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# SOME RESULTS OF (α, β) DERIVATIONS ON PRIME SEMIRINGS

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**Abstract**
This paper investigates the concept $(\alpha, \beta)$ derivation on semiring and extend a few results of this map on prime semiring. Also we establish the commutativity of prime semiring and investigate when $(\alpha, \beta)$ derivation becomes zero.

**Keywords**: Semirings, Prime Semirings, Semiprime Semirings, $(\alpha, \beta)$ Derivation.

# بعض النتائج للمشتقات (α, β) على اشباه الحلقات الاولية

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**الخلاصة**
في هذا البحث درسنا مفهوم المشتقات $(\alpha, \beta)$ على اشباه الحلقات وقمنا بتوسيع بعض النتائج على اشباه الحلقات الاولية. و حصلنا على ابدالية اشباه الحلقات الاولية وكذلك متى تصبح المشتقات $(\alpha, \beta)$ صفرا عليها.

## 1. Introduction

The notation of semiring was first introduced by Vandiver in 1934, then many researchers had been studying diverse kinds of semirings , it's properties and different types of derivations on it. A nonempty set say $S$ together with two binary operations (addition and multiplication), this triple is called semiring, if S with addition is a semigroup, $S$ with multiplication is also semigroup and addition distributive with respect to multiplication on S [1]. The only difference between ring and semiring conditions is there´s no additive invertible elements in semirings but this property exist in rings since the set together with addition define a group. If we suppose S any semiring and $D: S \rightarrow S$ be a map defined on $S$, then $D$ is called additive map if it preserves addition relation. Now, this additive map said to be derivation on $S$ if $D(xy) = D(x)y + xD(y)$ for all $x$ and y in $S$. Moreover $(\alpha, \beta)$ derivation introduced as d is derivation on $S$ and $\alpha, \beta$ are two automorphisms on S such that $d(xy) = \alpha(x)d(y) + d(x)\beta(y)$ for all $x$ and $y$ in $S$ [2]. We also used commutator which is defined in [3] as $[x, y] = xy - yx$ with $[x + y, z] = [x, z] + [y, z]$ and $[xy, z] = x[y, z] + [x, z]y$. We'll also present some necessary definitions for this paper in the preliminaries.

## 2. Preliminaries

**Definition 2.1**: - [4] A nonempty set $S$ with the binary operation $*$ said to be semigroup iff $x * (y * z) = (x * y) * z$ for all $x, y, z \in S$.
**Definition 2.2**: - [4] A semigroup $S$ called commutative iff $x * y = y * x$ for all $x, y \in S$.
**Definition 2.3**: - [4] A nonempty set $S$ with two binary operation + and . is said to be a semiring iff the following conditions satisfied:-
1- $(S, +)$ Semigroup.
2- $(S, .)$ Semigroup.

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3- $x. (y + z) = x. y + x. z$ and $(y + z) .x = y. x + z. x$ for all $x, y, z \in S$.

**Notation**: Throughout this paper we shall assume that S contains 0 and 1.

**Example 2.4**: - [4] Let $B = \{0, 1\}$ and the operations +, . defined on $B$ by the tables below:

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

| . | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

Then $(B, +, .)$ is semiring.

**Example 2.5**: - [4] Let $Z_0^+ = \{x \in Z: x \geq 0\}$, + and . are usual addition and usual multiplication, then $(Z_0^+, +, .)$ is semiring but not ring.

**Example 2.6**: - Let $S = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in Z_0^+ \}$ with usual matrices addition and multiplication of integers, then $S$ is semiring.

**Definition 2.7**: - [5] A semiring $S$ is called additively commutative iff $x + y = y + x$ for all $x, y \in S$, and is called multiplicatively commutative iff $x. y = y. x$ for all $x, y \in S$. Also $S$ is called commutative semiring iff it is both additively and multiplicatively commutative.

**Definition 2.8**: - [6] A semiring $S$ is called additively cancellative iff $x + y = x + z$ implies $y = z$ for all $x, y, z \in S$, and it is called multiplicatively cancellative iff $x. y = x. z$ implies $y = z$ for all $x, y, z \in S$. Also $S$ is called cancellative semiring iff it is both additively and multiplicatively cancellative.

**Definition 2.9**: - [3] Let $S$ be a semiring, the set $Z(S) = \{x \in S: x. y = y. x$, for all $x, y \in S\}$ is called the center of $S$.

**Lemma 2.10**:- If $S$ is multiplicatively commutative then $Z(S) = S$.

**Proof**

It´s clear that $Z(S) \subseteq S$, we need only to show that $S \subseteq Z(S)$.
Let $x \in S$, since $S$ is multiplicatively commutative then $x. y = y. x$, for all $y \in S$, so $x \in Z(S)$.
We get $S \subseteq Z(S)$, then $Z(S) = S$.

**Definition 2.11**: - [4] Let $(S, +, .)$ be a semiring, an element $0 \in S$ is called zero of $S$ iff $x + 0 = x = 0 + x$ for all $x \in S$, an element $1 \in S$ is called identity of $S$ iff $x.1 = x = 1.x$ for all $x \in S$.

**Definition 2.12**: - [1] Let $(S, +, .)$ be a semiring and $T$ nonempty proper subset of S, then $T$ is called subsemiring if it is semiring with the operations + and ., that is if $(T, +, .)$ is semiring itself.

**Remark 2.13**: - [2] If $S$ is semiring with 0 and 1 then any subset of $S$ which contain 0 and 1 is subsemiring of $S$.

**Definition 2.14**: - [7] Let $(S, +, .)$ be a semiring and $I$ be a nonempty subset of $S$, if:-

1- $1 \notin I$.
2- $a + b \in S$ for all $a, b \in I$.
3- $r.a \in S$ for all $a \in I$ and $r \in S$.

Then $I$ is called Left ideal of $S$. Similarly we can define right ideal.
If $I$ is both left and right idel then we call it an ideal.

**Example 2.15**: - Let $Z_0^+$ under usual addition and multiplication of integers is semiring, then $< 2 > = \{2n$: for some $n \in Z_0^+\}$ is an ideal of $Z_0^+$.

1- $1 \notin < 2 >$.
2- Since usual addition of integers closed under $Z_0^+$ then it is closed under $< 2 >$.
3- Let $2n_1 \in < 2 >$ and $n \in Z_0^+$ then $n. 2n_1 = (2.n)\, n_1$. Now since usual multiplication of integers is associative, hence $(2.n)\, n_1 \in < 2 >$.

Then $< 2 >$ is an ideal of $Z_0^+$.

**Definition 2.16**:- [1] Let $S$ be a semiring and $I$ be a nonzero ideal of $S$, then the set $Z(I) = \{ a \in I : a. b = b. a, \forall\, b \in I\}$ called the center of $I$ .

**Lemma 2.17**:- If $I$ is commutative as semiring then $Z(I) = I$.

**Proof (**Trivial).

**Definition 2.18**: - [8] A semiring $S$ is called Prime if whenever $x\ S\ y = 0$ implies either $x = 0$ or $y = 0$ for all $x, y \in S$.

**Definition 2.19**: - [8] A semiring $S$ is called semiprime if whenever $x\ S\ x = 0$ implies $x= 0$ for all $x \in S$.

**Definition 2.20**: - [8] A semiring $S$ is called $n$-torsion free iff whenever $nx = 0$ then $x = 0$ for all $x \in S$, where $n \neq 0$.

**Lemma 2.21**:- Let $S$ be a prime semiring and $I$ be a nonzero left (right) ideal of $S$, then $Z(I) \subseteq Z(S)$.

**Proof**

Let $0 \neq a \in Z(I)$, since $Z(I) \subseteq I$, then $a \in I$.
Let $x \in S$, then $x\,a \in I$ (By definition of left ideal).
Since $a \in Z(I)$, we have $[x\,a, a] = x\,[a, a] + [x, a]\,a = 0$ then,

$$[x, a]\,a = 0 \text{ for all } x \in S \qquad \dots (1)$$

Replace $x$ by $x\,y$ in (1), where $y \in S$ we get $[x\,y, a]\,a = x\,[y, a]\,a + [x, a]\,ya = 0$.
By using (1) we get $[x, a]\,y\,a = 0$ for all $x, y \in S$ and for all $a \in I$.
Then $[x, a]\,S\,I = 0$.
By primness and since $I$ is nonzero ideal of $S$ then $[x, a] = 0$ for all $x \in S$ and $a \in Z(I)$.
Then $Z(I) \subseteq Z(S)$.

**Lemma 2.22**: - Let $S$ be a semiring and $I$ be a nonzero ideal of $S$, if $I$ commutative as semiring then $I \subseteq Z(S)$. If $S$ is prime then $S$ is commutative.

**Proof**

Since $I$ is commutative as semiring then by (Lemma 2.17) we have $I = Z(I)$.
By (lemma 2.21) we have $Z(I) \subseteq Z(S)$, then $I \subseteq Z(S)$.
Now, If $S$ is prime, Let $x, y \in S$ and $a \in I$
Then $a\,x \in Z(S)$ that is$[a\,x, y] = 0$ for all $y \in S$.
$a\,[x, y] + [a, y]\,x = \mathrm{a}\,[x, y] = 0$ for all $a \in I$.
Then $I\,[x, y] = 0 \Rightarrow I\,S\,[x, y] = 0$.
By primness of S and since $I$ is nonzero ideal, then $[x, y] = 0$ for all $x, y \in S$.
Then $S$ is commutative.

**Definition 2.23**: - [2] Let $S$ be a semiring, then a function $f$: $S \rightarrow S$ is called additive map if it is preserve addition relation i.e. $f(x + y) = f(x) + f(y)$ for all x, $y \in S$.

**Definition 2.24**: - [2] An additive map $d$: $S \rightarrow S$ is called derivation on $S$ if $d(xy) = d(x)\,y + x\,d(y)$ for all $x, y \in S$

**Definition 2.25**: - [8] Let $S$ be a semiring and $\alpha, \beta$ are two automorphisms of $S$. An additive map $d$: $S \rightarrow S$ is called *(α, β)* derivation on S if $d(xy) = \alpha(\mathrm{x})\,d(y) + d(x)\,\beta(y)$ for all $x, y \in S$.

**Example 2.26**: - [2] Let $S = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in Z_0^+\}$ under usual matrices addition and multiplication is semiring. Suppose that $\alpha$: $S \rightarrow S$ defined by $\alpha(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ and β: S→S defined by $\beta(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}$ . Define a derivation $d$: $S \rightarrow S$ by $d(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$. Then $d$ is $(\alpha, \beta)$ derivation on $S$.
Since $d$ is derivation on $S$ (i.e. additive map), we will only check if $d(xy) = \alpha(x)\,d(y) + d(x)\,\beta(y)$.
Now, Let $x = \begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix}$ and $y = \begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix}$,
$d(x\,y) = \begin{pmatrix} 0 & a_1b_2 + b_1c_2 \\ 0 & 0 \end{pmatrix}$,
$\alpha(x)\,d(y) = \begin{pmatrix} 0 & a_1b_2 \\ 0 & 0 \end{pmatrix}$ and $d(x)\,\beta(y) = \begin{pmatrix} 0 & b_1c_2 \\ 0 & 0 \end{pmatrix}$.
Then $\alpha(x)\,\mathrm{d}(y) + d(x)\,\beta(y) = \begin{pmatrix} 0 & a_1b_2 + b_1c_2 \\ 0 & 0 \end{pmatrix}$.
We get $\mathrm{d}(x\,y) = \alpha(x)\,d(y) + d(\mathrm{x})\,\beta(y)$, then $d$ is $(\alpha, \beta)$ derivation on $S$.

**Lemma 2.27**:- Let $S$ be a prime semiring. Suppose that $\alpha$ and $\beta$ are two automorphisms of $S$ and $d$: $S \rightarrow S$ is $(\alpha, \beta)$ derivation such that for all $x \in S$ we have $a.\,d(x) = 0$ or $d(x).\,\mathrm{a} = 0$, where $a \in S$, then either $a = 0$ or $d = 0$.

**Proof**

Let $a.\,d(x) = 0$.

Since $d$ is $(\alpha, \beta)$ derivation of $S$, then $d(xy) = \alpha(x) d(y) + d(x) \beta(y)$ for all $x, y \in S$.
Then $a. (\alpha(x) d(y) + d(x) \beta(y)) = a . \alpha(x) d(y) + a. d(x) \beta(y) = a . \alpha(x) d(y) = 0$.
Since $\alpha$ is automorphism of $S$ we get, $a S d(y) = 0$.
By primness of $S$ either $a = 0$ or $d(x) = 0$ for all $x \in S$. i.e. either $a = 0$ or $d = 0$.
Similarly for $d(x). a = 0$.

## 3. Results

**Theorem 3.1**:- Let $S$ be a cancellative prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d$: $S \longrightarrow S$ is $(\alpha, \beta)$ derivation. If $d$ acts as homomorphism on $S$ then $d = 0$ on $S$.

**Proof**

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$$d(x y) = \alpha(x) d(y) + d(x) \beta(y) \text{ for all } x, y \in S \quad \text{... (1)}$$

Since $d$ acts as homomorphism on $S$ then,

$$d(x y) = d(x) d(y) \text{ for all } x, y \in S \quad \text{... (2)}$$

From (1) and (2) we get,

$$\alpha(x) d(y) + d(x) \beta(y) = d(x) d(y) \text{ for all } x, y \in S \quad \text{... (3)}$$

Replace $x$ by $x r$ in (3), where $r \in S$, we get,
$\alpha(x r) d(y) + d(x r) \beta(y) = d(x r) d(y)$
Since $d$ acts homomorphism on $S$ and $\alpha, \beta$ automorphisms of $S$ then

$$\begin{aligned}\alpha(x) \alpha(r) d(y) + d(x) d(r) \beta(y) &= d(x) d(r) d(y) \\ &= d(x) d(r y) \\ &= d(x) [\alpha(r) d(y) + d(r) \beta(y)] \\ &= d(x) \alpha(r) d(y) + d(x) d(r) \beta(y)\end{aligned}$$

By additively cancellative property we get,
$\alpha(x) \alpha(r) d(y) = d(x) \alpha(r) d(y)$
Now, by multiplicatively cancellative property we get,

$$\alpha(x) = d(x) \text{ for all } x \in S \quad \text{... (4)}$$

Replace $x$ by $x y$ in (4) we get,
$\alpha(x y) = d(x y)$
Since $\alpha$ is automorphism of $S$ then $\alpha(x) \alpha(y) = d(x y)$.
Since $\alpha = d$ from relation (4) then $\alpha(x) d(y) = \alpha(x) d(y) + d(x) \beta(y)$
By additively cancellative property we get $d(x) \beta(y) = 0$.
By (Lemma 2.27) and since $\beta \neq 0$ then $d = 0$ on $S$.

**Theorem 3.2**: - Let $S$ be a cancellative prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d$: $S \longrightarrow S$ is $(\alpha, \beta)$ derivation such that α and $\beta$ commute with d. If d acts as anti-homomorphism on $S$ then $d = 0$ on $S$.

**Proof**

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$$d(x y) = \alpha(x) d(y) + d(x) \beta(y) \text{ for all } x, y \in S \quad \text{... (1)}$$

Since $d$ acts as anti-homomorphism on $S$ then,

$$d(x y) = d(y) d(x) \text{ for all } x, y \in S \quad \text{... (2)}$$

From (1) and (2) we get,

$$\alpha(x) d(y) + d(x) \beta(y) = d(y) d(x) \text{ for all } x, y \in S \quad \text{... (3)}$$

Replace $x$ by $x y$ in (3) we get,
$\alpha(x y) d(y) + d(x y) \beta(y) = d(x y) d(y)$
Since $d$ acts homomorphism on $S$ and $\alpha, \beta$ automorphisms of $S$ then,

$$\begin{aligned}\alpha(x) \alpha(y) d(y) + d(y) d(x) \beta(y) &= d(y) d(x) d(y) \\ &= d(y) d(xy) \\ &= d(y) [\alpha(x) d(y) + d(x) \beta(y)] \\ &= d(y) \alpha(x) d(y) + d(y) d(x) \beta(y)\end{aligned}$$

By additively cancellative property we get,
$\alpha(x) \alpha(y) d(y) = d(y) \alpha(x) d(y)$
Now, since α commute with d and using multiplicatively cancellative property we get,

$$\alpha(y) = d(y) \text{ for all } y \in S \quad \text{... (4)}$$

Replace $y$ by $x y$ in (4) we get $\alpha(x y) = d(x y)$.

Since $\alpha$ is automorphism of $S$ then $\alpha(x)\,\alpha(y) = d(x\,y)$.
Since $\alpha = d$ from relation (4), then $\alpha(x)\,d(y) = \alpha(x)\,d(y) + d(x)\,\beta(y)$.
By additive cancellative property we get $d(x)\,\beta(y) = 0$.
By (Lemma 2.27) and since $\beta \neq 0$ then $d = 0$ on $S$.

**Lemma 3.3**:- Let $S$ be a semiring and $I$ be a nonzero ideal of $S$. Suppose that $\alpha$, $\beta$ are two automorphisms on $S$ and $d$: $S \rightarrow S$ is $(\alpha, \beta)$ derivation such that $d$ onto on $I$. If $[d(u), a] = 0$ for all $u \in I$ and $a \in S$, then $a \in Z(I)$.

**Proof**

Let $[d(u), a] = 0$ for all $u \in I$, then $[d(I), a] = 0$ for all $a \in S$.
Since $d$ is onto on $I$ then $d(I) = I$.
Then $[I, a] = 0$ for all $a \in S$.
Then $a \in Z(I)$.

**Theorem 3.4: -** Let $S$ be a 2-torsion free prime semiring and $I$ be a nonzero ideal of $S$. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d: S \longrightarrow S$ is $(\alpha, \beta)$ derivation on $S$ such that $d$ onto on $I$, and $d$ commute with $\alpha$ and $\beta$. If $d(x\,y) = d(y\,x)$ for all $x, y \in I$ then $S$ is commutative.

**Proof**

For any element $c \in I$ such that $c = [x, y]$ where $x, y \in I$ with $d(c) = 0$.
We have $d(z\,c) = d(c\,z)$.
Since $d$ is $(\alpha, \beta)$ derivation on $S$ then,
$\alpha(z)\,d(c) + d(z)\,\beta(c) = \alpha(c)\,d(z) + d(c)\,\beta(z)$
Since $d(c) = 0$ then,

$$d(z)\,\beta(c) = \alpha(c)\,d(z) \quad \ldots (1)$$

That is $[c, d(z)] = 0$ for all $z \in I$
By (Lemma 3.3) we get $c \in Z(I)$ for all $c \in I$, then $[x, y] \in Z(I)$.
Then $[a, [x, y]] = 0$ for all $a \in I$ … (2)
Replace $y$ by $x\,y$ in (2) we get $[a, [x, x\,y]] = 0$. Then,

$$[a, x]\,[a, y] = 0 \quad \ldots (3)$$

Replace $y$ by $y\,a$ in (3) we get,
$[a, x]\,[a, y\,a] = [a, x]\,y\,[x, a] = 0$ for all $x, y, a \in I$.
Then $[a, x]\,I\,[x, a] = 0$
Now $[a, x]\,S\,I\,[x, a] = 0$.
By primness of $S$, either $[a, x] = 0$ or $I\,[x, a] = 0$.
Case 1: If $[a, x] = 0$ for all $a, x \in I$ then $I$ is commutative and by (Lemma2.22) we get $S$ is commutative.
Case 2: If $I\,[x, a] = 0$, then $I\,S\,[x, a] = 0$.
By primness of $S$ and since $I$ is nonzero ideal we get $[x, a] = 0$ for all $x, a \in I$.
Then $I$ is commutative and by (Lemma2.22) we get $S$ is commutative.

**Lemma 3.5:-** Let $S$ be a prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d: S \longrightarrow S$ is $(\alpha, \beta)$ derivation on $S$. If $d$ acts as homomorphism on $I$ and $d = 0$ on $I$ then d = 0 on $S$.

**Proof**

Let $s\,v \in I$, where $s \in S$ and $v \in I$, then $d(s\,v) = 0$
Since $d$ acts as homomorphism then,

$$d(s)\,d(v) = 0 \quad \ldots (1)$$

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$$\alpha(s)\,d(v) + d(s)\,\beta(v) = 0 \quad \ldots (2)$$

From (1) and (2) we get,
$d(s)\,d(v) = \alpha(s)\,d(v) + d(s)\,\beta(v)$
Since $d = 0$ on $I$ then,

$$D(s)\,\beta(v) = 0 \text{ for all } s \in S \text{ and } v \in I \quad \ldots (3)$$

By (lemma 2.27) either $\beta(v) = 0$ for all $v \in I$ or $d(s) = 0$ for all $s \in S$
Since $\beta$ is nonzero, then $d(s) = 0$ for all $s \in S$.
Then $d = 0$ on $S$.

**Theorem 3.6**: - Let $S$ be a cancellative prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d: S \longrightarrow S$ is $(\alpha, \beta)$ derivation on $S$. If $d$ acts as homomorphism on $I$ then $d = 0$ on $S$.

**Proof**

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$d(uv) = \alpha(u)\, d(v) + d(u)\, \beta(v)$ for all $u, v \in I$ ... (1)

Since $d$ acts as homomorphism on $S$ then,

$d(u\,v) = d(u)\, d(v)$ for all u, v $\in I$ ... (2)

From (1) and (2) we get,

$\alpha(u)\, d(v) + d(u)\, \beta(v) = d(u)\, d(v)$ $u, v \in I$ ... (3)

Replace $v$ by $v\,t$ in (3), where $t \in$ I, we get,

$\alpha(u)\, d(v\,t) + d(u)\, \beta(v\,t) = d(u)\, d(v\,t)$

Since d acts homomorphism on $I$ and $\alpha$, $\beta$ are automorphisms of $I$ then,

$\alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t) = d(u)\, d(v)\, d(t)$

$= d(u\,v)\, d(t)$

$= [\alpha(u)\, d(v) + d(u)\, \beta(v)]\, d(t)$

$= \alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t)$

By additively cancellative property we get,

$d(u)\, \beta(v)\, d(t) = d(u)\, \beta(v)\, \beta(t)$

By multiplicatively cancellative property we get,

$d(t) = \beta(t)$ for all $t \in S$ … (4)

Substitute (4) in (3) we get:

$d(u)\, d(v) = \alpha(u)\, d(v) + d(u)\, d(v)$

Now by multiplicatively cancellative property we get,

$\alpha(u)\, d(v) = 0$

Replace $u$ in above equatin by $u\,r$, where r $\in S$, we get $\alpha(u\,r)\, d(v) = 0$.

Then $\alpha(u)\, \alpha(r)\, d(v) = 0$ for all $r \in S$ and $u,v \in I$.

Since $\alpha$ automorphisms of $S$ then $\alpha(u)\, S\, d(v) = 0$.

By primness and since α is nonzero, then $d(v) = 0$ for all $v \in I$.

Then $d = 0$ on $I$.

By (Lemma 3.5) we get $d = 0$ on $S$.

**Lemma 3.7:-** Let $S$ be a prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d: S \longrightarrow S$ is $(\alpha, \beta)$ derivation on $S$. If $d$ acts as anti-homomorphism on $I$ and $d = 0$ on $I$ then d = 0 on $S$.

**Proof**

Let $s\,v \in I$, where $s \in S$ and $v \in I$ , then $d(sv) = 0$.

Since $d$ acts as anti-homomorphism then,

$d(v)\, d(s) = 0$ … (1)

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$\alpha(s)\, d(v) + d(s)\, \beta(v) = 0$ … (2)

From (1) and (2) we get,

$d(v)\, d(s) = \alpha(s)\, d(v) + d(s)\, \beta(v)$

Since $d = 0$ on $I$ then,

$d(s)\, \beta(v) = 0$ for all $s \in S$ and $v \in I$ … (3)

By (Lemma 2.27) and since $\beta$ is nonzero, we get $d(s) = 0$ for all $s \in S$.

Then $d = 0$ on $S$.

**Theorem 3.8**: - Let $S$ be a cancellative prime semiring. Suppose that $\alpha$ and $\beta$ are two nonzero automorphisms of $S$ and $d: S \longrightarrow S$ is $(\alpha, \beta)$ derivation on $S$ such that d commute with $\alpha$ and $\beta$. If $d$ acts as anti-homomorphism on $I$ then $d = 0$ on $S$.

**Proof**

Since $d$ is $(\alpha, \beta)$ derivation of $S$ then,

$d(u\,v) = \alpha(u)\, d(v) + d(u)\, \beta(v)$ for all $u, v \in I$ ... (1)

Since $d$ acts as anti-homomorphism on $S$ then,

$d(u\,v) = d(v)\, d(u)$ for all u, v $\in I$ ... (2)

From (1) and (2) we get,

$\alpha(u)\, d(v) + d(u)\, \beta(v) = d(v)\, d(u) \quad u, v \in I$ ... (3)

Replace $v$ by $v\, t$ in (3), where $t \in I$, we get,

$\alpha(u)\, d(v\, t) + d(u)\, \beta(vt) = d(v\, t)\, d(u)$

Since d acts anti-homomorphism on $I$ then,

$\alpha(u)\, d(v)\, d(t) + d(u)\, \beta(v)\, d(t) = d(t)\, d(v)\, d(u)$

$= d(t)\, d(u\, v)$

$= d(t)\, [\alpha(u)\, d(v) + d(u)\, \beta(v)]\, d(u)$

$= d(t)\, \alpha(u)\, d(v) + d(t)\, d(u)\, \beta(v)$

Since $\alpha$ commute with $d$ and using additively cancellative property we get,

$d(t)\, \beta(v)\, d(u) = d(u)\, \beta(v)\, \beta(t)$

Since $\beta$ commute with $d$ and using multiplicatively cancellative property we get,

$d(t) = \beta(t) \quad$ for all $t \in S$ … (4)

Substitute (4) in (3) we get, $d(v)\, d(u) = \alpha(u)\, d(v) + d(v)\, d(u)$.

Now by multiplicatively cancellative propertywe get,

$\alpha(u)\, d(v) = 0$

By (Lemma 2.27) and since $\alpha$ is nonzero, then $d(v) = 0$ for all $v \in I$.

then $d = 0$ on $I$.

By (Lemma 3.7) we get $d = 0$ on $S$.

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