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On Γ-n- (Anti) Generalized Strong Commutativity Preserving Maps for Semiprime Γ-Rings

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Abstract.

In this study, we prove that let N be a fixed positive integer and R be a semiprime Γ-ring with extended centroid C_Γ. Suppose that additive maps f and $g: R \to R$ such that f is onto, satisfy one of the following conditions (i) f and g belong to Γ-N-generalized strong commutativity preserving for short; (Γ-N-GSCP) on R (ii) f and g belong to Γ-N-anti-generalized strong commutativity preserving for short; (Γ-N-AGSCP) on R. Then there exists an element $\lambda \in C_\Gamma$ and additive maps $\xi, \eta_1 and\ \eta_2 : R \to C_\Gamma$ such that is of the form $g(x^n) = \lambda\alpha x + \xi(x)$ and $f(x) = \lambda^{-1}\alpha x + \eta_1(x)$ when condition (i) is satisfied, and $f(x) = -\lambda^{-1}\alpha x + \eta_2(x)$ when condition (ii) is satisfied $for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Keywords: semiprime Γ-ring, extended centroid, Γ-N-anti-generalized strong commutativity preserving maps.

حول تعميم الدوال الحافظة للابدالية (المضادة) القويةΓ−N من حلقات كاما شبه الاوليه

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الخلاصة

في هذا البحث نثبت الاتي ، لنفترض أن N عددًا صحيحًا موجبًا ثابتًا وأن يكون R شبه اوليه حلقة−Γ مع النقطة الوسطى الممتدة C_Γ. افترض أن الدوال التجميعية $g: R \to R$ و f بحيث f شاملة ، تفي بأحد الشروط التالية:

(i) g وf هي Γ−N− الحافظة على التبادلية القوية المعممة وباختصار $\Gamma-N-GSCP$ على R (ii) g وf هي Γ−N − الحافظة على التبادلية القوية المعممة المضادة وباختصار $\Gamma-N-AGSCP$ على R. فانه يوجد عنصر $\lambda \in C_\Gamma$ والدوال التجميعيةξ, η_1 و $\eta_2 : R \to C_\Gamma$ بحيث من الشكل$f(x) = \lambda^{-1}\alpha x +$ $\eta_1(x)$ و $g(x^n) = \lambda\alpha x + \xi(x)$ عندما يتم استيفاء الشرط (i) و$f(x) = -\lambda^{-1}\alpha x + \eta_2(x)$ عندما يتم استيفاء الشرط (ii) لكل $x \in R$ و $\alpha \in \Gamma$.

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1. Introduction and Preliminaries

In 1964 the concept of a Γ-ring first introduced by Nobusawa [1]. In 1966 thisΓ-ring is generalized by Barnes [2]. Let R and Γ be additive abelian groups, if there exists a mapping $R \times \Gamma \times R \to R$, such that $(x, \alpha, y) \to x\alpha y$ which satisfies the conditions

(i) $\in R$,

(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha+\beta)z = x\alpha z + x\beta z, x\alpha(y+z) = x\alpha y + x\alpha z,$

(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$ **.**

Then R is called a Γ-ring. Let R be a Γ-ring, an additive subgroup A of R is called a right (left) ideal of R if $A\Gamma R \subset A$ ($R\Gamma A \subset A$). If A is both a right and a left ideal, then we say A is an ideal of R. A Γ-ring R is said to be prime if $x\Gamma R\Gamma y = (0)$ with $x, y \in R$, implies $x = 0$ or $y = 0$ and semiprime if $x\Gamma R\Gamma x = (0)$ with $x \in R$ implies $x = 0$. Let R be a Γ-ring and A be a subset of R, the subset $Ann_l(A) = \{ r \in R\text{: } A\alpha r = \langle 0\rangle \text{ } for\ all\ \alpha \in \Gamma \}$ is called a left annihilator of A. A right annihilator $Ann_r(A)$ can be defined similarly. If A is a left and right annihilator in R, then $Ann(A)$ denotes its annihilator. Moreover, if $= Ann(Ann(A))$, then an ideal A of R is closed and the annihilator of any ideal A of R is a closed ideal. The set $Z(R) = \{x \in R : x\alpha y = y\alpha x$ $for\ all\ \alpha \in \Gamma\ and\ y \in R\}$ is called the center of the Γ-ring R [3]. Let R be a Γ-ring and Q the quotient Γ-ring of R then a set $C_\Gamma = \{g \in Q\text{: } g\alpha f = f\alpha g \quad for\ all\ f \in Q\ and\ \alpha \in \Gamma\}$ is called the extended centroid of a Γ-ring R [4]. If R is a Γ-ring then $[x, y]_\alpha = x\alpha y - y\alpha x\ \ for\ all\ \ x, y \in R\ and\ \alpha \in \Gamma$ is called the commutator of x and y with respect to $\alpha \in \Gamma$. A mapping f of a Γ-ring R into itself is said to be commuting if $[f(x), x]_\alpha = 0\ \ for\ all\ x \in R\ and\ \alpha \in \Gamma.$ A mapping f of a Γ-ring R into itself is said to be centralizing if $[f(x), x]_\alpha$ lies in the center of R for every $x \in R\ and \alpha \in \Gamma$ [5]. The concept that strong commutativity preserving maps of semiprime rings (**SCP**) was first introduced by Bell and Mason in [6]. In a Γ-ring R, a map $f: R \to R$ is Γ-strong commutativity preserving (**Γ-SCP**) on a set $S \subseteq R$ if $[f(x), f(y)]_\alpha = [x, y]_\alpha$ $for\ all\ x, y \in S\ and\ \alpha \in \Gamma$ [7]. In [8] Hamil and Majeed introduced the concept of a generalized strong (co)commutativity preserving right centralizers on a subset of a Γ-ring. An additive mapping $d: R \to R$ is called a derivation of a Γ-ring R if $d(x, y) = d(x)\alpha y + x\alpha d(y)$ $for\ all\ x, y\ \in R\ and\ \alpha \in \Gamma$. Let R be a Γ-ring, an additive mapping $d: R \to R$ is called a semi-derivation associated with a map $g: R \to R$, if every $x, y \in R$ and $\alpha \in \Gamma$, then $d(x, y) = d(x)\alpha g(y) + x\alpha d(y) = d(x)\alpha y + g(x)\alpha d(y)$ and $d(g(x)) = g(d(x))$. Also Γ-ring R is said to be 2-torsion free if $2x = 0,\ x \in R$ implies that $x = 0$ [9]. In this study, assumption the identity.

Let R be a Γ-ring additive maps $f, g: R \to R$ then

$$f(x)\alpha y\beta g(z) = g(x)\alpha y\beta f(z)\ \ for\ all\ x, y, z \in R\ and\ \alpha, \beta \in \Gamma \qquad (*).$$

We will extend the results of Bresar and Miers [10] to semiprime Γ-ring.

Now, we will present some new definitions and proven results.

Definition 1.1: Let R be a Γ-Ring, two maps $f, g: R \to R$ are said to be Γ-generalized strong commutativity preserving for short; (Γ-GSCP) on a set $S \subseteq R$ if

$[f(x), g(y)]_\alpha = [x, y]_\alpha \quad for\ all\ x, y\ \in S\ and\ \alpha \in \Gamma.$

Definition 1.2: Let R be a Γ-Ring, two maps $f, g: R \to R$ are said to be Γ-anti-generalized strong commutativity preserving for short; (Γ-AGSCP) on a set $S \subseteq R$ if

$[f(x), g(y)]_\alpha = [y, x]_\alpha \quad for\ all\ x, y \in S\ and\ \alpha \in \Gamma.$

Definition 1.3: Let N be a fixed positive integer and R be a Γ- Ring, two maps $f, g: R \to R$ are said to be Γ-N- generalized strong commutativity preserving for short; (Γ-N-GSCP) mapping on a set $S \subseteq R$ if

$$[f(x), g(y^n)]_\alpha = [x, y]_\alpha \quad for\ all\ \ x, y \in S\ and\ \alpha \in \Gamma.$$

Definition 1.4: Let N be a fixed positive integer and R be a Γ-Ring, two maps $f, g: R \rightarrow R$ are said to be Γ-N- anti–generalized strong commutativity preserving for short; (Γ-N-AGSCP) mapping on a set $S \subseteq R$ if

$$[f(x), g(y^n)]_\alpha = [y, x]_\alpha \quad for\ all\ x, y\ \in S\ and\ \alpha \in \Gamma.$$

Definition 1.5: Let R be a Γ-ring, A biadditive mapping $B: R\text{x}R \rightarrow R$ is called a biderivation if $B(x\alpha y, z) = B(x, z)\alpha y + x\alpha B(y, z)$ and $B(x, y\alpha z) = B(x, y)\alpha z + y\alpha B(x, z)\ for\ all\ \ x, y\ \in R\ and\ \alpha \in \Gamma$.

Definition 1.6: Let R be a Γ-ring, an element $x\ \in R$ is called an idempotent if $\alpha \in \Gamma$ such that $x^2 = x\alpha x = x$.

Theorem 1.7 [11]: Let R be a semiprime Γ-ring with extended centroid C_Γ and S be a set. Suppose that additive maps $f, g: S \rightarrow R$, satisfy (*). Then there exist idempotents $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ such that $\varepsilon_i\alpha\varepsilon_j = 0\ , for\ i \neq j,\ \ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1\ \ , \varepsilon_1\alpha f(s) = 0,\ \ \varepsilon_2\alpha g(s) = 0$ and $\varepsilon_3\alpha f(s) = \lambda\beta\varepsilon_3\alpha g(s)\ for\ all\ s \in S, \alpha, \beta \in \Gamma$ and for some invertible $\lambda \in C_\Gamma$, where C_Γ is the extended centroid of R.

Corollary 1.8 [11]: Let R be a semiprime Γ-ring and $a, b \in R$ satisfy $a\alpha x\beta b = b\alpha x\beta a$ for all $x \in R$ and $\alpha, \beta \in \Gamma$. Then there exist idempotents $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ such that $\varepsilon_i\alpha\varepsilon_j = 0,\ for\ i \neq j,\ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1\ , \varepsilon_1\alpha a = 0\ , \varepsilon_2\alpha b = 0\ and\ \varepsilon_3\alpha a = \lambda\beta\varepsilon_3\alpha b$ for some invertible $\lambda \in C_\Gamma$, where C_Γ is the extended centroid of R.

2. The Main Results

Lemma 2.1: Let R be a Γ-ring, and $B: R\text{x}R \rightarrow R$ be a biderivation, then $B(x, y)\beta z\gamma[u, v]_\alpha = [x, y]_\alpha\beta z\gamma B(u, v)\ for\ all\ x, y, z, u, v \in R\ and\ \alpha, \beta, \gamma \in \Gamma$.

Proof: We compute $B(x\alpha u, y\beta v)$ in two different ways.

$$B(x\alpha u, y\beta v) = B(x, y\beta v)\alpha u + x\alpha B(u, y\beta v) \quad for\ all\ x, y, u, v \in R\ and\ \alpha, \beta \in \Gamma. \quad (2.1)$$

It follows from (2.1) that

$$B(x\alpha u, y\beta v) = B(x, y)\beta v\alpha u + y\beta B(x, v)\alpha u + x\alpha B(u, y)\beta v + x\alpha y\beta B(u, v).$$

Analogously, we obtain

$$B(x\alpha u, y\beta v) = B(x\alpha u, y)\beta v + y\beta B(x\alpha u, v)$$
$$= B(x, y)\alpha u\beta v + x\alpha B(u, y)\beta v + y\beta B(x, v)\alpha u + y\beta x\alpha B(u, v)$$

Comparing $B(x\alpha u, y\beta v)$ in both computations, we arrive at

$$B(x, y)\beta[u, v]_\alpha = [x, y]_\alpha\beta B(u, v)\ for\ all\ x, y, u, v \in R\ and\ \alpha, \beta \in \Gamma. \quad (2.2)$$

Replacing u by $z\gamma u$ and using the relations

$[z\gamma u, v]_\alpha = [z, v]_\alpha\gamma u + z\gamma[u, v]_\alpha$ and $B(z\gamma u, v) = B(z, v)\gamma u + z\gamma B(u, v)$.

$$B(x, y)\beta([z, v]_\alpha\gamma u + z\gamma[u, v]_\alpha) = [x, y]_\alpha\beta(B(z, v)\gamma u + z\gamma B(u, v))$$
$$B(x, y)\beta[z, v]_\alpha\gamma u + B(x, y)\beta z\gamma[u, v]_\alpha = [x, y]_\alpha\beta B(z, v)\gamma u + [x, y]_\alpha\beta z\gamma B(u, v)$$

By (2.2), we get

$$B(x, y)\beta z\gamma[u, v]_\alpha = [x, y]_\alpha\beta z\gamma B(u, v)\ for\ all\ x, y, z, u, v \in R\ and\ \alpha, \beta, \gamma \in \Gamma.$$

We obtain the assertion of the Lemma.

Theorem 2.2: Let R be a semiprime Γ-ring with an extended centroid C_Γ, and let $B: R\text{x}R \rightarrow R$ be a biderivation. Then there exist an idempotent $\varepsilon \in C_\Gamma$ and an element $\mu \in C_\Gamma$ such that $(1 - \varepsilon)\alpha R \subseteq C_\Gamma$ and $\varepsilon\beta B(x, y) = \mu\gamma\varepsilon\beta[x, y]_\alpha\ \ for\ all\ x, y \in R$ and $\alpha, \beta, \gamma \in \Gamma$.

Proof: By Lemma (2.1) $B(x,y)\beta z\gamma[u,v]_\alpha = [x,y]_\alpha\beta z\gamma B(u,v)$ $for\ all\ x,y,z,u,v \in R\ and\ \alpha,\beta,\gamma \in \Gamma$.
Let $x,y \in R$ and $e = (1-\varepsilon)$,then
$e\alpha y\beta e\alpha x = e\alpha(x\beta e\alpha y) = e\alpha x\beta e\alpha y$. We get
$(1-\varepsilon)\alpha y\beta(1-\varepsilon)\alpha x = (1-\varepsilon)\alpha x\beta(1-\varepsilon)\alpha y$ $for\ all\ x,y \in R$ and $\alpha,\beta \in \Gamma$.
Then, $(1-\varepsilon)\alpha R \subseteq C_\Gamma$.
Now, let $S = R\text{x}R$ and define $A: S \to R$ by $A(x,y) = [x,y]_\alpha$. Note that the mappings $A,B: S \to R$. By Theorem (1.7), there exist mutually orthogonal idempotents $\varepsilon_1,\varepsilon_2,\varepsilon_3 \in C_\Gamma$ with sum 1 such that $for\ some\ \lambda \in C_\Gamma$, we have, $\varepsilon_1\beta B(x,y) = 0, \varepsilon_2\beta[x,y]_\alpha = 0$ and $\varepsilon_3\beta B(x,y) = \lambda\gamma\varepsilon_3\beta[x,y]_\alpha$ $for\ all\ x,y \in R\ and\ \alpha,\beta,\gamma \in \Gamma$. We set $\varepsilon = \varepsilon_3 + \varepsilon_1$, $\mu = \lambda\gamma\varepsilon_3$, and note that ε and μ have desired properties.

Corollary 2.3: Let R be a semiprime Γ-ring with extended centroid C_Γ, and let $f: R \to R$ be a commuting additive mapping. Then there exists $\lambda \in C_\Gamma$ such that $f(x) = \lambda\alpha x + \xi(x)$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$ where an additive mapping $\xi: R \to C_\Gamma$.

Proof: Linearizing $[f(x),x]_\alpha = 0$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$, we obtain
$[f(x),y]_\alpha = [x,f(y)]_\alpha$. Hence, we see that the mapping $(x,y) \to [f(x),y]_\alpha$ is a biderivation. By Theorem (2.2) there exists an idempotent $\varepsilon \in C_\Gamma$ and an element $\mu \in C_\Gamma$ such that $(1-\varepsilon)\alpha R \subseteq C_\Gamma$, and $\varepsilon\alpha[f(x),y]_\alpha = \mu\gamma\varepsilon\alpha[x,y]_\alpha$ holds $for\ all\ x,y \in R\ and\ \alpha,\gamma \in \Gamma$. We have
$\varepsilon\alpha f(x)\alpha y - \varepsilon\alpha y\alpha f(x) = \mu\gamma\varepsilon\alpha x\alpha y - \mu\gamma\varepsilon\alpha y\alpha x$
$\varepsilon\alpha f(x)\alpha y - \mu\gamma\varepsilon\alpha x\alpha y = \varepsilon\alpha y\alpha f(x) - \mu\gamma\varepsilon\alpha y\alpha x$
$(\varepsilon\alpha f(x) - \mu\gamma\varepsilon\alpha x)\alpha y - y\alpha(\varepsilon\alpha f(x) - \mu\gamma\varepsilon\alpha x) = 0$
Thus, $\varepsilon\alpha f(x) - \mu\gamma\varepsilon\alpha x \in C_\Gamma$. Now, let $\lambda = \mu\gamma\varepsilon$ and define a mapping ξ by
$\xi(x) = (\varepsilon\alpha f(x) - \lambda\alpha x) + (1-\varepsilon)\alpha f(x)$. Note that ξ maps in C_Γ and that
$\xi(x) + \lambda\alpha x = \varepsilon\alpha f(x) + 1\alpha f(x) - \varepsilon\alpha f(x)$, then
$f(x) = \lambda\alpha x + \xi(x)$ holds for every $x \in R\ and\ \alpha \in \Gamma$.

Proposition 2.4: Let R be a 2-torsion free semiprime Γ-ring with extended centroid C_Γ, and S be a subring of R, if $f: R \to R$ a centralizing additive mapping of S, then f commuting of S .

Proof: A Linearizing of $[f(x),x]_\alpha \in Z$, we obtain
. In particular, $[f(x),x^2]_\alpha + [f(x),y]_\alpha + [f(y),x]_\alpha \in Z$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$
$[f(x^2),x]_\alpha \in Z$. Since $[f(x),x]_\alpha \in Z$, we have $[f(x),x^2]_\alpha = 2[f(x),x]_\alpha\alpha x$. So,
(2.3)$2[f(x),x]_\alpha\alpha x + [f(x^2),x]_\alpha \in Z$ $for\ all\ x \in S\ and\ \alpha \in \Gamma$.
By assumption, $[f(x^2),x^2]_\alpha \in Z$ $for\ all\ x \in S\ and\ \alpha \in \Gamma$. That is
Now $for\ all\ x \in S\ and\ \alpha \in \Gamma$. (2.4)$[f(x^2),x]_\alpha\alpha x + x\alpha[f(x^2),x]_\alpha \in Z$
fix $x \in U$ and let $z = [f(x),x]_\alpha \in Z$, $u = [f(x^2),x]_\alpha$. We must show that $z = 0$. By (2.3) we have
$0 = [f(x),2z\alpha x + u]_\alpha = [f(x),2z\alpha x]_\alpha + [f(x),u]_\alpha = 2z\alpha[f(x),x]_\alpha + [f(x),u]_\alpha = 2z^2 + [f(x),u]_\alpha$. So,
$[f(x),u]_\alpha = -2z^2$ $for\ all\ x \in U\ and\ \alpha \in \Gamma$. (2.5)
According to (2.4) .
We have $0 = [f(x),u\alpha x + x\alpha u]_\alpha = [f(x),u\alpha x]_\alpha + [f(x),x\alpha u]_\alpha =$
$[f(x),u]_\alpha\alpha x + u\alpha[f(x),x]_\alpha + [f(x),x]_\alpha\alpha u + x\alpha[f(x),u]_\alpha$, applying (2.5)
$-2z^2\alpha x + u\alpha z + z\alpha u - 2x\alpha z^2 = 0$
we then get $-4z^2\alpha x + 2z\alpha u = 0$. So, $z\alpha u = 2z^2\alpha x$. Multiplying (2.5) by z and using the last relation we obtain $-2z^3 = [f(x),2z^2\alpha x]_\alpha = 2z^3$. As result $z^3 = 0$. Since the center of a

semiprime Γ-ring contains no nonzero nilpotents, we conclude that $z = [f(x), x]_\alpha = 0$. then f commuting .

Corollary 2.5: Let R be a 2-torsion free semiprime Γ-ring with extended centroid C_Γ, and let $f: R \to R$ be a centralizing additive mapping. Then there exists $\lambda \in C_\Gamma$ such that $f(x) = \lambda\alpha x + \xi(x)$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$ where an additive mapping $\xi: R \to C_\Gamma$.

Proof: Combining Proposition (2.4) and Corollary (2.3), we get $f(x) = \lambda\alpha x + \xi(x)$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Corollary 2.6: Let R be a semiprime Γ-ring with extended centroid C_Γ, and let $f: R \to R$ be a centralizing additive mapping. If either R has a 2-torsion free or f is commuting on R. Then there exists $\lambda \in C_\Gamma$ such that $f(x) = \lambda\alpha x + \xi(x)$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$ where an additive mapping $\xi: R \to C_\Gamma$.
We begin with technical lemma.

Lemma 2.7: Let A be the ideal of Γ-ring R generated by all commutators in R. Suppose that $(\lambda_0\gamma\mu_0 - 1)\alpha A = 0$ for some $\lambda_0, \mu_0 \in C_\Gamma\ and\ \alpha, \gamma \in \Gamma$. Then there exists an invertible element $\lambda \in C_\Gamma$ such that $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$ and $(\lambda^{-1} - \mu_0)\alpha R \subseteq C_\Gamma$. Moreover, if $\lambda_0 = \mu_0$, then $\lambda = \lambda^{-1}$.

Proof: Since $Ann(A)$ be a closed ideal then there exists an idempotent $\varepsilon \in C_\Gamma$ such that $Ann(A) = \varepsilon\alpha R = \varepsilon\alpha Q \cap R$. Define $\lambda, \mu \in C_\Gamma$ by $\lambda = \lambda_0\alpha(1-\varepsilon) + \varepsilon$, $\mu = \mu_0\alpha(1-\varepsilon) + \varepsilon$. Whence $(\lambda\gamma\mu - 1) = (\lambda_0\gamma\mu_0 - 1)\alpha(1-\varepsilon)$ which yields $(\lambda\gamma\mu - 1)\alpha(A \oplus Ann(A)) = 0$, for some $\lambda_0, \mu_0 \in C_\Gamma\ and\ \alpha, \gamma \in \Gamma$ $(\lambda_0\gamma\mu_0 - 1)\alpha A = 0$ and $(1-\varepsilon)\alpha Ann(A) = 0$. Since $A \oplus Ann(A)$ is an essential ideal of Γ-ring R it follows that $\lambda\gamma\mu - 1 = 0$, that is, $\mu = \lambda^{-1}$. Clearly, $\lambda_0 = \mu_0$ implies $\lambda = \mu = \lambda^{-1}$. We claim that $\varepsilon\gamma R \subseteq C_\Gamma$. Indeed, there exists an essential ideal E such that $\varepsilon\alpha E \subseteq R$. So, $\varepsilon\alpha E \subseteq \varepsilon\alpha Q \cap R = Ann(A)$, that is, $A\gamma\varepsilon\alpha E = 0$ which gives $\varepsilon\gamma A = 0$; thus, $[\varepsilon\gamma R, R]_\alpha = \varepsilon\gamma[R, R]_\alpha = 0$ which shows that $\varepsilon\gamma R \subseteq C_\Gamma$ Therefore, as $\lambda - \lambda_0 = (1-\lambda_0)\alpha\varepsilon$, we see that $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$. Similarly, we have $(\lambda^{-1} - \mu_0)\alpha R \subseteq C_\Gamma$.

Theorem 2.8: Let R be a semiprime Γ-ring with extended centroid C_Γ. Suppose that an additive map $f: R \to R$ is Γ-SCP. Then $f(x) = \lambda\alpha x + \xi(x)$ where $\lambda \in C_\Gamma$, $\lambda^2 = 1$ and an additive mapping $\xi: R \to C_\Gamma$.

Proof: Our first goal is to prove that f is commuting. For $x, y \in R\ and\ \alpha, \beta \in \Gamma$, we have
$[f(y^2), [y, x]_\alpha]_\beta = [f(y^2), [f(y), f(x)]_\alpha]_\beta$
$= [f(x), [f(y), f(y^2)]_\alpha]_\beta + [f(y), [f(y^2), f(x)]_\alpha]_\beta$, by (Γ-SCP) map
$= [f(x), [y, y^2]_\alpha]_\beta + [f(y), [y^2, x]_\alpha]_\beta = [f(y), [y^2, x]_\alpha]_\beta$.Thus,

$$[f(y^2), [y, x]_\alpha]_\beta = [f(y), [y^2, x]_\alpha]_\beta \quad for\ all\ x, y \in R\ and\ \alpha, \beta \in \Gamma \tag{2.6}$$

Replacing x by $y\beta x$ in both sides (2.6), we get
$[f(y^2), [y, y\beta x]_\alpha]_\beta = [f(y^2), y\beta[y, x]_\alpha]_\beta = [f(y^2), y]_\beta\beta[y, x]_\alpha + y\beta[f(y^2), [y, x]_\alpha]_\beta$.
And
$[f(y), [y^2, y\beta x]_\alpha]_\beta = [f(y), y\beta[y^2, x]_\alpha]_\beta = [f(y), y]_\beta\beta[y^2, x]_\alpha + y\beta[f(y), [y^2, x]_\alpha]_\beta$.
Comparing both results and by using (2.6), we arrive at

$$[f(y^2), y]_\beta\beta[y, x]_\alpha = [f(y), y]_\beta\beta[y^2, x]_\alpha \quad for\ all\ x, y \in R\ and\ \alpha, \beta \in \Gamma \tag{2.7}$$

Replacing x by $x\alpha z$, $z \in R$ in (2.7),
$[f(y^2), y]_\beta\beta[y, x\alpha z]_\alpha = [f(y), y]_\beta\beta[y^2, x\alpha z]_\alpha \quad for\ all\ x, y, z \in R\ and\ \alpha, \beta \in \Gamma$.

We obtain
$[f(y^2),y]_\beta \beta x\alpha [y,z]_\alpha = [f(y),y]_\beta \beta x\alpha [y^2,z]_\alpha$
$for\ all\ x,y,z \in R\ and\ \alpha,\beta \in \Gamma$ (2.8)
Replacing y by $f(r)$, $r \in R$ in (2.6), thus we obtain
$[f(f(r)^2),f(r)]_\beta \beta x\alpha [f(r),z]_\alpha = [f(f(r)),f(r)]_\beta \beta x\alpha [f(r)^2,z]_\alpha$
According to (Γ-SCP) map, we get
$[f(r)^2,r]_\beta \beta x\alpha [f(r),z]_\alpha = [f(r),r]_\beta \beta x\alpha [f(r)^2,z]_\alpha$
$for\ all\ x,z,r \in R\ and\ \alpha,\beta \in \Gamma$ (2.9)
Now fix $r \in R$ and we show that $[f(r),r]_\alpha = 0$. As a special case of (2.9), we have

$[f(r)^2,r]_\beta \beta x\alpha [f(r),r]_\alpha = [f(r),r]_\beta \beta x\alpha [f(r)^2,r]_\alpha$
$for\ all\ x,r \in R\ and\ \alpha,\beta \in \Gamma$ (2.10)
Applying Corollary (1.8), we see that there are mutually orthogonal idempotents $\varepsilon_1,\varepsilon_2,\varepsilon_3 \in C_\Gamma$ such that $\varepsilon_1+\varepsilon_2+\varepsilon_3 = 1$, $\varepsilon_1\beta[f(r),r]_\alpha = 0$, $\varepsilon_2\beta[f(r)^2,r]_\alpha = 0$, $\varepsilon_3\beta[f(r)^2,r]_\alpha = v\alpha\varepsilon_3\beta[f(r),r]_\alpha$, for some invertible $v \in C_\Gamma$. By (2.9) we thus obtain

$$\begin{aligned}[f(r),r]_\beta \beta x\alpha [f(r)^2,z]_\alpha &= (\varepsilon_1+\varepsilon_2+\varepsilon_3)\beta[f(r),r]_\beta \beta x\alpha [f(r)^2,z]_\alpha \\ &= (\varepsilon_2+\varepsilon_3)\beta[f(r),r]_\beta \beta x\alpha [f(r)^2,z]_\alpha \\ &= (\varepsilon_2+\varepsilon_3)\beta[f(r)^2,r]_\beta \beta x\alpha [f(r),z]_\alpha \\ &= (\varepsilon_3)\beta[f(r)^2,r]_\beta \beta x\alpha [f(r),z]_\alpha \\ &= v\alpha\varepsilon_3\beta[f(r),r]_\beta \beta x\alpha [f(r),z]_\alpha\end{aligned}$$

Setting $\mu = v\alpha\varepsilon_3$, we thus have
$[f(r),r]_\alpha \beta x\alpha [f(r)^2 - \mu\beta f(r),z] = 0\ for\ all\ x,z \in R\ and\ \alpha,\beta \in \Gamma$.
That is, $[f(r)^2 - \mu\beta f(r),R]_\alpha \subseteq I$, where $I = \{q \in Q: [f(r),r]_\alpha Rq = 0\}$. Of course, I is a right ideal of Q.
Now, for any $z \in R$, we have
$\mu\beta[r,z]_\alpha - f(r)\beta[r,z]_\alpha - [r,z]_\alpha\beta f(r)$
$= \mu\beta[f(r),f(z)]_\alpha - f(r)\beta[f(r),f(z)]_\alpha - [f(r),f(z)]_\alpha\beta f(r)$
$= [\mu\beta f(r),f(z)]_\alpha - [f(r)^2,f(z)]_\alpha = [\mu\beta f(r) - f(r)^2,f(z)]_\alpha$
which shows that
$\mu\beta[r,z]_\alpha - f(r)\beta[r,z]_\alpha - [r,z]_\alpha\beta f(r) \in I\ .for\ all\ r,z \in R\ and\ \alpha,\beta \in \Gamma.$ (2.11)
Replacing z by $z\alpha r$ in (2.11), we get

$$\mu\beta[r,z]_\alpha\alpha r - f(r)\beta[r,z]_\alpha\alpha r - [r,z]_\alpha\alpha r\beta f(r) \in I.$$

On the other hand, since I is a right ideal, we have

$$\big(\mu\beta[r,z]_\alpha - f(r)\beta[r,z]_\alpha - [r,z]_\alpha\beta f(r)\big)\alpha r \in I.$$

Comparing the last two relations we get $[r,z]_\alpha\beta[f(r),r]_\alpha \in I\ \ for\ all\ r,z \in R\ and\ \alpha,\beta \in \Gamma$.
That is,
$[f(r),r]_\alpha\beta R\beta[r,z]_\alpha\beta[f(r),r]_\alpha = 0\ \ for\ all\ r,z \in R\ and\ \alpha,\beta \in \Gamma$. (2.12)
Replacing z by $f(r)\beta z$ and using $[r,f(r)\beta z]_\alpha = [r,f(r)]_\alpha\beta z + f(r)\beta[r,z]_\alpha$
it follows at once that
$[f(r),r]_\alpha\beta R\beta[r,f(r)]_\alpha\beta R\beta[f(r),r]_\alpha = 0$.
Since R is semiprime Γ-ring it follows that $[f(r),r]_\alpha = 0\ for\ all\ r \in R\ and\ \alpha \in \Gamma$. Thus we proved that f is commuting.

According to Corollary (2.3), we have $f(x) = \lambda_0 \alpha x + \xi_0(x),\ x \in R$ and $\in \Gamma$, where $\lambda_0 \in C_\Gamma$ and ξ_0 is an additive map of R into C_Γ. Therefore, the relation
$[f(x), f(y)]_\alpha = [x, y]_\alpha$ can be rewritten as
$({\lambda^2}_0 - 1)\alpha[x, y]_\alpha = 0$, which shows that $({\lambda^2}_0 - 1)\alpha A = 0$.
By the Lemma (2.7) there is $\lambda \in C_\Gamma$ such that $\lambda^2 = 1$ and $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$. For any $x \in R$ and $\alpha \in \Gamma$, we thus have
$f(x) = \lambda_0 \alpha x + \xi_0(x) = \lambda \alpha x + (\lambda_0 - \lambda)\alpha x + \xi_0(x) = \lambda \alpha x + \xi(x)$
where $\xi(x) = (\lambda_0 - \lambda)\alpha x + \xi_0(x) \in C_\Gamma$, This proves the theorem.
Assuming that f is onto then even a stronger result can be easily obtained.

Theorem 2.9: Let R be a semiprime Γ-ring with extended centroid C_Γ. Suppose that an additive maps $f, g: R \to R$ are Γ-GSCP. If f is onto, then there exists an invertible element $\lambda \in C_\Gamma$ and an additive maps $\xi, \eta: R \to C_\Gamma$. Such that $g(x) = \lambda \alpha x + \xi(x)$, $f(x) = \lambda^{-1} \alpha x + \eta(x)$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Proof: Define a biadditive map $B: R \times R \to R$ by $B(x, y) = [x, g(y)]_\alpha$. Clearly, B is a derivation in the first argument. Pick $x_0 \in R$; as f is onto, we have $x_0 = f(x_1)$ for some $x_1 \in R$. Thus $B(x_0, y) = [f(x_1), g(y)]_\alpha = [x_1, y]_\alpha$. This shows that B is a biderivation. By Theorem (2.2) there are $\varepsilon, \mu \in C_\Gamma$, ε an idempotent, such that $(1 - \varepsilon)\alpha R \subseteq C_\Gamma$, $\varepsilon\alpha[x, g(y)]_\alpha = \varepsilon\alpha\mu\beta[x, y]_\alpha$, $for\ all\ x, y \in R\ and\ \alpha, \beta \in \Gamma$. Thus,
$[R, \varepsilon\alpha g(y) - \varepsilon\alpha\mu\beta y]_\alpha = 0$ and $\varepsilon\alpha g(y) - \varepsilon\alpha\mu\beta y \in C_\Gamma$ $for\ all\ y \in R\ and\ \alpha, \beta \in \Gamma$. Whence
$g(y) - \varepsilon\alpha\mu\beta y = (\varepsilon\alpha g(y) - \varepsilon\alpha\mu\beta y) + (1 - \varepsilon)\alpha g(y) \in C_\Gamma$,
and so $g(y) = \lambda_0 \alpha y + \xi_0(y)$ where $\lambda_0 = \varepsilon\alpha\mu \in C_\Gamma$,
$\xi_0(y) = g(y) - \varepsilon\alpha\mu\beta y \in C_\Gamma$. By (Γ-GSCP) map it now follows that
$[x, f(x)]_\alpha = [f(x), g(f(x))]_\alpha = 0\ for\ all x \in R\ and\ \alpha \in \Gamma$; that is f is commuting. Therefore, f is of the form $f(x) = \mu_0 \alpha x + \eta_0(x)$, $\mu_0 \in C_\Gamma, \eta_0(x) \in C_\Gamma$.
By $[f(x), g(y)]_\alpha = [x, y]_\alpha$ it now follows at once that $(\lambda_0 \beta \mu_0 - 1)\alpha A = 0$.
By the Lemma (2.7), there is an invertible $\lambda \in C_\Gamma$ such that $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$,
$(\lambda^{-1} - \mu_0)\alpha R \subseteq C_\Gamma$. Whence
$f(x) = \mu_0 \alpha x + \eta_0(x) = \lambda^{-1}\alpha x + (\mu_0 - \lambda^{-1})\alpha x + \eta_0(x) = \lambda^{-1}\alpha x + \eta(x)$
$g(x) = \lambda_0 \alpha x + \xi_0(x) = \lambda \alpha x + (\lambda_0 - \lambda)\alpha x + \xi_0(x) = \lambda \alpha x + \xi(x)$
where $\eta(x) = (\mu_0 - \lambda^{-1})\alpha x + \eta_0(x) \in C_\Gamma$, $\xi(x) = (\lambda_0 - \lambda)\alpha x + \xi_0(x) \in C_\Gamma$.

Theorem 2.10: Let R be a semiprime Γ-ring with extended centroid C_Γ, N be a fixed positive integer and suppose that an additive maps $f\ and\ g: R \to R$ such that f is onto, if one of these conditions is fulfilled:
(*i*) $f\ and\ g$ *belong to Γ-N-GSCP on* R.
(*ii*) $f\ and\ g$ *belong to Γ-N-AGSCP on* R.
Then there exists an element $\lambda \in C_\Gamma$ and additive maps $\xi, \eta_1 and\ \eta_2 : R \to C_\Gamma$ such that $g(x^n) = \lambda \alpha x + \xi(x)$ and $f(x) = \lambda^{-1}\alpha x + \eta_1(x)$ when condition (*i*) is satisfied, and $f(x) = -\lambda^{-1}\alpha x + \eta_2(x)$ when condition (*ii*) is satisfied $for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Proof: Suppose that $f\ and\ g\ be$ Γ-N-GSCP $on\ R$.
Define a biadditive map $B: R \times R \to R$ by
$$B(x, y) = [x, g(y^n)]_\alpha\ for\ all\ x, y \in R\ and\ \alpha \in \Gamma. \quad (2.13)$$
It is clear that B is a derivation in the first argument. Pick $x_0 \in R$; $as\ f\ is\ onto$, we have $x_0 = f(x_1)\ \ for\ some\ x_1 \in R$.
Thus $B(x_0, y) = [f(x_1), g(y^n)]_\alpha = [x_1, y]_\alpha$, this shows that B is a derivation in the second argument, so B is a biderivation on R.

By Theorem (2.2), there are $\varepsilon, \mu \in C_\Gamma$, ε an idempotent , such that
$(1-\varepsilon)\alpha R \subseteq C_\Gamma$ and $\varepsilon\beta[x, g(y^n)]_\alpha = \varepsilon\beta\mu\gamma[x, y]_\alpha$ $for\ all\ x, y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. Thus $[R, \varepsilon\alpha g(y^n) - \varepsilon\beta\mu\gamma y] = 0$ and so $(\varepsilon\alpha g(y^n) - \varepsilon\beta\mu\gamma y) \in R$ $for\ all\ y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. When

$$g(y^n) - \varepsilon\beta\mu\gamma y = (\varepsilon\alpha g(y^n) - \varepsilon\beta\mu\gamma y) + (1-\varepsilon)g(y^n) \in C_\Gamma$$

$for\ all\ y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. (2.14)
So we have
$g(y^n) = \lambda_0\alpha y + \xi_0(y)$ where $\lambda_0 = \varepsilon\beta\mu$ and $\xi_0(y) = g(y^n) - \varepsilon\beta\mu\gamma y \in C_\Gamma$
$for\ all\ y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. (2.15)
By condition (i), we have
$[x, f(x)]_\alpha = [f(x), g(f(x^n))]_\alpha$ $for\ all\ x, y \in R\ and\ \alpha \in \Gamma$. (2.16)
From (2.15) and (2.16), we get

$$[x, f(x)]_\alpha = [f(x), \lambda_0\alpha f(x) + \xi_0(f(x))]_\alpha = 0\ for\ all\ x \in R\ and\ \alpha \in \Gamma .$$

That is f is commuting on R. So, by Corollary (2.6), f is of the form
$f(x) = \mu_0\alpha x + \eta_0(x)$, where μ_0 and $\eta_0(x) \in C_\Gamma$ $for\ all\ x \in R\ and\ \alpha \in \Gamma$.
Substituting (2.15) and (2.16) in condition (i), we get
$(\lambda_0\gamma\mu_0 - 1)\beta\, [x, y]_\alpha = 0$ $for\ all\ x, y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. (2.17)
It follows that $(\lambda_0\gamma\mu_0 - 1)\alpha A = 0$ where A be the ideal of R generated by all commutators in R, By Lemma (2.7) ,there is an invertible element $\lambda \in C_\Gamma$ such that $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$ and $(\lambda^{-1} - \mu_0)\alpha R \subseteq C_\Gamma$, when
$f(x) = \mu_0\alpha x + \eta_0(x) = \lambda^{-1}\alpha x + (\mu_0 - \lambda^{-1})\alpha x + \eta_0(x) = \lambda^{-1}\alpha x + \eta_1(x)$
$g(x^n) = \lambda_0\alpha y + \xi_0(x) = \lambda\alpha x + (\lambda_0 - \lambda)\alpha x + \xi_0(x) = \lambda\alpha x + \xi(x)$.
Where $\eta_1(x) = (\mu_0 - \lambda^{-1})\alpha x + \eta_0(x) \in C_\Gamma, \xi(x) = (\lambda_0 - \lambda)\alpha x + \xi_0(x) \in C_\Gamma$.
Suppose that $f\ and\ g\ be$ Γ-N-AGSCP $on\ R$.
Define a biadditive map $B: R \times R \to R$ by

$$B(x, y) = [g(y^n), x]_\alpha\ for\ all\ x, y \in R\ and\ \alpha \in \Gamma.$$

By similar argument then used to prove the theorem when condition (*i*) is satisfied, we can get $g(x^n) = \lambda_0\alpha y + \xi_0(x)\ where\ \lambda_0 \in C_\Gamma$, and $\xi_0(y) \in C_\Gamma$ $for\ all\ y \in R$, and $f(x) = \mu_0\alpha x + \eta_0(x)$
Where $\mu_0 \in C$, $\eta_0(x) \in C_\Gamma\ for\ all\ x \in R$. Thus from condition (*ii*), we get
$(\lambda_0\gamma\mu_0 - 1)\beta\, [x, y]_\alpha = 0$ $for\ all\ y \in R\ and\ \alpha, \beta, \gamma \in \Gamma$. By Lemma (2.7), there is an invertible element $\lambda \in C_\Gamma$ such that $(\lambda - \lambda_0)\alpha R \subseteq C_\Gamma$ and $(\lambda^{-1} + \mu_0)\alpha R \subseteq C_\Gamma$, when
$f(x) = \mu_0\alpha x + \eta_0(x) = -\lambda^{-1}\alpha x + (\mu_0 + \lambda^{-1})\alpha x + \eta_0(x) = -\lambda^{-1}\alpha x + \eta_2(x)$
$g(x^n) = \lambda_0\alpha x + \xi_0(x) = \lambda\alpha x + (\lambda_0 - \lambda)\alpha x + \xi_0(x) = \lambda\alpha x + \xi(x)$.
Where $\eta_2(x) = (\mu_0 + \lambda^{-1})\alpha x + \eta_0(x) \in C_\Gamma$ and $\xi(x) = (\lambda_0 - \lambda)\alpha x + \xi_0(x) \in C_\Gamma$.

3. Applications

Lemma 3.1: Let R be a semiprime Γ-ring and $a, b \in R$ such that $a\alpha x\beta b = b\alpha x\beta a$ $for\ all\ x \in R\ and\ \alpha, \beta \in \Gamma$. If $a \neq 0$, then $a = \lambda\alpha b$ for some λ in the extended centroid C_Γ of *R*.

Proof: Thus, elements a and b satisfy the requirements of Corollary (1.8). Therefore, there exist mutually orthogonal idempotents $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ such that for every $x \in R$ we have $\varepsilon_1\alpha g(x) = 0$, $\varepsilon_2\alpha f(x) = 0$ and $\varepsilon_3\alpha g(x) = \lambda\beta\varepsilon_3\alpha f(x)$ where an invertible element $\lambda \in C_\Gamma$. Let $\varepsilon_1 = 0, \varepsilon_2 = 0, \varepsilon_3 = 1$, note that $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ satisfies the assertion of the Corollary.

Lemma 3.2: Let R be a semiprime Γ-ring. If functions $f: R \to R$ and $g: R \to R$ are such that $(x)\alpha y\beta g(z) = g(x)\alpha y\beta f(z)\ for\ all\ x, y, z \in R\ and\ \alpha, \beta \in \Gamma$, if $f \neq 0$, then there exists λ in the extended centroid C_Γ of R such that $g(x) = \lambda\alpha f(x)\ for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Proof: By Theorem (1.7), there exist mutually orthogonal idempotents $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ such that for every $x \in R$ we have $\varepsilon_1 \alpha g(x) = 0, \varepsilon_2 \alpha f(x) = 0$ and $\varepsilon_3 \alpha g(x) = \lambda \beta \varepsilon_3 \alpha f(x)$ where λ is an invertible element in C_Γ. Let $\varepsilon_1 = 0, \varepsilon_2 = 0, \varepsilon_3 = 1$, note that $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in C_\Gamma$ satisfies assertion of the theorem.

Theorem 3.3: Let R be a semiprime Γ-ring. If f is a semiderivation of R (with associated function) then either f is a derivation or $f(x) = \lambda\alpha(1-g)(x)\ for\ all\ x \in R$ and $\alpha \in \Gamma, \lambda \in C_\Gamma$ where C_Γ the extended centroid of R and g is an endomorphism.

Proof: We may assume that $f \neq 0$. In this state, g is a Γ-ring endomorphism. Note that $f(x\alpha y) = f(x)\alpha g(y) + x\alpha f(y) = f(\mathrm{x})\alpha y + g(x)\alpha f(y)$
can be written in the form
$(1-g)(x)\alpha f(y) = f(\mathrm{x})\alpha(1-g)(y)\ \ for\ all\ x, y \in R\ and\ \alpha \in \Gamma.$
In particular, $(1-g)(x)\alpha f(y\beta z) = f(\mathrm{x})\alpha(1-g)(y\beta z)\ \ for\ all\ x, y, z \in R,$ and $\alpha, \beta \in \Gamma$.
But on the other hand,

$$(1-g)(x)\alpha f(y\beta z) = (1-g)(x)\alpha f(\mathrm{y})\beta g(z) + (1-g)(x)\alpha y\beta f(z),$$

and
$f(x)(1-g)(y\beta z) = f(x)\alpha(1-g)(\mathrm{y})\beta g(z) + f(x)\alpha y\beta(1-g)(z).$
Comparing the last two relations and applying
$(1-g)(x)\alpha f(y) = f(x)\alpha(1-g)(y).$
We get
$(1-g)(x)\alpha y\beta f(z) = f(x)\alpha y\beta(1-g)(z)\ \ \ for\ all\ x, y, z \in R\ and\ \alpha, \beta \in \Gamma.$
If $g = 1$ then f is a derivation; therefore, we may ssume that $1 - g \neq 0$ and so the assertion of the theorem follows immediately from the lemma (3.2), i.e., $(x) = \lambda\alpha f(x)\ for\ all\ x \in R\ and\ \alpha \in \Gamma$. Replacing $g(x)$ by $f(x)$ and $f(x)$ by $(1-g)(x)$ We get $f(x) = \lambda\alpha(1-g)(x)\ for\ all\ x \in R\ and\ \alpha \in \Gamma$.

Corollary 3.4: Let R be a semiprime Γ-ring, and let $f: R \to R$ be additive mapping. If g a Γ-ring endomorphism of R. Then there exists $\lambda \in C_\Gamma$ where C_Γ the extended centroid of R and an additive mapping $\xi: R \to C_\Gamma$ such that $f(x) = \lambda\alpha x + \xi(x)\ \ for\ all\ x \in R\ and\ \alpha \in \Gamma.$

Proof: Application (*), the identity

$$f(x)\alpha x\beta g(x) = g(x)\alpha x\beta f(x)\ \ for\ all\ x \in R\ and\ \alpha, \beta \in \Gamma.$$

By Theorem (3.3), every semi-derivation f of a prime Γ-ring R is either a derivation, or it is of the form
$f(x) = \lambda\alpha(1-g)(x),$ where $\lambda \in C_\Gamma$ and g a Γ-ring endomorphism of R.
We get

$$f(x) = \lambda\alpha(1-g)(x) = \lambda\alpha x + \lambda\alpha g(x) = \lambda\alpha x + \xi(x)$$

where $\xi(x) = \lambda\alpha g(x)$. Then

$$f(x) = \lambda\alpha x + \xi(x)\ \ for\ all\ x \in R\ and\ \alpha \in \Gamma.$$

References

[1] N. Nobusawa, "On A Generalization of The Ring Theory", *Osaka J.Math.*; vol. 1, pp. 81–89, 1964.
[2] W. E. Barnes, "On The Γ-rings of Nobusawa" *Pacific J. Math.*; vol. 18, no. 3, pp. 411–422, 1966.
[3] M. A. Öztürk, M. Sapanci, M. Soytürk, and K. H. Kim, "Symmetric Bi-Derivation on Prime Gamma Rings" *Scientiae Mathematicae*; vol. 3, no. 2, pp. 273-281, 2000.
[4] M. A. ¨Ozt¨urk and Y. B. Jun, "On The Centroid of The Prime Gamma Rings II" *Turkish J. Math.*; vol. 25, pp. 367-377, 2001.

[5] A. H. Majeed and S. k. Motashar, "Γ-Centralizing Mappings of Semiprime Γ-rings" *Iraqi Journal of Science*; vol. 53, no. 3, pp. 657-662, 2012.
[6] H. E. Bell and G.Mason, "On Derivations in Near Rings and Rings" *Math. J. Okayama Univ*.; vol. 34, pp. 135–144, 1992.
[7] O. Arslan and B. Arslan, "Strong Commutativity Preserving Derivations on Lie Ideals of Prime Γ-Rings" *Mathematica Moravica*; vol. 23 , no. 1, pp. 63–73, 2019.
[8] S. A. Hamil and A. H. Majeed, "Generalized Strong Commutativity Preserving Centralizers of Semiprime Γ- Rings" *Iraqi Journal of Science*; vol. 60, no. 10, pp. 2223-2228, 2019.
[9] K. H. KIM and Y. H. LEE, "On Generalized Semiderivations of Γ-Rings" *Electronic Journal of Mathematical Analysis and Applications*; vol. 5, no. 2, pp. 18-27, 2017.
[10] M. Brešar and C. R. Miers, "Strong Commutativity Preserving Maps of Semiprime Rings" *Canad. Math. Bull.*; vol. 37, no. 4, pp. 457–460, 1994.
[11] A. A. Abdulridha and A. H. Majeed, "N-(anti) Strong Commutativity Preserving Maps on Semiprime Γ-Rings" *AIP Conference Proceedings*; Accepted for publication.