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# Two-Component Generalization of a Generalized the Short Pulse Equation

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**Abstract**
In this article, we introduce a two-component generalization for a new generalization type of the short pulse equation was recently found by Hone and his collaborators. The coupled of nonlinear equations is analyzed from the viewpoint of Lie's method of a continuous group of point transformations. Our results show the symmetries that the system of nonlinear equations can admit, as well as the admitting of the three-dimensional Lie algebra. Moreover, the Lie brackets for the independent vectors field are presented. Similarity reduction for the system is also discussed.

**Keywords:** Generalized the short pulse equation, Two-component generalization, Lie symmetry analysis, Similarity reduction.

# تعميم مكون من مركبتين لتعميم من معادلة النبضة القصيرة

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**الخلاصة**
في هذا البحث، نقدم تعميم مكون من مركبتين لتعميم جديد من معادلة النبضة القصيرة التي وجدت مؤخرا من قبل هون وزملاؤه. يتم تحليل النظام المكون من مركبتين من المعادلات التفاضلية الجزئية غير الخطية من الرتبة الثانية من وجهة نظر طريقة لي لمجموعة مستمرة من التحويلات النقطية. تظهر نتائجنا التماثلات التي يمكن أن يعترف بها نظام المعادلات غير الخطية وكذلك جبر لي ثلاثي الابعاد. بالإضافة الى ذلك تباديل لي لحقل المتجهات المستقلة تم تقديمها والتخفيض المتماثل للنظام ايضا تم مناقشته.

## 1. Introduction

The short pulse equation appears in many fields of sciences, and it becomes a source of interest in nonlinear optics, among others. The derivation of the equation came out in the differential geometry in the work of [1, 2]. The equation is given by

$$u_{xt} = u + \frac{1}{6}(u^3)_{xx}\,, \tag{1}$$

where $u$ is a function of two independent variables $x$ and $t$, and the subscripts denote to the partial derivatives regarding independent variables. Sakovich [3] brought in a generalized version of equation (1), and later he gave a generalization for the generalized equation [4]. The super extensions of equation (1) are inspected in [5]. Pietrzyk el al. [6] suggested a vector generalization of the short pulse type equation, which is later studied in [7]. A long the same line, Mastsuno [8] came up with a multi-component type of equation (1), and the author also succeeded in getting some type of solutions. Feg [9] also derived an integrable coupled short pulse equations, and the multi-component generalization of modified type of equation (1) that obtained as a reduction of Feg's system is considered in [10]. The Lax representations of Mastuno's and Feg's systems were examined by Popowicz [11]. More recently,

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Hone el al. [12] classified a general form of partial differential equations of order two and they found a new equation that generalizing equation (1). This is given by

$$u_{xt} = u + (u^2 - 4u^2 u_x)_x \,, \tag{2}$$

where u is a field of one spatial dimension x plus time t. In the current work, we propose a generalization for equation (2) to two-component system of nonlinear equations, that is

$$u_{xt} = u + (v^2 - 4v^2 u_x)_x \,, \tag{3}$$
$$v_{xt} = v + (u^2 - 4u^2 v_x)_x \,, \tag{4}$$

where u and v are two fields of one spatial dimension x plus time t. Clearly, the system consists of six nonlinear terms $vv_x$, $vv_x u_x$, $v^2 u_{xx}$, $uu_x$, $uu_x v_x$ and $u^2 v_{xx}$, and four linear terms $u_{xt}, u, v_{xt}$ and v. As far as we know, the two-component system (3)-(4) that generalizes the generalized type of the short pulse equation that has introduced in the current work seen nowhere in the literature. We shall look at the system, which represents the main problem, in the coming sections. The remaining of the paper is lined up in such a way. In section two, the Lie group analysis is overviewed in order to make the work is a self-contained. We proceed, in section three, to analyse the two-component system of equations by Lie's method of a continuous group of point transformations. Section four is stressed to discuss the similarity reduction of the main problem. The conclusions are given in the last section.

**2. Lie Symmetry Analysis**

The Lie's method of a continuous group of point transformations is a well-known approach that has been applied to several types of differential equations. The start point of this approach goes back to Sophus Lie, and from since the method has been noticed and developed. The approach can be briefly summarized, following the same descriptions in [13] and also can be found in [14-18], as follows

Consider a system of equations

$$\Delta^n\left(x_i, u^p, u^p{}_{(i_1)}, u^p{}_{(i_2)}, u^p{}_{(i_1 i_2)}, \dots, u^p{}_{(i_1 i_2 \dots i_k)}\right) = 0, \; n = 1,2,3,\dots,N \tag{5}$$

where $x_i$, $i = 1,2,3,..,s$ are independent variables and $u^p, p = 1,2,3,..,r$ are dependent variables, and $u^p{}_{(i_1)} = \frac{\partial u^p}{\partial x_i^{i_1}}, \dots, u^p{}_{(i_1 i_2 \dots i_k)} = \frac{\partial^k u^p}{\partial x_i^{i_1} \dots \partial x_j^{i_k}}$, $i_m = 1,2,..,s$ for $m = 1,2,..,k$ refer to the partial derivatives regarding independent variables, $N$ is the number of equations, and $k$ is the number of the highest derivatives. A general form of group of point transformations [13], reads

$$x_i^{\#} = X_i(x_i, u^p; \alpha), \quad (u^p)^{\#} = U^p(x_i, u^p; \alpha), \tag{6}$$

where the parameter α is considered to be too small $\alpha \ll 1$. The linearization of the Lie group around the identity $(\alpha = 0)$ shapes the infinitesimals form of Lie group, this is given by [13],

$$x_i^{\#} = X_i(x_i, u^p; \alpha) \approx x_i + \alpha\xi_i(x_i, u^p) + O(\alpha^2), \tag{7}$$
$$(u^p)^{\#} = U^{\mu}(x_i, u^p; \alpha) \approx u^p + \alpha\eta_p(x_i, u^p) + O(\alpha^2), \tag{8}$$

and the infinitesimals $\xi_i$ and $\eta_p$ are therefore taken to be

$$\frac{dx_i^{\#}}{d\alpha} = \xi_i(x_i, u^p) \quad \text{and} \quad \frac{d(u^p)^{\#}}{d\alpha} = \eta_p(x_i, u^p),$$

with the initial conditions

$$(x_i^{\#}, (u^p)^{\#})|_{\alpha=0} = (x_i, u^p).$$

The infinitesimal generator of (6) is expressed by

$$Y = \xi_i(x_i, u^p)\frac{\partial}{\partial x_i} + \eta_p(x_i, u^p)\frac{\partial}{\partial u^p}$$

and the extension of the infinitesimal generator (the Prolongations) to include the derivatives is [13],

$$Pr^{(k)}Y = \xi_i(x_i, u^p)\frac{\partial}{\partial x_i} + \eta_p(x_i, u^p)\frac{\partial}{\partial u^p}\eta_p^{(k)}\left(x_i, u^p, u^p{}_{(i_1)}\right)\frac{\partial}{\partial u^p{}_{(i_1)}} + \cdots +$$
$$\eta^{(k)}_{p;i_1 i_2 \dots i_k}\left(x_i, u^p, u^p{}_{(i_1)}, u^p{}_{(i_2)}, u^p{}_{(i_1 i_2)}, \dots, u^p{}_{(i_1 i_2 \dots i_k)}\right)\frac{\partial}{\partial u^p_{(i_1 i_2 \dots i_k)}},$$

Where $\eta^{(k)}_{p;i_1i_2\ldots i_k} = D_{i_k}\eta^{(k-1)}_{p,i_1i_2\ldots i_k} - \left(D_{i_k}\xi_j\right)u^p_{(i_1i_2\ldots i_{k-1}j)}$ and $D_{i_k} = \frac{\partial}{\partial x_i} + u^p{}_{(i_1)}\frac{\partial}{\partial u^p{}_{(i_1)}} + \cdots u^p_{(i_1i_2\ldots i_k)}\frac{\partial}{\partial u^p_{(i_1i_2\ldots i_k)}} + \cdots$

The groups (7)-(8) is admitted by (5) if the following hold [13],

$$\mathrm{Pr}^{(k)}Y\Delta^n\left(x_i, u^p, u^p{}_{(i_1)}, u^p{}_{(i_2)}, u^p{}_{(i_1i_2)}, \ldots, u^p{}_{(i_1i_2\ldots i_k)}\right)\Big|_{\Delta^n\left(x_i,u^p,u^p{}_{(i_1)},u^p{}_{(i_2)},u^p{}_{(i_1i_2)},\ldots,u^p{}_{(i_1i_2\ldots i_k)}\right)=0} = 0.$$

When a group of symmetries acts upon a system of equations it maps it to another system of new variables and the new system preserves the original system form as well as it maps its solutions; which leads to getting a new solution from the known one. The similarity variables associated with the Lie symmetries can be obtained by solving the characteristics equation [13],

$$\frac{dx_2}{\xi_2} = \cdots = \frac{dx_s}{\xi_s} = \frac{du^1}{\eta_1} = \frac{du^2}{\eta_2} = \cdots = \frac{du^p}{\eta_p},$$

and the general solution can be expressed as

$\xi_1(x_I, u^p) = c_1, \xi_2(x_I, u^p) = c_2, \ldots, \xi_s(x_I, u^p) = c_s, \eta_1(x_I, u^p) = c_{s+1}, \eta_2(x_I, u^p) = c_{s+2} \ldots, \eta_{s+p-1}(x_I, u^p) = c_{s+p-1}$,

where $c's$ are arbitrary constants. In the next section, we analyse the two-component system under study by Lie symmetry analysis.

## 3. Lie symmetry analysis for the Main problem

We deal with the main problem (3)-(4) in this section. The Lie's method of a continuous group of point transformations is used to characterize all possible symmetry groups for the system of nonlinear equations under consideration.

Consider the system of nonlinear equations (3)-(4), that reads

$$\Delta^{(1)} = \Delta^{(1)}(x,t,u,v) = u_{xt} - (v^2 - 4v^2u_x)_x - u = 0, \quad (9)$$

$$\Delta^{(2)} = \Delta^{(2)}(x,t,u,v) = v_{xt} - (u^2 - 4u^2v_x)_x - v = 0, \quad (10)$$

we start off by setting group of infinitesimals for the system above as

$$x^{\#} = x + \alpha\xi_1(x,t,u,v),$$
$$t^{\#} = t + \alpha\xi_2(x,t,u,v),$$
$$u^{\#} = u + \alpha\eta_1(x,t,u,v),$$
$$v^{\#} = v + \alpha\eta_2(x,t,u,v),$$

where the parameter $\alpha \ll 1$, and $\xi_1, \xi_2, \eta_1$ and $\eta_2$ are functions of $x, t, u$ and $v$ are taken to be

$$\frac{dx^{\#}}{d\alpha} = \xi_1(x,t,u,v), \quad \frac{dt^{\#}}{d\alpha} = \xi_2(x,t,u,v),$$
$$\frac{du^{\#}}{d\alpha} = \eta_1(x,t,u,v), \quad \frac{dv^{\#}}{d\alpha} = \eta_2(x,t,u,v),$$

subject to the conditions

$$(x^{\#}, t^{\#}, u^{\#}, v^{\#})|_{\alpha=0} = (x,t,u,v),$$

and the corresponding infinitesimals generator is the linear first order differential operator

$$Y = \xi_1(x,t,u,v)\,\partial x + \xi_2(x,t,u,v)\,\partial t + \eta_1(x,t,u,v)\,\partial u + \eta_2(x,t,u,v)\,\partial v \quad (11)$$

and the extended of transformation to include derivatives is

$$\mathrm{Pr}^{(2)}Y = Y + \eta_1^{(t)}\,\partial u_t + \eta_2^{(t)}\,\partial v_t + \eta_1^{(x)}\,\partial u_x + \eta_2^{(x)}\,\partial v_x + \eta_1^{(xx)}\,\partial u_{xx} + \eta_2^{(xx)}\,\partial v_{xx} + \eta_1^{(xt)}\,\partial u_{xt} + \eta_2^{(xt)}\,\partial v_{xt}. \quad (12)$$

From acting the second prolongation (12) on the system of equations (9)-(10), this is written by

$$\mathrm{Pr}^{(2)}Y\left(\Delta^{(1)}(x,t,u,v)\right) = 0 \quad \text{whenever} \quad \Delta^{(1)}(x,t,u,v) = 0,$$
$$\mathrm{Pr}^{(2)}Y\left(\Delta^{(2)}(x,t,u,v)\right) = 0 \quad \text{whenever} \quad \Delta^{(2)}(x,t,u,v) = 0,$$

one can obtain the following nonlinear partial differential equations

$$\eta_1^{(xt)} - 2\eta_2 v_x - 2v\eta_2^{(x)} + 4v^2\eta_1^{(xx)} + 8\eta_2 v v_x u_{xx} + 8\eta_2^{(x)} v_x u_x + 8\eta_2^{(x)} v u_x + 8v v_x \eta_1^{(x)} - \eta_1 = 0, \quad (13)$$

$$\eta_2^{(xt)} - 2\eta_1 u_x - 2u\eta_2^{(x)} + 4u^2\eta_2^{(xx)} + 8\eta_1 u u_x v_{xx} + 8\eta_1^{(x)} u_x v_x + 8\eta_1^{(x)} u v_x + 8u u_x \eta_2^{(x)} - \eta_2 = 0, \quad (14)$$

where $\eta_1, \eta_2, \eta_1^{(t)}, \eta_1^{(x)}, \eta_2^{(x)}, \eta_1^{(xx)}, \eta_2^{(xx)}, \eta_1^{(xt)}$ and $\eta_2^{(xt)}$ are the coefficients of the second prolongation (12), and they are given by

$$\eta_1{}^{(t)} = D_t(\eta_1) - u_x D_t(\xi_1) - u_t D_t(\xi_2),$$
$$\eta_2{}^{(t)} = D_t(\eta_2) - v_x D_t(\xi_1) - v_t D_t(\xi_2),$$
$$\eta_1{}^{(x)} = D_x(\eta_1) - u_x D_x(\xi_1) - u_t D_x(\xi_2),$$
$$\eta_2{}^{(x)} = D_x(\eta_2) - v_x D_x(\xi_1) - v_t D_x(\xi_2),$$

and

$$\eta_1{}^{(Rx)} = D_x\left(\eta_1{}^{(R)}\right) - u_{Rx} D_x(\xi_1) - u_{Rt} D_x(\xi_2),$$
$$\eta_2{}^{(Rx)} = D_x\left(\eta_2{}^{(R)}\right) - v_{Rx} D_x(\xi_1) - v_{Rt} D_x(\xi_2),$$

the R′s are alternated by x′s and t′s, and

$$D_x = \partial_x + u_x\,\partial_u + u_{xx}\,\partial_{u_x} + u_{xt}\,\partial_{u_t} + \cdots + v_x\,\partial_v + v_{xx}\,\partial_{v_x} + u_{xt}\,\partial_{v_t} + \cdots$$

and

$$D_t = \partial_t + u_t\,\partial_u + u_{xt}\,\partial_{u_x} + u_{tt}\,\partial_{u_t} + \cdots + v_t\,\partial_v + v_{xt}\,\partial_{v_x} + v_{tt}\,\partial_{v_t} + \cdots.$$

Simplifying equations (13)-(14) with the benefit of equations (9)-(10), and due to the tremendous calculations, the computer software Maple 18 and the DesolvII package [19] on a PC with Cor i7 Intel 3.6 GHz and 32M RAM is used, to get the following twenty-one overdetermined equations

$$\xi_{2,x} = \xi_{1,u} = \xi_{2,u} = \xi_{1,v} = \xi_{2,v} = 0,$$
$$\eta_{1,v} = \eta_{2,u} = \eta_{1,uu} = \eta_{1,xu} = \eta_{2,vv} = \eta_{2,xv} = 0,$$
$$v\xi_{2,t} - v\xi_{1,t} + v\eta_{2,v} + \eta_2 = 0,$$
$$4u\eta_{2,x} + u\eta_{2,v} - \eta_2 - u\eta_{1,u} - u\xi_{2,t} = 0,$$
$$8\eta_2 u + 4u^2\xi_{2,t} - 4u^2\xi_{1,x} - \xi_{1,t} = 0,$$
$$-\eta_2 - v\xi_{1,x} - v\xi_{2,t} + \eta_{2,xt} + 4u^2\eta_{2,xx} + v\eta_{2,v} - 2u\eta_{1,x} = 0,$$
$$\eta_{2,tv} + 8u\eta_{1,x} - 4u^2\xi_{1,xx} - \xi_{1,xt} = 0,$$
$$8\eta_2 v + 4v^2\xi_{2,t} - 4v^2\xi_{1,x} - \xi_{1,t} = 0,$$
$$u\xi_{2,t} - u\xi_{1,t} + u\eta_{1,u} + \eta_1 = 0,$$
$$-v\eta_{2,v} + v\eta_{1,u} + 4v\eta_{1,x} - v\xi_{2,t} - \eta_2 = 0,$$
$$\eta_{1,tu} + 8v\eta_{2,x} - 4v^2\xi_{1,xx} - \xi_{1,xt} = 0,$$
$$-\eta_1 + \eta_{1,xt} + u\eta_{1,u} - u\xi_{2,t} + 4v^2\eta_{1,xx} - 2v\eta_{2,x} - u\xi_{1,x} = 0,$$

where $\xi_{i,\sigma} = \frac{\partial \xi_i}{\partial \sigma}$ , $\eta_{i,\sigma} = \frac{\partial \eta_i}{\partial \sigma}$, $\xi_{i,\sigma\rho} = \frac{\partial^2 \xi_i}{\partial\sigma\,\partial\rho}$ and $\eta_{i,\sigma\rho} = \frac{\partial^2 \eta_i}{\partial\sigma\,\partial\rho}$, for $i = 1,2$ and $\sigma, \rho = t, x, v, u$, Solving then the last equations to gain

$$\xi_1 = c_1 x + c_2\ ,\ \xi_2 = -c_1 t + c_3,\ \eta_1 = c_1 u \text{ and } \eta_2 = c_1 v,$$

where $c_i, i = 1,2,3$ are arbitrary constants; that give Lie algebra of three dimensions. The Lie algebra of point symmetry generators is spanned by the three vector fields

$$Y_1 = x\,\partial_x - t\,\partial_t + u\,\partial_u + v\,\partial_v, \quad Y_2 = \partial_x, \quad Y_3 = \partial_t, \tag{15}$$

and the Lie bracket (or Commutator) is given by [20], $[Y_i, Y_j] = Y_iY_j - Y_jY_i$ , $i, j = 1,2,3$. The Commentator of the Lie algebra is listed in Table-1.

**Table 1-**The Commentator table of the Lie algebra

| | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| $Y_1$ | **0** | $-Y_2$ | $Y_3$ |
| $Y_2$ | $Y_2$ | **0** | **0** |
| $Y_3$ | $-Y_3$ | **0** | **0** |

The transformed point that comes from the entries is $\exp(\alpha Y_i)\,(x, t, u, v) = (x^{\#}, t^{\#}, u^{\#}, v^{\#})$, and the groups of symmetries of the one parameter for the system (9)-(10) is therefore written by

$\Gamma_1^{\alpha}: (x, t, u, v) \to (x + \alpha\ , t, u, v),$ space translation,

$\Gamma_2^{\alpha}: (x, t, u, v) \to (x, t + \alpha, u, v),$ time translation,

$\Gamma_3^{\alpha}: (x, t, u, v) \to (e^{\alpha}x, e^{-\alpha}t, e^{\alpha}u, e^{\alpha}v),$ scaling.

Based on the relations[15], $\mathrm{Ad}[(exp(\alpha Y_i)]Y_j = \sum_{r=0}^{\infty}\frac{\alpha^r}{r!}(ad\, Y_i)^r Y_j$ , and $ad\, Y_i|_{Y_j} = [Y_j, Y_i] = -[Y_i, Y_j]$, $i, j = 1,2,3$. The adjoint construction for the Lie algebra is listed in Table-2

**Table 2-** The adjoint table of the Lie algebra

| Ad | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| $Y_1$ | $Y_1$ | $e^{\alpha} Y_2$ | $e^{-\alpha} Y_3$ |
| $Y_2$ | $Y_1 - \alpha Y_2$ | $Y_2$ | $Y_3$ |
| $Y_3$ | $Y_1 + \alpha Y_3$ | $Y_2$ | $Y_3$ |

We are now at the position to state the following, if $\{u(x,t), v(x,t)\}$ is a solution for the two-components system of equations (3)-(4) then

$$\Gamma_1^{\alpha}.u(x,t) = u^{\#}(x^{\#}-\alpha, t^{\#}),\quad \Gamma_1^{\alpha}.v(x,t) = v^{\#}(x^{\#}-\alpha, t^{\#}),$$
$$\Gamma_2^{\alpha}.u(x,t) = u^{\#}(x^{\#}, t^{\#}-\alpha),\quad \Gamma_2^{\alpha}.v(x,t) = v^{\#}(x^{\#}, t^{\#}-\alpha),$$
$$\Gamma_3^{\alpha}.u(x,t) = u^{\#}(e^{-\alpha}x^{\#}, e^{\alpha}t^{\#}),\quad \Gamma_3^{\alpha}.v(x,t) = v^{\#}(e^{-\alpha}x^{\#}, e^{\alpha}t^{\#}),$$

are also solutions. To clarify that, suppose that $\{u(x,t) = f(x,t),\ v(x,t) = g(x,t)\}$ is a solution for the two- component system (3)-(4) then the new solutions $\{u^{\#}(x^{\#},t^{\#}), v^{\#}(x^{\#},t^{\#})\}$ follow from groups $\Gamma_i^{\alpha}$, $i = 1,2,3$ actions, as follow

From $x^{\#} = x + \alpha$ and $t^{\#} = t$ that gives $x = x^{\#} - \alpha$ , $t = t^{\#}$ and $u^{\#}(x^{\#},t^{\#}) = u(x,t) = f(x,t) = f(x^{\#}-\alpha, t^{\#})$ and $v^{\#}(x^{\#},t^{\#}) = v(x,t) = g(x,t) = g(x^{\#}-\alpha, t^{\#})$.

From $x^{\#} = x$ and $t^{\#} = t + \alpha$ that gives $x = x^{\#}$, $t = t^{\#} - \alpha$ and $u^{\#}(x^{\#},t^{\#}) = u(x,t) = f(x,t) = f(x^{\#}, t^{\#}-\alpha)$ and $v^{\#}(x^{\#},t^{\#}) = v(x,t) = g(x,t) = g(x^{\#}, t^{\#}-\alpha)$.

From $x^{\#} = e^{\alpha}x$ and $t^{\#} = e^{-\alpha}t$ that gives $x = e^{-\alpha}x^{\#}$ , $t = e^{\alpha}t^{\#}$ and $u^{\#}(x^{\#},t^{\#}) = e^{\alpha}u(x,t) = e^{\alpha}f(x,t) = e^{\alpha}f(e^{-\alpha}x^{\#}, e^{\alpha}t^{\#})$ and $v^{\#}(x^{\#},t^{\#}) = e^{\alpha}v(x,t) = e^{\alpha}g(x,t) = e^{\alpha}g(e^{-\alpha}x^{\#}, e^{\alpha}t^{\#})$,

are also solutions satisfy the system of equations (3)-(4).

**4. Similarity reduction of the main problem**

We focus, in this section, on the reduction of the main problem (3)-(4) relying on similarity variables; these variables come from solving characteristics equations, and as a result a coupled of ordinary differential equations are formed.

In order to gain similarity variables related to the Lie symmetries (15) we solve the characteristics equations

$$\frac{dx}{x} = \frac{dt}{-t} = \frac{du}{u} = \frac{dv}{v}$$

Take $\frac{dx}{x} = \frac{dt}{-t}$ and solve it to have $\zeta = xt$. In the same way, one can have $ut = \vartheta$ and $vt = \varphi$, and that implies $u = \frac{1}{t}\vartheta(\zeta)$ and $v = \frac{1}{t}\varphi(\zeta)$. Substituting into the coupled of nonlinear partial differential equations (9)-(10) (or (3)-(4)) one can get the following nonlinear system of equations as a reduction

$$\zeta\vartheta_{\zeta\zeta} - 2\varphi\varphi_{\zeta} + 4\varphi^2\vartheta_{\zeta\zeta} + 8\varphi\varphi_{\zeta}\vartheta_{\zeta} - \vartheta = 0, \quad (16)$$

$$\zeta\varphi_{\zeta\zeta} - 2\vartheta\vartheta_{\zeta} + 4\vartheta^2\varphi_{\zeta\zeta} + 8\vartheta\vartheta_{\zeta}\varphi_{\zeta} - \varphi = 0, \quad (17)$$

where $\vartheta_{\zeta} = \frac{d\vartheta}{d\zeta}$, $\vartheta_{\zeta\zeta} = \frac{d^2\vartheta}{d\zeta^2}$ , $\varphi_{\zeta} = \frac{d\varphi}{d\zeta}$ and $\varphi_{\zeta\zeta} = \frac{d^2\varphi}{d\zeta^2}$ . That leads to state the following, if $\{\vartheta(\zeta), \varphi(\zeta)\}$ is a solution for the nonlinear equations (16)-(17) then *{u(x,t),v(x,t)}* is a solution for nonlinear equations (3)-(4).

**5. Conclusions**

To sum up, in the present work, we have proposed two-component generalization of a generalized the short pulse equation. The system of nonlinear equations have introduced here does not appear to have been considered before in the literature. Based on the Lie analysis we have characterized all possible symmetry groups that the two-component system of equations can admit; in terms of the space translation, the time translation and the scaling. The symmetry algebra of the two-component system of nonlinear equations is generated by the three vector fields. The Lie brackets for the vector fields are given. The similarity variable is used to get the reduction of the main problem to the coupled of nonlinear ordinary differential equations. To be clear, the main problem results are settled in sections three and four.

We would like to pinpoint that exact solution for the two-component system is an open problem needs to be explored. Studying the behaviour of solutions in a long and short period of time for the two fields is a good task one can carry on. In addition, the searching for Lax representation for the two-component system also needs to be considered. The integrability of nonlinear equations in terms of Painlevé analysis is an interesting piece of work we intend to examine in the near future.

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