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Modeling and Forecasting Periodic Time Series data with Fourier Autoregressive Model

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Abstract
Most frequently used models for modeling and forecasting periodic climatic time series do not have the capability of handling periodic variability that characterizes it. In this paper, the Fourier Autoregressive model with abilities to analyze periodic variability is implemented. From the results, FAR(1), FAR(2) and FAR(2) models were chosen based on Periodic Autocorrelation function (PeACF) and Periodic Partial Autocorrelation function (PePACF). The coefficients of the tentative model were estimated using a Discrete Fourier transform estimation method. FAR(1) models were chosen as the optimal model based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC). The residual of the fitted models was diagnosed to be white noise. The in-sample forecast showed a close reflection of the original rainfall series while the out-sample forecast exhibited a continuous periodic forecast from January 2019 to December 2020 with relatively small values of Periodic Root Mean Square Error (PRMSE), Periodic Mean Absolute Error (PMAE) and Periodic Mean Absolute Percentage Error (PMAPE). The comparison of FAR(1) model forecast with AR(3), ARMA(2,1), ARIMA(2,1,1) and SARIMA$(1,1,1)(1,1,1)_{12}$ model forecast indicated that FAR(1) outperformed the other models as it exhibited a continuous periodic forecast. The continuous monthly periodic rainfall forecast indicated that there will be rapid climate change in Nigeria in the coming yearly and Nigerian Government needs to put in place plans to curtail its effects.

Keywords: Fourier Autoregressive model, Climate change, Periodic time series, Forecasting, Rainfall Series

Introduction

Understanding the variability in climatic time series data of a region over a long period gives one an idea about the climate change of such region [1]. The change in the measures of climatic variables has been attributed to natural and man-made reasons [2].

This change has been termed climate change and [3] defined climate change as a change in the state of the climate that can be identified by changes in the mean and/or the variability of its properties, and that persists for an extended period, typically decades or longer.

Based on the growing consensus among several scientific kinds of literature that in the coming decades, there will be a rapid increase in climatic time series variability level worldwide. This will be unfavourable for crop growth and yields in many regions and countries [4-6]. Patterns of precipitation and storm events are also likely to change and intensity of rainfall events is likely to increase on the average as well [7, 8]. Nigeria like other countries in sub-Saharan Africa is highly vulnerable to the impacts of climate change [9]

The historical climatic record of Nigeria has shown considerable signals of a changing climate [10]. In hence, climate change in Nigeria can be a challenge to sustainable human development and this may lead to more frequent and more severe climate-related impacts that may deter efforts to achieve the country's development objectives, including the targets of the Nigeria Vision 20:2020 and the

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Millennium Development Goals (MDGs) [11, 12]. The challenges will be multifaceted (social, economic, environmental), and its impact on infrastructure will be significant [11]. It is also expected to negatively affect the already limited electrical power supply through impacts on the existing hydroelectric and thermal generation; service interruption [12].

This research article aims at modeling and forecasting Nigerian monthly rainfall series from January 1996 to December 2018 obtained from [13] since the variability changes in rainfall series is significant to climate change [14]. Several statistical methods like Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) had been used to model and forecast Nigerian Monthly Rainfall series but it seems inappropriate because these methods only take into consideration stationarity of the series and determining the presence of seasonal variation [15]. Hence, a Fourier Autoregressive (FAR) model proposed by [15] and has the ability to capture periodic changes from one period in a year to another year will be used to model and forecast Nigerian rainfall series.

Materials and Methods

Fourier autoregressive model

Fourier autoregressive (FAR) model is given as

$$y_{k\omega+v} = \varphi_0 + \sum_{i=1}^{p(v)} \left[\varphi_i(v) \cos 2\pi k/_{\omega} + \varphi_i^*(v) \sin 2\pi k/_{\omega}\right] y_{k\omega+v-i} + \varepsilon_{k\omega+v} \qquad (1)$$

where v is the period index ($v = 1, 2, \ldots, \omega$), k is the year index ($k = 0, \pm 1, \pm 2, \ldots$), $\varphi_i(v)$ is the periodic autoregressive coefficient, ω is the number of seasons and $\varepsilon_{k\omega+v}$ is a white noise with mean zero (0) and periodic variance, $\sigma_\varepsilon^2(v)$.

Model building in Fourier autoregressive model

The four basic steps in Fourier autoregressive model building will be carried out as follows:

Model identification for Fourier autoregressive (FAR) model

The identification of the FAR model will be determined using sample periodic Autocorrelation function ($PeACF$) given as

$$\Gamma_l(v) = \hat{\gamma}_l(v) \Big/ \sqrt{\hat{\gamma}_0(v)\hat{\gamma}_0(v-l)}, \qquad l \geq 0 \qquad (2)$$

where $\hat{\gamma}_l(v)$ is the sample periodic autocorrelation function and sample periodic Partial Autocorrelation function ($PePACF$) given as

$$\widehat{\emptyset}_{l+1,l+1} = \frac{\hat{\Gamma}_i(v) - \sum_{j=1}^{k} \widehat{\emptyset}_{kl}\, \hat{\Gamma}_{k\omega+v-l}(v)}{1 - \sum_{j=1}^{k} \widehat{\emptyset}_{kl}\, \hat{\Gamma}_{ll}} \qquad (3)$$

where

$$\widehat{\emptyset}_{k\omega+1,l} = \widehat{\emptyset}_{kl} - \widehat{\emptyset}_{k\omega+1,k\omega+l}\widehat{\emptyset}_{kl,k+1-l}$$

Model estimation of Fourier autoregressive model

The Fourier autoregressive coefficients will be estimated using the discrete Fourier transform estimation method where the discrete Fourier transform is assumed to be a periodic stationary process and this is expressed in matrix form as

$$\begin{pmatrix} y_{t1} \\ y_{t2} \\ \vdots \\ y_{t\omega} \end{pmatrix} = \begin{pmatrix} 1 & \frac{\cos 2\pi}{12} y_{t1-1} & \frac{\sin 2\pi}{12} y_{t1-1} & \cdots & \frac{\cos 2\pi k}{12} y_{t1-p(v)} & \frac{\sin 2\pi k}{12} y_{t1-p(v)} \\ 1 & \frac{\cos 2\pi(2)}{12} y_{t2-1} & \frac{\sin 2\pi(2)}{12} y_{t2-1} & \cdots & \frac{\cos 2\pi k}{12} y_{t2-p(v)} & \frac{\sin 2\pi k}{12} y_{t2-p(v)} \\ \vdots & \vdots & \vdots & & & \\ 1 & \frac{\cos 2\pi(\omega)}{12} y_{t\omega-p(v)} & \frac{\sin 2\pi(\omega)}{12} y_{t\omega-p(v)} & \cdots & \frac{\cos 2\pi k}{12} y_{t\omega-p(v)} & \frac{\sin 2\pi k}{12} y_{t\omega-p(v)} \end{pmatrix} \begin{pmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_1^* \\ \vdots \\ \varphi_{P(v)} \\ \varphi_{P(v)}^* \end{pmatrix} \qquad (4)$$

where $t = k\omega + v, for\ v = 0,1,2, \ldots, \omega\ and\ \omega = 12.$

After model estimation, the most appropriate model will be chosen based on the lowest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC) given as

$$\text{PAIC (P)} = n\, In\, \hat{\sigma}_\varepsilon^2(v) + 2P(v) \qquad (5)$$

and

$$PBIC = In\, \hat{\sigma}_\varepsilon^2(v) + \frac{lnN}{N}\, P(v) \qquad (6)$$

where $\widehat{\sigma}_{\varepsilon}^{2}(v)$ is the periodic estimator of $\sigma_{\varepsilon}^{2}(v)$ and $P(v)$ is the number of periodic autoregressive coefficients in the season respectively.

Diagnostic Checking in Fourier Autoregressive Model

After parameter estimation, the assessment of the model adequacy will be done by checking whether residuals $\{\varepsilon_t\}$ are white noise. Hence a careful analysis of the estimated residuals will be carried out by checking whether the residuals are white noise and this will be done by computing the sample PEACF and PEPACF of the residuals to determine whether they do not form any pattern and were found to be statistically significant within two standard deviations with $\alpha = 0.05$.

Forecasting in Fourier autoregressive model (FAR) model

Given a FAR(1) model as

$$y_{t1} = \mu + \emptyset_1 cosz(y_{t1-1}) - \emptyset_1^* sinz(y_{t1-1}) + \varepsilon_t$$

$$(1 - \emptyset_1 cosz - \emptyset_1^* sinz)(y_{t1} - \mu) = \varepsilon_t \qquad (7)$$

where $z = \frac{2\pi k}{\omega}$ and μ is a constant.

The general form of the forecast equation is given as

$$\hat{y}_{t1}(l) = \mu + [(\emptyset_1^l coszy_{t1}(l-1) - \mu) + (\emptyset_1^{*l} sinzy_{t1}(l-1) - \mu) \quad l \geq 1 \qquad (8)$$

Forecasting Evaluations

The forecast evaluations that will be used to measure the accuracy of FAR(1) model are Periodic Root Mean Square Error (PRMSE), Periodic Mean Absolute Error (PMAE) and Periodic Mean Absolute Percentage Error (PMAPE). The forecast evaluations are given as

$$PRMSE = \sqrt{\frac{1}{tv+1}\sum_{tv=1}^{p-1}(\hat{y}_{tv} - y_{tv})^2} \qquad (9)$$

$$PMAPE = \sum_{tv=1}^{p-1}\left|\frac{\hat{y}_{tv} - y_{tv}}{\hat{y}_{tv}}\right| \qquad (10)$$

$$PMAE = \frac{1}{tv+1}\sum_{tv=1}^{p-1}(\hat{y}_{tv} - y_{tv})^2 \qquad (11)$$

where $tv = 1, 2, \ldots, p-1$ $(Taiwo, 2017)$. The actual and predicted values for corresponding tv values are denoted by $\hat{y}_{tv}$ and y_{tv} respectively. The smaller the values of PRMSE, PMAPE and PMAE, the better the forecasting performance of the model.

Results and Discussion

In order to ascertain the efficiency of Fourier autoregressive models, Nigerian monthly rainfall series from January 1996 to December 2018 collected from [13] was analyzed. The results obtained from Autoregressive (AR), Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) models will be compared with FAR model results to ascertain the reasons while FAR is the better model for Nigerian rainfall series.

The rainfall series exhibited a cyclical or periodic variation and this informs the use of Fourier Autoregressive Model. A critical look at the $PeACF$ and $PePACF$ for January to December series showed that the $PeACF$ was stable and $PePACF$ cut off at lag 2, so tentative FAR(1), FAR(2) and FAR(3) were chosen for January to December rainfall series. Discrete Fourier transform estimation method was used to obtain the coefficients of FAR(1), FAR(2) and FAR(3). FAR(1) models estimation is given in equation (12) were chosen as the optimal models based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC) given in table 1.

$$\left.\begin{aligned}
y_{jan} &= 3.612765 - 0.339286\cos(t)\,y_{t-1} - 0.871348\sin(t)\,y_{t-1}\\
y_{feb} &= 8.376009 + 2.040730\cos(t)\,y_{t-1} + 0.697592\sin(t)\,y_{t-1}\\
y_{mar} &= 26.85521 - 2.915318\cos(t)\,y_{t-1} + 1.072262\sin(t)\,y_{t-1}\\
y_{apr} &= 64.12574 + 7.189011\cos(t)\,y_{t-1} - 7.315425\sin(t)\,y_{t-1}\\
y_{may} &= 115.6598 + 5.108940\cos(t)\,y_{t-1} + 0.519816\sin(t)\,y_{t-1}\\
y_{jun} &= 157.4872 - 6.767992\cos(t)\,y_{t-1} - 9.866734\sin(t)\,y_{t-1}\\
y_{jul} &= 212.0417 - 11.81382\cos(t)\,y_{t-1} - 11.87484\sin(t)\,y_{t-1}\\
y_{aug} &= 228.8587 - 3.177316\cos(t)\,y_{t-1} - 6.206400\sin(t)\,y_{t-1}\\
y_{sep} &= 201.2373 + 18.82645\cos(t)\,y_{t-1} + 3.634644\sin(t)\,y_{t-1}\\
y_{oct} &= 107.7105 - 0.157149\cos(t)\,y_{t-1} - 0.629027\sin(t)\,y_{t-1}\\
y_{nov} &= 109.7640 - 0.656149\cos(t)\,y_{t-1} - 0.529527\sin(t)\,y_{t-1}\\
y_{dec} &= 3.594377 + 0.717928\cos(t)\,y_{t-1} + 1.261494\sin(t)\,y_{t-1}
\end{aligned}\right\} \quad (12)$$

where $t = \frac{2\pi k}{\omega}$.

The residual of the fitted FAR(1) models was diagnosed using periodic residual autocorrelation. The periodic residual autocorrelation for FAR(1) models showed the residual was white noise, hence the models can be used to forecast Nigerian monthly rainfall series. The in-sample and out-sample forecast were obtained for Nigerian monthly rainfall series based on the FAR(1) models from January to December. The in-sample forecast for FAR(1) model from January to December showed a close look to the original series and out-sample forecast values for FAR(1) in Table-2 and Figure-1 showed a continuous periodic and close reflection of the original series from January 2019 to December 2020.

Table 1-Information Criteria for January to December Series

Month(s)	Inf. Criteria	FAR(1)	FAR(2)	FAR(3)	Momth(s)	Inf. Criteria	FAR(1)	FAR(2)	FAR(3)
january	paic	3.430102*	3.455248	3.541566	july	aic	10.21054*	10.34706	10.52046
	pbic	3.578210*	3.702095	3.887152		bic	10.35865*	10.59391	10.86605
february	paic	5.621061*	5.684433	5.839269	august	aic	9.501169*	9.638035	9.67652
	pbic	5.769169*	5.931279	6.184854		bic	9.649277*	9.884881	10.02211
march	paic	7.344116*	7.479084	7.503036	september	aic	9.421283*	9.584354	9.701643
	pbic	7.492224*	7.725931	7.848621		bic	9.569391*	9.8312	10.04723
april	paic	8.621772*	8.7137	8.721801	october	aic	9.011679*	9.250466	9.054205
	pbic	8.769880*	8.960547	9.067386		bic	9.27657*	9.497312	9.39979
may	paic	8.962050*	9.014371	9.144934	november	aic	9.030679*	9.250466	9.054205
	pbic	9.110158*	9.261218	9.490519		bic	9.278787*	9.497312	9.39979
june	aic	8.862177*	8.929468	9.097053	december	aic	4.626287*	4.75582	4.907723
	bic	9.010285*	9.176315	9.442639		bic	4.774395*	5.002667	5.253309

Table 2- Forecast of Nigerian Rainfall series from January 2019 to December 2020

Month	AR	ARMA	ARIMA	SARIMA	FAR
Jan-19	37.83854	32.35587	37.2978	2.94521	3.8648
Feb-19	76.19937	72.97246	74.81843	7.97545	13.5153
Mar-19	105.9529	116.4021	103.7225	23.1458	132.2314
Apr-19	120.5798	150.8834	117.7292	93.6134	34.0749
May-19	121.2371	167.9942	118.1123	121.723	649.8104
Jun-19	113.2972	164.6139	110.2251	125.624	689.5175
Jul-19	102.8612	143.3183	100.0514	165.923	474.2523
Aug-19	94.42364	111.2287	91.93392	218.061	388.8888
Sep-19	90.01039	77.7493	87.7716	245.435	821.4758
Oct-19	89.48844	51.88299	87.36449	98.9342	105.5695
Nov-19	91.47476	39.87402	89.32715	9.9567	95.0692
Dec-19	94.29991	43.77599	92.03366	2.43258	2.9225
Jan-20	96.68187	61.24982	94.26269	1.62353	5.7203
Feb-20	98.0003	86.54453	95.44619	10.6348	20.9143
Mar-20	98.2394	112.3036	95.5976	50.7535	133.261

Apr-20	97.75119	131.6498	95.07289	29.0345	33.0023
May-20	96.99036	139.9716	94.3125	81.3456	689.7604
Jun-20	96.32086	135.9625	93.6588	165.234	669.5021
Jul-20	95.93037	121.6967	93.28018	130.192	474.2523
Aug-20	95.83797	101.802	93.18441	255.46	388.8888
Sep-20	95.95526	82.02162	93.28076	180.43	921.5758
Oct-20	96.15902	67.59805	93.45031	115.423	114.2595
Nov-20	96.34639	61.92219	93.5974	11.5454	96.0132
Dec-20	96.46117	65.7846	93.67248	2.56701	3.0225

Table 3- Forecast Evaluation for Nigerian Rainfall Series FAR(1)

Month(s)	Mean Absolute Percent Error of FAR(1)	Root Mean Square Error of FAR(1)	Mean absolute Error of FAR(1)
january	8.24128	1.180193	0.885314
february	35.60173	2.484298	1.95046
march	37.45815	8.353536	6.794034
april	20.06252	15.82376	12.14949
may	13.29114	18.75859	15.12987
june	8.275527	17.84486	12.49376
july	12.02743	35.01917	26.36026
august	8.782356	24.56226	19.63751
september	9.291498	23.60051	17.65652
october	16.2637	20.4088	16.2637

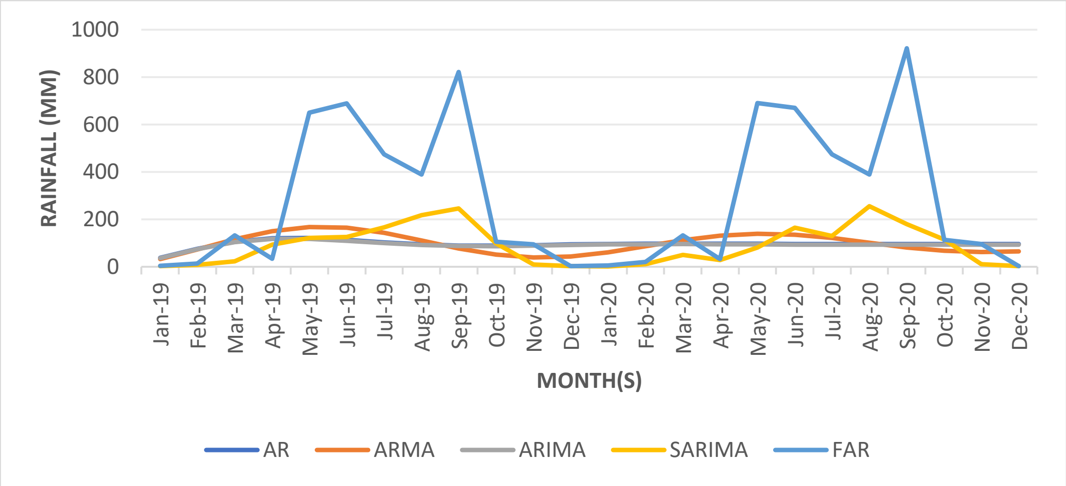

Figure 1- time plot of Nigerian monthly rainfall from january, 2019 to december, 2020.

In order to ascertain the reasons while Fourier Autoregressive model is the better model for forecasting Nigeria rainfall, the rainfall forecasted values from AR(3), ARMA(2,1), ARIMA(2,1,1) and SARIMA$(1,1,1)(1,1,1)_{12}$ given in Table-2 and Figure-1 is compared with that of FAR(1) model. Based on the comparison, AR(3), ARMA(2,1) and ARIMA(2,1,1) models are not appropriate for forecasting Nigerian rainfall since the forecast values from these models did not reflect the seasonality and periodicity that is usually present in rainfall series. While SARIMA$(1,1,1)(1,1,1)_{12}$ model captured and exhibited the seasonality in Nigerian rainfall series but the periodicities in the series were not resolved. But, FAR(1) model forecast in Table-2 and Figure-1 captured and exhibited the seasonality and periodicity that is presented in the rainfall series. As well, FAR(1) model showed a continuous periodic movement and close reflection to the original series from January 2019 to December 2020. The values of the forecast evaluation for FAR(1) model in Table-3 showed as well the consistent of the forecast since their values were relatively low. Hence, the Fourier Autoregressive model is adequate and suitable for modeling and forecasting Nigerian rainfall series.

Conclusion

The Fourier autoregressive model was used to analyze Nigerian monthly rainfall series collected from [13] from January 1996 to December 2018. FAR(1), FAR(2) and FAR(2) models were chosen based on $PeACF$ and $PeACF$. The coefficients of the tentative model were estimated using a Discrete Fourier transform estimation method. The FAR(1) models were chosen as the optimal model based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC). The residual of the fitted models was diagnosed to be white noise. The in-sample forecast showed a close reflection of the original rainfall series while the out-sample forecast exhibited a continuous periodic forecast from January 2019 to December 2020 with relatively small values of PRMSE, PMAE, and PMAPE. The comparison of FAR(1) model forecast with AR(3), ARMA(2,1), ARIMA(2,1,1) and $\text{SARIMA}(1,1,1)(1,1,1)_{12}$ showed that AR(3), ARMA(2,1), ARIMA(2,1,1) models are not appropriate for forecasting Nigerian rainfall since the forecast values from these models did not reflect the seasonality and periodicity that is usually present in rainfall series. While $\text{SARIMA}(1,1,1)(1,1,1)_{12}$ model captured and exhibited the seasonality in Nigerian rainfall series but the periodicities in the series were not resolved. But, FAR(1) model captured and exhibited the seasonality and periodicity that is presented in the rainfall series. FAR(1) model showed a continuous periodic movement and close reflection to the original series from January 2019 to December 2020. As well, the values of the forecast evaluation for FAR(1) model showed consistent of the forecast since their values were relatively low. Hence, Fourier Autoregressive model is adequate and suitable for modeling and forecasting Nigerian rainfall series. In conclusion, the continuous monthly periodic forecast indicated that there will be rapid climate change in Nigeria in the coming yearly and Nigerian Government needs to put in place plans to curtail its effects.

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