Abdulateef et al. Iraqi Journal of Science, 2023, Vol. 64, No. 10, pp: 5173-5181
DOI: 10.24996/ijs.2023.64.10.25

ISSN: 0067-2904

Employ Stress-Strength Reliability Technique in Case the Inverse Chen Distribution

Eman Ahmed Abdulateef *, Abbas Najim Salman *, Hadeel Hussein Luaibi
Department of Mathematics, College of Education for Pure Sciences (Ibn Al – Haitham), Baghdad University, Baghdad, Iraq

Received: 2/8/2022 Accepted: 21/11/2022 Published: 30/10/2023

Abstract
This paper uses classical and shrinkage estimators to estimate the system reliability (R) in the stress-strength model when the stress and strength follow the Inverse Chen distribution (ICD). The comparisons of the proposed estimators have been presented using a simulation that depends on the mean squared error (MSE) criteria.

Keywords: Inverse Chen distribution, Stress–Strength reliability model, Maximum likelihood estimator, Percentile estimator, Shrinkage estimator, Mean squared error.

توظيف تقنية موثوقية الإجهاد–المتانة في حالة توزيع Chen المعكوس

ايمان احمد عبد اللطيف [1*], عباس نجم سلمان [2], هديل حسين لعيبي [3]
قسم الرياضيات, كلية التربية للعلوم الصرفة–ابن الهيثم ,جامعة بغداد, بغداد,العراق

الخلاصة
يتعلق البحث بتوظيف المقدرات الكلاسيكية ومقدرات التقلص لتقدير نظام المعولية (R) في نموذج الاجهاد – المتانة ، عندما يتبع الإجهاد والمتانة توزيع Chen العكسي (ICD). تم تقديم مقارنات بين المقدرات المقترحة باستخدام المحاكاة اعتمادًا على معيار متوسط مربعات الخطأ (MSE).

1. Introduction

For the importance of the reliability of the stress-strength model, the researchers have studied systems by the different distributions, in the context of some of these studies, Rao and others (2019) estimated the reliability stress-strength model $R = P(Y < X)$, when X and Y follow the Exponentiated Inverse Rayleigh Distribution [1]. In the same year, Bareq and Alaa estimated R when X and Y follow the power distribution by different methods, namely the Maximum likelihood method, Shrinkage estimation method, Least square method and Moment method. They concluded that the constant shrinkage method is the best estimator [1]. In 2022, Eman and Abbas estimated R when the X and Y follow the odd Frechet inverse exponential distribution via different methods; Maximum likelihood method, Shrinkage estimation methods they concluded that the Preliminary Test Single Stage was the best estimator [2].

*Email: eman.a.al@ihcoedu.uobaghdad.edu.iq

Our aim is to derive the reliability of the stress-strength system for the Inverse Chen Distribution when the parameter is known and unknown, then the reliability is estimated using some estimations method; Maximum Likelihood Estimator (MLE), Percentile Estimator (PCE), Shrinkage Estimators. In addition, we make a comparison among the proposed estimation methods via simulation depending on the mean squared error. Chen in 2000 proposed a new two parameters lifetime distribution with a bathtub-shaped or increasing failure rate function with the following probability distribution function (PDF) and the cumulative distribution function (CDF) [3],

$$f(x) = \lambda\beta(x)^{\beta-1}e^{x^{\beta}}e^{\lambda(1-e^{x^{\beta}})};\ x,\lambda,\beta > 0 \quad (1)$$

$$F(x) = 1 - e^{\lambda(1-e^{x^{\beta}})};\ x,\lambda,\beta > 0 \quad (2)$$

Where λ and β are shape parameters, X is a random variable following the Chen distribution, then Y=1/X follows the Inverse Chen distribution with the parameters λ *and* β . The PDF and CDF of y are respectively given as follows [4];

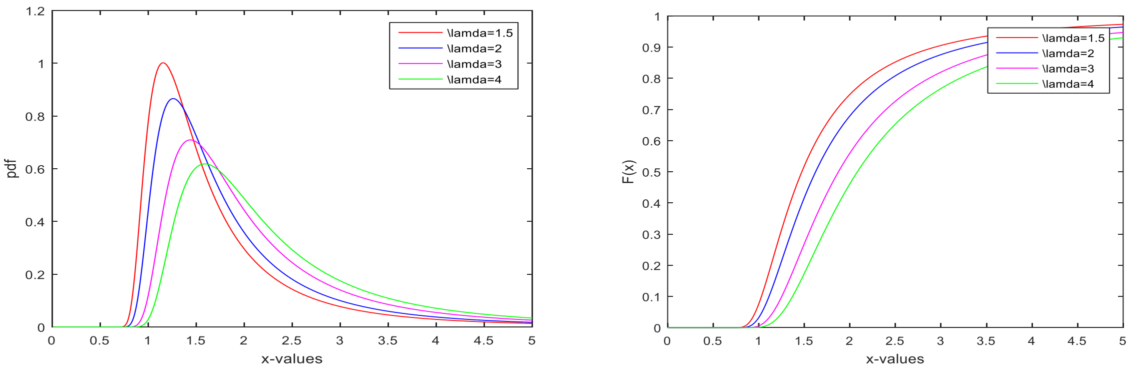
$$f(y) = \lambda\beta(y)^{-(\beta+1)}e^{y^{-\beta}}e^{\lambda(1-e^{y^{-\beta}})};\ y,\lambda,\beta > 0, \quad (3)$$

$$F(y) = e^{\lambda(1-e^{y^{-\beta}})};\ y,\lambda,\beta > 0. \quad (4)$$

As well, the reliability and hazard functions are given as follows:

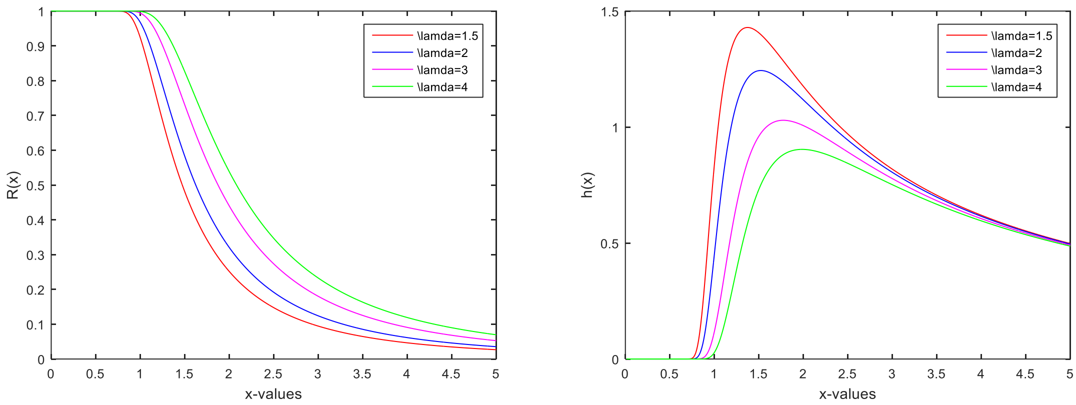
$$\mathrm{R(y)} = 1 - F(y) = 1 - e^{\lambda(1-e^{y^{-\beta}})}, \quad (5)$$

$$\mathrm{h(y)} = \frac{f(y)}{\mathrm{R(y)}} = \frac{\lambda\beta(y)^{-(\beta+1)}e^{y^{-\beta}}e^{\lambda(1-e^{y^{-\beta}})}}{1-e^{\lambda(1-e^{y^{-\beta}})}}, \quad (6)$$



(a) (b)

Figure 1: The plots of pdf and cdf, respectively of random variable Y follows ICE (y;λ,β) for some special choices of the parameter λ= (1.5, 2, 3, 4) and β=2.5, (a) Frame 1 (b) Frame 2.



(a) **(b)**

Figure 2 : The plots of the reliability function and hazard function respectively of random variable Y follows ICE (y;λ,β) for some special choices of the parameter λ= (1.5, 2, 3, 4) and β=2.5, (a) Frame 1 (b) Frame 2.

Throughout this paper, we assume that the random variable Y represents the stress, and the random variable X mentions the strength in the model of stress-strength (S-S) which are considered to distribute ICD (λ_1,β) and ICD (λ_2,β), respectively. When the strength exceeds the stress in the distribution, the system will work and this indicates that the reliability system R = P(Y < X) in the model of stress-strength is performance.

Now, the system reliability of this stress-strength (S-S) model can be derived as follows:

R = P(Y < X) = $\int_0^{\infty} F_y\,(x) f(x) dx$

$$= \int_0^{\infty} e^{\lambda_2(1-e^{x^{-\beta}})} \lambda_1 \beta (x)^{-(\beta+1)} e^{x^{-\beta}} e^{\lambda_1(1-e^{x^{-\beta}})} dx.$$

Assume that, w= $1 - e^{x^{-\beta}}$, we get $x = [\ln(1-w)]^{-\frac{1}{\beta}}$

So $dx = \frac{1}{\beta} * [\ln(1-w)]^{-\frac{1}{\beta}-1} * \frac{1}{1-w} dw$.

Then we obtain

$$R = \lambda_1 \int_{-\infty}^{0} e^{(\lambda_1+\lambda_2)w} dw, R = \frac{\lambda_1}{\lambda_1+\lambda_2} \quad , \tag{7}$$

2. Estimation Methods of R= P(Y < X)

2.1. Maximum Likelihood Estimator (MLE), [4]

Let $x_1, x_2, \ldots, x_n$ from ICE(λ_1, β) and $y_1, y_2, \ldots, y_m$ from ICE(λ_2, β). Then, the likelihood function turns into**:**

L= L$(\lambda_1, \lambda_2, \beta; x, y)$ =$\prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(\,y_j)$

$$=\prod_{i=1}^{n} \lambda_1 \beta (x_i)^{-(\beta+1)} e^{x_i^{-\beta}} e^{\lambda_1(1-e^{x_i^{-\beta}})} \prod_{j=1}^{m} \lambda_2 \beta (\,y_j)^{-(\beta+1)} e^{\,y_j^{-\beta}} e^{\lambda_2(1-e^{\,y_j^{-\beta}})}$$

$$= \lambda_1{}^{n} \lambda_2{}^{m} \beta^{(n+m)} \prod_{i=1}^{n}(x_i)^{-(\beta+1)} \prod_{i=1}^{n} e^{x_i^{-\beta}} \prod_{i=1}^{n} e^{\lambda_1(1-e^{x_i^{-\beta}})} \prod_{j=1}^{m}(\,y_j)^{-(\beta+1)} \prod_{j=1}^{m} e^{\,y_j^{-\beta}} \prod_{j=1}^{m} e^{\lambda_2(1-e^{\,y_j^{-\beta}})}$$

$$\ln(l) \;= n ln\lambda_1 + n ln\lambda_2 + (n+m) ln\beta - (\beta+1)\sum_{i=1}^{n} \ln\,(x_i) + \sum_{i=1}^{n} x_i{}^{-\beta} + \sum_{i=1}^{n} \lambda_1 \left(1 - e^{x_i^{-\beta}}\right) + -(\beta+1)\sum_{j=1}^{m} \ln\,(y_j) + \sum_{j=1}^{m} y_j{}^{-\beta} + \sum_{j=1}^{m} \lambda_2 \left(1 - e^{y_j^{-\beta}}\right)$$

$$\frac{\partial \mathrm{Ln}(l)}{\partial \lambda_1} = \frac{n}{\lambda_1} + n - \sum_{i=1}^{n} \left(e^{x_i^{-\beta}}\right)$$

$$\frac{\partial \mathrm{Ln}(l)}{\partial \lambda_2} = \frac{m}{\lambda_2} + m - \sum_{j=1}^{m} \left(e^{y_j^{-\beta}}\right)$$

The maximum likelihood estimator of the parameters λ_1 and λ_2 becomes, respectively as follows:

$$\widehat{\lambda_1}_{mle} = \frac{n}{\sum_{j=1}^{m}\left(e^{x_i^{-\beta}}\right)-n}, \tag{8}$$

$$\widehat{\lambda_2}_{mle} = \frac{m}{\sum_{j=1}^{m}\left(e^{y_j^{-\beta}}\right)-m}. \tag{9}$$

By substituting $\widehat{\lambda_1}_{mle}$ and $\widehat{\lambda_2}_{mle}$ in equation (7), the reliability estimation model $\widehat{R}_{\mathrm{mle}}$ has been getting:

$$\widehat{\mathrm{R}}_{mle} = \frac{\widehat{\lambda_1}_{mle}}{\widehat{\lambda_1}_{mle}+\widehat{\lambda_2}_{mle}}. \tag{10}$$

2.2 Percentile Estimator Method:[5]

Let X ~ ICD (λ_1, β) and Y ~ ICD (λ_2, β), then the Percentile estimator is obtained by minimizing the
sum of squared between the value and expected value of CDF as follows:

$P = \sum_{i=1}^{n}\left[E\left(F(x_i)\right) - F(x_i)\right]^{2}$.

$F(x_i)= e^{\lambda(1-e^{x_i^{-\beta}})}$ And, $E(F(x_i))$ =P_i. Such that; $P_i = \frac{i}{n+1}$; i=1, 2… n.

Now, the percentile estimator of λ_1 is obtained by minimizing the following:

$lnP = \sum_{i=1}^{n}[\mathrm{Ln}P_i - \mathrm{Ln}(e^{\lambda_1(1-e^{x_i^{-\beta}})})]^{2}$.

$\frac{\partial lnP}{\partial \lambda_1} = 2\sum_{i=1}^{n}\left[\mathrm{Ln}P_i - \lambda_1(1-e^{x_i^{-\beta}})\right](-\left(1-e^{x_i^{-\beta}}\right))$, then we

$2\sum_{i=1}^{n}\left[\mathrm{Ln}P_i - \lambda_1(1-e^{x_i^{-\beta}})\right](-\left(1-e^{x_i^{-\beta}}\right)) = 0$

Hence, the percentile estimator of $\lambda_1$is:

$$\widehat{\lambda_1}_{PCE} = \frac{\sum_{i=1}^{n} \mathrm{Ln}P_i(1-e^{x_i^{-\beta}})}{\sum_{i=1}^{n}(1-e^{x_i^{-\beta}})^2} \;;\; P_i=\frac{i}{n+1}, \text{ i=1, 2,… , n} \tag{11}$$

We obtain the percentile estimator of λ_2, $\widehat{\lambda_2}_{PCE} = \frac{\sum_{j=1}^{m} \mathrm{Ln}P_j(1-e^{y_j^{-\beta}})}{\sum_{j=1}^{m}(1-e^{y_j^{-\beta}})^2}$; $P_j=\frac{j}{m+1}$, j=1, 2… m

By substituting $\widehat{\lambda_1}_{PCE}$, $\widehat{\lambda_2}_{PCE}$ in equation (7), we obtain $\widehat{\lambda_1}_{PCE}$, $\widehat{\lambda_2}_{PCE}$ in equation (12),

$$\widehat{R}_{PCE} = \frac{\widehat{\lambda_1}_{PCE}}{\widehat{\lambda_1}_{PCE}+\widehat{\lambda_2}_{PCE}} . \tag{12}$$

2.3 Shrinkage Estimation Method

In the year 1968, J.R. Thompson recommended the problem of shrink a traditional estimator $\hat{\lambda}$ of the parameter λ to earlier estimate λ_0 via shrinkage weight factor $k(\hat{\lambda})$, where $0 \le k(\hat{\lambda}) \le 1$. He trusts that λ_0 is very neighboring to the actual value of λ. Consequently, the formula of - Type shrinkage estimator considered by Thompson (1968) for λ say $\hat{\lambda}_{sh}$ becomes [6].

$$\hat{\lambda}_{sh} = k\hat{\lambda}_{mle} + (1-k)\lambda_0. \tag{13}$$

2.3.1 The Shrinkage Weight Function estimators (Sh1).

In this subsection, the shrinkage weight factor is suggested as a function of sample sizes *n* and *m* respectively that is considered as the form below.

i.e. $k_1(\hat{\lambda}_1) = \frac{e^{-n}}{n}$, and $k_2(\hat{\lambda}_2) = \frac{e^{-m}}{m}$.

The shrinkage estimator of $\lambda_1$and $\lambda_2$using the previously considered shrinkage weight function is

$$\widehat{\lambda}_{i_{sh1}} = k_i(\hat{\lambda}_i)\widehat{\lambda}_{i_{mle}} + (1 - K_i(\hat{\lambda}_i))\lambda_{i_0} \text{ ,i=1, 2 .} \tag{14}$$

The shrinkage estimation$\hat{R}_{sh1}$ in equation (7) using shrinkage weight function estimators is to be:-

$$\hat{R}_{sh1}=\frac{\widehat{\lambda_1}_{sh1}}{\widehat{\lambda_1}_{sh1}+\widehat{\lambda_2}_{sh1}}. \tag{15}$$

2.3.2. Beta Shrinkage Factor estimator (Sh2): [7]

In this case, the assumption of k $(\hat{\lambda})$ for the Beta shrinkage weight factor has been taken as $k_1(\hat{\lambda}_1)$= (1,n), and $k_2(\hat{\lambda}_2)$= $\beta(1,m)$ and this implies the following shrinkage estimators:

$$\widehat{\lambda_1}_{sh2}= \beta\ (1,n)\ \widehat{\lambda_1}_{mle}+(1-\beta(1,m))\ \lambda_{1_0}\ , \tag{16}$$

$$\widehat{\lambda_2}_{sh2}= \beta\ (1,n)\ \widehat{\lambda_2}_{mle}+(1-\beta(1,m))\ \lambda_{2_0}\ . \tag{17}$$

Substituting $\widehat{\lambda_1}_{sh2}$, $\widehat{\lambda_2}_{sh2}$in equation (7), then the reliability estimation of the stress-strength model using Beta shrinkage factor estimator will become as follows:

$$\hat{R}_{sh2}=\frac{\widehat{\lambda_1}_{sh2}}{\widehat{\lambda_1}_{sh2}+\widehat{\lambda_2}_{sh2}}. \tag{18}$$

3. Simulation Experiments

In this section, numerical consequences were premeditated to compare the performance of the unalike estimators of system reliability consuming numerous sample sizes (30, 70 and 100) created on 1000 replication through criteria of mean squared error MSE. For this aim, Monte Carlo simulation was considered in creating the random sample from the uniform distribution over (0, 1) interval as u_1, u_2,…, u_n and v_1, v_2,…, v_m. The generating uniform random samples transform to follow IC distributions for different random sample n depending on (c. d. f.), [8].

$$F(x_i) = e^{\lambda_1(1-e^{x_i^{-\beta}})}\ ,$$

$$U_i = e^{\lambda_1(1-e^{x_i^{-\beta}})}\ ,$$

$$x_i = [\ln\ (1 - \frac{lnU_i}{\lambda_1}]^{-\frac{1}{\beta}},$$

by the same method, we get y_j, $y_j = [\ln\ (1 - \frac{lnV_J}{\lambda_2}]^{-\frac{1}{\beta}}$.

The following steps, compute the real value of R in equation (7) and the value of estimation methods of all proposal method $\hat{R}_{mle}$,$\hat{R}_{PCE}$, $\hat{R}_{sh1}$ and $\hat{R}_{sh2}$ in equations (10), (12), (15) and (18) , respectively.

Based on (L=1000) replication, the MSE is calculated for all proposed estimation methods as follows:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i - R\right)^2 .$$

For this situation, the estimation of the reliability system of stress- strength model for specific values of the parameters $of\ \lambda_1\ and\ \lambda_2$ was placed in the resulting tables below:

Table 1: Values of the $\hat{R}$ when $R = 0.40000, \lambda_1 = 2\ , \lambda_2 = 3, and\ \beta = 2.5$

(n,m)	$\hat{R}_{mle}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{pce}$
(30,30)	0.50148	0.39856	0.39855	0.43788
(30,70)	0.50824	0.39128	0.39217	0.55133
(30,100)	0.50976	0.38935	0.39077	0.62928
(70,30)	0.49397	0.40692	0.40586	0.29855
(70,70)	0.50059	0.39958	0.39944	0.43227
(70,100)	0.50213	0.39763	0.39804	0.47435
(100,30)	0.49239	0.40914	0.40746	0.25979
(100,70)	0.49895	0.40178	0.40103	0.38737
(100,100)	0.50047	0.39983	0.39963	0.43028

Table 2: Values MSE of the $\hat{R}$ when $R = 0.40000, \lambda_1 = 2, \lambda_2 = 3, and\ \beta = 2.5$

(n,m)	$MSE\hat{R}_{mle}$	$MSE\hat{R}_{sh1}$	$MSE\hat{R}_{sh2}$	$MSE\hat{R}_{pce}$	Best
(30,30)	0.010298806	0.000002062	0.000002106	0.083625868	$MSE\hat{R}_{sh1}$
(30,70)	0.011715908	0.000076091	0.000061245	0.106860122	$MSE\hat{R}_{sh2}$
(30,100)	0.012047305	0.000113481	0.000085141	0.126183106	$MSE\hat{R}_{sh2}$
(70,30)	0.008831102	0.000047926	0.000034322	0.073676632	$MSE\hat{R}_{sh2}$
(70,70)	0.010117432	0.000000177	0.000000311	0.077513935	$MSE\hat{R}_{sh1}$
(70,100)	0.010430688	0.000005611	0.000003852	0.087157404	$MSE\hat{R}_{sh2}$
(100,30)	0.008536710	0.000083535	0.000055611	0.083359354	$MSE\hat{R}_{sh2}$
(100,70)	0.009790329	0.000003182	0.000001068	0.078948749	$MSE\hat{R}_{sh2}$
(100,100)	0.010093985	0.000000028	0.000000139	0.081620432	$MSE\hat{R}_{sh1}$

Table 3: Values of the $\hat{R}$ when $R = 0.60000, \lambda_1 = 3, \lambda_2 = 2, and\ \beta = 2.5$.

(n,m)	$\hat{R}_{mle}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{pce}$
(30,30)	0.49853	0.60143	0.60145	0.56461
(30,70)	0.50580	0.59308	0.59414	0.70799
(30,100)	0.50759	0.59086	0.59254	0.73994
(70,30)	0.49192	0.60872	0.60783	0.42596
(70,70)	0.49939	0.60042	0.60056	0.57214
(70,100)	0.50114	0.59822	0.59897	0.61200
(100,30)	0.48997	0.61066	0.60923	0.39722
(100,70)	0.49881	0.60237	0.60196	0.49744
(100,100)	0.49999	0.60017	0.60037	0.56076

Table 4: Values MSE of the $\hat{R}$ when $R = 0.60000, \lambda_1 = 3, \lambda_2 = 2, and\ \beta = 2.5$

(n,m)	$MSE\hat{R}_{mle}$	$MSE\hat{R}_{sh1}$	$MSE\hat{R}_{sh2}$	$MSE\hat{R}_{pce}$	Best
(30,30)	0.010296325	0.000002055	0.000002109	0.079196945	$MSE\hat{R}_{sh1}$
(30,70)	0.008872918	0.000047930	0.000034324	0.077968615	$MSE\hat{R}_{sh2}$
(30,100)	0.008540302	0.000083509	0.000055595	0.081547240	$MSE\hat{R}_{sh2}$
(70,30)	0.011680822	0.000076105	0.000061272	0.108200204	$MSE\hat{R}_{sh2}$
(70,70)	0.010121691	0.000000177	0.000000311	0.081075662	$MSE\hat{R}_{sh1}$
(70,100)	0.009772895	0.000003182	0.000001068	0.077367120	$MSE\hat{R}_{sh2}$
(100,30)	0.012105653	0.000113628	0.000085105	0.121546666	$MSE\hat{R}_{sh2}$
(100,70)	0.010239457	0.000005608	0.000003848	0.094422167	$MSE\hat{R}_{sh2}$
(100,100)	0.010000202	0.000000028	0.000000139	0.080397278	$MSE\hat{R}_{sh1}$

Table 5: Values of the $\hat{R}$ when $R = 0.42857, \lambda_1 = 1.5, \lambda_2 = 2, and\ \beta = 2.5$.

(n,m)	$\hat{R}_{mle}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{pce}$
(30,30)	0.49435	0.42708	0.42706	0.45475
(30,70)	0.50822	0.41862	0.41965	0.60467
(30,100)	0.52101	0.41637	0.41802	0.64788
(70,30)	0.49576	0.43666	0.43545	0.30247
(70,70)	0.50720	0.42814	0.42799	0.45529
(70,100)	0.49967	0.42589	0.42637	0.50339
(100,30)	0.49335	0.43918	0.43728	0.27439
(100,70)	0.49591	0.43065	0.42981	0.39452
(100,100)	0.49966	0.42840	0.42819	0.45426

Table6: Values MSE of the $\hat{R}$ when $R = 0.42857, \lambda_1 = 1.5, \lambda_2 = 2, and\ \beta = 2.5$

(n,m)	$MSE\hat{R}_{mle}$	$MSE\hat{R}_{sh1}$	$MSE\hat{R}_{sh2}$	$MSE\hat{R}_{pce}$	Best
(30,30)	0.004326815	0.000002229	0.000002288	0.078072571	$MSE\hat{R}_{sh1}$
(30,70)	0.006344499	0.000099130	0.000079578	0.110734471	$MSE\hat{R}_{sh2}$
(30,100)	0.008545349	0.000148915	0.000111274	0.123165597	$MSE\hat{R}_{sh2}$
(70,30)	0.004514068	0.000065410	0.000047326	0.082695078	$MSE\hat{R}_{sh2}$
(70,70)	0.006182145	0.000000187	0.000000329	0.078876381	$MSE\hat{R}_{sh1}$
(70,100)	0.005055343	0.000007182	0.000004830	0.087571118	$MSE\hat{R}_{sh2}$
(100,30)	0.004196122	0.000112577	0.000075754	0.085299915	$MSE\hat{R}_{sh2}$
(100,70)	0.004534515	0.000004318	0.000001542	0.079085030	$MSE\hat{R}_{sh2}$
(100,100)	0.005053347	0.000000029	0.000000146	0.079139840	$MSE\hat{R}_{sh1}$

Table7: Values of the $\hat{R}$ when $R = 0.57143, \lambda_1 = 2, \lambda_2 = 1.5, and\ \beta = 2.5$.

(n,m)	$\hat{R}_{mle}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{pce}$
(30,30)	0.49114	0.57293	0.57295	0.54749
(30,70)	0.51016	0.56334	0.56455	0.66947
(30,100)	0.51085	0.56082	0.56273	0.72820
(70,30)	0.49644	0.58139	0.58036	0.41644
(70,70)	0.49964	0.57186	0.57200	0.54273
(70,100)	0.49907	0.56935	0.57019	0.61876
(100,30)	0.49025	0.58362	0.58197	0.34784
(100,70)	0.49803	0.57411	0.57362	0.47593
(100,100)	0.50044	0.57160	0.57181	0.55846

Table8: Values MSE of the $\hat{R}$ when $R = 0.57143, \lambda_1 = 2, \lambda_2 = 1.5, and\ \beta = 2.5$

(n,m)	$MSE\hat{R}_{mle}$	$MSE\hat{R}_{sh1}$	$MSE\hat{R}_{sh2}$	$MSE\hat{R}_{pce}$	Best
(30,30)	0.006445707	0.000002243	0.000002302	0.081200625	$MSE\hat{R}_{sh1}$
(30,70)	0.003753322	0.000065461	0.000047379	0.083534771	$MSE\hat{R}_{sh2}$
(30,100)	0.003669884	0.000112523	0.000075702	0.089719795	$MSE\hat{R}_{sh2}$
(70,30)	0.005623561	0.000099288	0.000079721	0.102906765	$MSE\hat{R}_{sh2}$
(70,70)	0.005153253	0.000000188	0.000000331	0.083826490	$MSE\hat{R}_{sh1}$
(70,100)	0.005235144	0.000004317	0.000001541	0.076698918	$MSE\hat{R}_{sh2}$
(100,30)	0.006589959	0.000148720	0.000111097	0.124942486	$MSE\hat{R}_{sh2}$
(100,70)	0.005387812	0.000007171	0.000004818	0.089070884	$MSE\hat{R}_{sh2}$
(100,100)	0.005039983	0.000000030	0.000000146	0.084073218	$MSE\hat{R}_{sh1}$

4. Numerical Results Analysis

From the previous tables of the simulation experiment, for all n and m =30, 70,100, we conclude that the best estimators by the following:

1- When $n \neq m$, the minimum (MSE) for $\hat{R}_{sh2}$ held using the Beta shrinkage estimator it is the best and follows the shrinkage weight factor estimator $sh1$.

2- When n= m, the minimum (MSE) for $\hat{R}_{sh1}$ held using the shrinkage weight factor estimator it is the best and follows the Beta shrinkage estimator$sh2$.

3- For all n and m, the third of the best estimator is the Maximum Likelihood Estimator and follows the Percentile Estimator.

5. Conclusion

In the absence of real data, we study the performance of the estimator obtained from simulated and the tables of simulation show that the estimate of the reliability system using shrinkage estimators method Sh_1 was the best performance when *n*=*m* . However, the shrinkage weight factor estimator Sh_2 was the best in the other cases for *n* and *m*.

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