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The Solar Attraction Effect on Orbital Elements of the Moon

Abdulrahman H. Saleh*, Taif A. Damin
Department of Astronomy and Space, College of Science, University of Baghdad, Baghdad, Iraq

Abstract

In this paper, the solar attraction effect on the Moon orbital elements had been studied. The solar attraction considered a third body perturbation. The initial values of the Moon orbital elements at 2015-1-21.5 which are used (*semi major axis*=380963.78 km, *e*ccentricity =0.056°, Eular angles (Ω=30°, ω=40°, , i=18.4°). The Moon in a perigee of its orbit at the initial time. A program which designed, was used to calculate the Moon position ,velocity and orbital elements through the years (2015-2035). The position and velocity components of the Moon were calculated by solving the equation of Kepler for elliptical orbit using Newton Raphson's method for (1000) periods and hundred steps of time for each period. The results show a secular changes for the Moon orbital elements with solar attraction, and they will deviate from the initial values with many years.

Keywords: Moon orbit, solar attraction, anomalistic month.

تأثير جذب الشمس على العناصر المدارية للقمر

عبد الرحمن حسين صالح*، طيف عادل ضامن
قسم الفلك والفضاء، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة:

في هذا البحث، تم دراسة تأثير جذب الشمس على العناصر المدارية للقمر، و هذه التأثير يعد من نوع اضطراب الجسم الثالث. القيم الابتدائية لعناصر مدار القمر المستخدمة كانت للتاريخ 2015-1-21.5 ، وكانت وفق الآتي: (*a*=380963.78 km, e=0.056°, Ω=30°, *w*=40°, , *i*=18.4) . وفي تلك اللحظة يكون القمر في الحضيض من مداره . البرنامج الذي صمم تم استخدامه لحساب موضع وسرعة القمر وعناصره المدارية للفترة بين (2015-2035) . تم حساب مركبات الموضع والسرعة للقمر من خلال حل معادلة كبلر لمدار القطع الناقص باستخدام طريقة نيوتن رافسن ولمدة 1000 دورة باعتماد مئة قراءة لكل دورة. أظهرت النتائج أن تغير العناصر المدارية للقمر تكون صغيرة بوجود جذب الشمس، و تنحرف عن القيم الابتدائية لها بمرور عدة سنوات.

1. Introduction

The moon is the only natural satellite of the Earth, the mass of the Moon is 7.35*10^{22} kg., compared with the Earth's which is 5.9742*10^{24} kg. with a mean distance 384700 km. The Sun mass is 1.9891*10^{30} kg, and the mean distance from the Earth 149.6*10^{6} km[1]. The Earth and the Moon are still close enough in size and close enough together to exert powerful influence over each other. Many events on the Earth depend on the Moon position phase as the solar and lunar eclipse, and affect activities such as deep-sea fishing and navigation [2], and for some people the Moon birth used to determine the dates as (Hegree date for Islamic countries).

Babylonian astronomy discovered the three main periods of the Moon's motion and used data analysis to build lunar calendars. The daily motion of the Moon through the sky has many unusual features that a careful observer can show without instruments [3].

*Email : Abdrahman29@yahoo.com

2. Moon Orbit and Phases:

The Earth revolves around the Sun and around its axis, The Moon orbit around Earth, this is called lunation [4]. The Moons orbit is not a perfect circle, its closest approach to Earth at a point known a perigee (353980 km), while the maximum distant called apogee (405468 km) [5]. These distances are vary every month, because the Sun's gravity and to a minor extent gravity of a few other planets. A celestial observer out the Solar system would see the Moon making loops like spiral about the Earth orbit. It is seen in orbit around the Sun, the effect of the Earth's influence is to make the Moon's orbit wiggle as Figure-1 [6].

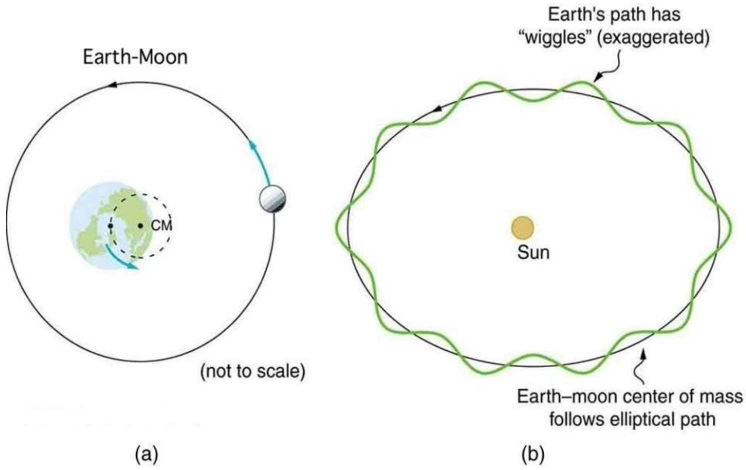


Figure 1- The Moon's orbit around the Earths orbit [6]

The Moon changing appearance by the observer in a lunar phases. The new moon is that instant when the Moon is in conjunction with the Sun, it is slightly north or south of the Sun because the lunar orbit inclined 5^{o} with the ecliptic. Up one day after the new moon, the crescent moon can be seen in the western evening sky. At one week after the new moon, the first quarter follows, when the longitudes of the Moon and the Sun differ by around 90. The full moon appears two weeks after the new moon, and one week after this the last quarter after that the Moon rise some hours before sun rise. Finally the waning crescent moon disappears in the east of the morning sky [7, 8].

When the Moon gets into the cone of Earth shadow, one can observe a lunar eclipse on Earth more specifically, The repetition of lunar eclipses is roughly and approximately periodic, and conforms the Saros cycle. A Saros is a period of time of approximately 18 years or 18 years and 10 days [9], 18.03 [10]. During a particular June solstice every 18.6 years (called by Babylonian Saros period) [11]. After one Saros, the Moon is back to the same phase it had when the cycle began, and the Sun returned to the same place it occupied with respect to the nodes of the Moon's orbit.

3. The Equation of the Moon motion:

The Ecliptic Coordinate system is convenient in studying the movements of the planets, asteroids and in describing the Solar System. The ecliptic longitude (λ) measures the angular distance of an object along the ecliptic from the vernal equinox point. The ecliptic latitude (β), measures the angular distance of an object from the ecliptic towards the north (positive) or south (negative) ecliptic pole [10].

The Moon orbit is affected by the relative positions of the Earth and the Sun. These effects are so significant that the orbital elements that are considered constants for the Earth are now all have the secular. the Moon orbit is inclined to ecliptic plane at an angle of around 5°9' that varies periodically up to ± 9 arc minutes [12]. The term " third-body " refers to any other body in space besides, including the perturbations by the gravitational influence of the Sun, and planets. The most significant influences come from the Sun. The gravitational forces of the Sun cause periodic variations in all of the Moon elements. To calculate the effect of the solar attraction, Kepler's equation of the Moon motion should be solved as the following :

The eccentric anomaly for the orbit calculated by solving Kepler's equation as:

$$\boldsymbol{E = M + e_o \sin E} \quad (1)$$

Where e_o the eccentricity convert from radians into degrees, M is the mean anomaly, and E is the eccentric anomaly of the Moon on the orbit at epoch.
Kepler's equation was solved by re-iteration or using Newton-Raphson method as a high accuracy [13].
The position and velocity components, calculated as:

$$x_w = a(\cos E - e)$$
$$y_w = a\sqrt{1-e^2}\sin E \qquad \textbf{(2)}$$
$$zw=0$$

$$\dot{x}_w = \frac{\sqrt{\mu a}}{r}\sin E$$
$$\dot{y}_w = \frac{\sqrt{\mu a\,(1-e^2)}}{r}\cos E \qquad \textbf{(3)}$$
$$\dot{z}_w = 0$$

The conversion of position and velocity of the Moon from the orbital plane to the Earth equatorial plane (x,y,z) can be used by Gaussian matrix (conversion matrix), which content Euler angles (i, Ω, ω) [13-15].
Where: the inclination (***i***) is the angle of intersection between the orbital plane and the equator (0 deg<i < 180 deg).
The argument of perigee (***ω***): is the angle between the direction of the ascending node and the direction of the perigee (0 deg< ω < 360 deg).
The right ascension of the ascending node (**Ω**): the angle between the vernal equinox and the point on the orbit at which the Moon crosses the equator from south to north (0 deg<Ω< 360 deg).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{-1}\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \qquad (4)$$

Where R^{-1} is the inverse of Gauss matrix

$$R^{-1} = \begin{bmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{bmatrix}$$

Where:

$$P_x = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i$$
$$P_y = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i$$
$$P_z = \sin\omega\sin i$$
$$Q_x = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i$$
$$Q_y = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i$$
$$Q_z = \cos\omega\sin i$$
$$W_x = \sin\Omega\sin i$$
$$W_y = -\cos\Omega\sin i$$
$$W_z = \cos i$$

Thus the position and the velocity components in the equatorial system are:

$$\left.\begin{aligned} x &= P_x x_w + Q_x y_w + W_x z_w \\ y &= P_y x_w + Q_y y_w + W_y z_w \\ z &= P_z x_w + Q_z y_w + W_z z_w \end{aligned}\right\} \qquad (5)$$
$$r = \left(x^2+y^2+z^2\right)^{1/2} \qquad (6)$$
$$\left.\begin{aligned} \dot{x} &= P_x \dot{x}_w + Q_x \dot{y}_w + W_x \dot{z}_w \\ \dot{y} &= P_y \dot{x}_w + Q_y \dot{y}_w + W_y \dot{z}_w \\ \dot{z} &= P_z \dot{x}_w + Q_z \dot{y}_w + W_z \dot{z}_w \end{aligned}\right\} \qquad (7)$$
$$\dot{r} = \left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)^{1/2} \qquad (8)$$

4. The Orbital Elements Calculation:

The elliptical orbital elements in general are (i, Ω, ω, a, e, M) can be calculated from the components of position, velocity and angular momentum vectors (h_x, h_y, h_z) which are normal to orbital plane as follows [13, 16]:

$$\vec{h} = \vec{r} \times \vec{v} \tag{9}$$

Or:

$$h_x = y\dot{z} - z\dot{y}\ ,\ h_y = z\dot{x} - x\dot{z}\ ,\ h_z = x\dot{y} - y\dot{x}\ ,\ h = \sqrt{h_x^2 + h_y^2 + h_z^2}$$

The inclination (i) of the orbit from the equatorial plane is given by:

$$\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z} \tag{10}$$

The longitude of ascending node (Ω) is calculated as:

$$\tan \Omega = \frac{h_x}{-h_y} \tag{11}$$

The semi-major axis (a) of the orbit calculated as :

$$a = \frac{rp+ra}{2} \tag{12}$$

Where: r_a is the apogee distance and r_p the perigee distance.

For elliptic orbits a will always be positive. The eccentricity (e) of the orbit is calculated as:

$$e = \frac{ra-rp}{2a} \tag{13}$$

The eccentric anomaly(E) is calculated as:

$$\tan E = \left(\frac{1-\frac{r}{a}}{x\dot{x} + y\dot{y} + z\dot{z}}\right)\sqrt{a\mu} \tag{14}$$

The mean anomaly (M) The true anomaly (f) are calculated as:

$$M = E - \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{a\mu}} \tag{15}$$

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \tag{16}$$

The argument of the latitude (u) defines the position of the satellite relative to the node line is calculated as:

$$\tan u = \frac{zh}{-xh_y + yh_x} \tag{17}$$

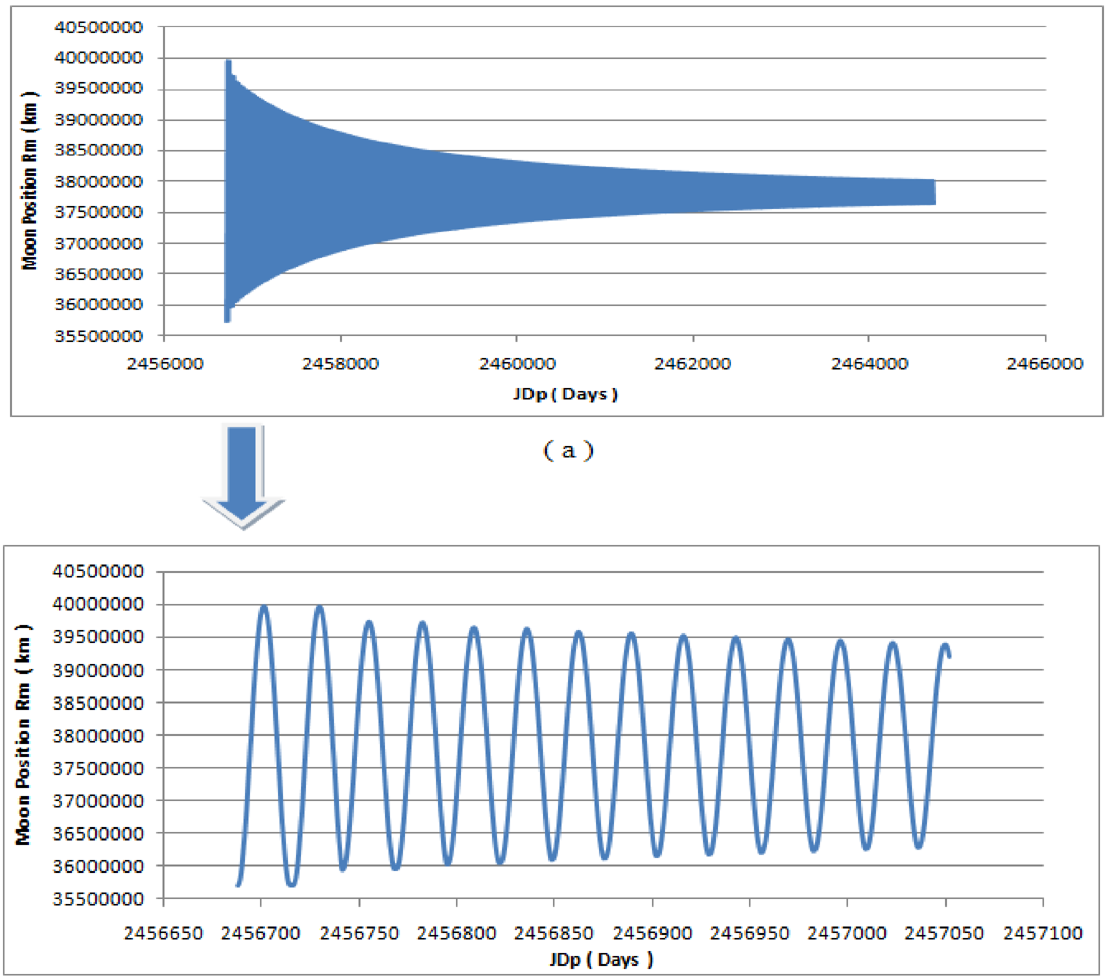
The argument of perigee (ω) can be found as:

$$\omega = u - f \tag{18}$$

5. Results and Discussion:

The Moon position and the velocity were calculated by solving Kepler's equation to show the effect of the solar attraction on them As in Figure-1a for position, it shows that it is changing through the time period (2015-2035). It started at the perigee with minimum distance of (357225757.20881 km) then increase to maximum distance at the apogee (406000000.01051 km.). The minimum values were increase to (376512311.34205 km.) and the maximum values decreasing to (399608475.55161 km.) and the mean value was (380329505.70282 km). The results shows the solar attraction effect when approach the moon orbit to circle.

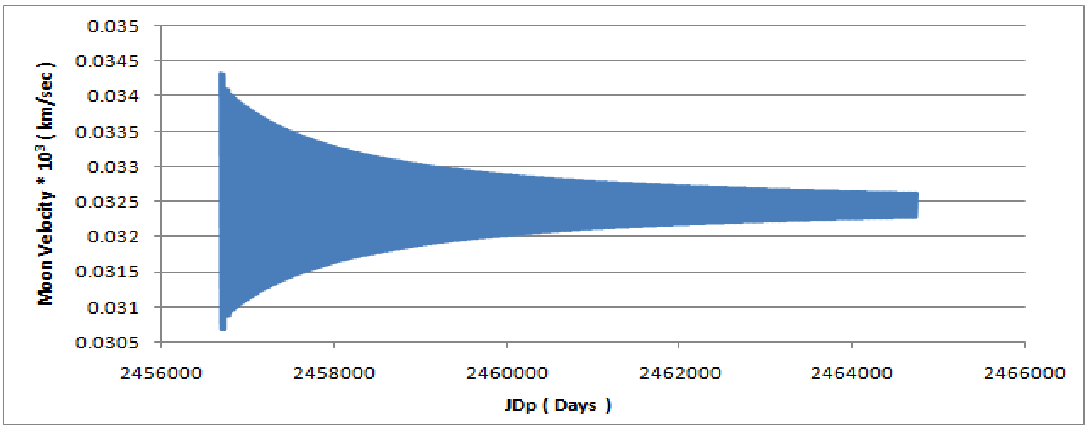
Figure-1b which is a scope of Figure-1a it shows the variation of the Moon position during the first years only to clear the changing through the period.



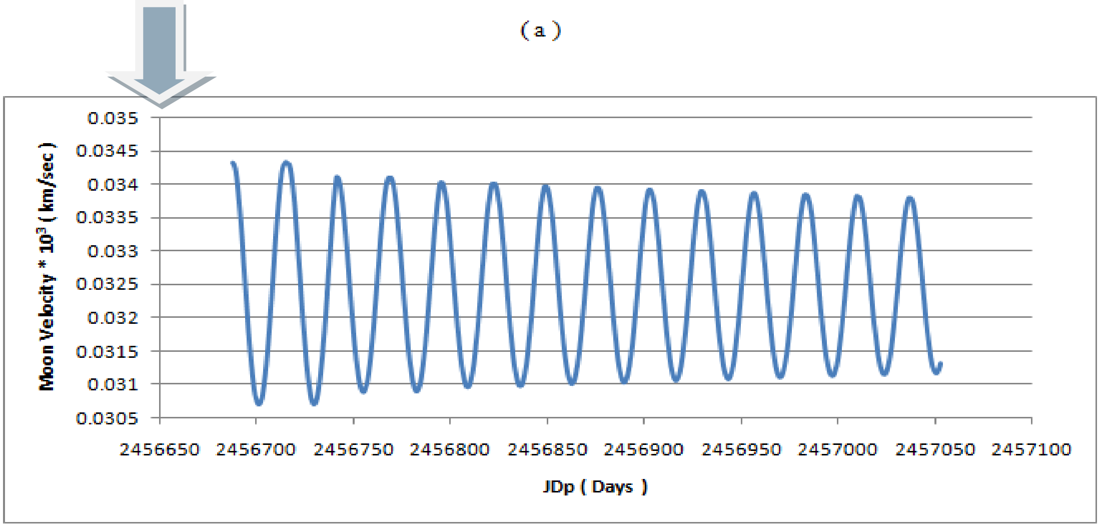
(b)

Figure 1-a,b Shows the variation of the Moon position with time included solar attraction.

Figure-2a shows that the velocity of the Moon changing between maximum and minimum values through any anomalistic month and these peaks are approach to other by the effect of the solar attraction only. The first maximum value at the apogee point (34.33 * 10^3 km/sec), then decreasing to reach its minimum value at the perigee (30.69 * 10^3 km/sec), and the last maximum and minimum values (32.62 * 10^3 km/sec), (32.38 * 10^3 km/sec), respectively. The variation is small from period to another, these variations are accumulated and became clear and important after periods greater than 12 periods.



(a)



(b)

Figure 2-a,b: Shows the variation of the Moon velocity.

Figures-3 to 10 show the orbital elements variation with time, with the effect of solar attraction through 1000 anomalistic periods, where the solar attraction was calculated by using the first method which solved Kepler's equation through using Newton-Raphson method.

The inclination (i), showed in Figure-3, the value of (i) is linearly increasing from (18.39999961849deg.) to (18.40000014168 deg.) through the 1000 periods, (27218) days. Knowing that (i) was subtracted by (i-30), to get more suitable plot.

Figure-4a show the linear decreasing in semi major axis (a) from (382470025.1-382470023.3) through the (27218) day, Figure-4b explains the behavior during the first periods.

Figure-5 shows the decreasing in the eccentricity (e) by the effect of solar perturbation from (0.05587679^o) to (0.00618853^o). It is gradually decreasing, to almost approach to zero, but it never reach it. Both (a) and (e) define the shape of the Moon orbit, thus, by the solar attraction, the Moon orbit became close to the circular orbit.

The longitude of ascending node (Ω) showed in Figure-6, the plot shows that (Ω) takes a concave behavior, increasing from (30.0000036) to (30.006188533) through 1000 periods, (27218) day. It is necessary to notice that the values of (Ω) on the plot were showed by points values, the actual values was by (30 * 10^{-5}), to be able to plot them easily.

Figure-7 showed the variation of the argument of perigee (ω) and how it takes a convex behavior decreasing with time, from maximum value (23.587931177^o) to minimum value (2.68635543 ^{o}). The mean anomaly and true anomaly showed increasing go more like a concave, in Figures-8 and-9. The variation take the shape of the exponential. True anomaly take the same behavior to the mean anomaly, increasing too, but in different values, (M) is ranging between minimum value (337.798078565^o) and maximum value (357.46292666^o), while (ta) is ranging between minimum value (336.412072229 ^{o}) and maximum value (357.313644476^o).

Figure-10 shows the variation of anomalistic month by the solar attraction, it takes linear decreasing. The real values don't appears on the plot, because the changing lied in the 6^{th} or 7^{th} after the separator shall. The result is that the orbit of the Moon was virtual orbit there for it's can't be compared with observations, but the results prove that the Sun is the dominate perturbed body on the Moon orbit.

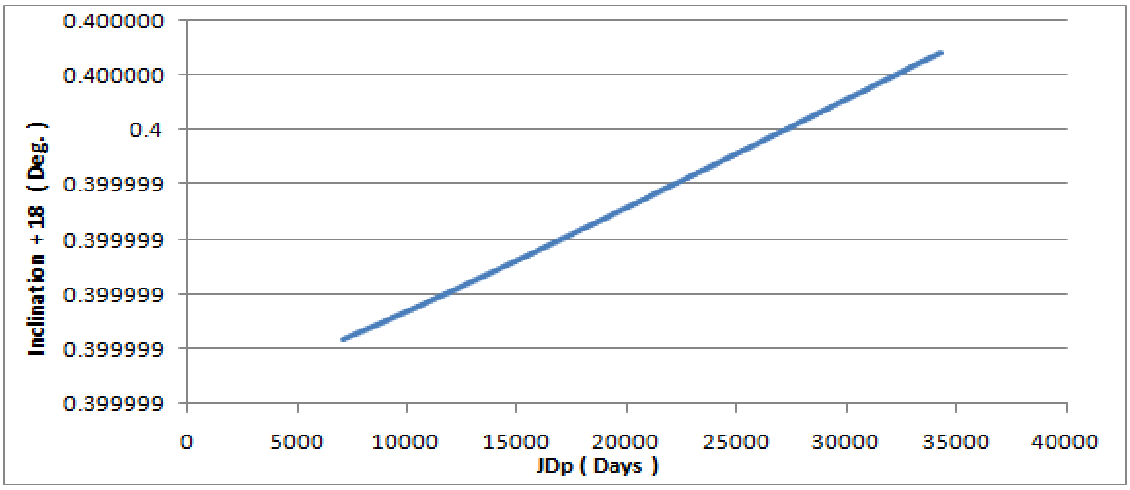
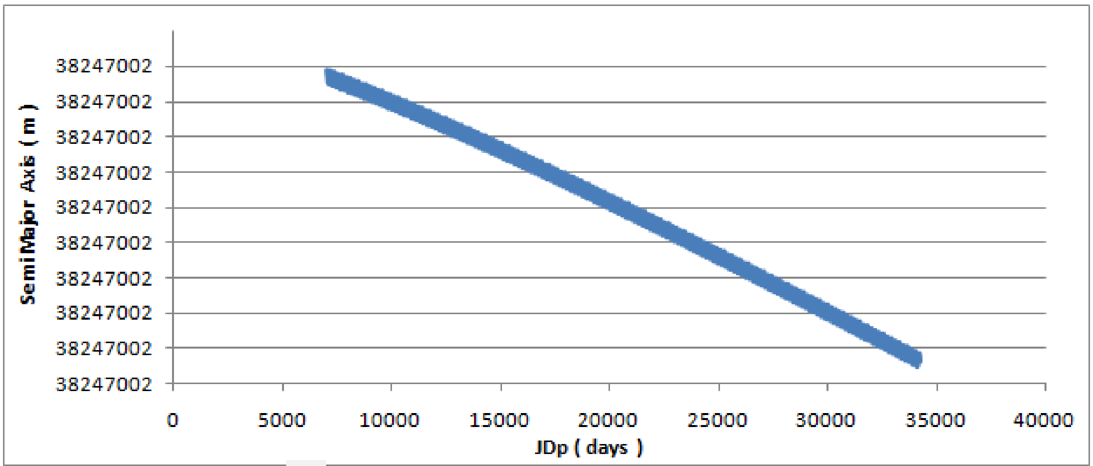
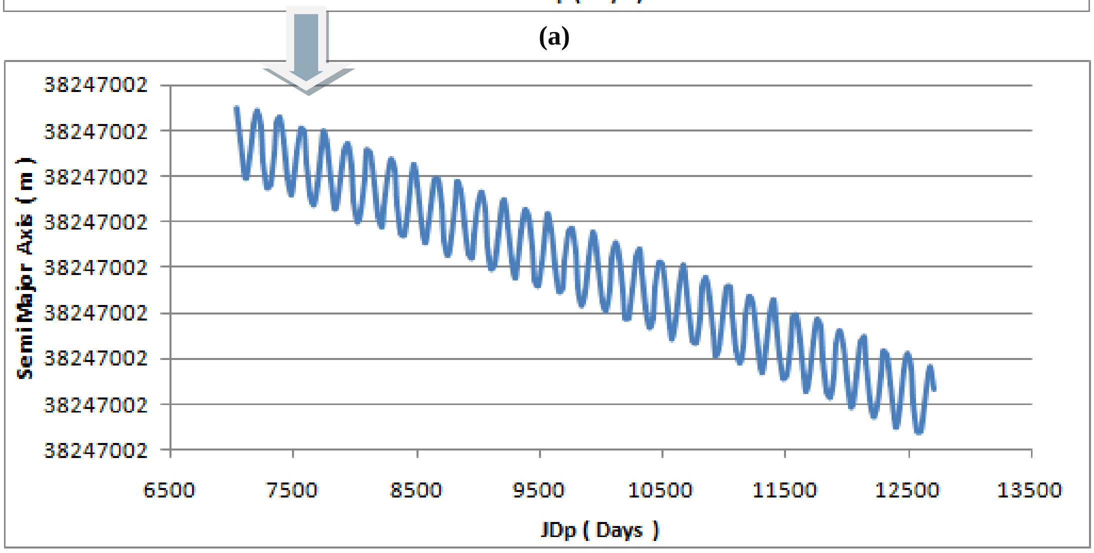


Figure 3- Shows the variation of the Moon inclination with time.



(a)



(b)

Figure 4- a.b shows the variation of the Moon semi major axis with time.

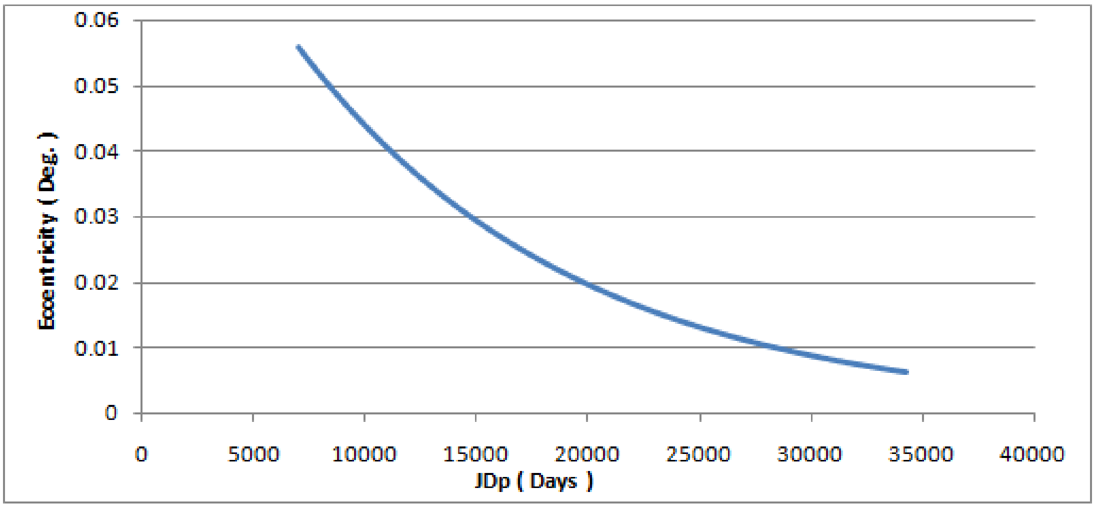


Figure 5- Shows the variation of the Moon eccentricity with time.

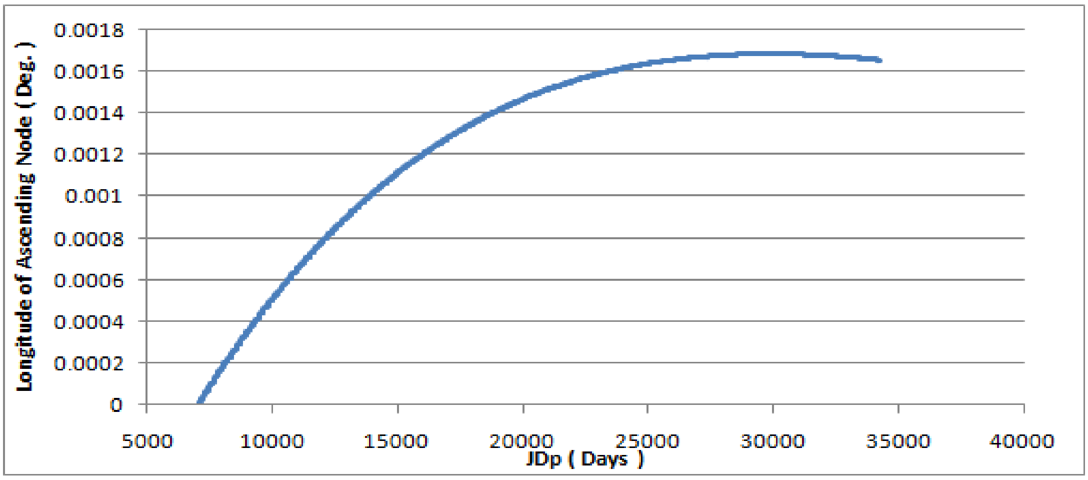


Figure 6- Shows the variation of the Moon longitude of ascending node with time.

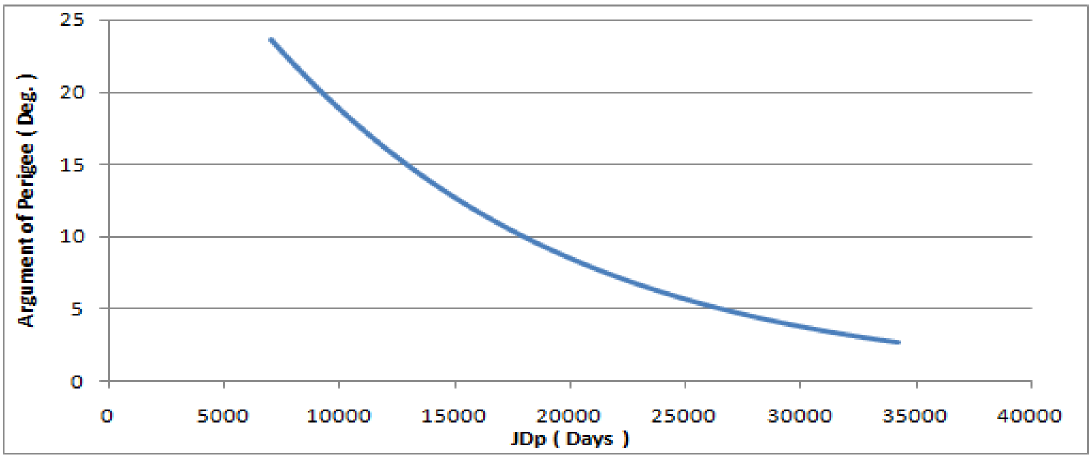


Figure 7- Shows the variation of the Moon argument of perigee with time.

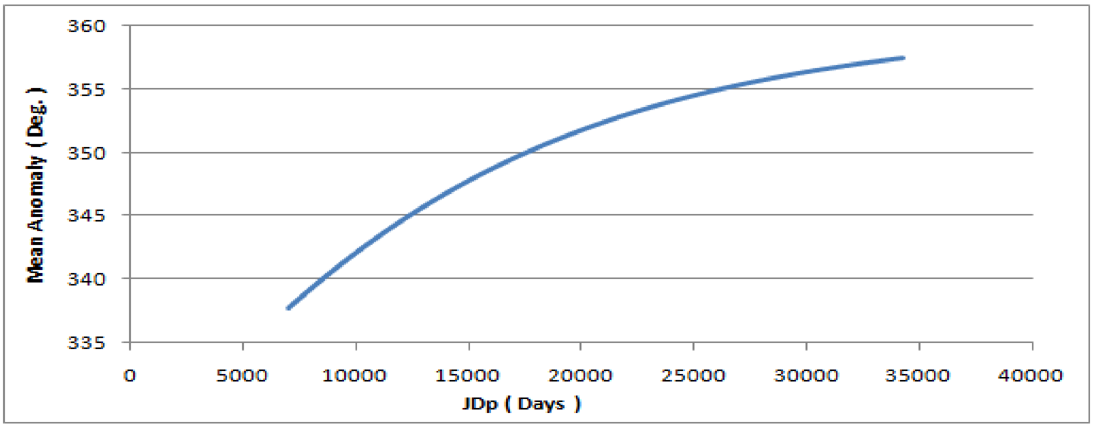


Figure 8- Shows the variation of the Moon mean anomaly with time.

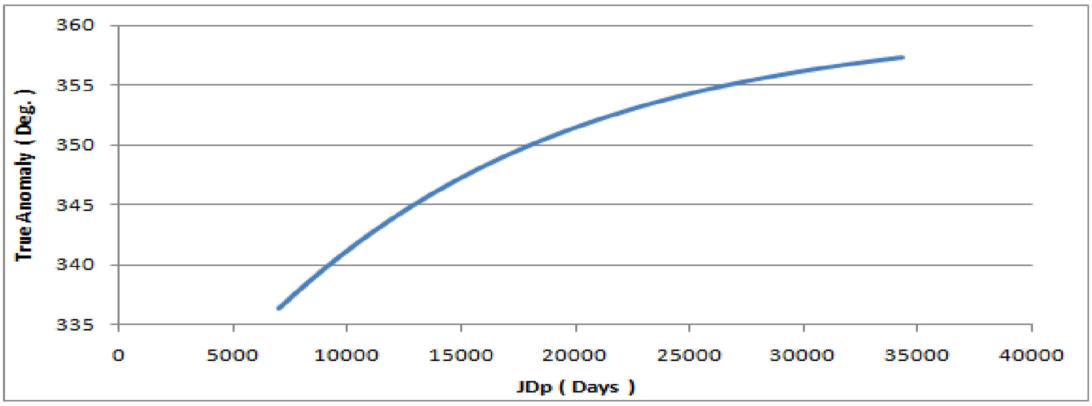
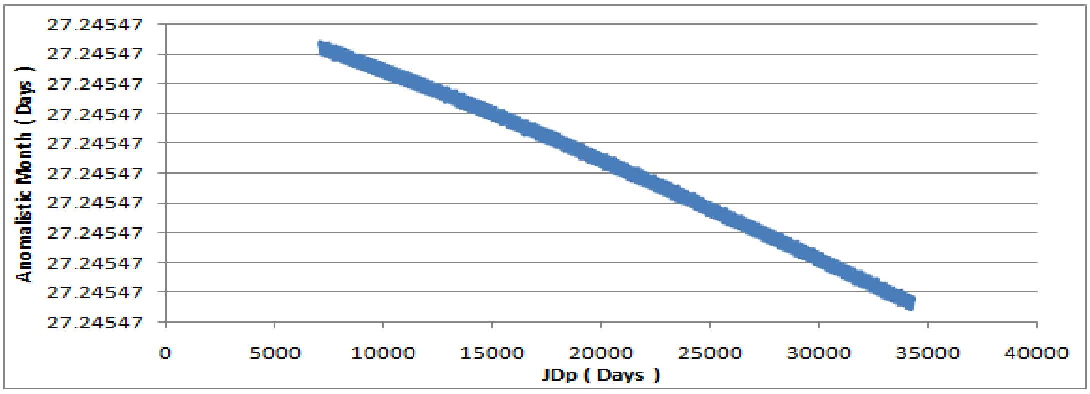
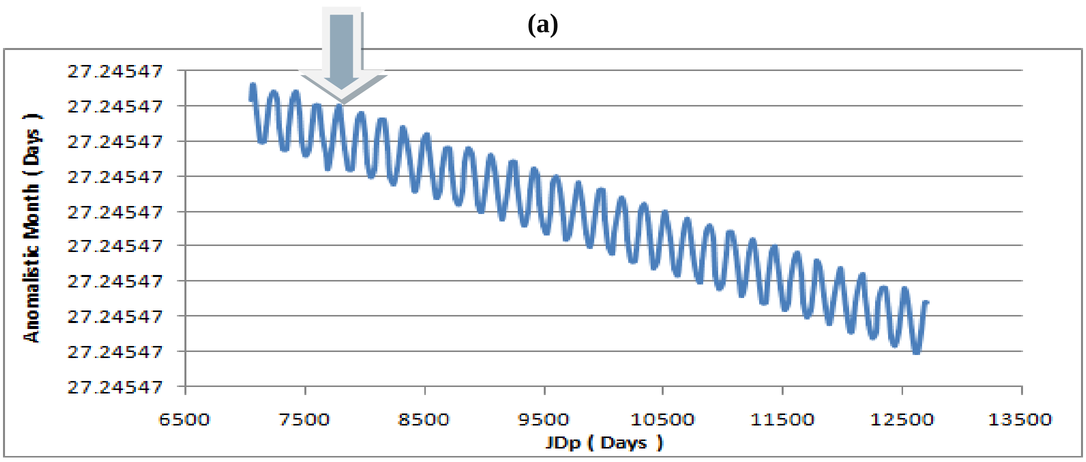


Figure 9- Shows the variation of the Moon true anomaly with time.



(a)



(b)

Figure 10-a.b Shows the variation of the Moon anomalistic month with time.

6. Conclusions

1. The Moon distance variation in the range (35722.5757-40600.0101) km, and the mean value (38161.2929 km).
2. The method that was used add the effect of the perturbation to all orbital elements of the Moon.
3. The Moon orbital elements have a secular change with solar attraction.
4. The solar attraction on the Moon made a balance with a planets attraction to make a stability on the Moon's orbit.

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