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Calculation of the Best Stability Orbit of the Satellite around the Earth before Transferring to Orbit around Mars

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Abstract:
In this research, the eccentricity will be calculated as well as the best height of satellite orbit that can used to transfer from that orbit around the Earth to construct an interplanetary trajectory, for example Mars, when the transfer can be accomplished by a simple impulse, that means the transfer consists of an elliptical orbit from the inner orbit (at a perigee point) to the outer orbit (at apogee point). We will determine Keplerian equation to find the value of a mean anomaly(M) by Rung-Cutta method.
There are several types of satellites orbits around the Earth, but by this study, we find that the best stable orbit to the satellite that is used to inter its orbit around Mars is the Medium Earth Orbit (MEO) at a height about 3500km above the Earth surface. (MEO) comprises a wide range of orbits anywhere between LEO and GEO, its height above the Earth surface is about 2000-35700 km, and it is very commonly used by navigation satellites.
The values of Euler angles (i=23.45°, Ω=20°, w=60°) were installed and the values of each of the height of the perigee point were changed by controlling the height from the ground surface (hp=500, 3500, 35800) km and the eccentricity of the orbit from e=0.1 to 0.9. We found that the best stable orbit of the satellite around the Earth in preparation for its transition to another orbit around Mars is at an altitude of 3500 km, with eccentricity of approximately 0.6, because this orbit was the most stable according to the program results shown in the graphs (no.10) because at the perigee of the orbit we obtained the highest escape speed was approximately 8 km/sec, which means the possibility of transfer with the lowest possible energy.

Keywords: Eccentricity, Semi-major axis, Escape velocity, Orbital elements

حساب افضل مدار مستقر للقمر الاصطناعي حول الارض قبل اطلاقه للدخول في مداره حول المريخ

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الخلاصة:
في هذا البحث، سيتم حساب الانحراف المركزي (e) وكذلك أفضل ارتفاع (hp) لمدار الأقمار الصناعية الذي يمكن استخدامه للانتقال من ذلك المدار حول الأرض تمهيدا للانتقال لبناء مسار حول الكواكب، على سبيل المثال المريخ، وعندما يمكن تحقيق النقل بواسطة دفعة بسيطة، حيث يكون النقل من مدار بيضاوي الشكل من المدار الداخلي (عند نقطة الحضيض) إلى المدار الخارجي (عند نقطة الأوج).

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في هذا البحث قمنا أولا بحل معادلة كبلر لغرض إيجاد قيمة متوسط الشذوذ للمدار (M) مع تثبيت قيم بعض العناصر المدارية وتم استخدام طريقة رونج-كوتا في حل المعادلة.

هناك عدة أنواع من الأقمار الصناعية التي تدور حول الأرض، ولكن من خلال هذه الدراسة وجدنا أن أفضل مدار للقمر الصناعي الذي يراد استخدامه كمرحلة مؤقتة للانتقال الى مدار حول المريخ هو المدار الأرضي المتوسط (MEO) حيث وجدنا أن أفضل ارتفاع كان حوالي 3500 كم فوق سطح الأرض، وتضم (MEO) مجموعة واسعة من المدارات تقع في أي مكان بين المدار الأرضي المنخفض والمدار الأرضي المتوسط، حيث يبلغ ارتفاعه فوق سطح الأرض حوالي بين 2000-35700 كم، ويستخدم بشكل شائع جدا من قبل أقمار الملاحة الصناعية.

تم تثبيت قيم زوايا اويلر ($i=23.45°$, $\Omega=20°$, $w=60°$) و القيام بتغيير قيم كل من ارتفاع نقطة الحضيض بواسطة التحكم بالارتفاع عن سطح الارض km (35800 , 3500 , 500)=hp والشذوذ المركزي للمدار (e) للقيم من 0.1 الى 0.9

و قد وجدنا بأن افضل مدار مستقر للقمر الصناعي حول الارض تمهيدا لانتقاله لمدار آخر حول المريخ هو عند الارتفاع 3500 km، و بشذوذ مركزي مقداره تقريبا 0.6 و ذلك لانه هذا المدار كان الاكثر استقرارا حسب نتائج البرنامج الموضحة بالرسم البياني رقم 10 كذلك لانه عند نقطة الحضيض للمدار حصلنا على اعلى سرعة هروب تمثلت بالقيمة 8 km/sec تقريبا مما يعني إمكانية الانتقال بأقل طاقة ممكنة.

1. Introduction:

Orbital elements are the investigation of how articles move affected by gravity. Much numerical and actual disclosures have been made by individuals attempting to tackle issues in orbital dynamics, including the greater part of Newton's advancement of mathematics and insight physical sciences, after that Einstein's hypothesis of general relativity [1].

In the 20th century, individuals started dispatching rockets into space. Spaceships opened up an entirely different series of issues, since they could change their circles. Just foreseeing where a spaceship would go depending on its present circle was at this point not adequate. The issues of deciding instructions to change from one circle to an alternate circle became intriguing [2].

Early space apparatus used to high push compound rocket motors to move starting with one circle orbit onto the next, the push from these rockets is regularly approximated, where it is that they produce a big push within a short time [3].

The law of force requires a rocket to launch fuel to create a big push used by engineers. The power delivered by the rocket must be corresponding to the speed, the successful escape velocity from orbit is called V_{esc}, rockets have a particular motivation around 250-400 second, this makes V_{esc} about (2.4 − 4.6) km/sec[4,5].

In this research, many perturbations that affect the orbit of satellite around the Earth were studied, such as (the Earth oblate, the atmospheric drag, solar radiation pressure, sun gravity and the third body attraction (for example: Moon)) [6].

2. The Important Orbital Elements:

2-1 *Semi-major axis (a) and Escape Velocity:* it is describing half the distance across the orbit's long axis, we can find it by the relationship:

$$a = -\frac{\mu}{2\varepsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where: a is the semi-major axis (km), μ is the gravitational parameter of the central body (km^3/s^2), ε is specific mechanical energy (km^2/s^2) [7].
From Kepler's third law, we could evaluate the semi major axis of the satellite orbits, as follows:

$$a = \sqrt[3]{\frac{GM}{4\pi^2} T^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where, G is the gravitational constant, M is the mass of earth, and T is the orbital round trip time. The semi major axis, a is measured using AU unit which denotes the average radius of the earth's orbit and is equal to 1 AU=1.5*10^11 m or in km. unit. [8,9]

On the other hand, an important factor in the solar system calculations which is the escape velocity of the object from the Earth surface. This measure could be evaluated using the equation:

$$v_{esc.} = \sqrt{\frac{2GM}{R_{E+h}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where, R_E is the Earth's radius, h is the station height, M is mass of the Earth and μ is the gravitational factor of the Earth(μ=GM).

The escape velocity defines the necessary speed to escape from the Earth's surface to space when the satellite transfers from orbit to another, the velocity must change sometimes and is called the escape velocity of orbit [8].

2-2 *Orbital Eccentricity:* The known planets in the planetary group have almost round circles, with an eccentricity (e) less than 0.04. Besides, a number of actual cycles can clammy or invigorate orbital unconventionalities. The orbit elliptical eccentricity can be found using the equation:

$$e = \sqrt{1 - \frac{b^2}{a^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Where, a, and b are the semi major and minor axis of the elliptic orbit respectively. Figure 1 shows the elliptical orbit of a satellite around any planet of the solar system. Now we have two important elements of the satellite orbital around the Earth: it is size by (a), and it is shape b (e) [1,2]. Actually, values of the eccentricity, (e), varies between (0, and 1). If e=0, then a=b, else if e<0, then the semi major axis a ≥ b for ellipse [7,9]. And it is specifying the shape of an orbit by the ratio of the distance between the two foci (2c) and the major axis(a), it can be found by:

$$e = \frac{2c}{2a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

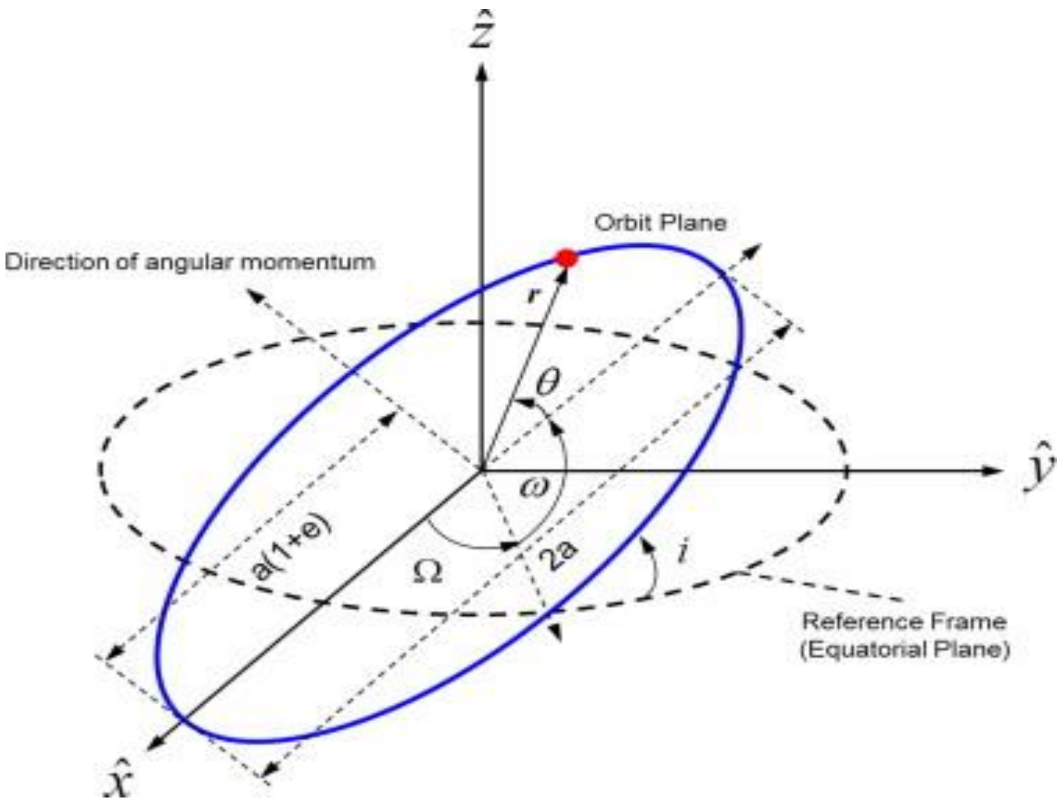

Figure 1: Is definition of the orbital elements of the satellite orbit around the planet as a Keplerian orbit [11].

2.3 Euler Angles: That contents of the (i, w, Ω) shown in Figure 1, that angles can be used to transformation of the satellite orbit plane to the Earth equatorial plane by using the inverse of Gauss matrix [10,11].

2.3.1 Inclination Angle (i): It is the angle between the satellite's orbit plane and the equatorial plane of the Earth, this angle value is between (0-180 degree).

2.3.2 The Argument of Perigee (w): This is given the displacement angle from the ascending n ode to the line between the Earth's center and the perigee, its value changes between (0-360 degree).

2.3.3 Right Ascension of Ascending Node (Ω): This is the angle determined from the direction of the vernal equinox to the ascending node, which is calculated by transformation from south to north, its value is between (0-360 degree).

3. Calculations of Perturbations:

3-1 Atmospheric Drag Acceleration: Drag depends on the properties of the fluid and it depends on the size, speed and the shape of the satellite, to express this acceleration by the following equation:

$$F_D = (\frac{1}{2})(pv^2\ C_D\ A) \qquad (6)$$

Where: (F_D) the drag force, (P) the fluid’s density, (v) the object velocity relative to the fluid, (A) the cross section area, (C_D) drag coefficient [4].

3-2 *Solar, Lunar and J_2 Perturbations Acceleration:*

High apogee, highly eccentric orbits are extremely sensitive to lunisolar perturbations. In many cases, they cause perigee to decay to the point of impact with the Earth's surface [4,5].

The perturbing effects of Sun, Moon, and the second harmonic in the field of Earth expressions by equations [10]:

$$asrp = p_s\left(\frac{A}{m}\right)x\left(\frac{r_{sat}}{R_{s-e}}\right)^2 \quad \text{………………} (7)$$

Solar radiation pressure is the force acting on the satellite due to the pressure of the solar wind.

Where: (asrp): acceleration of solar radiation pressure, (A) is a cross section of satellite, (m) is a mass, (p_s) is a solar radiation pressure = 4.56 x 10^{-6} N/m^2, (r_{sat}) is a distance of satellite from the Earth center, (R_{s-e}) is the distance between Sun and Earth, it's not constant.

The moon has a gravitational influence on the satellite, obviously, the sun and the moon are the main sources of perturbations, the attraction perturbation calculated by:

$$amp = \mu_m\left(\frac{r_m - r_{sat}}{(r_m - r_{sat})^3} - \frac{r_m}{r_m^3}\right) \quad \text{……………...}(8)$$

Where: (amp): acceleration of moon pressure, (μ_m)=G*m_m =4.9122*10^{12} m^3/sec^2, (r_m): the position vector of moon from the Earth, (r_{sat}): the position vector of satellite [13].

$$aop = 3\left[\ J_2\left(\frac{R}{r}\right)^2\right] \quad \text{……………….}(9)$$

One of the types of disturbances affecting on satellites is an oblates perturbation or non-spherical of the Earth.

(aop): acceleration of oblates perturbation, (J_2) = 1082.6283*10^{-6} for the Earth.

4. The Satellite Stability:

These laws explain a deviation of a satellite from ellipse orbit then a correction must happen to stay in the orbit.

Law (1): A satellite would tend to go off in a straight line if no force was applied to it.

Law (2): An attractive force makes the satellite deviate from a straight line and orbit Earth.

Law of Gravitation: This attractive force is the gravitational force between the Earth and the satellite. Gravity provides the inward pull that keeps the satellite in orbit.

Assuming a circular orbit, the gravitational force must equal the centripetal force:

$$\frac{mv^2}{r} = \frac{GM_E}{r^2} \quad \text{………………….}(10)$$

Where: v = tangential velocity, r = orbit radius = R_E + h (i.e. not the altitude of the orbit), R_E = radius of the Earth,

h = altitude of orbit = height above the Earth's surface,

m = mass of satellite, M_E = mass of the Earth.

$$v = \sqrt{\frac{GM_E}{r}} \quad \text{……………….}(11)$$

So we find that (v) depends only on the altitude of the orbit and mass of the Earth (not on the satellite's mass) [3].

5. Low Push Orbital Transfer Control:

Tracking down the least v move between circles utilizing constant pushing will permit huge mass reserve funds for future missions. Dispatch vehicles fit for placing substantial rocket into space are costlier than dispatch vehicles that can just place light space apparatus into space such as satellite, so that the strategies used to advance orbital directions are something similar for all employments of low-push motors. The issue considered in this work is raising satellites to GEO. This issue has the promptest application and is additionally the easiest one to address. Since the whole mission is near the Earth, just a single focal body should be thought of, while the multi-planet missions require thinking about the Earth, the Sun, and different planets. In the wake of finishing work with just a single focal body, it is feasible to extend it to incorporate numerous focal bodies in the future [6,7].

When deciding circles from perceptions, it is helpful to portray the circle utilizing mathematical boundaries. All circles are conic segments, so their shape can be portrayed by semi-major axis (a) and eccentricity (e) [12,13].

6. Simulation Results:

In this project, by implementing the equations illustrated in this research utilizing MatLab2017 simulation program, the results of the semi-major axis, eccentricity, inclination, longetude of ascending node, and perigee angle, as well as distance and velocity have been successfully obtained and plotted in the below figures.

Practical Side and Discussion of the Results:

When fixing the high (hp) value of the satellite for different altitudes (500, 3500, 35800) km and by changing the values of the central anomaly of the proposed orbit and observing the maximum speed at the perigee because it represents the best starting point for the satellite, we found the following:

1. At the low altitude of the orbit 500 km above the Earth surface and by changing the central anomalies (eccentricity) of the values e=0.1, 0.2, 0.4, 0.6, 0.8, by graphs analysis we note the futility of using the Low Earth Orbit at this height (500km) from sea level because of the instability of this orbit, and this leads to the possibility of losing control of the satellite is very high. Also, it shows that the suitable eccentricity with this height is 0.4.

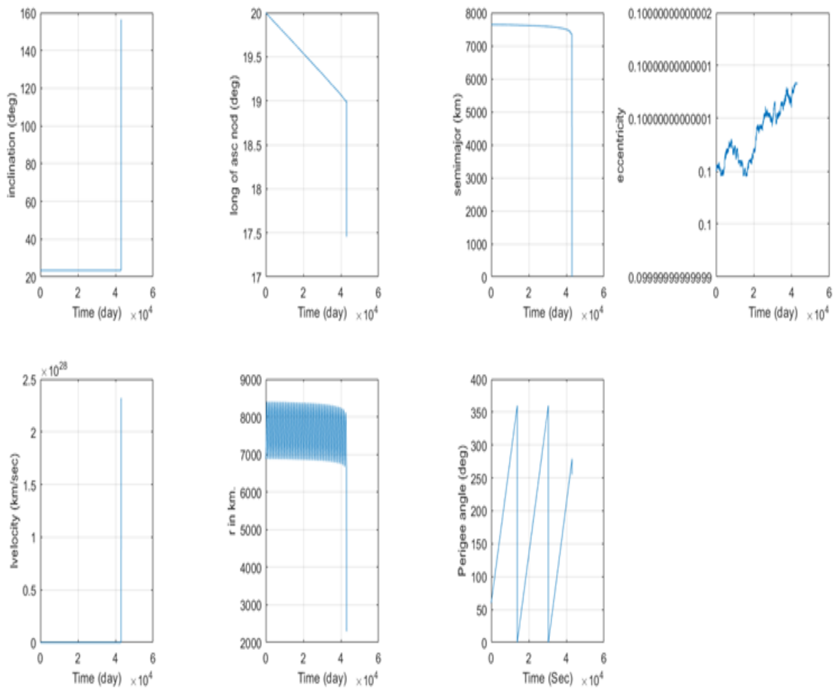

Figure 2: The variation of orbital elements and satellite distance through 990 period for orbit hp=500km, e=0.1.

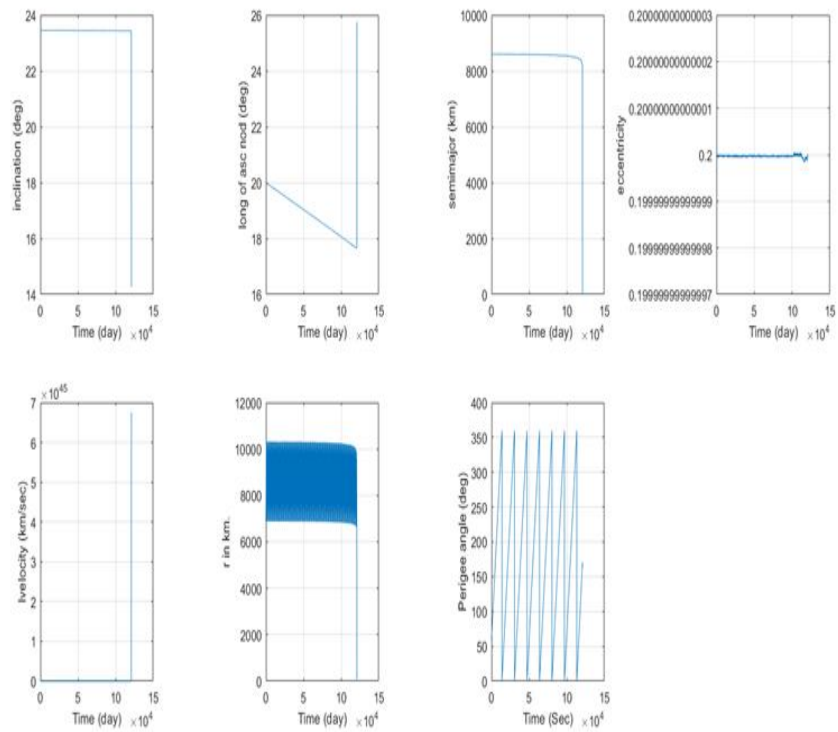

Figure 3: The variation of orbital elements and satellite distance through 990 period for orbit hp=500km, e=0.2

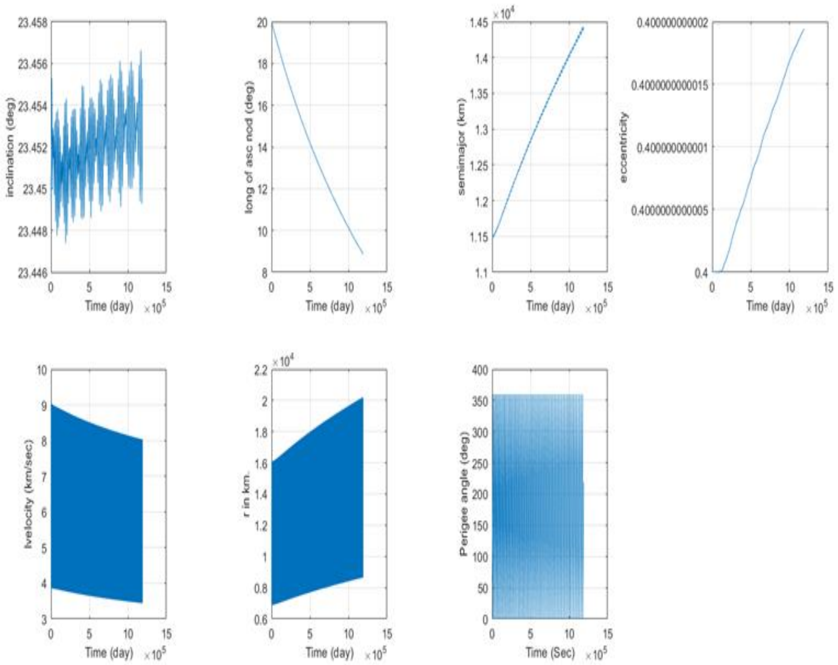

Figure 4: The variation of orbital elements and satellite distance through 990 period for orbit hp=500km, e=0.4

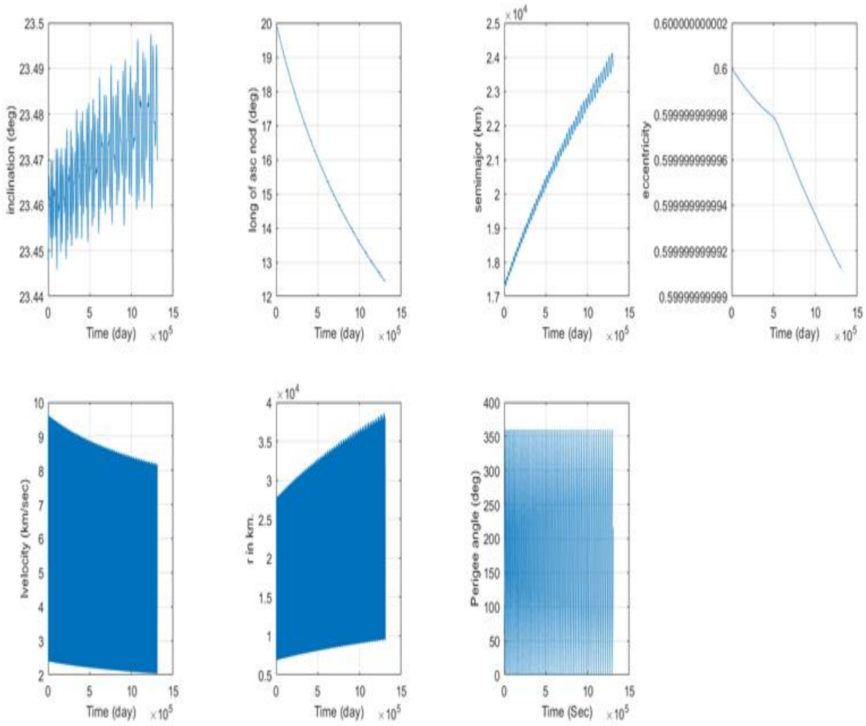

Figure 5: The variation of orbital elements and satellite distance through 990 period for orbit hp=500km, e=0.6

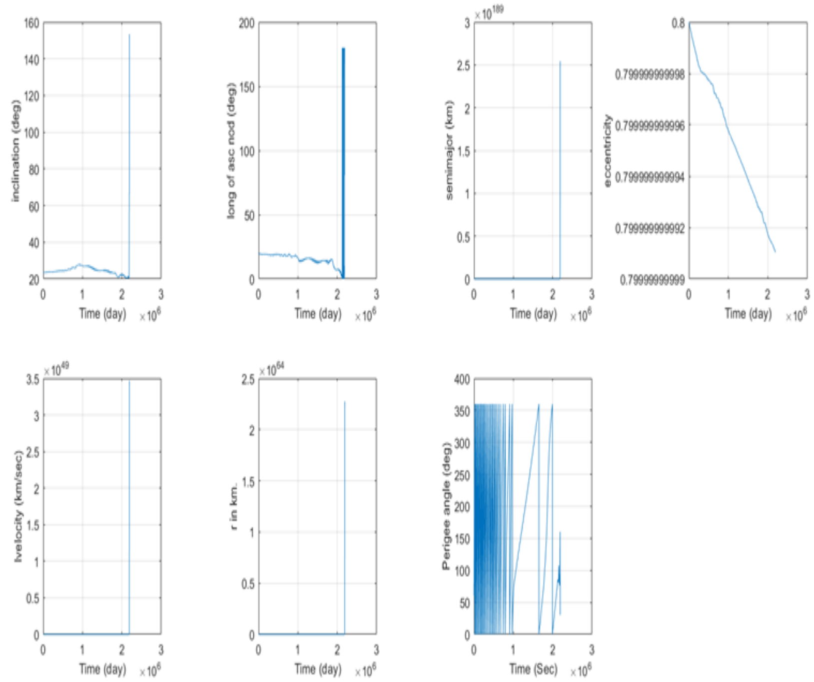

Figure 6: The variation of orbital elements and satellite distance through 990 period for orbit hp=500km, e=0.8

2. When fixing the value of the orbit height to the average altitude of 3500 km, and changing the values of the central anomaly of the orbit (eccentricity) to the values between e=0.1-0.9, it was found by analyzing the graphs of the results that some variable of orbital elements (hp & ecc) of this Medium Earth Orbit with other fixed values are good in terms of stability of the satellite and this is what we need in the aim of this research.

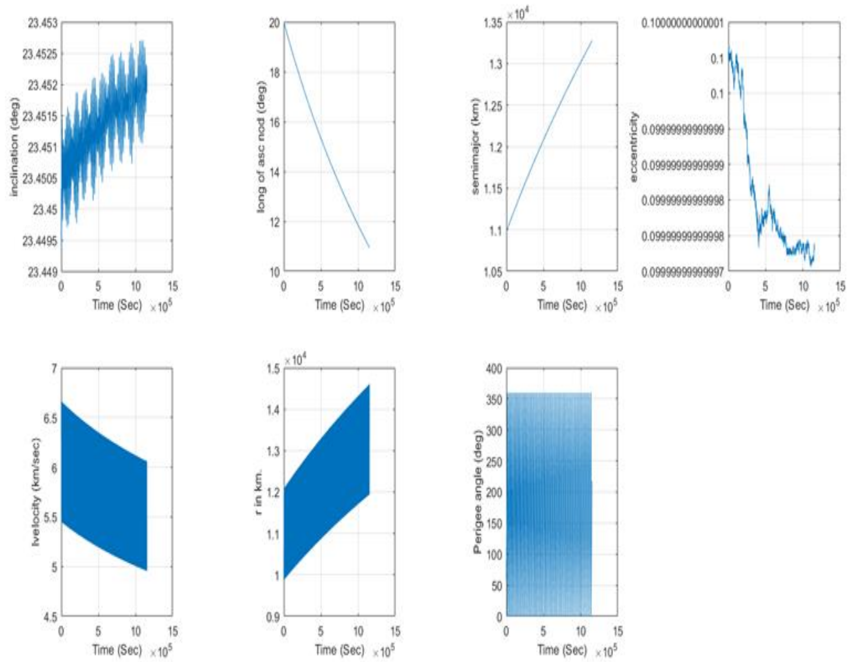

Figure 7: The variation of orbital elements and satellite distance through 990 period for orbit hp=3500km, e=0.1

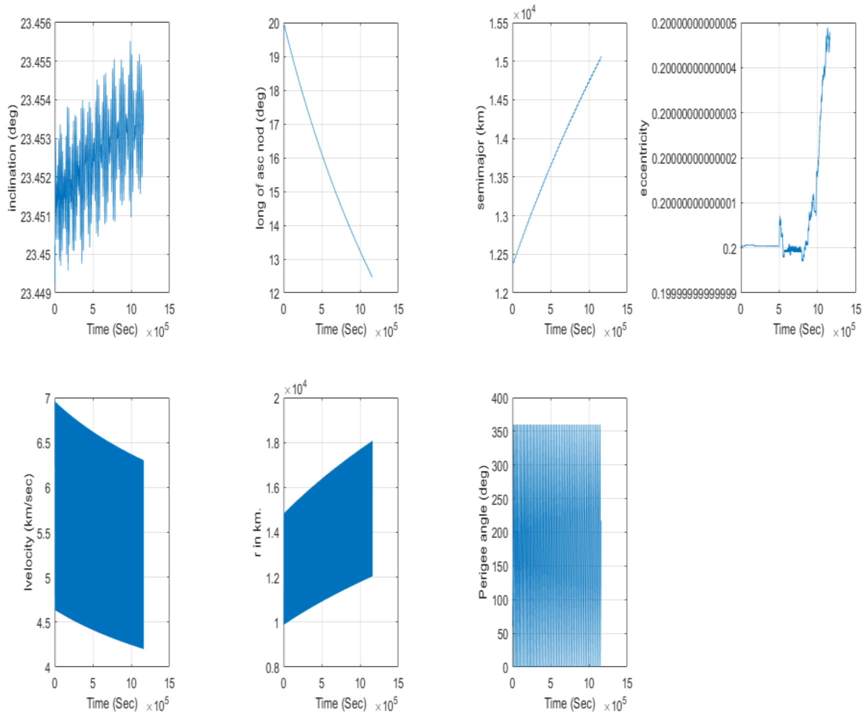

Figure 8: The variation of orbital elements and satellite distance through 990 period for orbit hp=3500km, e=0.2

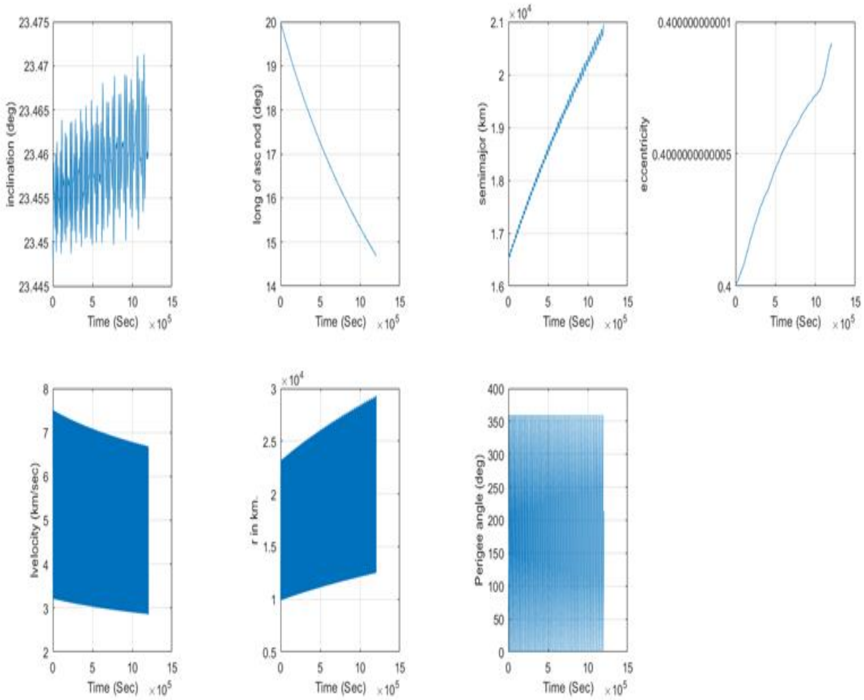

Figure 9: The variation of orbital elements and satellite distance through 990 period for orbit hp=3500km, e=0.4

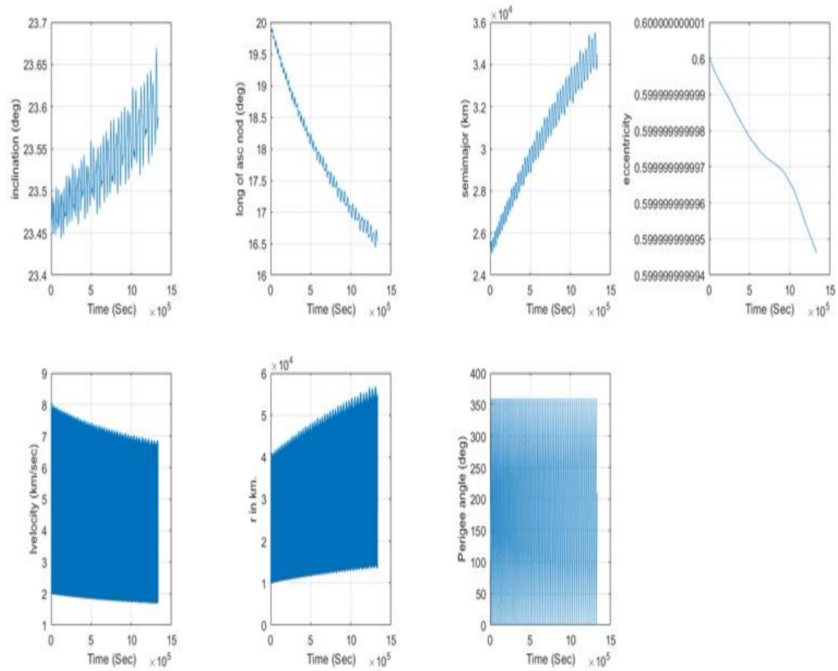

Figure 10: The variation of orbital elements and satellite distance through 990 period for orbit hp=3500km, e=0.6

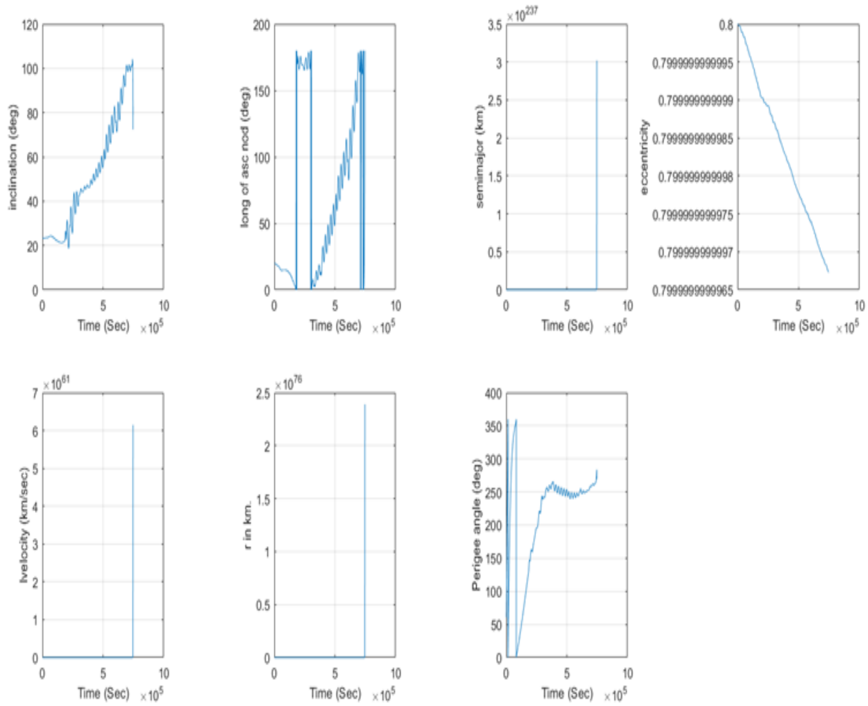

Figure 11: The variation of orbital elements and satellite distance through 990 period for orbit hp=3500km, e=0.8

3. The altitude of the orbit at the high altitude was fixed at 35800 km, and by changing the eccentricity of this orbit to values between e=0.1-0.9, and calculating the escape velocity values at the perigee each time, by analyzing the graphs of the orbital elements (variable and fixed) at an altitude of 35800 km within the High Earth Orbit, it was found that the satellite is less stable

and the velocity variation needed to transfer is big in this orbit, therefor, the possibility of losing it increases at this height above sea level.

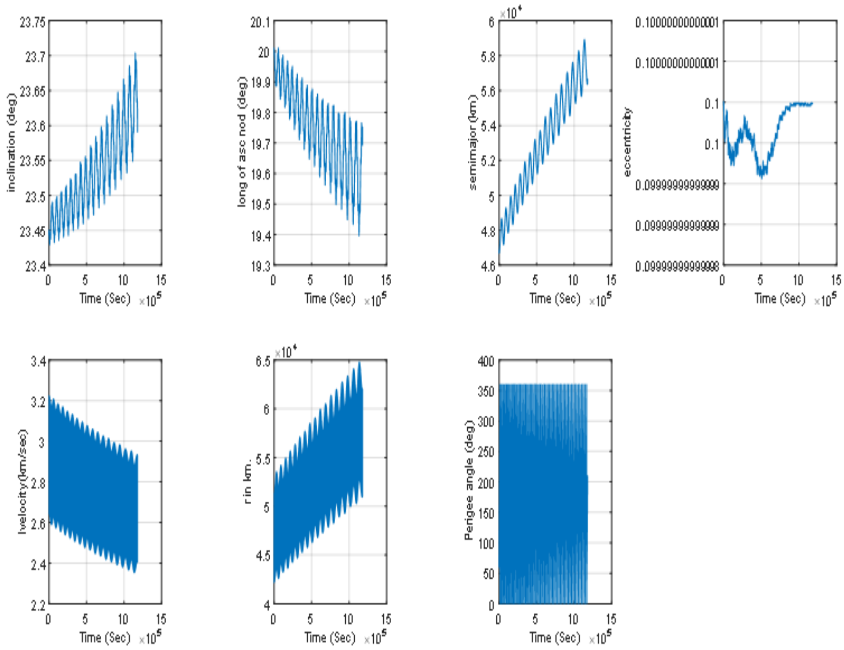

Figure 12: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.1

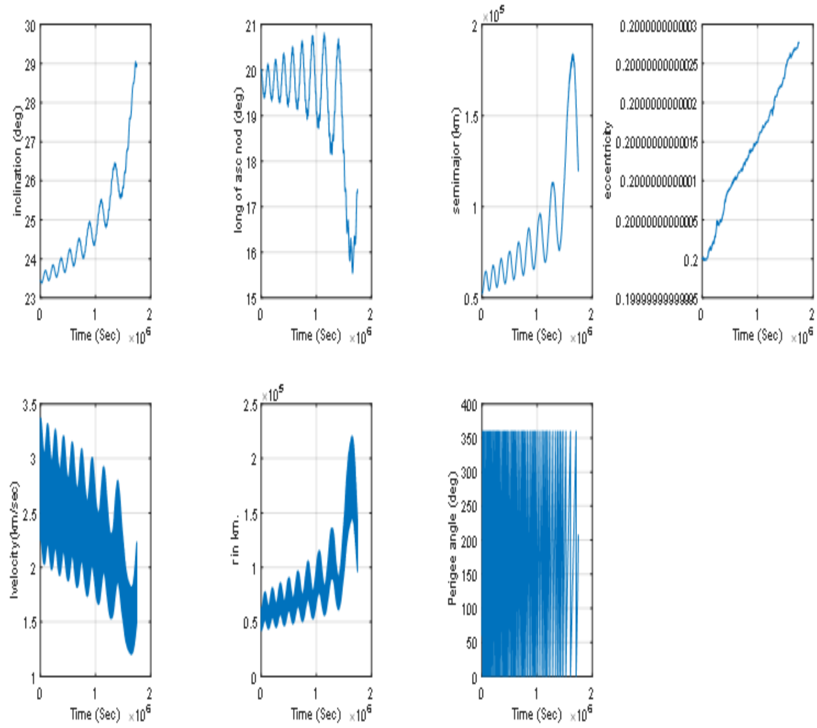

Figure 13: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.2

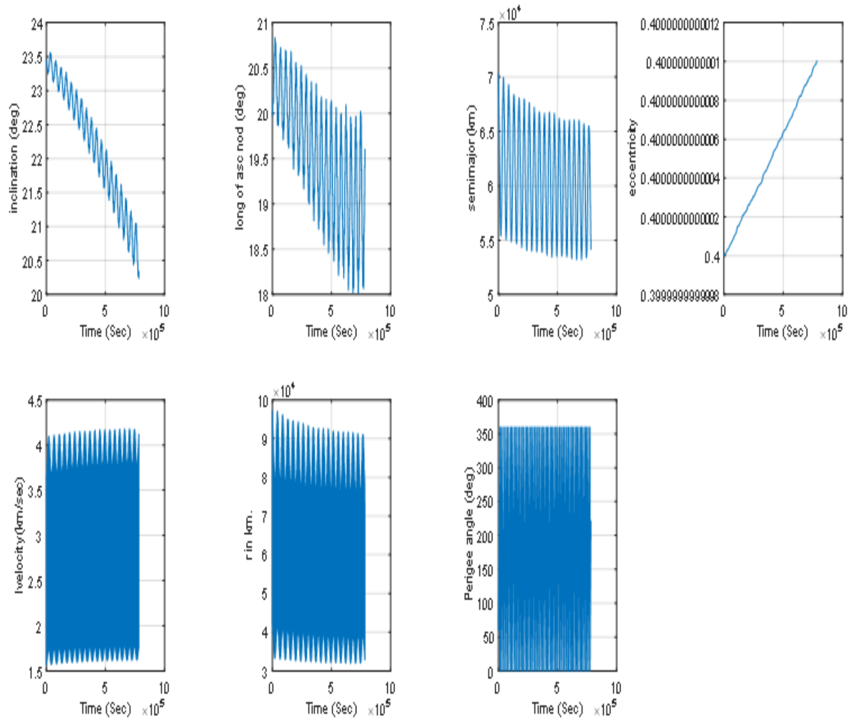

Figure 14: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.4

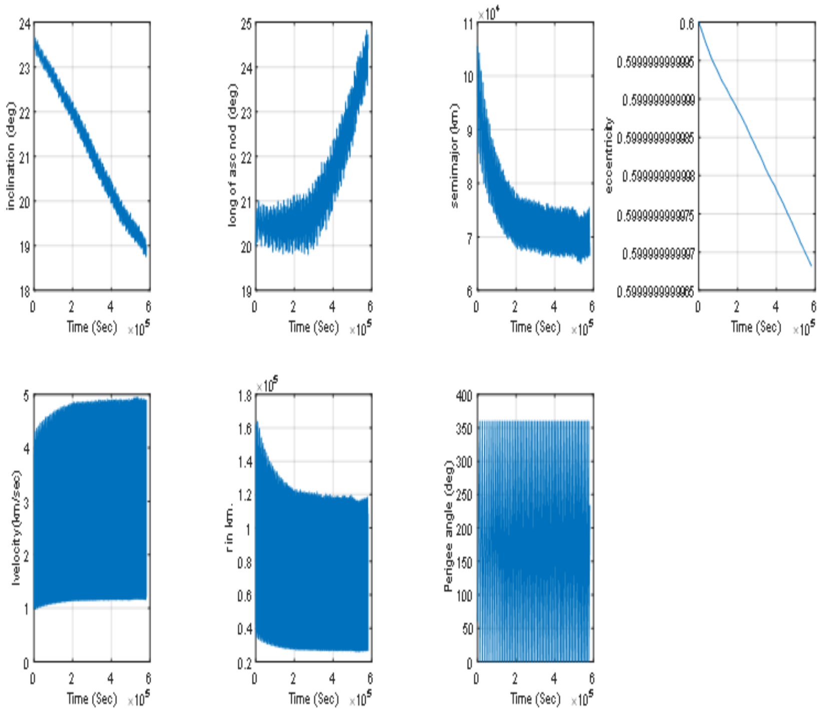

Figure 15: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.6

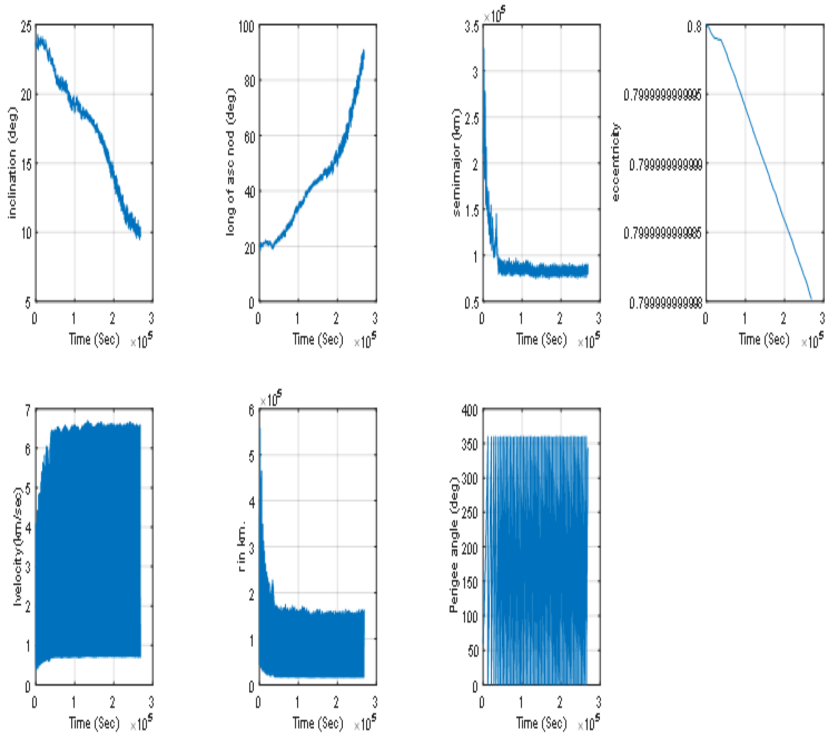

Figure 16: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.8

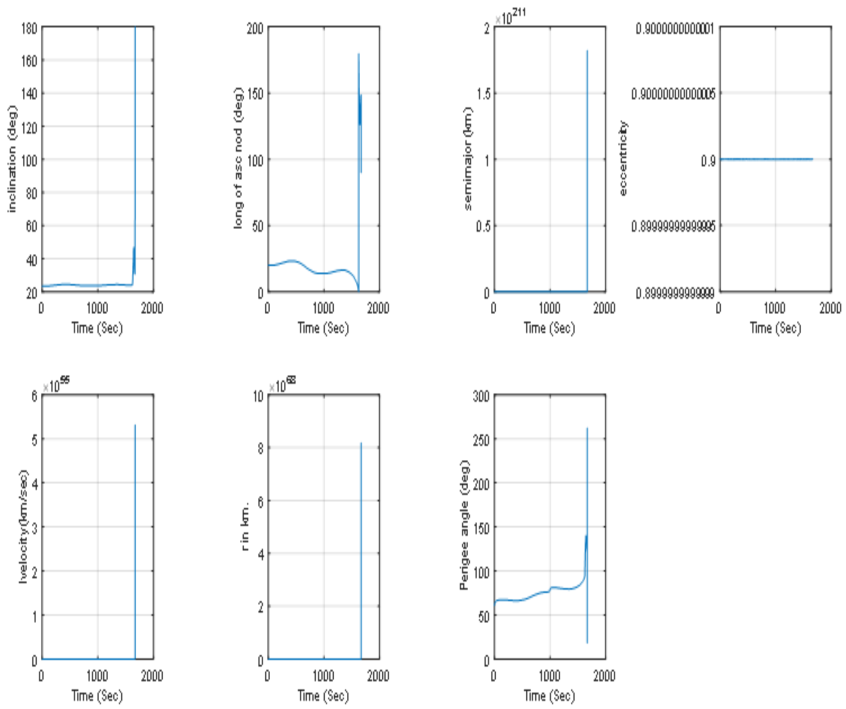

Figure 17: The variation of orbital elements and satellite distance through 990 period for orbit hp=35800km, e=0.9

4.Conclusions:

Through this research and when studying the best stable orbit of a satellite around the Earth in preparation for its transition to its orbit around Mars in the future and by fixing the values of some orbital elements for orbits of different values around the Earth and by changing the values of the height from the surface of the Earth and the central anomaly of the orbit found through the figures. The above graphs showed that the best value for the escape velocity at perigee point was for the height of the middle orbit at hp = 3500 km above sea level, and for the values of deviation of the eccentricity (e = 0.4, 0.6), this means that the best stable orbit in preparation for the stage of obtaining the best value for the expected escape velocity to move to orbit around Mars was at the MEO orbit around the Earth at an altitude of 3500 km, by eccentricity ranging between 0.4-0.6. Since at this height (hp) from the Earth surface and with this value of the eccentricity of the satellite's orbit, and with keeping the rest of the orbital elements constant, we found that the best stable orbit for the satellite before transfer to another orbit outside the Earth orbits and enter to planetary orbit (such as Mars) is the Medium Earth Orbit and the value of velocity needed to escape is minimum at the perigee point of satellite orbit.

The second zonal harmonic factor (j2) affects the longitude of the ascending node (Ω) as e it causes the orbit to stagger, the highest amount of change is at the equatorial orbit and the lowest at the polar orbit, and it is almost absent at the orbit inclination approximately 65 degrees where the rate of oscillation of the orbit is almost zero.

The acceleration resulting from this turbulence leads to a change in the location of the apogee point, while its effect on the trough point is very small.
The lifetime of the satellite under the influence of turbulence can be increased by increasing the altitude of the orbit's perigee, at the expense of the imaging efficiency of the Earth's surface.

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