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Role Of ψ-Operator in the Study of Minimal Open Sets

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ABSTRACT

The operator ψ has been introduced as an associated set-valued set function. Although it has importance for the study of minimal open sets as well as minimal I-open sets. As a result of this study, we introduce minimal I^*-open sets . In this study, several characterizations of minimal I^*-open sets are also investigated. This study also discusses the role of minimal I^*-open sets in the $*$-locally finite spaces. In an aspect of topological invariant, the homeomorphic images of minimal I^*-open set has been discussed here.

Keywords: $*$-open set, Minimal I^*-open set, $()^*$-operator, ψ-operator, Homeomorphism.
Mathematics Subject Classification: Primary 54A10, Secondary 54A05; 54C08.

1. INTRODUCTION

Recently Selim and Modak [*1*] introduced associated set-valued set function (in short associated function) in literature. The operator ψ [2, 3] on an ideal topological space is an example of an associated function. This associated function ψ has the association with the local function [*4*] of the ideal topological spaces and they are related by the following relation $\psi(A) = X\setminus(X\setminus A)^*$ [*3*,*5*]. Interior and closure operators of a topological space are also an example of associated functions. On the other hand, in [*6*], Nakaoka and Oda have introduced minimal open sets in topological space. Further, they discussed its various properties in topological spaces and in locally finite spaces. In [*7*], Jiang, Xiago-long and Young-bae have introduced minimal I-open sets in ideal topological spaces and characterized its various properties in 2018. In [*8*], Rashid and Hussein have introduced maximal and minimal regular β-open sets in topological spaces and discussed its related properties.

In this paper, we have jointly studied associated functions and minimal I-open sets. As an extraction, we have found minimal I^*-open sets in the topological spaces with ideals. These sets were played an important role in the study of local functions and set operators ψ. Homeomorphisms in the topological spaces also play a remarkable role for the studying of minimal I^*-open sets. The above ideas will also be introduced in terms of the recent topology $\tau_{Cl^*_\gamma}$ introduced by Tormet, Yavina and Brown in [*9*] and topologies σ_ξ and σ_{ξ_0} introduced by Yalaz and Kaymakci in [*10*].

2. Historical Background

The study of generalization of the limit points was introduced by Kuratowski [*11*] and

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Vaidyanathswamy [***12***] with the assistance of local function of the topological spaces. The local function has been defined with the assistance of ideal: A collection I of subsets of a set X is called an ideal on X if I is closed under hereditary and finite unions. If I is an ideal on a topological space (X,τ), then it is called ideal topological space and we call it a 'IT space' throughout this paper. Throughout this paper, the triplicate (X,τ,I) is denoted as XIT where the topological space (X,τ) will be denoted as XT and we write 'T space' instead of topological space, i.e., we have $XIT=(X,\tau,I)$ and $XT=(X,\tau)$. According to the modern notation of local function we have, $M^*(XIT)=\{p\in X: M\cap V\notin I \text{ for every } V\in\tau(p)\}$, where $\tau(p)=\{V\in\tau: p\in V\}$, when there is no scope of misunderstanding, we will write M^* or simply M^{*I} or $M^*(I,\tau)$ or M_τ^{*I} and call it the "local function of M".

This local function helps to determine a new topology on X, it is called $*$-topology and member of the $*$-topology are called the $*$-open sets. One of the most useful basis of the $*$-topology is, $\beta(I,\tau)=\{M\backslash I_1: M\in\tau, I_1\in I\}$ [***4***]. One of the most important question is, when $\beta(I,\tau)$ and $*$-topology are equal?. Njåstad [***13***,***14***] has given the answer to this question with the help of "Compatibility". The ideal I is called compatible with τ, denoted $I\sim\tau$, if the following is true for each $M\subset X$: if for all $p\in M$, there exists $V\in\tau(p)$, where $\tau(p)=\{V\in\tau: p\in V\}$ such that $V\cap M\in I$, then $M\in I$. The operator $\psi: \wp(X)\to\tau$ which has been defined as an associated function of local function $()^*$, that is $\psi(M)=X\backslash(X\backslash M)^*$, we will write simply $\psi_\tau(M)$ or $\psi_\tau^I(M)$. In this context, for $I\sim\tau$, $\psi(\psi(M))=\psi(M)$ [*2*] and $\psi(M)\backslash M\in I$ for every $M\subset X$. For the $*$-topological space, we denoted 'Cl^*', 'Int^*' and '$\aleph^*(p)$' as the closure operator, interior operator and collection of all $*$- open neighbourhoods of an element p respectively and $Cl^*(M)=X\backslash Int^*(X\backslash M)$. Furthermore, in the IT space XIT, mathematicians handles two structures on X, thus the condition $\tau\cap I=\{\emptyset\}$ is useful for the study of the same field. These concepts introduced by Newcomb [***15***] by the name of τ-boundary. As an application of local function, I-open set has been defined in literature. For a XIT space: A subset M of X is called I-open if $M\subset Int(M^*)$ [***16***]. The set of all I-open sets in a topological space with ideal XIT is denoted by $IO(XIT)$ or written simply as $IO(X)$ when there is no scope of misunderstanding.

3. MINIMAL I^*-OPEN SET

In this part, We now have introduced minimal I^*-open set and many features and characterizations of minimal I^*-open sets have been studied:

Definition 3.1 [*7*] Let XIT be an IT space. A subset $M(\neq\emptyset)$ of X is said to be a minimal I-open set if it is an I-open set satisfying $(\forall A\in IO(X))(A\subset M\Rightarrow A=\emptyset \text{ or } A=M)$.

Definition 3.2 A $*$-open set M in the IT space XIT is called minimal I^*-open set if and only if each and every $*$-open set contained in $\psi(M)$ is either $\emptyset$ or $\psi(M)$.
The set of all minimal I^*-open sets in a IT space XIT is denoted by $MI^*O(XIT)$ or written simply as $MI^*O(X)$ when there is no scope of misunderstanding.

Example 3.3 Let $X=\{e^1,e^2,e^3\}, \tau=\{\emptyset,X,\{e^1\},\{e^2\},\{e^1,e^2\}\}$ and $I=\{\emptyset,\{e^1\}\}$. Therefore all the $*$-open sets are $\emptyset,X,\{e^1\},\{e^2\},\{e^1,e^2\},\{e^2,e^3\}$. Here $\psi(\emptyset)=X\backslash X^*=X\backslash\{e^2,e^3\}=\{e^1\}$. Then $\emptyset$ is an element of $MI^*O(X)$. If we take $M=\{e^1\}$. Since $M\in I$, then $\psi(M)=X\backslash X^*=X\backslash\{e^2,e^3\}=\{e^1\}$. Hence the non-empty $*$-open set $M=\{e^1\}$ which is contained in $\psi(M)$ is $\psi(M)$. Thus $M=\{e^1\}$ is an element of $MI^*O(X)$.
If we take $N=\{e^1,e^2\}$. Then $\psi(N)=X\backslash(X\backslash N)^*=X\backslash\{e^3\}^*=X\backslash\{e^3\}=\{e^1,e^2\}$. Here the non empty $*$-open sets $\{e^1\}$ and $\{e^2\}$ which are contained in $\psi(N)$ but not equal to $\psi(N)$.

Hence $N = \{e^1, e^2\}$ is not an element of $MI^*O(X)$.

Example 3.4 Consider that XIT is an IT space where $X = \mathbb{N}$, set of all natural numbers, $\tau = \{\emptyset, X, \{1\}, \{1,2\}, \{1,2,3\}, \dots, \{1,2,3, \dots, n\}, \dots\}$ and $I = \{\emptyset, \{1\}\}$. Take $M = \{1\}$, then $\psi(M) = M$. This shows that $M = \{1\}$ is a member of $MI^*O(X)$.

Definition 3.5 Let XIT be an IT space. Then XIT is called ideal topological space with minimal I^*-open set(s) (Simply $ITSMI^*O$), if there exists a minimal I^*-open set in XIT.

Example 3.6 Consider the example $X = \{e^1, e^2, e^3\}, \tau = \{\emptyset, X, \{e^1\}, \{e^2\}, \{e^1, e^2\}\}$ and $I = \{\emptyset, \{e^1\}, \{e^2\}, \{e^1, e^2\}\}$. Therefore all the $*$ -open sets are: $\emptyset, X, \{e^1\}, \{e^2\}, \{e^3\}$, $\{e^1, e^2\}, \{e^2, e^3\}, \{e^1, e^3\}$. If we take $M = \emptyset \in I$, then $\psi(M) = X \backslash X^* = X \backslash \{e^3\} = \{e^1, e^2\}$. This implies $M = \emptyset$ is not a minimal I^*-open set. If we take $M = \{e^1\} \in I$, then $\psi(M) = X \backslash X^* = X \backslash \{e^3\} = \{e^1, e^2\}$. This implies $M = \{e^1\}$ is not a member of $MI^*O(X)$. If we take $M = \{e^2\} \in I$, then $\psi(M) = \{e^1, e^2\}$ and hence $M = \{e^2\}$ is not a a member of $MI^*O(X)$. If we take $M = \{e^3\}$, then $\psi(M) = X$ and hence $M = \{e^3\}$ is not a member of $MI^*O(X)$. If we
3
take $M = \{e^1, e^2\} \in I$, then $\psi(M) = \{e^1, e^2\}$ and hence $M = \{e^1, e^2\}$ is not a member of $MI^*O(X)$. If we take $M = \{e^2, e^3\}$, then $\psi(M) = X$ and hence $M = \{e^2, e^3\}$ is not a member of $MI^*O(X)$. If we take $M = \{e^1, e^3\}$, then $\psi(M) = X$ and hence $M = \{e^1, e^3\}$ is not a member of $MI^*O(X)$. If we take $M = X$, then $\psi(M) = X$ and hence $M = X$ is not a member of $MI^*O(X)$. Therefore, there is no member in $MI^*O(X)$ in this IT space XIT.

Note 3.7 In Example 3.3, $\emptyset$ is a member of $MI^*O(X)$. But in Example 3.6, $\emptyset$ is not a member of $MI^*O(X)$. If an ideal I is τ-boundary on a space XT, then $\psi(\emptyset) = \emptyset$ and hence $\emptyset$ is always a member of $MI^*O(X)$ in this IT space.

Note 3.8 X itself becomes a member of $MI^*O(X)$ in the IT space XIT only when the topology τ is indiscrete and the ideal I is trivial, otherwise X is never to be a member of $MI^*O(X)$ since always $\psi(X) = X$.

Note 3.9 If M is a member of $MI^*O(X)$ in the IT space XIT, then $X \backslash M$ need not be a member of $MI^*O(X)$. Here $X \backslash M$ is either a $*$-open set or not. Assume that $X \backslash M$ is a $*$-open set. In Example 3.3, $M = \{e^1\}$ is a member of $MI^*O(X)$ but $X \backslash M = \{e^2, e^3\}$ is not a member of $MI^*O(X)$, since $\psi(X \backslash M) = X$. Also if τ is an indiscrete topology and the ideal I is trivial, then $X \backslash M$ is always a member of $MI^*O(X)$ when M is a member of $MI^*O(X)$. If $X \backslash M$ is not a $*$-open set, then there is no chance to be a member of $MI^*O(X)$.
By the following example, we are to show that a minimal I-open set M in XIT may or may not be a member of $MI^*O(X)$.

Example 3.10 [15] Consider a IT space XIT, where $X = \{e^1, e^2, e^3, e^4\}$, $\tau = \{\emptyset, X, \{e^3\}, \{e^1, e^2\}, \{e^1, e^2, e^3\}\}$ and $I = \{\emptyset, \{e^2\}\}$, we have, $IO(X) = \{\emptyset, \{e^1\}, \{e^3\}, \{e^1, e^2\}, \{e^1, e^3\}, \{e^1, e^2, e^3\}, \{e^1, e^3, e^4\}, X\}$. Thus $\{e^1\}$ and $\{e^3\}$ are minimal I-open sets. The $*$-open sets are $\emptyset, X, \{e^1\}, \{e^3\}, \{e^1, e^2\}, \{e^1, e^3\}, \{e^1, e^2, e^3\}, \{e^1, e^3, e^4\}$. Take $M = \{e^1\}$, then $\psi(M) = \{e^1, e^2\}$. Hence M is not a member of $MI^*O(X)$ but a minimal I-open set. Also if we take $N = \{e^3\}$, then $\psi(N) = \{e^3\}$. Thus $\{e^3\}$ is a member of $MI^*O(X)$ as well as a minimal I-open set. Consequently we say that there is no relation between minimal I-open set and a member of $MI^*O(X)$.

Theorem 3.11 Let XIT be an $ITSMI^*O$. If M be a member of $MI^*O(X)$, then $M = \psi(M)$.

Proof. Let M be a member of $MI^*O(X)$. Since M is a member of $MI^*O(X)$, then $M \in \tau^*$. This implies $M \subset \psi(M)$. So $M \cap \psi(M) \subset \psi(M)$. Since M is a member of $MI^*O(X)$ and $M \cap \psi(M) \neq \emptyset$. Therefore, $\psi(M) \subset M$. Hence $M = \psi(M)$.
Consequently, we say that every member of $MI^*O(X)$ is an open set, $*$-open set and I-open set.

Corollary 3.12 Let XIT be an $ITSMI^*O$. If M be a member of $MI^*O(X)$, then $X\backslash M=(X\backslash M)^*$

Proof. The proof is obvious and hence omitted.
By the following example, we are to show that any $*$-open set M in an IT space XIT with $M = \psi(M)$ does not imply M is always a member of $MI^*O(X)$.

Example 3.13 We consider Example 3.3 and let us take a $*$-open set $M = \{e^1, e^2\}$. Then $M = \psi(M)$. But M is not a member of $MI^*O(X)$.

Note 3.14 Thus, Theorem 3.11 is a sufficient condition for the equality $\psi(M) = M$.

Lemma 3.15 Let XIT be an $ITSMI^*O$. Let M be a member of $MI^*O(X)$ and N be a $*$-open set. Then either $M \cap N = \emptyset$ or $\psi(M) \subset N$.

Proof. Let M be a member of $MI^*O(X)$and N be a $*$-open set such that $M \cap N \neq \emptyset$. Since M is member of $MI^*O(X)$ and $M \cap N \subset M = \psi(M)$ (by Theorem 3.11), then we have $M \cap N = \psi(M)$. This implies $\psi(M) \subset N$.

Corollary 3.16 Let XIT be an $ITSMI^*O$. Let M be a member of $MI^*O(X)$ and N be a $*$-open set. Then either $M \cap N = \emptyset$ or $X\backslash N \subset (X\backslash M)^*$.

Proof. The proof is obvious by Lemma 3.15 and hence omitted.

Lemma 3.17 Let XIT be an $ITSMI^*O$ and M, N be two members of $MI^*O(X)$. Then either $M \cap N = \emptyset$ or $M = N$.

Proof. If $M \cap N \neq \emptyset$, then $\psi(M) \subset N$ and $\psi(N) \subset M$ (by Lemma 3.15). This implies $M \subset N$ and $N \subset M$ (by Theorem 3.11). Hence $M = N$

Corollary 3.18 Let XIT be an $ITSMI^*O$. Then finite intersection of members of $MI^*O(X)$ in XIT is a members of $MI^*O(X)$.

By the following example, we are to show that the union of two members of $MI^*O(X)$ may not be a member of $MI^*O(X)$:

Example 3.19 Consider the example $X = \{e^1, e^2, e^3\}$, $\tau = \{\emptyset, X, \{e^1\}, \{e^2\}, \{e^1, e^2\}\}$ and $I = \{\emptyset, \{e^3\}\}$. Therefore all the $*$-open sets are $\emptyset, X, \{e^1\}, \{e^2\}, \{e^1, e^2\}$. Take $M = \{e^1\}$ and $N = \{e^2\}$. Then $\psi(M) = \{e^1\}$ and $\psi(N) = \{e^2\}$. Hence M and N are members of $MI^*O(X)$. Now $M \cup N = \{e^1, e^2\}$ and hence $\psi(M \cup N) = \{e^1, e^2\}$. This shows that $M \cup N$ is not a member of $MI^*O(X)$.

Theorem 3.20 Let XIT be an $ITSMI^*O$. Then intersection of a member of $MI^*O(X)$ and an open set is a member of $MI^*O(X)$.

Proof. Let M be a member of $MI^*O(X)$ and N be an open set. Then both M and N are $*$-open sets. This implies $M \cap N$ is a $*$-open set. Now $M \cap N \subset M = \psi(M)$. If $M \cap N = \emptyset$, then $M \cap N = \psi(M) = M$, which is a member of $MI^*O(X)$. This proves the theorem.

Proposition 3.21 Let XIT be an $ITSMI^*O$ and let M be a non trivial member of $MI^*O(X)$. If p be an element of M, then $M \subset N$ for any $N \in \aleph^*(p)$.

Proof. Let N be any member of $\aleph^*(p)$ such that $M \not\subseteq N$. Then $M \cap N$ is a non-empty proper $*$-open subset of $M = \psi(M)$ (by Theorem 3.11 and since $N \in \tau^*(p)$). This contradicts our assumption that M is a member of $MI^*O(X)$. Hence the result.

Proposition 3.22 Let XIT be an $ITSMI^*O$ and let M be a non trivial member of $MI^*O(X)$.Then $M = \cap \{N : N \in \aleph^*(p)\}$ for any element p of M.

Proof. By Proposition 3.21 and the fact that $M \in \aleph^*(p)$, we have $M \subset \cap \{N : N \in \aleph^*(p)\} \subset M$. Then $M = \cap \{N : N \in \aleph^*(p)\}$.

Theorem 3.23 Let XIT be an $ITSMI^*O$ and $M (\neq \emptyset)$ be a $*$-open set. Then the three criteria listed below are equivalent:
1. M is a member of $MI^*O(X)$;
2. $\psi(M) \subset Cl^*(P)$ for any subset $P(\neq \emptyset)$ of $\psi(M)$;
3. $Cl^*(\psi(M)) = Cl^*(P)$ for any subset $P(\neq \emptyset)$ of $\psi(M)$.

Proof. (1) ⇒ (2): Let $P(\neq \emptyset)$ be any subset of $\psi(M)$. Since M is a member of $MI^*O(X)$, then $\psi(M) = M$ (By Theorem 3.11). Then by Lemma 3.15, for any element p of $M = \psi(M)$ and any $N \in \aleph^*(p)$, we have $P = \psi(M) \cap P \subset N \cap P$. Then we have,
$N \cap P \neq \emptyset$ and hence p is an element of $Cl^*(P)$, since N is any $*$-open set. It follows that $\psi(M) \subset Cl^*(P)$.
(2) ⇒ (3): For any subset $P(\neq \emptyset)$ of $\psi(M)$, we have $Cl^*(P) \subset Cl^*(\psi(M))$. Also $\psi(M) \subset Cl^*(P)$, this implies $Cl^*(\psi(M)) \subset Cl^*(Cl^*(P)) = Cl^*(P)$. This implies $Cl^*(\psi(M)) \subset Cl^*(P)$. Thus $Cl^*(\psi(M)) = Cl^*(P)$ for any subset $P(\neq \emptyset)$ of $\psi(M)$.
(3) ⇒ (1): Let $Cl^*(\psi(M)) = Cl^*(P)$ is satisfied for any subset $P(\neq \emptyset)$ of $\psi(M)$. Suppose that M is not a member of $MI^*O(X)$. Then there is a non-empty $*$-open set N such that $N \subsetneq \psi(M)$, as a result, there is a member $p \in \psi(M)$ such that $p \notin N$. Then we have $Cl^*(\{p\}) \subset X \setminus N$, it follows that $Cl^*(\{p\}) \neq Cl^*(\psi(M))$ which contradicts the given condition. Hence M is a member of $MI^*O(X)$.

Corollary 3.24 Let XIT be an $ITSMI^*O$ and $M(\neq \emptyset)$ be a $*$-open set. Then the three criteria listed below are equivalent:
1. M is a member of $MI^*O(X)$; 5
2. $Int^*(P) \subset (X \setminus M)^*$ for any subset $P(\neq \emptyset)$ of $\psi(M)$;
3. $Int^*(X \setminus M)^* = Int^*(X \setminus P)$ for any subset $P(\neq \emptyset)$ of $\psi(M)$.

Proof. The proof is obvious and hence omitted.
Theorem 3.25 Let XIT be an $ITSMI^*O$ with $\tau \cap I = \{\emptyset\}$ in which every member of $MI^*O(X)$

is contained in I. If M is a member of $MI^*O(X)$, then $\psi(N) \cap M = \emptyset$ for any $N \subset X$.
Proof. Let M be a member of $MI^*O(X)$ and let N be a subset of X. Suppose that $\psi(N) \cap M \neq \emptyset$. Then there exists an element $p \in \psi(N) \cap M$, i.e., $p \in \psi(N)$ and $p \in M$. Since every member of $MI^*O(X)$ is contained in I, then $M \in I$ and so $\psi(N) \cap M \in I$, by heredity. This implies that $p \notin (\psi(N))^*$, by Corollary 3.4 in [*2*]. Since $\psi(N) \in \tau$ and $\tau \cap I = \{\emptyset\}$ then $\psi(N) \subset (\psi(N))^*$. This implies $p \notin \psi(N)$. This contradicts the fact that $p \in \psi(N)$. Therefore $\psi(N) \cap M = \emptyset$.

Corollary 3.26 Let XIT be an $ITSMI^*O$ with $\tau \cap I = \{\emptyset\}$ in which every member of $MI^*O(X)$ is contained in I. If M is a member of $MI^*O(X)$ of X, then $M \subset (X \backslash N)^*$ for any $N \subset X$.
A subset M of a T space (X, τ) is called a pre-open set if $M \subset Int(Cl(M))$ [*17*]. For an IT space XIT, the collection of all pre-open sets in $(X, \tau^*(I))$ is denoted as $PO^*(X, \tau^*(I))$.
By the following example, we are to show that every subset of a pre-open set need not be a pre-open set.

Example 3.27 Consider a topological space XT, where $X = \{e^1, e^2, e^3, e^4\}$, $\tau = \{\emptyset, X, \{e^1\}, \{e^3\}, \{e^1, e^2\}, \{e^1, e^3\}, \{e^1, e^2, e^3\}\}$. Therefore, the τ-closed sets are: $\emptyset, X, \{e^2, e^3, e^4\}, \{e^1, e^2, e^4\}, \{e^3, e^4\}, \{e^2, e^4\}, \{e^4\}$. Take $A = \{e^1, e^2\}$. Then $Cl(A) = \{e^1, e^2, e^4\}$ and hence $Int(Cl(A)) = \{e^1, e^2\}$. Thus $A \subset Int(Cl(A))$. So A is a pre-open set. But if we take $A_1 = \{e^2\}$. Then $Int(Cl(A_1)) = \emptyset$ and so $A_1 \not\subset Int(Cl(A_1))$. Hence A_1 is not a pre-open set. Thus any subset of a pre-open set may not be again a pre-open set.
We know that every I-open set is a pre-open set [*18*] and hence every pre-open set is also a pre-open set in the $*$-topology. Also any subset of a pre-open set is not again a pre-open set. In this respect following theorem is interesting.

Theorem 3.28 Let M be a member of $MI^*O(X)$ of an IT space XIT. Then for any subset $P(\neq \emptyset)$ of M, $P \in PO^*(X, \tau^*(I))$.
Proof. Let M be a member of $MI^*O(X)$ and $P(\neq \emptyset)$ be any subset of M. We have to prove that $P \in PO^*(X, \tau^*(I))$. Since M is a member of $MI^*O(X)$ and $P(\neq \emptyset)$ is any subset of M. Then $\psi(M) = M$ (By Theorem 3.11) and $\psi(M) \subset Cl^*(P)$ (by Theorem 3.23). This implies $M \subset Cl^*(P)$ and hence $Int^*(M) \subset Int^*(Cl^*(P))$. Since M is a $*$-open set, we have $P \subset M = Int^*(M) \subset Int^*Cl^*(P)$. This shows that $P \in PO^*(T, \tau^*(I))$.

Theorem 3.29 Let XIT be an IT space with minimal I^*-open sets and let M be a non trivial member of $MI^*O(X)$ and $P(\neq \emptyset)$ be a subset of X. If there exists a $W \in \aleph^*(P)$ such that $W \subset Cl^*(P \cup M)$, then $P \cup S \in PO^*(X, \tau^*(I))$ for any subset $S(\neq \emptyset)$ of M where $\aleph^*(P)$ denotes collection of all $*$-open sets containing P.
Proof. Since M is a member of $MI^*O(X)$. Then $\psi(M) = M$ (By Theorem 3.11). Also, by Theorem 3.23, $Cl^*(M) = Cl^*(S)$ since $S(\neq \emptyset)$ is a subset of M. Thus we have $Cl^*(P \cup S) = Cl^*(P) \cup Cl^*(S) = Cl^*(P) \cup Cl^*(M) = Cl^*(P \cup M)$. Since $W \subset Cl^*(P \cup M)$, then $W \subset Cl^*(P \cup S)$. Then $Int^*(W) \subset Int^*Cl^*(P \cup S)$. Since $W \in \aleph^*(P)$, then W is a $*$-open set such that $P \subset W$ and hence we have $P \subset W = Int^*(W) \subset Int^*Cl^*(P \cup S)$ by assumption. Also since $Int^*(M) = M \subset Cl^*(M) \subset Cl^*(P) \cup Cl^*(M) = Cl^*(P \cup M)$, then $Int^*(M) \subset Int^*Cl^*(P \cup M)$. Again as M is a $*$-open set, we have $S \subset M = Int^*(M) \subset Int^*Cl^*(P \cup M) = Int^*Cl^*(P \cup S)$. Thus $P \cup S \subset Int^*Cl^*(P \cup S)$. This proves that $P \cup S \in PO^*(X, \tau^*(I))$.

Corollary 3.30 Let XIT be an $ITSMI^*O$ and let M be a non trivial member of $MI^*O(X)$ and
6

$P(\neq \emptyset)$ be a subset of X. If $\exists$ a $W \in \aleph^*(P)$ such that $W \subset Cl^*(M)$, then $P \cup S$ is a pre-open set in $(X, \tau^*(I))$ for any subset $S(\neq \emptyset)$ of M.

Proof. By the given condition, $W \subset Cl^*(M) \subset Cl^*(P) \cup Cl^*(M) = Cl^*(P \cup M)$. So by Theorem 3.29, $P \cup S$ is a pre-open set in $(X, \tau^*(I))$.

By the following example, we are to show that the condition of Theorem 3.29, does not necessarily imply the condition of Corollary 3.30.

Example 3.31 We consider Example 3.3 and take $M = \{e^1\}$ and $P = W = \{e^2, e^3\}$. Then $P \cup M = \{e^1, e^2, e^3\}$ and $(P \cup M)^* = \{e^2, e^3\}$. Therefore $Cl^*(P \cup M) = \{e^1, e^2, e^3\}$. Hence $W \subset Cl^*(P \cup M)$. Also $M^* = \emptyset$ and hence $Cl^*(M) = M \cup M^* = \{e^1\}$. Thus $W = \{e^2, e^3\} \not\subseteq \{e^1\} = Cl^*(M)$.

Corollary 3.32 Let XIT be an $ITSMI^*O$ and let M be a non trivial member of $MI^*O(X)$ and p be an element of X. Define $M_p \equiv \cap \{W: W \in \aleph^*(p)\}$. Then $M_p \cap M = \emptyset$ or $M \subset M_p$.

Proof. If $M \subset W$ for any $W \in \aleph^*(p)$, then $M \subset \cap \{W: W \in \aleph^*(p)\}$. This implies $M \subset M_p$. Otherwise, there exists $W \in \aleph^*(p)$ such that $W \cap M = \emptyset$. Hence $M \cap M_p = \emptyset$.

4. FINITE $*$-OPEN SETS

In this part, we have studied certain features of members of $MI^*O(X)$ in finite $*$-open sets and $*$-locally finite spaces.

Theorem 4.1 Let XIT be an IT space satisfying $I \sim \tau$ and let $N(\neq \emptyset)$ be a finite $*$-open set. Then there exists at least one (finite) non trivial member M of $MI^*O(X)$ such that $M \subset \psi(N)$.

Proof. If N is a member of $MI^*O(X)$, we may write $M = N = \psi(N)$. If N is not a member of $MI^*O(X)$, then there exists a finite $*$-open set N_1 such that $\emptyset \neq N_1 \subsetneq \psi(N)$. If N_1 is member of $MI^*O(X)$, we may write $M = N_1$. If N_1 is not a member of $MI^*O(X)$, then there exists a finite $*$-open set N_2 such that $\emptyset \neq N_2 \subsetneq \psi(N_1)$. This implies $\emptyset \neq N_2 \subsetneq \psi(N_1) \subsetneq \psi(N)$, since $I \sim \tau$. We have a sequence of $*$-open sets satisfying $\psi(N) \supsetneq \psi(N_1) \supsetneq \psi(N_2) \ldots \supsetneq \psi(N_k) \supsetneq \ldots$ if we continue this approach. Since N is a finite set, then $\psi(N)$ is finite and therefore the aforementioned process occurs only finitely. Finally, we get a member M of $MI^*O(X)$, $M = N_t$ for some $t \in \mathbb{Z}_+$.

Corollary 4.2 Let XIT be an IT space satisfying $I \sim \tau$ and let $N(\neq \emptyset)$ be a finite $*$-open set. Then there exists at least one (finite) non trivial member M of $MI^*O(X)$ such that $(X \setminus N)^* \subset X \setminus M$.

Definition 4.3 Let XIT be an IT space. If each element of $\tau^*(I)$ is contained in a finite $*$-open set, then $(X, \tau^*(I))$ is called a $*$-locally finite space.

Corollary 4.4 Let X be a $*$-locally finite space satisfying $I \sim \tau$ and $N(\neq \emptyset)$ be a $*$-open set. Then $\exists$ at least one (finite) non trivial member M of $MI^*O(X)$ such that $M \subset \psi(N)$.

Proof. Since $N \neq \emptyset$, then there exists an element p (say) of N. Since X is a $*$-locally finite space, then there exists a finite $*$-open set N_p such that $p \in N_p$. Since $N \cap N_p (\neq \emptyset)$ is a finite $*$-open set and $I \sim \tau$, then we get a member M of $MI^*O(X)$ such that $M \subset \psi(N \cap N_p) \subset \psi(N)$.

Corollary 4.5 Let X be a $*$-locally finite space satisfying $I \sim \tau$ and $N(\neq \emptyset)$ be a $*$-open set. Then $\exists$ at least one (finite) non trivial member M of $MI^*O(X)$ such that $(X\setminus N)^* \subset X\setminus M$.

Application: Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set. Then by Lemma 3.15 and Theorem 4.1, we see that there exists $k \in \mathbb{Z}_+$ such that $\{N_1, N_2, N_3, \ldots, N_k\}$ is the set of all members of $MI^*O(X)$ in $\psi(M)$. Then it meets the next two conditions:-
(i) $N_i \cap N_j = \emptyset$ for any i, j with $1 \leq i, j \leq k$ and $i \neq j$.

7
(ii) If M' is a member of $MI^*O(X)$ in $\psi(M)$, then there exists i with $1 \leq i \leq k$ such that $M' = M_i$.

Theorem 4.6 Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set and $M \notin MI^*O(X)$. Let $\{M_1, M_2, M_3, \ldots, M_n\}$ be the set of all members of $MI^*O(\psi(M))$ and p be an element of $\psi(M)\setminus(M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_n)$.
In this case, define $M_p \equiv \cap \{W: W \in \aleph^*(p)\}$. Then $\exists\, i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$ such that $M_i \subset \psi(M_p)$.

Proof. If possible, let $M_i \not\subset \psi(M_p)$ for any $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$. Since $\psi(M_p)$ is a $*$-open set, then we have $M_i \cap \psi(M_p) = \emptyset$ for any member M_i of $MI^*O(\psi(M))$ by Lemma 3.15. Since $M_p(\neq \emptyset)$ is a finite $*$-open set and $I \sim \tau$, then there exists a member M' of $MI^*O(\psi(M))$ such that $M' \subset \psi(M_p)$ by Theorem 4.1. Since $M' \subset \psi(M_p) \subset \psi(M)$, then M' is a member of $MI^*O(\psi(M))$. By assumption, we have $M_i \cap M' \subset M_i \cap \psi(M_p) = \emptyset$ for any member M_i of $MI^*O(\psi(M))$. Then $M' \neq M_i$ for any $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$. This contradicts our assumption. Hence the result is got.

Corollary 4.7 Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set and $M \notin MI^*O(X)$. Let $\{M_1, M_2, M_3, \ldots, M_n\}$ be the set of all members of $MI^*O(\psi(M))$ and p be an element of $\psi(M)\setminus(M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_n)$. Define $M_p \equiv \cap \{W: W \in \aleph^*(p)\}$. Then, there exists $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$ such that $(X\setminus M_p)^* \subset X\setminus M_i$.

Proposition 4.8 Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set and $M \notin MI^*O(X)$. Let $\{M_1, M_2, M_3, \ldots, M_n\}$ be the set of all members of $MI^*O(\psi(M))$ and p be an element of $\psi(M)\setminus(M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_n)$. Then, there exists $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$ such that $M_i \subset \psi(W_p)$ for any $W_p \in \aleph^*(p)$.

Proof. Since $W_p \supset \cap \{W: W \in \aleph^*(p)\}$ then by Theorem 4.6 we have the required result.
Corollary 4.9 Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set and $M \notin MI^*O(X)$. Let $\{M_1, M_2, M_3, \ldots, M_n\}$ be the set of all members of $MI^*O(\psi(M))$ and p be an element of $\psi(M)\setminus(M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_n)$. Then, there exists $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$ such that $(X\setminus W_p)^* \subset X\setminus M_i$ for any $W_p \in \aleph^*(p)$.

Note 4.10 Let XIT be an IT space satisfying $I \sim \tau$ and let $M(\neq \emptyset)$ be a finite $*$-open set and $M \notin MI^*O(X)$. Let $\{M_1, M_2, M_3, \ldots, M_n\}$ be the set of all members of $MI^*O(\psi(M))$ and p be an element of $\psi(M)\setminus(M_1 \cup M_2 \cup M_3 \cup \ldots \cup M_n)$. Then there does not always exist a $i \in \mathbb{Z}_+$ of $\{1,2,3,\ldots,n\}$ such that $p \in Cl^*(M_i)$.

We are now giving an example in support of the Note 4.10:
Example 4.11 Consider the example $X=\{e^1,e^2\}$, $\tau=\{\emptyset,X,\{e^1\},\{e^2\}\}$ and $I=\{\emptyset,\{e^1\}\}$. Therefore all the $*$-open sets are $\emptyset,X,\{e^1\},\{e^2\}$.Then $\psi(\emptyset)=X\setminus X^*=\{e^1,e^2\}\setminus\{e^2\}=\{e^1\}$, $\psi(\{e^1\})=X\setminus(X\setminus\{e^1\})^*=X\setminus\{e^2\}^*=X\setminus\{e^2\}=\{e^1\}$, $\psi(\{e^2\})=X\setminus(X\setminus\{e^2\})^*=X\setminus\{e^1\}^*=X\setminus\emptyset=X$, $\psi(X)=X\setminus\emptyset^*=X\setminus\emptyset=X$. Then we see that $\psi(\emptyset)\setminus\emptyset=\{e^1\}\in I$, $\psi(\{e^2\})\setminus\{e^2\}=X\setminus\{e^2\}=\{e^1\}\in I$, $\psi(\{e^1\})\setminus\{e^1\}=\{e^1\}\setminus\{e^1\}=\emptyset\in I$ and $\psi(X)\setminus X=X\setminus X=\emptyset\in I$. So, $\tau\sim I$. If we take $M=\{e^2\}$, then $\psi(M)=X\setminus(X\setminus\{e^2\})^*=X\setminus\{e^1\}^*=X\setminus\emptyset=X$. This implies M is not a member of $MI^*O(X)$. Take $M_1=\{e^1\}\in I$, then $\psi(M_1)=X\setminus X^*=X\setminus\{e^2\}=\{e^1\}$. This implies M_1 is a member of $MI^*O(X)$. Now member of $MI^*O(\psi(M))$ is $M_1=\{e^1\}$. Hence $\psi(M)\setminus M_1=\{e^2\}$ and $Cl^*\{e^1\}=\{e^1\}\cup\{e^1\}^*=\{e^1\}$. So $e^2\notin Cl^*\{e^1\}$. Hence the result is obtained.

5. IMAGE OF MINIMAL I^*-OPEN SETS

In this part, we have studied homeomorphic images of members of $MI^*O(X)$ in the topological spaces with an ideal.

Lemma 5.1 [*15*] Let $f: X\to Y$ be a function. If I is an ideal on X, then $f(I)=\{f(I_1): I_1\in I\}$
8
is also an ideal on Y.

Lemma 5.2 [*15*] Let $f: X\to Y$ be an injective function. If I is an ideal on Y, then the set $f^{-1}(I)=\{f^{-1}(I_1): I_1\in I\}$ is also an ideal on X.
We shall consider the following four results which will help us for discussing the image of members of $MI^*O(X)$. Although these four results have been considered in [*19*], but their proofs have been made with the help of filters. Here we shall prove these results with the help of ideals.

Result 5.3 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on X. If $f: X\to Y$ is a homeomorphism, then for any $M\in\wp(X)$, $(f(M))^{*f(I)}=f(M^{*I})$.

Proof. Let $p\in(f(M))^{*f(I)}$. Then $f(M)\cap V\notin f(I)$ for every $V\in\tau_2(p)$. Since f is a homeomorphism, then there exists $q\in X$ such that $f(q)=p$. Since f^{-1} is continuous for any $U\in\tau_1(q)$, there exists $V_1\in\tau_2(p)$ such that $f^{-1}(V_1)\subset U$. Thus for any $U\in\tau_1(q)$, there exists $V_1\in\tau_2(p)$ such that $V_1\subset f(U)$. This implies $f(M)\cap f(U)\notin f(I)$ for every $U\in\tau_1(q)$. Thus $f(M\cap U)\notin f(I)$ for every $U\in\tau_1(q)$, since f is an injective. Then by definition of $f(I)$, $M\cap U\notin I$ for every $U\in\tau_1(q)$. Hence, $q\in(M)^{*I}$ and consequently $f(q)\in f((M)^{*I})$. Thus $p\in f((M)^{*I})$. Since p is an arbitrary member of $(f(M))^{*f(I)}$, then $(f(M))^{*f(I)}\subset f((M)^{*I})$.
Conversely let $r\in f((M)^{*I})$. Then there exists $s\in(M)^{*I}$ such that $f(s)=r$. Since $s\in(M)^{*I}$, then $M\cap U\notin I$ for every $U\in\tau_1(s)$. This implies $f(M\cap U)\notin f(I)$ for every $U\in\tau_1(s)$ and hence $f(M)\cap f(U)\notin f(I)$ for every $U\in\tau_1(s)$. Since f is continuous, then for every $V\in\tau_2(r)$ there exists $U_1\in\tau_1(s)$ such that $f(U_1)\subset V$. Thus $f(M)\cap V\notin f(I)$ for every $V\in\tau_2(r)$. Consequently $r\in(f(M))^{*f(I)}$. Since r is an arbitrary member of $f((M)^{*I})$, then $f((M)^{*I})\subset(f(M))^{*f(I)}$. Combining, we get the required result.

Result 5.4 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on X. If
$f: X\to Y$ is a homeomorphism, then for any $M\in\wp(X)$, $\psi_{\tau_2}^{f(I)}(f(M))=f(\psi_{\tau_1}^{I}(M))$.

Proof. We have, $\psi_{\tau_2}^{f(I)}(f(M))=Y\setminus(Y\setminus f(M)$. This implies $\psi_{\tau_2}^{f(I)}(f(M))=f(X)\setminus(f(X)\setminus$

$f(M))^{*f(I)} = f(X)\backslash(f(X\backslash M))^{*f(I)}$, since f is bijective. By Result 5.3, $\psi_{\tau_2}^{f(I)}(f(M)) = f(X)\backslash f((X\backslash M)^{*I})$ and hence $\psi_{\tau_2}^{f(I)}(f(M)) = f(X\backslash(X\backslash M)^{*I}) = f(\psi_{\tau_1}^{I}(M))$. Hence the result.

Result 5.5 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on Y. If $f: X \rightarrow Y$ is a homeomorphism, then for any $M \in \wp(Y)$, $f^{-1}(M^{*I}) = (f^{-1}(M))^{*f^{-1}(I)}$.

Proof. Follows from Result 5.3.

Result 5.6 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on Y. If $f: X \rightarrow Y$ is a homeomorphism, then for any $M \in \wp(Y)$, $\psi_{\tau_1}^{f^{-1}(I)}(f^{-1}(M)) = f^{-1}(\psi_{\tau_2}^{I}(M))$.

Proof. Follows from Result 5.4.

Theorem 5.7 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on X. If $f: X \rightarrow Y$ is a homeomorphism, then for any member M of $MI^*O(X)$, $f(M)$ is a member of $M(f(I))^*O(Y)$.

Proof. If possible, let $f(M)$ is not a member of $Mf(I)^*O(Y)$, then there exists $\emptyset \neq V \in \tau_2^*(f(I))$ such that $V \subsetneq \psi_{\tau_2}^{f(I)}(f(M))$. This implies $V \subsetneq f(\psi_{\tau_1}^{I}(M))$ (by Result 5.4). Since f is homeomorphism, $f^{-1}(V) \subsetneq \psi_{\tau_1}^{I}(M) = M$ (by Theorem 3.11) and hence $\emptyset \neq f^{-1}(V) \subsetneq M$. Since f is a homeomorphism and $V \in \tau_2^*(f(I))$, then $f^{-1}(V) \in \tau_1^*(I)$ and hence $\emptyset \neq f^{-1}(V) \subsetneq M$ leads a contradiction as M is a member of $MI^*O(X)$. Hence the result.

Theorem 5.8 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on Y. If $f: X \rightarrow Y$ is a homeomorphism, then for any member M of $MI^*O(Y)$, $f^{-1}(M)$ is a member of $M(f^{-1}(I))^*O(X)$.

Proof. It follows from Result 5.7.

Note 5.9 Let (X,τ_1) and (Y,τ_2) be two T spaces and I be an ideal on X. If $f: X \rightarrow Y$ is a
9
function, then for any member M of $MI^*O(X)$, $f(M)$ is not always a member of $M(f(I))^*O(Y)$.
We are now giving an example in support of the Note 5.10:

Example 5.10 Consider (X,τ_1,I) be a T space with an ideal where $X = \{p,q,r\}$, $\tau_1 = \{\emptyset, X, \{p\}, \{q\}, \{p,q\}\}$ and $I = \{\emptyset, \{r\}\}$. Clearly $M = \{p\}$ is a member of $MI^*O(X)$. Again consider (Y,τ_2) be another topological space where $Y = \{a,b\}$ and $\tau_2 = \{\emptyset, \{a\}, Y\}$. Let us define a function $f: X \rightarrow Y$ defined by $f(p) = b$, $f(q) = b$ and $f(r) = a$. Then $f(I) = \{f(I_1): I_1 \in I\} = \{\emptyset, \{a\}\}$ is an ideal on Y. Thus $\tau_2^*(f(I)) = \{\emptyset, \{a\}, \{b\}, S\}$. Now $f(M) = \{f(x): x \in M\} = \{b\}$. Therefore $\psi_{\tau_2}^{f(I)}(f(M)) = Y\backslash(Y\backslash\{b\})^{*f(I)} = Y\backslash\{a\}^* = Y\backslash\emptyset$. This implies $f(M)$ is not a member of $M(f(I))^*O(Y)$.

Note 5.11 [*20*] Let $f: X \rightarrow Y$ be a function. If I is an ideal on Y, then $\overleftarrow{f}(I) = \{A: A \subset f^{-1}(I_1) \in f^{-1}(I)\}$ is also an ideal on X.

Note 5.12 Let (X,τ_1) and (Y,τ_2) be two topological spaces and I be an ideal on Y. If $f: X \rightarrow Y$ is a function, then for any member M of $MI^*O(Y)$, $f^{-1}(M)$ is not always a member $M(\overleftarrow{f}(I))^*O(X)$.

We are now giving an example in support of the Note 5.12:

Example 5.13 Consider (Y,τ_2,I) be an IT space where $Y = \{e^1, e^2, e^3\}$, $\tau_2 = \{\emptyset, Y, \{e^1\}, \{e^2\}, \{e^1, e^2\}\}$ and $I = \{\emptyset, \{e^1\}\}$. Clearly $M = \{e^1\}$ is a membe of $MI^*O(Y)$. Again consider

(X, τ_1) be another T space where $X = \{a, b\}$ and $\tau_1 = \{\emptyset, \{a\}, \{b\}, X\}$.
Let us define a function $f: X \to Y$ defined by $f(a) = e^1$ and $f(b) = e^1$. Then $f^{-1}(I) = \{f^{-1}(I_1): I_1 \in I\} = \{\emptyset, \{a, b\}\}$ and hence $\overleftarrow{f}(I) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ is an ideal on X by Note 5.13. Thus $\tau_1^*(\overleftarrow{f}(I)) = \{\emptyset, \{a\}, \{b\}, X\}$. Now $f^{-1}(M) = \{a, b\}$. Therefore $\psi_{\tau_1}^{\overleftarrow{f}(I)}(f^{-1}(M)) = X \setminus (X \setminus \{a, b\})^{*\overleftarrow{f}(I)} = X \setminus \emptyset = X \setminus \emptyset = X$. This implies $f^{-1}(M)$ is not a member of $M(\overleftarrow{f}(I))^*O(X)$.

CONCLUSION

In this writeup, we have added a new kind of open set called minimal I^*-open set in IT spaces and discussed its various properties. Using this idea, we have discussed relationship of pre-open sets, minimal open sets, minimal I-open sets and minimal I^*-open sets. Also we have studied minimal I^*- open sets in locally finite and finite spaces. Furthermore, images of minimal I^*-open sets under homoemorphisms have been discussed here. One can study on the continuity in topological spaces using minimal I^*-open set.The other properties of these type of sets can be found and one can introduce some other relations on these type of sets to develop the skills of learning mathematics. This research work can be extended to compactness, paracompactness, connectedness etc. In future, one may be interested for working on minimal I^*-closed sets, separation axioms of minimal I^*-open set and connectedness in IT spaces using these sets.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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10

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