الجوي القياسي الدولي. يختلف نموذج الغلاف الجوي المفترض اختلافًا كبيرًا عن الغلاف الجوي الفعلي لأن تقلبات الطقس لم تؤخذ في نظر الاعتبار في هذا النموذج.

---

*Email: waleedib1972cnc@gmail.com

1. Introduction

A few highly concentrated gases, including nitrogen, oxygen, and argon, as well as several trace gases, including water vapor, carbon dioxide, methane, and ozone, are found in the atmosphere [1]. These gases are all components of air. The ionosphere is formed in the upper part of the atmosphere by absorbing the incoming solar radiation and creating the ion – electron pairs [2] [3]. The temperature, density, and pressure of air are significant properties.

The equation of state links these variables, which change with altitude, latitude, longitude, and season. The Clausius-Clapeyron equation [4] and the basic law of thermodynamics [5] are two other fundamental equations that can be applied to the atmosphere. Temperature and the amount of water vapor covering a surface at saturation are related by the Clausius-Clapeyron equation. The first law of thermodynamics links energy transmission and changes in work to a gas's changing temperature [4]. The atmosphere of the Earth is a dynamic system that is always in flux. Altitude, longitude and latitude, time of day, season, and even the sunspot activity of the sun all affect the atmosphere's pressure and temperature. It is impractical to take into consideration all these variances when analyzing the design and functionality of flying vehicles and the research of meteorites, as a result, a standard environment is established in order to establish a point of comparison for flight testing, wind tunnel data, and meteorite studies [6].

Air pressure, temperature, and density all vary with altitude in the Earth's atmosphere. The mass of air divided by the volume of air is known as air density [7], where the mass of air is the total of all gases, aerosol particles, and hydrometeor particles. The ideal definition of air pressure is the weight (force) of air above a horizontal plane divided by the surface area of the plane [8]. The pressure that results entirely from the weight of the air in a column above a specific altitude is known as hydrostatic air pressure, and it is this form of pressure that is being discussed here. Fluid that is at rest is said to be as hydrostatic. Air pressure is only hydrostatic when there is no vertical acceleration, which can happen when the air is at rest or moving at a constant vertical speed [9].

When air pressure is averaged across a broad horizontal region with a diameter greater than 3 km and outside of a storm system, it is fair to assume that the pressure is hydrostatic because the vertical accelerations are often negligible. Vertical accelerations can be significant over tiny regions with a diameter of less than 3 km and within individual clouds. Air pressure does not increase hydrostatically as it accelerates vertically [10].

The typical atmosphere provides mean values for pressure, temperature, density, and other attributes as functions of altitude; these values were acquired using an experimental balloon and sounding rocket observations in conjunction with a mathematical model of the atmosphere. The standard atmosphere roughly represents ordinary atmospheric conditions, however, this is not its primary significance. Instead, it serves as a resource that all aeronautical engineers may utilize to arrange their usage of tables of standard reference conditions. Itineraries for the work are shown in Figure 1 [6].

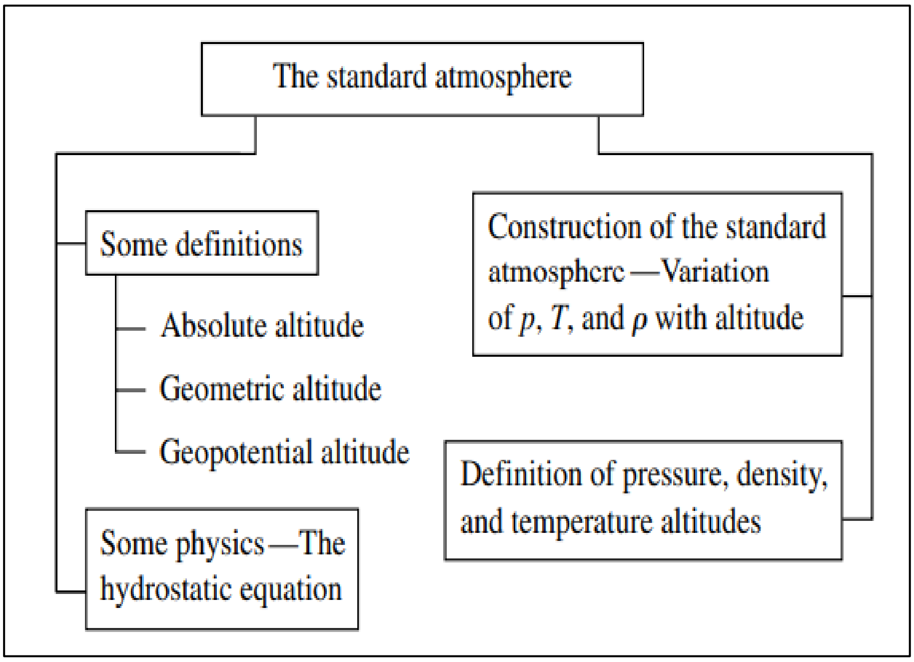

Figure 1: work's itineraries [6].

2. Equilibrium Assumptions

At altitudes sufficiently below 86 km, the atmosphere is considered to be homogeneously mixed with a relative-volume composition that maintains a constant mean molecular weight M. The air is believed to be dry. The air is regarded as though it were a perfect gas, and the total pressure p, temperature T, and total density, at any location in the atmosphere, are connected by the equation of state. The equation of state, also known as the perfect gas law, takes the following form [11]:

$$p = \frac{\rho R T}{M} \tag{1}$$

R is the gaseous universal constant [12]. In terms of the total number density N and the Avogadro constant N_A, the equation of state may also be expressed as follows:

$$p = \frac{NRT}{N_A} \tag{2}$$

This form represents the summation of p_i, the partial pressures of the individual gas species, where p_i is related to n_i the number density of the ith gas species in the following expression [13]:

$$p_i = n_i \ k \ T \tag{3}$$

k stands for the Boltzmann constant. The atmosphere is considered to be in hydrostatic equilibrium and to be horizontally stratified within the height zone of full mixing, where dp, the differential of pressure, and dz, the differential of geometric height are connected by the relationship:

$$dp = -g \, \rho \, dz \tag{4}$$

Where g is the acceleration of gravity that varies with height. Another variation of the hydrostatic equation, which forms the foundation for calculating low-altitude pressure, is produced by subtracting the variable ρ between Eq. (1) and Eq. (4);

$$\mathrm{lnp} = \frac{dp}{p} = \frac{-\mathrm{g\ M}}{\mathrm{RT}}dz \tag{5}$$

3. Hydrostatic Equation

The temperature is taken to be constant and the air is taken to be a perfect gas in order to determine the standard pressure, denoted by p, at a specific height. The altitude is determined by taking a pressure (pA) measurement is referred to as pressure altitude [14]. The hydrostatic equation, perfect gas law, and temperature lapse rate equation may all be used to compute pressure variations. Think about an air column that is vertical and has the cross-sectional area A=1 shown in Figure 2. Between (z) and (z+dz), the air column's mass (dm) is:

$$\mathrm{dm} = \rho(\mathrm{z})\mathrm{Adz} \tag{6}$$

where ρ (z) is the density of air at height (z) as illustrated in Figure 2. The force acting on this column as a result of the weight of the air area is:

$$\mathrm{fd_G} = \mathrm{dm\,g(z)} = \mathrm{g(z)}\rho(\mathrm{z})\mathrm{Adz} \tag{7}$$

where g (z) is the acceleration due to gravity at height (z). The changes in pressures (p→Δp) lead to higher pressure:

$$\Delta \mathrm{f_p} = \mathrm{A\ p\ \Delta p} \rightarrow \mathrm{df_p} = \mathrm{A[p-(p-dp)]}$$

$$\mathrm{df_p} = -\mathrm{A\ dp} \tag{8}$$

(-dp) is a positive quantity and the balance of forces [15] $\mathrm{df_G} = \mathrm{df_p}$:

$$\frac{dp}{dz} = -\mathrm{g(z)}\rho(\mathrm{z}) \tag{9}$$

Where g (z) is the acceleration of gravity-dependent on height. The law of gravitation provides an expression for gas as a function of altitude [16].

4. Definition of Altitude

Everyone can have a basic understanding of what altitude means. To be used quantitatively in engineering, it must, like so many other broad words, be defined with greater specificity. The six main types of altitudes are absolute, geometric, geopotential, pressure, temperature, and density.

When the Earth is at sea level, the measurement by definition is the geometric altitude z, which means the geometric height is above sea level [17].

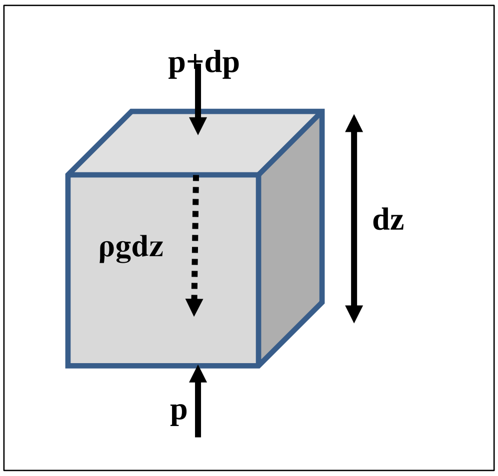

Figure 2: Hydrostatic balance of forces.

The measurement from the center of the Earth would equal the absolute altitude ($h_a = z + r$) if r is the radius of the Earth. Figure 3 provides an illustration of this. In particular, the absolute altitude is crucial for space travel since the local acceleration of gravity, g, changes with ha. g changes inversely as the square of the distance from the Earth's center, according to Newton's law of gravity. The local gravitational acceleration g at a specific absolute height ha is determined by choosing g_o to be the gravitational acceleration at sea level.

$$g = g_o\left(\frac{r}{h_a}\right)^2 = g_o\left(\frac{r}{r+z}\right)^2 \tag{10}$$

When working with mathematical models of the atmosphere, it is important to consider how g changes with height [18].

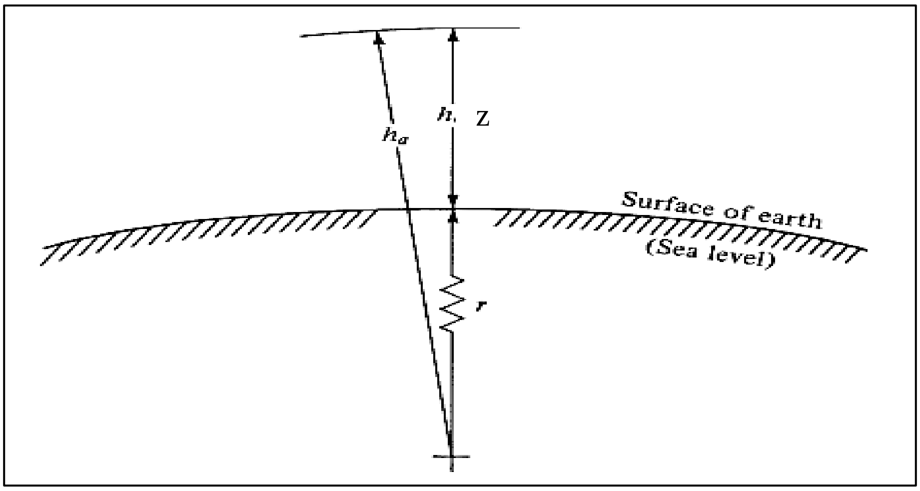

Figure 3: Definition of altitude [17].

Integrating Eq. (5) from z_o to z:

$$p = p_o e^{\left[\frac{-gM}{RT}(z-z_o)\right]} \tag{11}$$

Scale height H is defined as [19]:

$$H = \frac{R^*T}{Mg} \tag{12}$$

Eq. (11) becomes:

$$p = \mathrm{p}_o \mathrm{e}^{\left[-\frac{(z-\mathrm{z}_o)}{\mathrm{H}}\right]} \tag{13}$$

The speed of sound is symbolized by C and it is calculated using the equation [20]:

$$\mathrm{C} = \left(\frac{\gamma \mathrm{R}T}{M}\right)^{1/2} \tag{14}$$

Where γ is ratio of thermal capacities (specific heat) for air in presence of constant pressure and volume respectively [21]:

$$\gamma = \frac{\mathrm{C_p}}{\mathrm{C_v}} = 1.4 \tag{15}$$

5. Results and Discussion

The data in Figure 5 shows that near-surface tropospheric temperatures drop from the Equator to high latitudes, which is what one would expect given that this is where the Earth receives the most incident solar radiation.

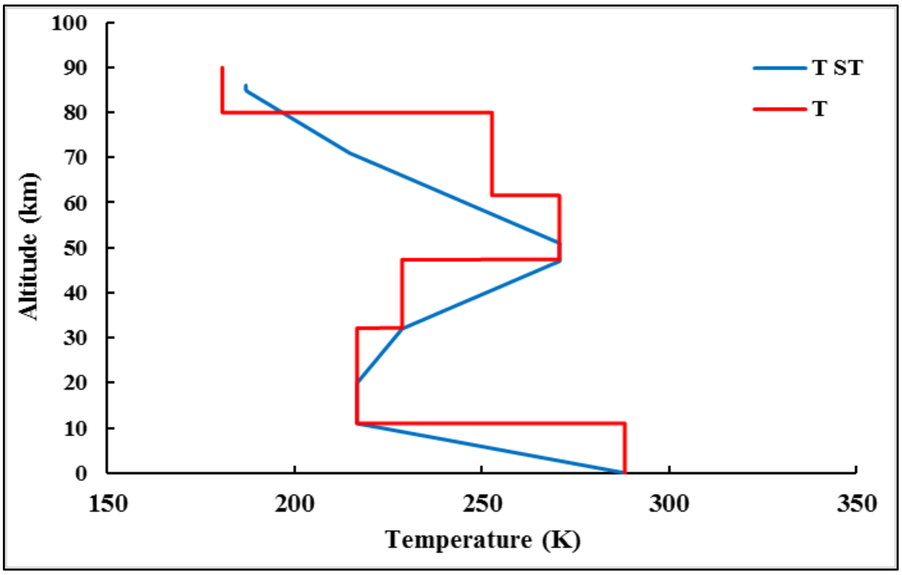

Figure 5: Temperature with altitude for atmospheric model (red line) and compared with international standard atmosphere (blue line).

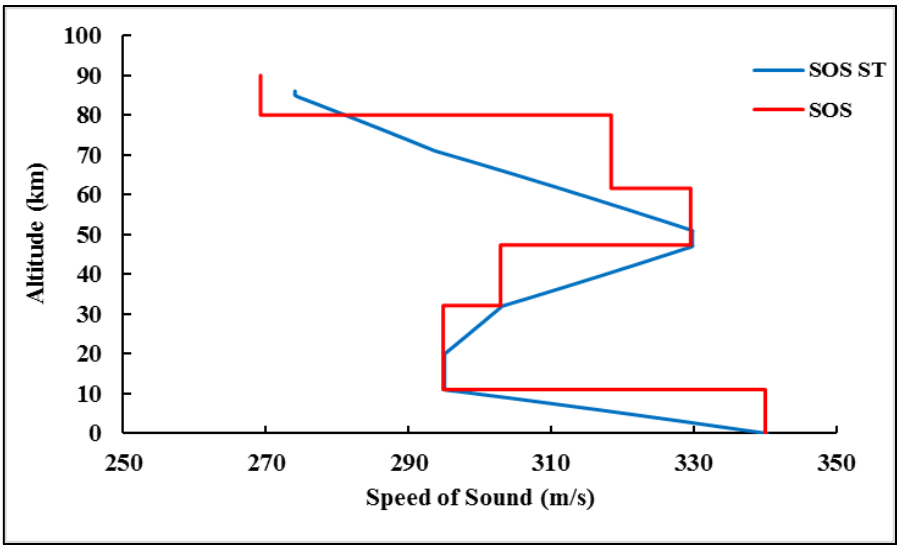

Figure 6: Speed of sound with Altitude for atmospheric model (red line) and compared with international standard atmosphere (blue line).

The tropopause is the troposphere's upper boundary. It is defined by the World Meteorological Organization (WMO) as the lowest altitude at which the lapse rate (rate of temperature decrease with height) decreases to 2 K at 1 km or less, and at which the lapse rate, averaged between this altitude and any altitude within the following 2 km, does not exceed 2 K 1km. Before it rises with increasing height in the stratosphere, the temperature is isothermal with increasing altitude above the tropopause base. The temperature varies with altitude for the atmospheric model (red line) and is compared with the international standard atmosphere (blue line), this agrees with [14].

The calculated sound speed is depicted in Figure 6 and varied with altitude, illustrating the concept's limits owing to high attenuation. The attenuation which exists at sea level for high frequencies applies to increasingly lower frequencies as air pressure lowers, or as the mean free path rises. Due to this, the idea of sound speed gradually loses its application at high altitudes (apart from frequencies close to zero). This figure is the same behavior of the temperature figure because the speed of sound depends on the temperature according to Eq. 14, this result agrees with Vladimir [22].

Figure 7 illustrates how the air pressure above high altitudes reduces as height increases. Because of the density used to calculate air pressure, according to Eq. (11). This figure shows that air pressure decreases exponentially with height. It also demonstrates that 50% of the mass of the atmosphere is found between sea level and 5.5 km. Nearly 48 kilometers is where 99.9% of mass is located. Sea level surface pressure is typically 760mmHg, or 1 atm.
Normal deviations from normal sea-level pressure range from +7.5 to 15 mmHg depending on the place and time. The atmospheric model for pressure (red line) is identical with standard atmosphere (blue line), this agrees with Waleed [23].

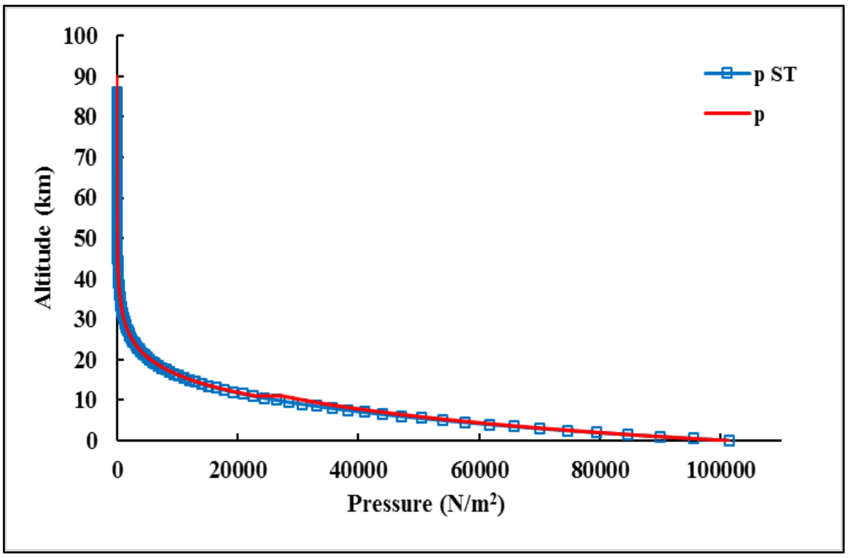

Figure 7: Pressure varied with Altitude for atmospheric model (red line) and compared with international standard atmosphere (blue line).

According to Figure 8, the density drops exponentially with height as a result of the decreased air molecule density. Although the air molecules are relatively low over 30 km in height, the atmospheric impacts are thought to be significantly below 30 km. The atmospheric model for density (red line) is identical with standard atmosphere (blue line), this agrees with Waleed [23].

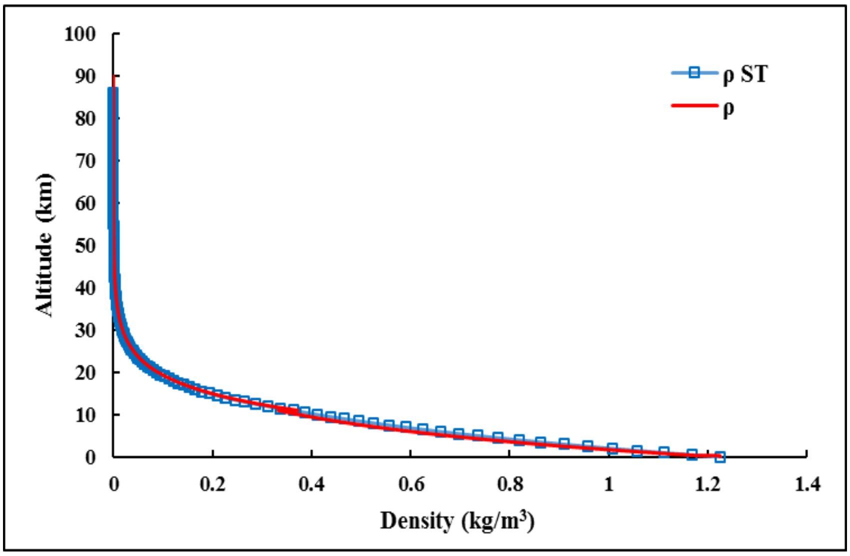

Figure 8: Air density varied with Altitude for atmospheric model (red line) and compared with international standard atmosphere (blue line).

The gravitational acceleration decreases with altitude, as shown in Figure 9. g is equal to 9.65 m/s^2 at 50 km altitude, which is 9% lower than 9.8 m/s^2 at sea level. However, at 86 km altitude, g is close to 9.51 m/s^2, which is close to 15% smaller than 9.8 m/s^2.
The gravitational acceleration has a linear relationship with altitude and it is calculated using Eq. (10).

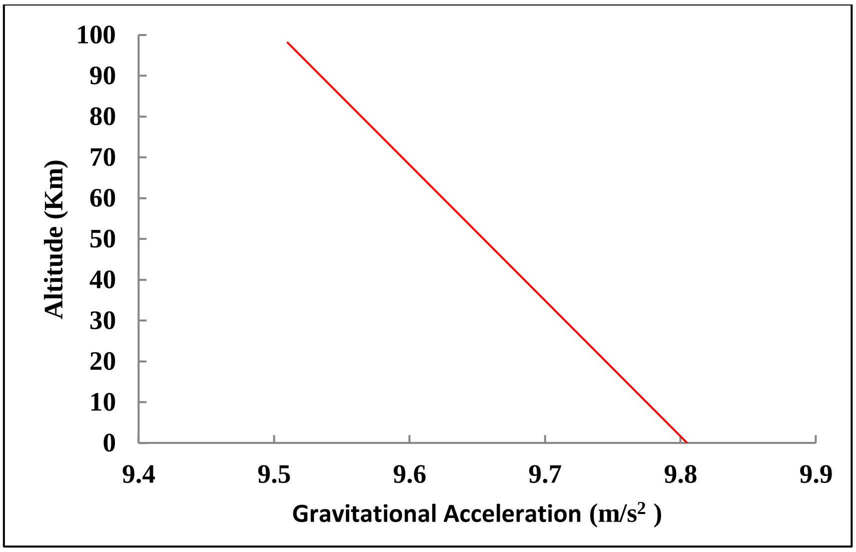

Figure 9: Acceleration of gravity varied with Altitude.

Conclusions

Calculating the Earth's atmosphere model parameters using the turbo C language is successfully completed. In this work, the atmospheric model is compared with the standard atmosphere and the results of pressure, density, and gravity acceleration are identical to those of the standard atmosphere but the temperature is slightly different. The model is built from sea level to 86 km because this region is important for the bodies entering the Earth from space.

The presumed atmosphere model differs significantly from the actual atmosphere because weather fluctuations are not taken into consideration in this model. And the model that has been built can be useful for aircraft and missile designs, ballistic trajectories, and other uses. References

References

[1] John E. Frederick, *Principles of Atmospheric Science, Jones & Bartlett Learning*, 2008 .
[2] A. Aseel A. Temur, "Plasma Characteristics of the Earth's Ionosphere in F-layer," *Iraqi Journal of Science*, vol. 63, no. 7, pp. 3225-3235, 2022.
[3] K. A. H. Mahmood K. Mardan, "Study the Influence of Solar Activity on the Ionospheric Electron, Ion and Neutral Particle Temperatures over Iraqi Region Using Ionospheric Models," *Iraqi Journal of Science*, vol. 59, no. 1A, pp. 209-217, 2018.
[4] M. S. Suzuki, *Clausius-Clapeyron equation, Physics, SUNY at Binghamton*, 2017, pp. 1-37.
[5] Mcdanie, B. Zohuri and a. Patrick, "Thermodynamics in Nuclear Power Plant Systems," pp. 99-148., 2019.
[6] J. Anderson, *Introduction to Flight, McGraw Hill*, 2011, pp. 101-120.
[7] S. Davidson, "Air Density Measurement for Mass Calibration," *Measurement - Supports Science - Improves Technology*, pp. 25-28, 2000.
[8] D. F. Young, B. R. Munson, T. H. Okiishi and a. W. W. Huebsch, *Introduction to Fluid Mechanics, John Wiley & Sons, Inc*., 2011, pp. 33-60.
[9] G. C. Nihous, "Notes on hydrostatic pressure," *Journal Ocean Eng. Mar. Energy*, vol. 2, pp. 105-109, 2016.
[10] X. Chen, "A Comparison of Hydrostatic and Non hydrostatic Pressure Components in Seiche Oscillations," *Mathematical and Computer Modelling*, vol. 41, pp. 887-902, 2005.
[11] S. Angrist, L. Helper, G. L. Order and a. Chaos, *Laws of Energy and Entropy, New York*,1967.
[12] F. S. L. Pitre, L. Risegari, C. Guianvarc'h, C. Martin, M. Himbert, M. Plimmer, B. M. A. Allard, P. G. Albo, B. Gao, M. Moldover and a. J. Mehl, "New measurement of the Boltzmann constant k by acoustic thermometry of helium-4 gas," *Metrologia*, vol. 54, pp. 856–873 , 2017.
[13] J. Arnaud, L. Chusseau and and F. Philippe, On the Ideal Gas Law, *arXiv*, 2011, pp. 1-14.
[14] Airbus, getting to Grips with Aircraft Performance, *Airbus Industrie*, 2000, pp. 11-16.
[15] W. I. Yaseen, "Determination of Skip Entry Trajectories for Space Vehicles at Circular and Super Circular Speeds," *Iraqi Journal of Physics*, vol. 8, no. 12, pp. 29-34, 2010.
[16] Y. Deng, A. J. Ridley and a. W. Wang, "Effect of the altitudinal variation of the gravitational acceleration on the thermosphere simulation," *Journal of Geophysical Research*, vol. 113, pp. 1-8, 2008.
[17] T. H. Meyer, D. R. Roman and a. D. B. Zilkoski, "What does height really mean? Part III: Height Systems," *Surveying and Land Information Science* , vol. 66, no. 2, pp. 149-160, 2006.
[18] J. Ruskin, *Aerospace Vehicles Can Be Divided into Two Basic Categories: Atmospheric*, 2004.
[19] K. M. Lee and a. D. M. Ryan, " "Scale height — A parameter for characterizing atmosphere," *The Physics Teacher*, vol. 53, pp. 122-123, 2015.
[20] K. A. Krout and a. S. H. Sohrab, "On the Speed of Sound", *International Journal of Thermodynamics*," IJOT, vol. 19, no. 1, pp. 29-34, 2016.
[21] D. H. P. M. C. M. G. Santostasia, N. T. Nguyesnb, S. Poudelb, S. Pradhanb, D. Roshanb and a. M. Waglec, "A student designed experiment measuring the speed of sound as a function of altitude," *Arxiv Physics*, pp. 1-8, 2008.
[22] V. G. Kirtskhalia, "Speed of Sound in Atmosphere of the Earth," *Open Journal of Acoustics* , vol. 2, pp. 80-85, 2012.
[23] Y. C. a., M. N. A. and a. W. I. Yaseen, "Calculation of The Parameters for Atmospheric Model for The Earth," *Journal of Al-Nahrain University*, vol. 12, no. 3, pp. 64-69, 2009.