DOI: 10.24996/ijs.2019.60.6.23

ISSN: 0067-2904

Fixed Point Theorems of Fuzzy $\mathcal{T}^*$-Cone Metric Space and Their Integral Type Application

Sarim H. Hadi
Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Basrah, Iraq

Abstract
The objective of this work is to study the concept of a fuzzy $\mathcal{T}^*$-cone metric space And some related definitions in space. Also, we discuss some new results of fixed point theorems. Finally, we apply the theory of fixed point achieved in the research on an integral type.

Keyword: Fuzzy $\mathcal{T}^*$-cone metric space**,** Fuzzy-complete, Fuzzy-continuous, Fuzzy-sequentially convergent and fixed point with application of integral type.

مبرهنات النقطة الصامدة في الفضاء المتري الضبابي المخروطي من النوع $\mathcal{T}^*$ مع تطبيقاتها التكاملية

صارم حازم هادي
قسم الرياضيات، كلية التربية للعلوم الصرفة، جامعة البصرة، البصرة، العراق

الخلاصة
الهدف من هذا العمل هو دراسة مفهوم الفضاء المتري الضبابي المخروطي من النوع $\mathcal{T}^*$ وبعض التعاريف المتعلقة بهذا الفضاء. أيضا، سنثبت بعض النتائج الجديدة لنظريات نقطة ثابتة. وأخيرًا، نطبق نظرية النقطة الثابتة التي تم تحقيقها حول دالة تكاملية.

1. Introduction

The Cone metric space and the fixed point this space was presented by L. G. Huang and X. Zhang [1]. The Cone metric space is a generalization of metric space. Further, there are many researchers discussed the generalization of the fixed point theorem in Cone of these space [2-7]. In 2008, Bari and Vetro introduced a new concept on the fixed point in the field of φ- mapping and concluded some of the results and examples achieved.

One of their interested in the applications in the fuzzy Cone metric space was introduced by Bag [8] and studied many subjects that are related to it. After that, many authors touched on the concept of the fixed point in this space. Oner, Kandemir, and Tanay provided another definition of the concept of fuzzy cone metric space and concluded many of the results related to it (see[4-6], [10] and[11]).

In this work, we will define fuzzy $\mathcal{T}^*$-Cone metric space. We also mentioned some definitions related to this space and proved some of the properties achieved. In addition, we show fa ixed point in fuzzy $\mathcal{T}^*$-Cone metric space. The range of fuzzy Cone metric is considered as $C^*(I)$, where $C^*(I)$ is defined by the set of all fuzzy real number defined on C where C is a given real Banach space),[9].

2. MAIN RESULTS

The definitions of the fuzzy number, convex fuzzy real number, and normal fuzzy number as mentioned in ([12]), as well as for the arithmetic operations $\oplus$, $\ominus$, $\odot$ on $C \times C$ were presented by

Email: sarim.h2014@yahoo.com

([11]). There are also important definitions such as fuzzy real Banach space, interior point and fuzzy closed subset of $C^*(I)$ Which was put forward by Bag [9].
A subset ρ of $C^*(I)$ is called fuzzy Cone if
(1) ρ fuzzy closed, nonempty and $\rho \neq \{0\}$.
(2) $a, b \in R$, $a, b \succcurlyeq 0$, $v, w \in \rho \Longrightarrow av \oplus bw \in \rho$
(3) $v \in \rho$ and $\ominus v \in \rho \Longrightarrow v = 0$
Given a fuzzy Cone $\rho \subset C^*(I)$, we define a partial ordering $\preccurlyeq$ with respect to ρ by $v \preccurlyeq w$ iff $w \ominus v \in \rho$. On the other side $v \prec w$ s.t $v \neq w$ while v $\preccurlyeq$ w will stand $w \ominus v \in$ Int $\rho^{(*)}$
A fuzzy Cone ρ is said to be normal if there is a number $k \succ 0$ s.t for all $x, y \in \rho$, with $0 \preccurlyeq v \preccurlyeq w \Longrightarrow v \preccurlyeq kw$.
A fuzzy Cone ρ is said to be regular if $\{x_n\}$ is an increasing sequence which is bounded above is convergent, that is $x_1 \preccurlyeq x_2 \preccurlyeq \cdots \preccurlyeq x_n \preccurlyeq \cdots \preccurlyeq w$ for some$w \in C^*(I)$, then there is $x \in C^*(I)$ such that $\|x_n - x\| \longrightarrow 0$ as $\longrightarrow \infty$.
Equivalently, ρ is called regular if $\{x_n\}$ is decreasing sequence which is bounded below is convergent [9], [11].
Definition 2.1 The function $\mathcal{T}^*: X \times X \times X \to C^*(I)$ is called fuzzy $\mathcal{T}^*$-Cone metric space if satisfies conditions:
(I) $\mathcal{T}^*(x, y, z) \succcurlyeq 0$ for all $x, y, z \in X$.
(II) $\mathcal{T}^*(x, y, z) = 0 \Leftrightarrow x = y = z$.
(III) $\mathcal{T}^*(x, y, z) = \mathcal{T}^*(x, z, y) = \mathcal{T}^*(y, z, x) = \cdots$ for all $x, y, z \in X$.
(IV) $\mathcal{T}^*(x, y, z) \preccurlyeq \mathcal{T}^*(x, y, a) \oplus \mathcal{T}^*(a, a, z)$ for all $x, y, z, a \in X$
Definition 2.2 Let $(X, \mathcal{T}^*)$ be a fuzzy $\mathcal{T}^*$-Cone metric space, take $\{x_n\}$ be a sequence in X and $x \in X$. If for every $e \in C(I^*)$ there is a positive $k \in N$ such that $\mathcal{T}^*(x_n, x, x) \preccurlyeq \|e\|$ for all $n \succ k$, then $\{x_n\}$ is said to be fuzzy-converges to x.
Theorem 2.3 Let $(X, \mathcal{T}^*)$ be a fuzzy $\mathcal{T}^*$-Cone metric space and $\{x_n\}$ be a sequence in X, then
(1) $\{x_n\}$ is fuzzy-converges to x iff $\mathcal{T}^*(x_n, x, x) \to 0$ as $n \to \infty$.
(2) If $\{x_n\}$ is fuzzy-converges to x and y, then $x = y$.
Proof:
(1) suppose$\{x_n\}$ fuzzy- converge to x, for every $\varepsilon \succ 0$, choose $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ there exist positive integer k such that $k\|e\| \prec \varepsilon$
Then there exist $k_2 \in N$ such that $\mathcal{T}^*(x_n, x, x) \preccurlyeq \|e\|$ for all $n \succ k_2$
$\mathcal{T}^*(x_n, x, x) \preccurlyeq k_2\|e\| \prec \varepsilon$ (since ρ is normal)
$\Longrightarrow \mathcal{T}^{*,1}{}_{\alpha}(x_n, x, x) \prec \varepsilon$ and $\mathcal{T}^{*,2}{}_{\alpha}(x_n, x, x) \prec \varepsilon$, for all $\alpha \in (0,1]$
$\Longrightarrow lim_{n\to\infty}\mathcal{T}^{*,1}{}_{\alpha}(x_n, x, x) = 0$ and $lim_{n\to\infty}\mathcal{T}^{*,2}{}_{\alpha}(x_n, x, x) = 0$
$\Longrightarrow lim_{n\to\infty}\mathcal{T}^*(x_n, x, x) = 0 \Longrightarrow \mathcal{T}^*(x_n, x, x) \longrightarrow 0$.
Conversely: suppose that $\mathcal{T}^*(x_n, x, x) \longrightarrow 0$ as $n \longrightarrow \infty$.
For $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ there is $\delta \succ 0$ such that $\|x\| \prec \delta$
Then $\|e\| \ominus \|x\| \in Int\, \rho$ [by condition $^{(*)}$]
For this $\delta \succ 0$, $k \in N$ such that $\mathcal{T}^*(x_n, x, x) \prec \delta$ for all $n \succ k$
Let $\mathcal{T}^*(x_n, x, x) = \|y_n\| \Longrightarrow \|y_n\| \prec \delta$
$\|e\| \ominus \|y_n\| \in Int\, \rho$ for all $n \succ k$
Leads to $\|y_n\| \preccurlyeq \|e\|$ for all $n \succ k$
Then, we have $\mathcal{T}^*(x_n, x, x) \preccurlyeq \|e\|$ for all $n \succ k \Longrightarrow x_n \to x\ as\ n \to \infty$.
(2) suppose that $x_n \to x$ and $x_n \to y$
Thus for any $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ there exist $k \in N$ such that $\mathcal{T}^*(x_n, x, x) \preccurlyeq \|e\|$ and $\mathcal{T}^*(x_n, y, y) \preccurlyeq \|e\|$ for all $n \succ k$
Now:
$$\mathcal{T}^*(x, x, y) \preccurlyeq \mathcal{T}^*(x_n, x, x) \oplus \mathcal{T}^*(x_n, y, y) \preccurlyeq 2\|e\|$$
Hence $\mathcal{T}^*(x, x, y) \preccurlyeq 2k\|e\|$
Since e is arbitrary , we have $\mathcal{T}^*(x, x, y) = 0 \Longrightarrow x = y$.
Definition 2.4 Let $(X, \mathcal{T}^*)$ be a fuzzy $\mathcal{T}^*$-Cone metric space and let $\{x_n\}$ be a sequence. If for every $e \in C(I^*)$ there is a positive $k \in N$ such that $\mathcal{T}^*(x_n, x_m, x) \preccurlyeq \|e\|$ for all $n, m \succ k$, then $\{x_n\}$ is called fuzzy-Cauchy seq. in X .

Definition 2.5 Let $(X,\mathcal{T}^*)$ be a fuzzy $\mathcal{T}^*$-Cone metric space. If every fuzzy-Cauchy sequence is fuzzy-convergent in X, it is said to be fuzzy-complete fuzzy $\mathcal{T}^*$-Cone metric space .

Theorem 2.6 Let $(X,\mathcal{T}^*)$ be a fuzzy $\mathcal{T}^*$-cone metric space and $\{x_n\}$ be a sequence in X. If $\{x_n\}$ is f-convergent then $\{x_n\}$ is f-Cauchy sequence.

Proof: Let $\{x_n\}$ fuzzy-converge to x, for any $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ there exist $k \in N$ such that $\mathcal{T}^*(x_n,x,x) \preccurlyeq \left\|\frac{e}{2}\right\|$ and $\mathcal{T}^*(x_m,x,x) \preccurlyeq \left\|\frac{e}{2}\right\|$ for all $n,m \succ k$

Now : $\mathcal{T}^*(x_n,x_m,x) \preccurlyeq \mathcal{T}^*(x_n,x,x) \oplus \mathcal{T}^*(x_m,x,x) \preccurlyeq \|e\|$ for all $n,m \succ k$

Thus $\{x_n\}$ is fuzzy- Cauchy sequence.

Theorem 2.7 Let $(X,\mathcal{T}^*)$ be a fuzzy cone metric space and ρ be a normal fuzzy cone with constant k, $\{x_n\}$ be a sequence in X, then $\{x_n\}$ is fuzzy-Cauchy sequence if only if $\mathcal{T}^*(x_n,x_m,x) \to 0$ as $m,n \to \infty$.

Proof: Let $\{x_n\}$ be a fuzzy-Cauchy sequence in X. For every $n,m \succ 0$, choose $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ and $k\|e\| \prec \varepsilon$. Then there is $k \in N$ such that $\mathcal{T}^*(x_n,x_m,x) \preccurlyeq \|e\|$ for all $n,m \succ k$

So that when, $\succ k$, $\mathcal{T}^*(x_n,x_m,x) \preccurlyeq k\|e\| \prec \varepsilon$ (since ρ is normal)

Since $\varepsilon > 0$ is arbitrary, it follows that $\mathcal{T}^*(x_n,x_m,x) \to 0$ as $m,n \to \infty$.

Conversely: suppose that $\mathcal{T}^*(x_n,x_m,x) \to 0$ as $m,n \to \infty$

For $e \in C(I^*)$ with $0 \preccurlyeq \|e\|$ there is $\delta \succ 0$ such that $\|x\| \prec \delta$

$\|e\| \ominus (\|e\| \ominus \|x\|) \prec \delta \implies \|e\| \ominus \|x\| \in Int\,\rho$. For this $\delta \succ 0$ there is $k \in N$ such that $\mathcal{T}^*(x_n,x_m,x) \prec \delta$ for all $n,m \succ k$

suppose $\mathcal{T}^*(x_n,x_m,x) = \|z_{n,m}\| \implies \|z_{n,m}\| \prec \delta$

$\implies \|e\| \ominus \|z_{n,m}\| \in Int\,\rho$ for all $n,m \succ k$

$\implies \|z_{n,m}\| \preccurlyeq \|e\|$ for all $n,m \succ k$

$\implies \mathcal{T}^*(x_n,x_m,x) \preccurlyeq \|e\|$ for all $n,m \succ k$

$\implies \{x_n\}$ is fuzzy-Cauchy

Definition 2.8. A sub additive function is a function $\wp: \rho \to \rho$ with the following property, for all $v,u \in \rho$, $\wp(v+u) \preccurlyeq \wp(v) + \wp(u)$.

Definition 2.9 Let $(X,\mathcal{T}^*)$ be a fuzzy Cone metric space, $\mathrm{F}: X \to X$. Then

(i) F is said to be fuzzy-continuous if $lim_{n\to\infty} x_n = x \implies lim_{n\to\infty} \mathrm{F}x_n = \mathrm{F}x$.

(ii) Fρ is said to be fuzzy-sub sequentially convergent, if we have a for every sequence $\{x_n\}$, and F$\{x_n\}$ is fuzzy- convergent $\implies \{x_n\}$ has a fuzzy- convergent subsequence.

(iii) F is said to be fuzzy-sequentially convergent, if for every sequence $\{x_n\}$, and F$\{x_n\}$ is fuzzy-convergent $\implies \{x_n\}$ has also fuzzy-convergent.

Definition 2.10 let ρ be a fuzzy Cone, we denote $\mathcal{G} = \{\wp / \wp: \rho \to \rho$ is non decreasing, sub additive, fuzzy-continuous, fuzzy-sequentially convergent with $\wp(\tau) = 0 \Leftrightarrow \tau = 0\}$.

Theorem 2.11 Let $(X,\mathcal{T}^*)$ be a complete fuzzy $\mathcal{T}^*$-cone metric space, $\hbar: X \to X$ continuous function satisfying condition

$$\mathcal{G}\mathcal{T}^*(\hbar x,\hbar y,\hbar z) \preccurlyeq u\mathcal{G}[\mathcal{T}^*(x,y,z)] \oplus v\mathcal{G}[\mathcal{T}^*(\hbar x,x,x)] \oplus w\mathcal{G}[\mathcal{T}^*(\hbar y,y,y)] \oplus e\mathcal{G}[\mathcal{T}^*(\hbar z,z,z)]$$

Where $\mathcal{G}$ as defined in (2.10) and u,v,w,e are constant satisfying $0 \prec u+v+w+e \prec 1$, then $\hbar$ has a unique fixed point.

Proof: Let $x_0 \in X$ and let $\{x_n\}$ be a sequences in X

If $x_n = x_{n+1}$ for some n, then $\hbar$ has fixed point in X

Suppose $x_n \neq x_{n+1}$ such that $\hbar x_n = x_{n+1}$

$$\mathcal{G}[\mathcal{T}^*(\hbar x_{n-1},\hbar x_n,\hbar x_n)] = \mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})]$$

$$\preccurlyeq u\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)] \oplus v\mathcal{G}[\mathcal{T}^*(x_{n-1},\hbar x_{n-1},\hbar x_{n-1})] \oplus w\mathcal{G}[\mathcal{T}^*(x_n,\hbar x_n,\hbar x_n)] \oplus e\mathcal{G}[\mathcal{T}^*(x_n,\hbar x_n,\hbar x_n)]$$

$$= u\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)] \oplus v\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)] \oplus w\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})] \oplus e\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})]$$

$$\implies \mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})] \preccurlyeq (u+v)[\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)]] \oplus (w+e)[\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})]$$

$$\big(1-(w+e)\big)[\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})] \preccurlyeq (u+v)[\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)]]$$

Hence $$\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})] \preccurlyeq \frac{(u+v)}{(1-(w+e))}[\mathcal{G}[\mathcal{T}^*(x_{n-1},x_n,x_n)]]$$

$$\implies \mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})] \preccurlyeq \mathcal{L}^n[\mathcal{G}[\mathcal{T}^*(x_0,x_1,x_1)]]$$

Where $\mathcal{L} = \frac{(u+v)}{(1-(w+e))}$. Now since $\mathcal{L} \prec 1$ and ρ is fuzzy closed, we have for $\varpi \in \rho$ there exist $\zeta \succ 0$ such that $\varpi \ominus a \in \rho$ when $\|a\| \prec \zeta$. By properties fuzzy cone and by normality of fuzzy cone we have $lim_{n\to\infty}\,\mathcal{G}[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})]=0$ when $\mathcal{G}$ is a fuzzy-sequentially convergent hence $\mathcal{T}^*(x_n,x_{n+1},x_{n+1})$ is also convergent and $\mathcal{G}$ is fuzzy-continuous therefore

$$lim_{n\to\infty}\mathcal{T}^*(x_n,x_{n+1},x_{n+1})=0.$$

Now, we have to prove $\{x_n\}$ is a fuzzy-Cauchy sequence

Now since $lim_{n\to\infty}\mathcal{T}^*(x_n,x_{n+1},x_{n+1})=0$ and ρ is fuzzy closed $\mathcal{T}^*(x_n,x_{n+1},x_{n+1})\preccurlyeq\frac{\varpi}{2}$ for every $\varpi\in\rho$ hence

$$\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\preccurlyeq\mathcal{T}^*(x_n,x_{n+1},x_{n+1})+\mathcal{T}^*(x_{n+1},x_{n+2},x_{n+2})+\cdots+\mathcal{T}^*(x_{n+m-1},x_{n+m},x_{n+m})$$
$$\preccurlyeq(\mathcal{L}^n+\mathcal{L}^{n+1}+\cdots+\mathcal{L}^{n+m-1})\mathcal{T}^*(x_0,x_1,x_1)$$
$$\preccurlyeq\frac{\mathcal{L}^n}{1-\mathcal{L}}\mathcal{T}^*(x_0,x_1,x_1)\longrightarrow 0\ as\ n\longrightarrow\infty.$$

Hence $\{x_n\}$ is a fuzzy-Cauchy sequence, since X is fuzzy-complete $\Longrightarrow lim_{n\to\infty}x_n=a.$

We have $a\in X$.

$$\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\preccurlyeq\mathcal{G}[\mathcal{T}^*(\hbar a,\hbar a,\hbar x_n)]\oplus\mathcal{G}[\mathcal{T}^*(\hbar x_n,a,a)]$$
$$\preccurlyeq u\mathcal{G}[\mathcal{T}^*(a,a,x_n)]\oplus v\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus w\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(\hbar x_n,x_n,x_n)]\oplus\mathcal{G}[\mathcal{T}^*(\hbar x_n,a,a)]$$
$$=u\mathcal{G}[\mathcal{T}^*(a,a,x_n)]\oplus v\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus w\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(x_{n+1},x_n,x_n)]\oplus\mathcal{G}[\mathcal{T}^*(x_{n+1},a,a)]$$
$$\preccurlyeq u\mathcal{G}[\mathcal{T}^*(a,a,x_n)]\oplus(v+w)\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus(e+1)\mathcal{G}[\mathcal{T}^*(x_{n+1},a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(x_n,x_n,a)]$$
$$1-(v+w)\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\preccurlyeq u\mathcal{G}[\mathcal{T}^*(a,a,x_n)]\oplus(e+1)\mathcal{G}[\mathcal{T}^*(x_{n+1},a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(x_n,x_n,a)]$$
$$\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\preccurlyeq\frac{1}{1-(v+w)}\{u\mathcal{G}[\mathcal{T}^*(a,a,x_n)]\oplus(e+1)\mathcal{G}[\mathcal{T}^*(x_{n+1},a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(x_n,x_n,a)]\}$$

Sine $\frac{1}{1-(v+w)}\prec 1$ and as $n\to\infty$, $\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]=0$ and so $\mathcal{T}^*(\hbar a,a,a)=0\Longrightarrow\hbar a=a$

Now, let q be another fixed point of $\hbar$ that is $\hbar q=q$.

$$\mathcal{G}[\mathcal{T}^*(\hbar a,\hbar a,\hbar q)]\preccurlyeq u\mathcal{G}[\mathcal{T}^*(a,a,q)]\oplus v\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus w\mathcal{G}[\mathcal{T}^*(\hbar a,a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(\hbar q,q,q)]$$
$$=u\mathcal{G}[\mathcal{T}^*(a,a,q)]\oplus v\mathcal{G}[\mathcal{T}^*(a,a,a)]\oplus w\mathcal{G}[\mathcal{T}^*(a,a,a)]\oplus e\mathcal{G}[\mathcal{T}^*(q,q,q)]$$
$$\Longrightarrow\ \mathcal{G}[\mathcal{T}^*(\hbar a,\hbar a,\hbar q)]\preccurlyeq u\mathcal{G}[\mathcal{T}^*(a,a,q)]$$

We have $(u-1)\mathcal{G}[\mathcal{T}^*(\hbar a,\hbar a,\hbar q)]\in\rho$, by properties of fuzzy cone ρ.

Hence $(u-1)\mathcal{G}[\mathcal{T}^*(\hbar a,\hbar a,\hbar q)]=0$

$\Longrightarrow\mathcal{T}^*(\hbar a,\hbar a,\hbar q)=0\Longrightarrow\hbar a=\hbar q\Longrightarrow a=q.$

Corollary 2.12 Let $(X,\mathcal{T}^*)$ be a complete fuzzy $\mathcal{T}^*$-cone metric space, $\hbar:X\to X$, satisfy

$\mathcal{T}^*(\hbar x,\hbar y,\hbar z)\preccurlyeq u[\mathcal{T}^*(x,y,z)]\oplus v[\mathcal{T}^*(\hbar x,x,x)]\oplus w[\mathcal{T}^*(\hbar y,y,y)]\oplus e[\mathcal{T}^*(\hbar z,z,z)]$

Where u,v,w,e are constant satisfying $0\prec u+v+w+e\prec 1$, then $\hbar$ has a unique fixed point.

Theorem 2.13. Let $(X,\mathcal{T}^*)$ be a complete fuzzy $\mathcal{T}^*$-cone metric space, $\hbar:X\to X$, continuous function satisfy condition

$\mathcal{T}^*(\hbar x,\hbar y,\hbar z)$

$\preccurlyeq u\,max\{\mathcal{T}^*(x,y,z),\mathcal{T}^*(x,\hbar x,\hbar y),\mathcal{T}^*(y,\hbar y,\hbar x),\mathcal{T}^*(z,\hbar z,\hbar x),\mathcal{T}^*(\hbar x,x,x),\mathcal{T}^*(\hbar y,y,y),\mathcal{T}^*(\hbar z,z,z)\}$

Where $u$are constant satisfying $0\prec u\prec 1$, then $\hbar$ has a unique fixed point.

Proof: Let $x_0\in X$ and let $\{x_n\}$ be a sequences in X

If $x_n=x_{n+1}$ for some n, then $\hbar$ has fixed point in X

Suppose $x_n\neq x_{n+1}$ such that $\hbar x_n=x_{n+1}$

$$\mathcal{T}^*(\hbar x_{n-1},\hbar x_n,\hbar x_{n+1})=\mathcal{T}^*(x_n,x_{n+1},x_{n+2})$$
$$\preccurlyeq u\max\{\mathcal{T}^*(x_{n-1},x_n,x_{n+1}),\mathcal{T}^*(x_{n-1},\hbar x_{n-1},\hbar x_n),\mathcal{T}^*(x_n,\hbar x_n,\hbar x_{n-1})$$
$$,\mathcal{T}^*(x_{n+1},\hbar x_{n+1},\hbar x_{n-1}),\mathcal{T}^*(\hbar x_{n-1},x_{n-1},x_{n-1}),\mathcal{T}^*(\hbar x_n,x_n,x_n),\mathcal{T}^*(\hbar x_{n+1},x_{n+1},x_{n+1})\}$$
$$=u\max\{\mathcal{T}^*(x_{n-1},x_n,x_{n+1}),\mathcal{T}^*(x_{n-1},x_n,x_{n+1}),\mathcal{T}^*(x_n,x_{n+1},x_n)$$
$$,\mathcal{T}^*(x_{n+1},x_{n+2},x_n),\mathcal{T}^*(x_n,x_{n-1},x_{n-1}),\mathcal{T}^*(x_{n+1},x_n,x_n),\mathcal{T}^*(x_{n+2},x_{n+1},x_{n+1})\}$$

Denote $\mathcal{T}^*{}_n=\mathcal{T}^*(x_{n+1},x_{n+2},x_n)$

$$=u\max\{\mathcal{T}^*{}_{n-1},\mathcal{T}^*{}_{n-1},\mathcal{T}^*{}_n,\mathcal{T}^*{}_n,\mathcal{T}^*{}_{n-1},\mathcal{T}^*{}_{n-1},\mathcal{T}^*{}_n\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If $\mathcal{T}^*{}_n\succ\mathcal{T}^*{}_{n-1}$, we have $\mathcal{T}^*{}_n\preccurlyeq u\mathcal{T}^*{}_n\prec\mathcal{T}^*{}_n$. [it is a contradiction]

Hence $\mathcal{T}^*{}_n\preccurlyeq\mathcal{T}^*{}_{n-1}$, from (1) $\mathcal{T}^*{}_n\preccurlyeq u\,\mathcal{T}^*{}_{n-1}$ ……………………………….(2)

From (1) and (2), we get

$$\mathcal{T}^*(x_n,x_{n+1},x_{n+2})\preccurlyeq u\mathcal{T}^*(x_{n-1},x_n,x_{n+1})$$
$$\preccurlyeq u^2\mathcal{T}^*(x_{n-2},x_{n-1},x_n)\preccurlyeq\cdots\preccurlyeq u^n\mathcal{T}^*(x_0,x_1,x_2)$$

Now, we have to prove $\{x_n\}$ is a fuzzy-Cauchy sequence

$$\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\preccurlyeq\mathcal{T}^*(x_n,x_n,x_{n+1})+\mathcal{T}^*(x_{n+1},x_{n+1},x_{n+2})+\cdots+\mathcal{T}^*(x_{n+m-1},x_{n+m-1},x_{n+m})$$
$$\preccurlyeq u^n\mathcal{T}^*(x_0,x_1,x_1)+u^{n+1}\mathcal{T}^*(x_0,x_1,x_1)+\cdots+u^{n+m-1}\mathcal{T}^*(x_0,x_1,x_1)$$

$$\preccurlyeq (u^n + u^{n+1} + \cdots + u^{n+m-1})\mathcal{T}^*(x_0, x_1, x_1)$$
$$\preccurlyeq \frac{u^n}{1-u}\mathcal{T}^*(x_0, x_1, x_1) \longrightarrow 0 \; as \; n \longrightarrow \infty.$$

Hence $\{x_n\}$ is a fuzzy-Cauchy sequence, since X is f-complete $\Longrightarrow lim_{n\to\infty} x_n = a$,

$$\mathcal{T}^*(\hbar a, a, a) \preccurlyeq \mathcal{T}^*(\hbar a, \hbar a, \hbar x_n) \oplus \mathcal{T}^*(\hbar x_n, a, a)$$
$$\preccurlyeq [u \, max\{\mathcal{T}^*(a, a, x_n), \mathcal{T}^*(a, \hbar a, \hbar a), \mathcal{T}^*(a, \hbar a, \hbar a)$$
$$, \mathcal{T}^*(x_n, \hbar x_n, \hbar a), \mathcal{T}^*(\hbar a, a, a), \mathcal{T}^*(\hbar a, a, a), \mathcal{T}^*(\hbar x_n, x_n, x_n)\}] \oplus \mathcal{T}^*(\hbar x_n, a, a)$$
$$= u \max\{\mathcal{T}^*(\hbar a, a, a), \mathcal{T}^*(\hbar x_n, a, a)\}$$
$$\mathcal{T}^*(\hbar a, a, a) \preccurlyeq u\mathcal{T}^*(\hbar a, a, a) \oplus u\mathcal{T}^*(\hbar x_n, a, a)$$
$$\mathcal{T}^*(\hbar a, a, a) \preccurlyeq \frac{u}{1-u}\mathcal{T}^*(\hbar x_n, a, a)$$

as $n \to \infty$, $\Longrightarrow \mathcal{T}^*(\hbar a, a, a) = 0 \Longrightarrow \hbar a = a$.

Now, let q be another fixed point of $\hbar$ that is $\hbar q = q$.

$$\mathcal{T}^*(\hbar a, \hbar a, \hbar q) \preccurlyeq u \max\{\mathcal{T}^*(a, a, q), \mathcal{T}^*(a, \hbar a, \hbar a), \mathcal{T}^*(a, \hbar a, \hbar a), \mathcal{T}^*(q, \hbar q, \hbar a)$$
$$, \mathcal{T}^*(\hbar a, a, a), \mathcal{T}^*(\hbar a, a, a), \mathcal{T}^*(\hbar q, q, q)\}$$
$$\preccurlyeq u \, \mathcal{T}^*(\hbar q, \hbar q, \hbar a) \prec \mathcal{T}^*(\hbar q, \hbar q, \hbar a)$$

[This is contradiction] $\Longrightarrow a = q$

Therefore $\hbar$ has a unique fixed point.

3. APPLICATION OF INTEGRAL TYPE

The idea of this part of the research will apply the theory of fixed point achieved in the research on an integral type as follows:

$$\int_0^{\mathcal{T}^*(\hbar x, \hbar y, \hbar z)} \Upsilon(\tau)d\tau \preccurlyeq \mathbb{a}(\mathcal{T}^*(x,y,z)) \int_0^{\mathcal{T}^*(x,y,z)} \Upsilon(\tau)d\tau \oplus \mathbb{b}(\mathcal{T}^*(x,y,z)) \int_0^{\mathcal{T}^*(\hbar x,x,x)} \Upsilon(\tau)d\tau \oplus$$
$$\mathbb{c}(\mathcal{T}^*(x,y,z)) \int_0^{\mathcal{T}^*(\hbar y,y,y)} \Upsilon(\tau)d\tau \oplus \mathbb{d}(\mathcal{T}^*(x,y,z)) \int_0^{\mathcal{T}^*(\hbar z,z,z)} \Upsilon(\tau)d\tau \quad \ldots\ldots\ldots\ldots(3^*)$$

Where $\mathbb{a}, \mathbb{b}, \mathbb{c}, \mathbb{d}: [0,\infty) \to [0,1)$ are four functions with

$$\mathbb{a}(\tau) \oplus \mathbb{b}(\tau) \oplus \mathbb{c}(\tau) \oplus \mathbb{d}(\tau) \prec 1 \;\; \forall \tau \in [0,1) \qquad (3.1)$$
$$lim_{\eta\to\tau} \sup\{\mathbb{a}(\eta) \oplus \mathbb{b}(\eta) \oplus \mathbb{c}(\eta) \oplus \mathbb{d}(\eta)\} \prec 1 \;\; \forall \tau \in [0,1) \qquad (3.2)$$
$$min\{ lim_{\eta\to 0} \sup \mathbb{a}(\eta), \; lim_{\eta\to 0} \sup \mathbb{b}(\eta), \; lim_{\eta\to 0} \sup \mathbb{c}(\eta), \; lim_{\eta\to 0} \sup \mathbb{d}(\eta)\} \prec 1 \qquad (3.3)$$

The integral function is defined as follows:

$\Upsilon: (0,\infty) \to (0,\infty)$ is a Lebesgue-integrable function which is nonnegative and such that $\int_0^{\varepsilon} \Upsilon(\tau)d\tau \succ 0$ for each $\varepsilon \succ 0$. (3.4)

Theorem 3.1 Let $(X, \mathcal{T}^*)$ be a complete fuzzy $\mathcal{T}^*$-cone metric space, $\hbar: X \to X$ satisfying the conditions (3*), (3.1), (3.2) , (3.3) and (3.4), then $\hbar$ has a unique fixed point.

Proof: Let $x_0 \in X$ and let $\{x_n\}$ be a sequences in X

If $x_n = x_{n+1}$ for some n, then $\hbar$ has fixed point in X

Suppose $x_n \neq x_{n+1}$ such that $\hbar x_n = x_{n+1}$

$$\int_0^{\mathcal{T}^*(\hbar x_{n-1}, \hbar x_n, \hbar x_n)} \Upsilon(\tau)d\tau \preccurlyeq$$
$$\mathbb{a}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau \oplus \mathbb{b}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(\hbar x_{n-1},x_{n-1},x_{n-1})} \Upsilon(\tau)d\tau \oplus$$
$$\mathbb{c}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(\hbar x_n,x_n,x_n)} \Upsilon(\tau)d\tau \oplus \mathbb{d}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(\hbar x_n,x_n,x_n)} \Upsilon(\tau)d\tau$$
$$= \mathbb{a}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau \oplus \mathbb{b}(\mathcal{T}^*(x_{n-1}, x_n, x_n)) \int_0^{\mathcal{T}^*(x_n,x_{n-1},x_{n-1})} \Upsilon(\tau)d\tau \oplus$$
$$\mathbb{c}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right) \int_0^{\mathcal{T}^*(x_{n+1},x_n,x_n)} \Upsilon(\tau)d\tau \oplus \mathbb{d}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right) \int_0^{\mathcal{T}^*(x_{n+1},x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau \preccurlyeq \left[\mathbb{a}(\mathcal{T}^*(x_{n-1}, x_n, x_n) \oplus \mathbb{b}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right)\right] \int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\oplus [\mathbb{c}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right) \oplus \mathbb{d}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right)] \int_0^{\mathcal{T}^*(x_{n+1},x_n,x_n)} \Upsilon(\tau)d\tau$$
$$1 - \left[\mathbb{c}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right) \oplus \mathbb{d}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right)\right] \int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau$$
$$\preccurlyeq \left[\mathbb{a}(\mathcal{T}^*(x_{n-1}, x_n, x_n) \oplus \mathbb{b}\left(\mathcal{T}^*(x_{n-1}, x_n, x_n)\right)\right] \int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau$$

By (3.1), we will get

$\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau \preccurlyeq$
$\frac{[\mathbb{a}(\mathcal{T}^*(x_{n-1},x_n,x_n))\oplus\mathbb{b}(\mathcal{T}^*(x_{n-1},x_n,x_n))]}{1-[\mathbb{c}(\mathcal{T}^*(x_{n-1},x_n,x_n))\oplus\mathbb{d}(\mathcal{T}^*(x_{n-1},x_n,x_n))]}\int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau \prec \int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau \ (*)$

What we got from $(*)$ shows us that the sequence $[\mathcal{T}^*(x_n,x_{n+1},x_{n+1})]$ is decreasing, so there is $\delta \succcurlyeq 0$ with $lim_{n\to\infty}\mathcal{T}^*(x_n,x_{n+1},x_{n+1}) = 0$.
We find that $\delta = 0$. (otherwise $\delta \succ 0$). By $(*)$ we conclude that

$$\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau \preccurlyeq \left[\mathbb{a}(\mathcal{T}^*(x_{n-1},x_n,x_n)\oplus\mathbb{b}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\right]\int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\oplus\left[\mathbb{c}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\oplus\mathbb{d}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\right]\int_0^{\mathcal{T}^*(x_{n+1},x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\preccurlyeq [\mathbb{a}(\mathcal{T}^*(x_{n-1},x_n,x_n)\oplus\mathbb{b}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\oplus\mathbb{c}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)$$
$$\oplus\mathbb{d}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)]\int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau$$

Taking $lim_{\eta\to\tau}$sup using (3.2) we have reached an

$$0 \prec \int_0^{\delta} \Upsilon(\tau)d\tau = lim_{n\to\infty}\sup\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau$$
$$\preccurlyeq lim_{n\to\infty}\sup\left[\mathbb{a}(\mathcal{T}^*(x_{n-1},x_n,x_n)\oplus\mathbb{b}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\oplus\mathbb{c}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\oplus\mathbb{d}\left(\mathcal{T}^*(x_{n-1},x_n,x_n)\right)\right]$$
$$lim_{n\to\infty}\sup\int_0^{\mathcal{T}^*(x_{n-1},x_n,x_n)} \Upsilon(\tau)d\tau \prec \int_0^{\delta} \Upsilon(\tau)d\tau$$

Which is contradiction .
$\Longrightarrow lim_{n\to\infty}\mathcal{T}^*(x_n,x_{n+1},x_{n+1}) = 0$
Now we must prove that $\{x_n\}$ is fuzzy-Cauchy seq.

$$0 \prec \int_0^{\delta} \Upsilon(\tau)d\tau = lim_{n\to\infty}sup\int_0^{\mathcal{T}^*(x_n,x_{n+m},x_{n+m})} \Upsilon(\tau)d\tau \preccurlyeq$$
$$lim_{n\to\infty}\sup\left[\mathbb{a}(\mathcal{T}^*(x_n,x_{n+m},x_{n+m}))\right)\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{b}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)\int_0^{\mathcal{T}^*(x_{n+1},x_{n+2},x_{n+2})} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{c}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)\int_0^{\mathcal{T}^*(x_{n+m-2},x_{n+m-1},x_{n+m-1})} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{d}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)\int_0^{\mathcal{T}^*(x_{n+m-1},x_{n+m},x_{n+m})} \Upsilon(\tau)d\tau]$$
$$\preccurlyeq lim_{n\to\infty}\sup\left[\mathbb{a}(\mathcal{T}^*(x_n,x_{n+m},x_{n+m}))\right)\oplus\mathbb{b}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)\oplus\mathbb{c}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)$$
$$\oplus\mathbb{d}\left(\mathcal{T}^*(x_n,x_{n+m},x_{n+m})\right)]$$
$$lim_{n\to\infty}\sup\{\int_0^{\mathcal{T}^*(x_n,x_{n+1},x_{n+1})} \Upsilon(\tau)d\tau, \int_0^{\mathcal{T}^*(x_{n+1},x_{n+2},x_{n+2})} \Upsilon(\tau)d\tau,$$
$$\int_0^{\mathcal{T}^*(x_{n+m-2},x_{n+m-1},x_{n+m-1})} \Upsilon(\tau)d\tau, \int_0^{\mathcal{T}^*(x_{n+m-1},x_{n+m},x_{n+m})} \Upsilon(\tau)d\tau\} \prec \int_0^{\delta} \Upsilon(\tau)d\tau$$

Hence $\{x_n\}$ is f-Cauchy seq. since $(X,\mathcal{T}^*)$ is complete fuzzy $\mathcal{T}^*$-cone metric space
$\Longrightarrow lim_{n\to\infty}x_n = a$

$$0 \prec \int_0^{\mathcal{T}^*(\hbar a,a,a)} \Upsilon(\tau)d\tau = \int_0^{\mathcal{T}^*(\hbar a,x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\preccurlyeq lim_{n\to\infty}sup[\mathbb{a}(\mathcal{T}^*(a,x_n,x_n))\int_0^{\mathcal{T}^*(a,x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{b}(\mathcal{T}^*(a,x_n,x_n))\int_0^{\mathcal{T}^*(\hbar a,a,a)} \Upsilon(\tau)d\tau\oplus\mathbb{c}\left(\mathcal{T}^*(a,x_n,x_n)\right)\int_0^{\mathcal{T}^*(\hbar x_n,x_n,x_n)} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{d}\left(\mathcal{T}^*(a,x_n,x_n)\right)\int_0^{\mathcal{T}^*(\hbar x_n,x_n,x_n)} \Upsilon(\tau)d\tau] \qquad (3.5)$$

by taking ($lim_{n\to\infty}sup$) for (3.5) $\Longrightarrow \hbar a = a$ and to prove that the point a is unique. let q be another fixed point of $\hbar$ that is $\hbar q = q$.

$$0 \prec \int_0^{\mathcal{T}^*(a,a,q)} \Upsilon(\tau)d\tau = \int_0^{\mathcal{T}^*(\hbar a,\hbar a,\hbar q)} \Upsilon(\tau)d\tau$$
$$\preccurlyeq \mathbb{a}(\mathcal{T}^*(a,a,q))\int_0^{\mathcal{T}^*(a,a,q)} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{b}(\mathcal{T}^*(a,a,q))\int_0^{\mathcal{T}^*(\hbar a,a,a)} \Upsilon(\tau)d\tau\oplus\mathbb{c}\left(\mathcal{T}^*(a,a,q)\right)\int_0^{\mathcal{T}^*(\hbar a,a,a)} \Upsilon(\tau)d\tau$$
$$\oplus\mathbb{d}\left(\mathcal{T}^*(a,a,q)\right)\int_0^{\mathcal{T}^*(\hbar q,q,q)} \Upsilon(\tau)d\tau \prec \int_0^{\mathcal{T}^*(a,a,q)} \Upsilon(\tau)d\tau$$

Which is contradiction $\Longrightarrow a = q$.

4. Conclusion

In this work, we have proposed a new study of the fuzzy cone metric space, which is the generalization of this space, it is the fuzzy $\mathcal{T}^*$-cone metric space.

Where many definitions and theorems were made regarding this space. The completion of the fuzzy metric space and the fuzzy cone metric space, and we have proved many of the fixed points of view in space. Finally, we have applied these theorems to some kind of integration.

5. Acknowledgment

A Special thanks to the editors and employees of the magazine for their continuous efforts and work in the magazine. The author is also thankful to the Editor-in-Chief of the Journal for their valuable comments which helped us to revise the paper.

6. References

1. Huang L. G. and Zhang, X**. 2007.** "Cone metric spaces and fixed point theorems of contractive mappings", *Journal of Mathematical Analysis and Applications*, **332**(2): 1468–1476.
2. Abbas M. and Jungck, G. **2008.** "Common fixed point results for noncommuting mappings without continuity in cone metric spaces", *Journal of Mathematical Analysis and Applications*, **341**(1): 416–420.
3. Bari C. Di and Vetro, P. **2008.** "ϕ-pairs and common fixed points in cone metric spaces", Rendiconti del Circolo Matematico di Palermo, **57**(2): 279– 285.
4. Choudhury, B. S. and Metiya, N. **2010.** "The point of coincidence and common fixed point for a pair of mappings in cone metric spaces", *Computers and Mathematics with Applications*, **60**: 1686-1695.
5. Rezapour S. and Hamlbarani, R. **2008.**"Some notes on the paper: "Cone metric spaces and fixed point theorems of contractive mappings", *Journal of Mathematical Analysis and Applications*, **345**(2): 719–724.
6. Sabetghadam F., Masiha, H. P. and Sanatpour, A. **2009.** "Some coupled fixed point theorems in cone metric spaces", *Fixed Point Theory and Application*, vol. **2009**, Article ID 125426, 8 pages
7. Vetro P. **2007.** "Common fixed points in cone metric spaces", *Rendiconti del Circolo Matematico di Palermo*, **56**(3): 464–468.
8. Bag T. **2013.** "Fuzzy Cone Metric Spaces and Fixed Point Theorems of Contractive Mappings", *Annl. Fuzzy Math. Inform*. **6**(3): 657-668.
9. Bag T. **2015.** "Fuzzy Cone Metric Spaces and Fixed point Theorems on Fuzzy T-Kannan & Fuzzy T-Chatterjea Type Contractive Mappings*", Fuzzy Inform. Eng.,* **7**: 305-315.
10. Bag T. **2012**. "Some results on D*-fuzzy metric spaces*", International Journal of Mathematics and Scientific Computing*, **2**(1): 29-33.
11. Oner T., M., Kandemir, B. and Tanay, B. **2015.** "Fuzzy cone metric spaces", *J. Nonlinear Sci. Appl*., **8**(2015): 610-616.
12. Mizumoto M. and Tanaka, J. **1979.**"Some properties of fuzzy numbers in M. M. Gupta et al. Editors, *Advances in Fuzzy Set Theory and Applications*", North-Holland, New-York, 153-164.