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Multi-Objective Shortest Path Model for Optimal Route between Commercial Cities on America

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Abstract
The traditional shortest path problem is mainly concerned with identifying the associated paths in the transportation network that represent the shortest distance between the source and the destination in the transportation network by finding either cost or distance. As for the problem of research under study it is to find the shortest optimal path of multi-objective (cost, distance and time) at the same time has been clarified through the application of a proposed practical model of the problem of multi-objective shortest path to solve the problem of the most important 25 commercial US cities by travel in the car or plane. The proposed model was also solved using the lexicographic method through package program Win-QSB 2.0 for operational research applications.

Keywords: shortest path problem, multi-objective optimization, multi-objective shortest path problem, lexicographic method

استخدام نموذج المسار الأقصر متعدد الأهداف لإيجاد أمثل طريق بين المدن التجارية في أمريكا

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الخلاصة
تهتم مشكلة أقصر طريق التقليدية أساساً بتحديد الطرق المتصلة في شبكة النقل والتي تمثل أقصر مسافة بين المصدر، ومكان الوصول في شبكة النقل عن طريق إيجاد إما الكلفة أو المسافة. أما فيما يخص مشكلة البحث قيد الدراسة فهي تتمثل بإيجاد أقصر طريق أمثل متعدد الأهداف لكل من (الكلفة، المسافة والوقت) في آن واحد وقد تم توضيح ذلك من خلال تطبيق نموذج عملي مقترح لمشكلة أقصر طريق متعدد الأهداف لحل مشكلة أهم 25 مدينة تجارية امريكية عن طريق السفر بالسيارة أو بالطائرة، كما وتم حل النموذج المقترح باستخدام الطريقة المعجمية من خلال البرنامج الجاهز (Win-QSB 2.0) الخاص بتطبيقات بحوث العمليات.

1. Introduction and Reference Review

Multi-objective combinatorial optimization MOCO, also called multi-criteria optimization, is a well-studied branch of optimization, where the goal is to find optimal solutions based on multi-objective. Many real-life problems can be represented as networks, such as transportation networks, communication networks, pipeline distribution networks, and neural networks. The primary aim of these network models is to optimize the performance with respect to predefined objectives. Multi-objectives such as optimization of cost, distance, time, delay, risk, reliability, quality of service and environment impact etc. may arise in such problems. This problem can be formulated as a multi-objective shortest path problem MOSPP. There are many studies to treating the problem of the multi-

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objective shortest path, where White [1] used the multi-objective linear programming model to identify the optimal path with an effective set of efficiently policies and paths. Arthur [2] also studied a range of approximate methods of Pareto optimal paths in multi-objective shortest path problems. John et al. [3] proposed an interactive method to generate an approximation of the non-inferior solution set for two objective shortest path problems, the goal of this approach is to assist the decision maker DM in selecting the preferred or best compromise solution from among the noninferior solutions. Coutinho-Rodrigues, Clímaco, and Current [4] presented a proposal for a new interactive approach to search for unsupported non-dominated solutions (lying inside duality gaps) based on the k-shortest path procedure. Guerriero and Musmanno [5] presented a method for finding an optimal solution for a multi-criteria shortest path problem by using a class of labeling methods to generate a complete set of Pareto optimal path-length POPL vectors from an origin node (source) to all other nodes in a multi-criteria network. Tarapata [6] compared the effectiveness of solving the shortest multi-objective path problem defined as a mathematical programming problem by using the CPLEX 7.0 solver, and the multiple-weighted graph problems using the modified Dijkstra's algorithm. Raith and Ehrgott [7] were able to make an effective comparison of the different strategies to solve the bi-objective shortest path problem of complex networks (i.e., many nodes and arcs). Paixão and Santos [8] studied traditional labeling technology to solve the multi-objective shortest path problem, taking into account the presence of more than a cost of arcs. Duque, Lozano, and Medaglia [9] presented a precise iterative method based on the implicit census to solve a bi-objective shortest path problem to a network consist of 1.2 million nodes and 2.8 million arcs. Thomas, Twan, and Wilcovanden [10] presented a new fully polynomial time approximation scheme for the multi-objective shortest path problem with non-negative and integer arc costs.

1.1. Research Problem

The problem of this paper is how to choose the (DM or businessman) for the shortest possible way to travel by car or plane among the most important 25 US commercial cities from Los Angeles to New York City, taking into consideration achieve three objectives, (cost, distance and time. The **goal of this research** is to find an optimal multi-objective solution to the problem of the shortest path to the most important US commercial cities, while making a comparison between the preference of travel by car or plane, taking into account the minimization of the objectives of cost, distance and time together**.**

2. Multi-Objective Linear Programming Problem MOLPP

MOLPP is one of the most important mathematical optimization models and is an extension of traditional one-objective programming, where consideration is given more than one objective is sought by the DM to achieve [11]. Often these objectives are conflicting with each other, and these ones of the most important difficulties facing us in the multi-objective optimization models. The concept of the optimal solution is no longer logical. In general, there is no possible solution that can optimize all objective functions at the same time, where Zeleny [12] has demonstrated in a very expressive manner the fundamental differences between traditional single-objective optimization models and multi-objective optimization models. The multi-objective linear programming model can now be represented as follows [13]:

$$\begin{aligned} &MOLP = Min\ Z(x) = ((z_1(x), z_2(x), \dots, z_n(x)) \\ &s.t. \quad x \in D \end{aligned} \tag{2.1}$$

Where the represents: $n \geq 2$ the number of objective functions; $x = (x_1, x_2, ..., x_r)$ the vector of decision variables; $D = Ax \geq b$ a space of feasible solution; and Z(x) vector of objective functions, respectively. The solution obtained from the multi-objective linear programming model is a set of non-dominant solutions called the Pareto set PS.

2. Shortest Path Problem SPP

Let's consider that we have a flow network that has a certain starting point and a certain endpoint and that the arrows that connect the network points take many paths to reach the starting point and the access point. We try through the shortest path problem, to get the shorter of these arrows or lines that connect the starting point and the end point. For example, this model is used to obtain the shortest distance or path that can be taken between one city and another through a network of paths. The length of each arc is a function of that distance, travel time, cost of travel or any other measure. In other words, we try to find the shortest path between the start and access point. One of the most important issues in network flow is that of assigning the shortest path between the source node and the destination node. Consider that we have the network composed of n the node (1, 2, …,n) so that each arc (i, j) corresponds to a non-negative number d_{ij} , called distance, or transit time from node i to node j. If there is no direct route between i and j, the distance is: $d_{ij} = +\infty$. The distance d_{ij} can vary from a distance d_{ji} (i.e. $d_{ij} \neq d_{ji}$). The problem lies in how to find the length of the shortest path, and the shorter route, from the source node 1, to the destination node n. As one way to solve this problem, we can interpret it as a network represented by a guided graph G(V, E), where V represents the set of headers (nodes) and E represents the set of links (arcs). The link between node i and node j is expressed by (i, j)$\in$E; and d_{ij} is the cost of the link between i and j; as well as x_{ij}^{pq} (whereas $0 \le x_{ij}^{pq} \le 1$) represents the amount of traffic from node p $\in$ V to node q $\in$ V by routing it through (i, j) $\in$E [14].

2.1. A General Model of Single Objective Shortest Path Problem SOSPP

Generally, the problem of finding the shortest path can be formulated as a linear programming problem as shown below:

$$Min\ Z(x) = \sum_{(i,j)\in E} d_{ij} x_{ij} \qquad (3.1.1)$$

$$s.t.\ \sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = 1, \quad if\ \ i = p \qquad (3.1.2)$$

$$\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = 0, \quad \forall i \neq p\ , q \in V \qquad (3.1.3)$$

$$x_{ij} = 0\ or 1, \qquad \forall (i,j) \in E. \qquad (3.1.4)$$

x_{ij} and d_{ij} is the decision variable and the link cost (i, j), respectively [14]. The equation (3.1.1) represents the objective function that reduces the cost of the path from the node p to node q and x_{ij} is the amount flow from node p to node q through (i, j). Equations (3.1.2) to (3.1.4) are constraints of the model, where equations (3.1.2) - (3.1.3) are represented the conditions for maintaining flow. And Equation (3.1.2) represents the maintenance of flows in the source node, node p. The difference between the amount of traffic received and the amount of traffic issued, $\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji}$, is equal to one. Here, the amount of outgoing traffic at node p equals one. Equation (3.1.3) maintains flows at the intermediate node i, such that i ≠ p, q. The amount of traffic issued at the node i, $\sum_{j:(i,j)\in E} x_{ij}$, equal to the amount of incoming traffic at the node i, $\sum_{j:(j,i)\in E} x_{ji}$. The equation (3.1.4) represents the range of decision variables x_{ij} . At the destination node, node q, the condition to maintain flows is,

$$\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = -1, \quad if\ \ i = q \qquad (3.1.5)$$

The equation (3.1.5) must be met. However, Equation (3.1.5) is discounted using equations (3.1.2)-(3.1.3). Therefore, Equation (3.1.5) is guaranteed by Equations (3.1.2) and (3.1.3) [14].

4. Multi-Objective Shortest Path Problem MOSPP

The MOSPP is one of the most important and common problems. The problem is how to find the shortest path of any network based on a certain set of objectives, like cost, time, distance, etc. Consider that the DM looks at how to choose the feasible shortest path to travel by car or plane to the most important 25 US cities, taking into consideration three main objectives: cost, distance and time. It can be said that the main objective of the DM is to find an efficient solution for MOSPP to find an of the most important US commercial cities.

4.1. The Proposed Mathematical Model to Solve MOSPP

In this paragraph, a proposed mathematical model will be formulated to solve MOSPP based on the original model of SOSPP mentioned above in paragraph 3.1. Let's consider that there are three objectives that the DM seeks to achieve together of travel using the car (the first, second and third objectives) is to minimization (cost Z_1, distance Z_2, and time Z_3), respectively. There are also three other objectives that the DM seeks to achieve in order to travel with the aircraft (Target 1, 2 and 3):

minimization (cost W_1, distance W_2, and time W_3), respectively. The mathematical model can now be formulated for the MOSPP as shown below:
Minimized of all objective functions

$$\text{Cost function by car}: \quad Z_1 = \sum\nolimits_{all\ arcs} c_{ij} x_{ij} \qquad (4.1.1)$$

$$\text{Distance function by car}: \quad Z_2 = \sum\nolimits_{all\ arcs} d_{ij} x_{ij} \qquad (4.1.2)$$

$$\text{Time function by car}: \quad Z_3 = \sum\nolimits_{all\ arcs} t_{ij} x_{ij} \qquad (4.1.3)$$

$$Cost function\ by\ airplane: W_1 = \sum\nolimits_{allarcs} \alpha_{ij} x_{ij} \qquad (4.1.4)$$

$$\text{Distance} function\ by\ airplane: W_2 = \sum\nolimits_{allarcs} \beta_{ij} x_{ij} \qquad (4.1.5)$$

$$Time function\ by\ airplane: W_3 = \sum\nolimits_{allarcs} \delta_{ij} x_{ij} \qquad (4.1.6)$$

$$s.t. \quad \sum\nolimits_{arcs\ out} x_{ij} - \sum\nolimits_{arcs\ in} x_{ji} = 1, \ \text{Origin node (i)} \qquad (4.1.7)$$

$$\sum\nolimits_{arcs\ out} x_{ij} - \sum\nolimits_{arcs\ in} x_{ji} = 0, \ \text{Intermediate nodes} \qquad (4.1.8)$$

$$\sum\nolimits_{arcs\ in} x_{ij} - \sum\nolimits_{arcs\ out} x_{ji} = -1, \ \text{Destination node} \qquad (4.1.9)$$

$$0 \le x_{ij} \le 1. \qquad (4.1.10)$$

There are many approaches used to solve the above model. We will use the lexicographic method that described in the below.

4.2. Lexicographic Method

In this method, the (DM) has to arrange the objective functions according to its importance. The preferred solution of the problem (2.1), in this case, is the solution that minimizes the vales of the most important objective functions as $\left[z_1(x), z_2(x), \ldots, z_n(x)\right]$ which is a vector of objective functions which is arranged according to the importance of the functions of the (DM), such that $z_1(x)$ is the most important objective function among the other and $z_2(x)$ is the objective function followed by the importance and so on [15]. The first problem to be solved is: Find a vector that minimizes,

$$\left.\begin{array}{l} z_1(x) \\ s.t. \ \ x \in D \end{array}\right\} \qquad (4.2.1)$$

Where $x \in D$ represents a feasible region solution, and x_1^* is a solution of this problem as well $z_1(x_1^*) = z_1^*$; if this solution is unique then x_1^* will be a solution to a problem (2.1), but if there is more than one solution, the second problem that must be solved is: Find a vector x that minimizes,

$$\left.\begin{array}{l} z_2(x) \\ s.t. \ \ x \in D \ : \ z_1(x_1^*) = z_1^* \end{array}\right\} \qquad (4.2.2)$$

Let x_2^* represents a solution to this problem and $z_2(x_2^*) = z_2^*$. Now, if x_2^* is a unique solution, the problem (2.1) has this solution, otherwise, this frequency is repeated until we get a unique solution to one of these problems or complete all the objective functions and x_n^*, which is the solution to the problem n, is the solution to the problem (2.1). In general, the problem that must be resolved are: Find a vector x that minimizes,

$$\left.\begin{array}{l} z_j(x) \\ s.t. \ x \in D : z_\ell(x) = z_\ell^*, \ \ell = 1, 2, \ldots, j-1. \end{array}\right\} \qquad (4.2.3)$$

And since the steps of the solution are stopped when we obtained a unique solution of the problem j, this solution is the solution to the problem (2.1), and the functions which have the ranks are less than j neglected.

5. Application for Real Case Study

Consider that, there is a businessman who wants to take the shortest path by using the car or plane of the most important 25 US commercial cities to reach New York City from the hometown city Los Angeles, taking into account the minimization (cost, distance and time) together. Cost, distance and time data were collected among the major US commercial cities (https://www.distance-cities.com/).

Figure-1 illustrates the network diagram of the link among the most important 25 US commercial cities. Table-1 shows the names of the most important US commercial cities; Table-2 shows the cost, distance and time of travel by car, and finally, Table-3 shows the cost, distance and time of travel by plane.

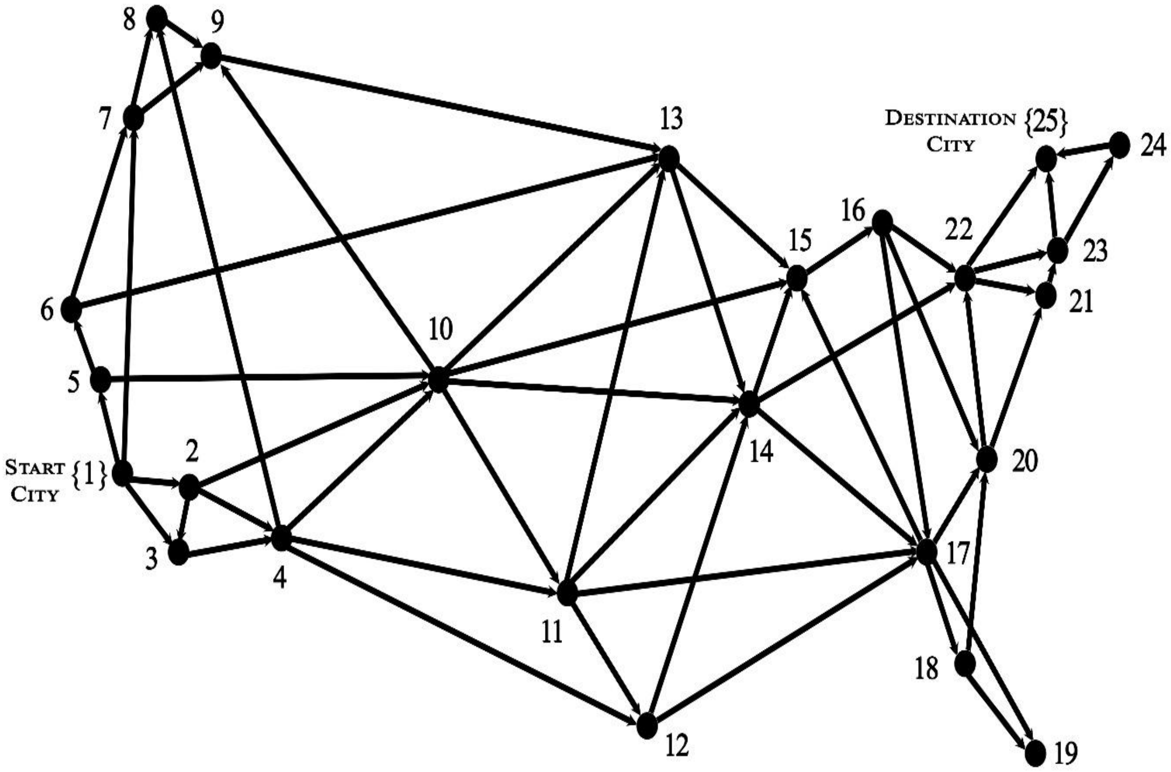

Figure 1- Map of networking among the top 25 US commercial cities

Table 1- Names of the most important of (25) American commercial city

Node	City	Node	City
1	Los Angeles, California	14	St Louis, Missouri
2	Riverside, California	15	Chicago, Illinois
3	San Diego, California	16	Detroit, Michigan
4	Phoenix, Arizona	17	Atlanta, Georgia
5	San Jose, California	18	Tampa, Florida
6	San Francisco, California	19	Miami, Florida
7	Portland, Oregon	20	Charlotte, N.C.
8	Seattle, Washington	21	Washington, D.C.
9	Washington, State	22	Pittsburgh, Pennsylvania
10	Denver, Colorado	23	Philadelphia, Pennsylvania
11	Dallas, Texas	24	Boston, Massachusetts
12	Houston, Texas	25	New York, N.Y.
13	Minneapolis, Minnesota		

Table 2- Cost (C) in dollars ($), distance (D) in miles (Miles) and time (T) in minutes to travel among the most important (25) American commercial cities by using the car

Start - Destination node	Travel using the car			Start - Destination node	Travel using the car		
	Cost	Distance	Time		Cost	Distance	Time
1 – 2	7.15	57.3	60	11 – 17	72.01	782	672
1 – 3	14.67	120	124	12 – 14	69.90	779	726
1 – 5	41.33	340	324	12 – 17	72.93	793	673
1 – 7	113.17	963	882	13 – 14	52.92	559	505
2 – 3	12.24	101	115	13 – 15	40.64	408	371
2 – 4	34.83	326	287	14 – 15	28.16	297	280
2 – 10	108.47	984	859	14 – 17	50.92	554	505
3 – 4	38.28	355	323	14 – 22	60.64	602	545
4 – 8	153.10	1421	1306	15 – 16	28.53	283	257
4 – 10	83.41	821	759	16 – 17	69.82	733	674
4 – 11	98.24	1065	910	16 – 20	64.44	639	621
4 – 12	108.75	1176	994	16 – 22	31.07	289	270
5 – 6	5.82	48.4	58	17 – 18	43.42	456	380
5 – 10	142.94	1273	1122	17 – 19	62.89	663	561
6 – 7	74.67	635	599	17 – 15	69.48	716	629
6 – 13	217.21	1966	1740	17 – 20	23.33	244	222
7 – 8	19.90	173	198	18 – 19	26.74	280	247
7 – 9	22.66	269	289	18 – 20	55.71	580	520
8 – 9	5.12	103	122	20 – 21	40.25	399	380
9 – 13	175.27	1585	1401	20 – 22	46.56	447	419
10 – 9	140.04	1299	1148	21 – 23	15.49	139	148
10 – 11	74.48	791	724	22 – 21	26.72	242	238
10 – 13	90.75	914	774	22 – 23	33.78	304	283
10 – 14	80.30	850	718	22 – 25	40.65	371	354
10 – 15	99.78	1004	858	23 – 24	32.84	310	321
11 – 12	21.51	239	214	23 – 25	10.30	96.7	109
11 – 13	93.71	942	840	24 – 25	22.67	215	229
11 – 14	56.69	631	582				

Table 3- Cost (C) in dollars ($), distance (D) in miles (Miles) and time (T) in minutes to travel among the top 25 US commercial cities by using the plane

Start - Destination node	**Travel using the airplane**			**Start - Destination node**	**Travel using the airplane**		
	Cost	**Distance**	**Time**		**Cost**	**Distance**	**Time**
1 – 2	300	49	36	11 – 17	126	721	99
1 – 3	182	112	31	12 – 14	239.5	679	96
1 – 5	121.5	306	50	12 – 17	103	702	95
1 – 7	142	825	117	13 – 14	235.5	466	68
2 – 3	385.5	86	40	13 – 15	79.5	355	66
2 – 4	153.5	309	50	14 – 15	190.5	262	45
2 – 10	216	794	105	14 – 17	64.5	467	67
3 – 4	124	299	50	14 – 22	121.5	559	97
4 – 8	232	1114	156	15 – 16	122	238	45
4 – 10	135	586	80	16 – 17	134.5	596	90
4 – 11	107.5	886	115	16 – 20	229.5	504	78
4 – 12	222.5	1016	128	16 – 22	211	205	37
5 – 6	190.5	42	35	17 – 18	100.5	416	62
5 – 10	155.5	929	122	17 – 19	121.5	604	88
6 – 7	118	535	76	17 – 15	123	588	88
6 – 13	221	1585	186	17 – 20	214.5	226	42
7 – 8	152.5	145	42	18 – 19	101.5	205	39
7 – 9	143.5	139	40	18 – 20	243	510	78
8 – 9	117	39	35	20 – 21	159	329	54
9 – 13	231	1360	161	20 – 22	265.5	363	58
10 – 9	178.5	983	140	21 – 23	297	124	34
10 – 11	81	662	96	22 – 21	108	190	40
10 – 13	112	700	93	22 – 23	210.5	258	46
10 – 14	111	797	100	22 – 25	216	316	55
10 – 15	105	920	117	23 – 24	133.5	271	49
11 – 12	171.5	225	42	23 – 25	208	81	23
11 – 13	96.5	862	116	24 – 25	110	190	48
11 – 14	205.5	548	78				

5.1. MOSPP by Using Car or Airplane

Based on the model proposed in paragraph 4.1 and using the above Tables-(2-3) that mentioned (2 and 3), a multi-objective linear programming model will be built to solve the shortest path problem among the most important 25 US commercial cities by traveling either by car or by plane as shown below:

$$\underset{(Travel\ by\ Car)}{Minimization}\begin{cases} Z_1(\text{x}) = 7.15x_{1,2} + 14.67x_{1,3} + 41.33x_{1,5} + 113.17x_{1,7} + \dots + 22.67x_{24,25} & (5.1.1) \\ Z_2(\text{x}) = 57.3x_{1,2} + 120x_{1,3} + 340x_{1,5} + 963x_{1,7} + \dots + 215x_{24,25} & (5.1.2) \\ Z_3(\text{x}) = 60x_{1,2} + 124x_{1,3} + 324x_{1,5} + 882x_{1,7} + \dots + 229x_{24,25} & (5.1.3) \end{cases}$$

$$\underset{(Travel\ by\ Plane)}{Minimization}\begin{cases} W_1(x) = 300x_{1,2} + 182x_{1,3} + 121.5x_{1,5} + 142x_{1,7} + ... + 110x_{24,25} & (5.1.4) \\ W_2(x) = 49x_{1,2} + 112x_{1,3} + 306x_{1,5} + 825x_{1,7} + ... + 190x_{24,25} & (5.1.5) \\ W_3(x) = 36x_{1,2} + 31x_{1,3} + 50x_{1,5} + 117x_{1,7} + ... + 48x_{24,25} & (5.1.6) \end{cases}$$

Subject to:

$x_{1,2} + x_{1,3} + x_{1,5} + x_{1,7} = 1$ (5.1.7), $x_{1,2} = x_{2,3} + x_{2,4} + x_{2,10}$ (5.1.8)

$x_{1,3} + x_{2,3} = x_{3,4}$ (5.1.9), $x_{2,4} + x_{3,4} = x_{4,8} + x_{4,10} + x_{4,11} + x_{4,12}$ (5.1.10)

$x_{1,5} = x_{5,6} + x_{5,10}$ (5.1.11), $x_{5,6} = x_{6,7} + x_{6,13}$ (5.1.12)

$x_{1,7} + x_{6,7} = x_{7,8} + x_{7,9}$ (5.1.13), $x_{4,8} + x_{7,8} = x_{8,9}$ (5.1.14)

$x_{7,9} + x_{8,9} + x_{10,9} = x_{9,13}$ (5.1.15), $x_{2,10} + x_{4,10} + x_{5,10} = x_{10,9} + x_{10,11} + x_{10,13} + x_{10,14} + x_{10,15}$ (5.1.16)

$x_{4,11} + x_{10,11} = x_{11,12} + x_{11,13} + x_{11,14} + x_{11,17}$ (5.1.17), $x_{4,12} + x_{11,12} = x_{12,14} + x_{12,17}$ (5.1.18)

$x_{6,13} + x_{9,13} + x_{10,13} + x_{11,13} = x_{13,14} + x_{13,15}$ (5.1.19)

$x_{10,14} + x_{11,14} + x_{12,14} + x_{13,14} = x_{14,15} + x_{14,17} + x_{14,22}$ (5.1.20)

$x_{10,15} + x_{13,15} + x_{14,15} + x_{17,15} = x_{15,16}$ (5.1.21), $x_{15,16} = x_{16,17} + x_{16,20} + x_{16,22}$ (5.1.22)

$x_{11,17} + x_{12,17} + x_{14,17} + x_{16,17} = x_{17,18} + x_{17,19} + x_{17,20}$ (5.1.23), $x_{17,18} = x_{18,19} + x_{18,20}$ (5.1.24)

$x_{16,20} + x_{17,20} + x_{18,20} = x_{20,21} + x_{20,22}$ (5.1.25), $x_{20,21} + x_{22,21} = x_{21,23}$ (5.1.26)

$x_{14,22} + x_{16,22} + x_{20,22} = x_{22,21} + x_{22,23} + x_{22,25}$ (5.1.27), $x_{21,23} + x_{22,23} = x_{23,24} + x_{23,25}$ (5.1.28)

$x_{23,24} = x_{24,25}$ (5.1.29), $x_{22,25} + x_{23,25} + x_{24,25} = 1$ (5.1.30)

All decision variables (≥ 0 and ≤ 1) (5.1.31)

Where: Z_1(x), Z_2(x), Z_3(x); represents the function of (cost, distance and time) respectively, when businessman traveling by car, and W_1(x), W_2(x), W_3(x); represents the function of (cost, distance and time) respectively, when businessman traveling by plane. The equations from (5.1.7) to (5.1.31) represent the constraints of the problem to which we will refer briefly $x \in D$.

5.2. Solve the Model of MOSPP by Using the Car or Airplane

In this paragraph, the above model will be resolved by using the package program (Win-QSB 2.0) [16]. Firstly, we will solve each objective function individually with the problem constraints as a standard linear programming model to obtain the values of Z_1(x), Z_2(x) and Z_3(x), where the travel model by using the car; and the values of W_1(x), W_2(x) and W_3(x) for the travel model by using the aircraft, respectively. So that we can use the Lexicographic method that mentioned above in paragraph 4.2 to find the final optimal solution for the travel either by car or by plane and as follows:

$$\left.\begin{matrix} Min\ \ Z_1(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (5.2.1) \qquad \left.\begin{matrix} Min\ \ Z_2(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (5.2.2) \qquad \left.\begin{matrix} Min\ \ Z_3(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (4.2.3)$$

- After we solved the models (5.2.1-5.2.3), were obtained the preliminary results shown below,

$$Z_1(x) = 297.2099;\ Z_2(x) = 2864.30;\ Z_3(x) = 2536.$$

$$\left.\begin{matrix} Min\ \ W_1(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (5.2.4) \qquad \left.\begin{matrix} Min\ \ W_2(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (5.2.5) \qquad \left.\begin{matrix} Min\ \ W_3(x) \\ s.t.\ \ x \in D \end{matrix}\right\} \ (4.2.6)$$

- After we solved the models (5.2.4-5.2.6), were obtained the preliminary results shown below,

$$W_1(x) = 964.500; W_2(x) = 2515; W_3(x) = 393.$$

We will now use the Lexicographic method to obtain the optimal solution of travel by using the car or plane as follows:

- (The final model of travel by car)

$$\left.\begin{aligned}
&Min\ \ Z_1(x) = 7.15x_{1,2} + 14.67x_{1,3} + 41.33x_{1,5} + 113.17x_{1,7} + \ldots + 22.67x_{24,25} \\
&s.t.\ \ Z_2(\mathrm{x}) = 57.3x_{1,2} + 120x_{1,3} + 340x_{1,5} + 963x_{1,7} + \ldots + 215x_{24,25} = 2864.30 \\
&\qquad Z_3(\mathrm{x}) = 60x_{1,2} + 124x_{1,3} + 324x_{1,5} + 882x_{1,7} + \ldots + 229x_{24,25} = 2536 \\
&\qquad x \in D
\end{aligned}\right\} \quad (5.2.7)$$

After we solving the model (5.2.7), the final optimal solution was obtained for the three objectives as shown below,

$$Z_1^*(\mathrm{x}) = 297.21/\$;\ Z_2^*(\mathrm{x}) = 2864.30/Miles;\ Z_3^*(\mathrm{x}) = 2536\,Minutes\ /\ 42.27\,Hour;$$

$$\left\{x^*_{1,2}, x^*_{2,10}, x^*_{10,14}, x^*_{14,22}, x^*_{22,25}\right\} = 1.$$

- (The final model of travel by plane)

$$\left.\begin{aligned}
&Min\ W_1(x) = 300x_{1,2} + 182x_{1,3} + 121.5x_{1,5} + 142x_{1,7} + \ldots + 110x_{24,25} \\
&s.t.\ \ W_2(\mathrm{x}) = 49x_{1,2} + 112x_{1,3} + 306x_{1,5} + 825x_{1,7} + \ldots + 190x_{24,25} = 2515 \\
&\qquad W_3(\mathrm{x}) = 36x_{1,2} + 31x_{1,3} + 50x_{1,5} + 117x_{1,7} + \ldots + 48x_{24,25} = 393 \\
&\qquad x \in D
\end{aligned}\right\} \quad (5.2.8)$$

After solving the model (5.2.8), the final optimal solution was obtained for the three objectives as shown below,

$$W_1(\mathrm{x}) = 964.5/\$; W_2(\mathrm{x}) = 2515/Miles; W_3(\mathrm{x}) = 393\,Minutes\ /\ 6.55\,Hour;$$

$$\left\{x^*_{1,2}, x^*_{2,10}, x^*_{10,14}, x^*_{14,22}, x^*_{22,25}\right\} = 1.$$

The above results show that, if the DM is looking for minimization cost only, the traveling by car is the optimal solution; and if the DM is looking to minimization the distance and time, the travel by plane is optimal. The optimal path is represented in Table (4).

Table 4- The optimal path has the minimization of the objective functions simultaneously, for travel by car or plane

City of America	Optimal path	Travel using the car			Travel using the airplane		
		Cost	Distance	Time	Cost	Distance	Time
Los Angeles, California - Riverside, California	$1 \rightarrow 2$	7.15	57.3	60	300	49	36
Riverside, California - Denver, Colorado	$2 \rightarrow 10$	108.47	984	859	216	794	105
Denver, Colorado - St Louis, Missouri	$10 \rightarrow 14$	80.30	850	718	111	797	100
St Louis, Missouri - Pittsburgh, Pennsylvania	$14 \rightarrow 22$	60.64	602	545	121.5	559	97
Pittsburgh, Pennsylvania - New York, N.Y.	$22 \rightarrow 25$	40.65	371	354	216	316	55
Total		297.21	2864.3	2536	964.5	2515	393

6. Conclusions

In this paper, we have focused on how to solve the problem of the shortest path problem of the most important 25 commercial cities in the United State of America with real data analysis, also how to find a final optimal solution that satisfies the DM ambition and obtain the maximum budget achieved in terms of the minimization cost, distance and time. Therefore, a multi-objective linear programming model was proposed to solve the shortest path problem. This model proved the efficiency and effectiveness of solving the problem, were taken into account the objective functions (cost, distance and time) the minimization cost only, is the optimal solution when traveling is by car; and minimization the distance and time is optimal when the travel is by plane. The lexicographic method provided a definitive solution for all objective functions with one optimal path. The use of this type of mathematical model is a new technique that will raise the quality of rational decision making in institutions that are interested in logistics transportation and looking for real solutions to the problems of multi-objective optimization.

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