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Calculation of the effects of the sun, moon and satellite position on the perturbation forces of the low retrograde orbits

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Abstract

the actual position and velocity of the sun and the moon were calculated through one year , and the satellite position and velocity components (x,y,z, v_x, v_y, v_z) were calculated as well as the momentum component at inclination (116.5˚) , argument of perigee (30˚), longitude node angle (40˚), eccentricity (0.01), for different perigee heights (200,300,..,1000 km). The acceleration of perturbations calculated in this work is the sun and the moon attraction on the satellite, the solar radiation pressure, the atmospheric drag as well as the earth oblateness. The result shows that the perturbation forces of atmospheric drag acceleration are affected by altitude and the sun, moon attraction does not depend on distance from satellite but depend on the angle between (sun – earth – satellite) and (moon - earth – satellite). the earth oblateness acceleration does not depend on altitude of satellite and time but it depends on the position on its orbit and orbital inclination. It is found that the solar radiation pressure acceleration is depending on the angle (sun - earth – satellite) with a minimum value at (180˚).

Keywords: *Retrograde Satellite orbit, perturbation, atmospheric drag*

حساب تأثير مواقع الشمس والقمر و القمر الصناعي على قوى الاضطراب للمدارات التراجعية الواطئة

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الخلاصة

تم حساب الموقع والسرعة الفعلية للشمس والقمر خلال سنة واحدة ، و حساب مركبات الموقع والسرعة (v_z ،v_y ،v_x ،z ،y ،x) بالإضافة إلى مركبات الزخم عند الميل (116.5˚) ، دالة مثابة الحضيض (30˚) ، زاوية خط طول العقدة الصاعدة (40˚) ، الشذوذ المركزي (0.01) ، لارتفاعات مختلفة للحضيض (200, 300 ،..... ، 1000 كم). في هذا العمل تم حساب تعجيل اضطراب جذب الشمس والقمر على القمر الصناعي ، وضغط الإشعاع الشمسي ، وكبح الغلاف الجوي ، فضلا عن عدم كروية الأرض. تبينت النتائج قوى الاضطراب في تعجيل كبح الغلاف الجوي تعتمد على الارتفاع ، لا تعتمد جاذبية الشمس والقمر على البعد من القمر الصناعي بل تعتمد على الزاوية (شمس – أرض – قمر الصناعي) و (قمر– أرض– قمر صناعي). لا يعتمد تعجيل تفلطح الأرض على ارتفاع القمر الصناعي عن مركز الأرض ولا على الزمن ، بل يعتمد على موقعه في مداره وميل المدار . يعتمد تعجيل ضغط الإشعاع الشمسي على الزاوية (شمس – أرض – قمر الصناعي) والقيمة الدنيا له عند الزاوية (180˚).

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1. Introduction

The orbital perturbation may be defined as the deviation of elements from their mean values due to external forces. These forces may be classified into two types: gravitational and non-gravitational forces. The gravitational forces arise from the mutual gravitational attraction between various bodies such as earth-satellite or earth-moon. The non-gravitational forces arise from space environment such as radiation pressure, atmospheric drag, and geomagnetic field, etc. [1]

The perturbing forces that cause the satellite orbit to deviate from a theoretically regular orbital motion can be divided into two categories, conservative perturbing forces and non-conservative perturbing forces, the non conservative perturbing forces are due to other celestial bodies such as Moon, Sun and etc, while the solar pressure, atmospheric drag, non-homogeneity, and oblateness of the Earth are conservative perturbing forces. In this research, all perturbations types will be calculated and studied the effect of these perturbations on the satellite orbital elements through many periods for low retrograde satellite orbits (inclination > 90˚). [2- 4]

To include the effects of the perturbations, the equations of motion can be written in a general form as [2]

$$\ddot{r} = -\mu \frac{\vec{r}}{r^3} + \vec{a}_p \tag{1}$$

Where:

$\ddot{r}$ is the acceleration of the satellite, $\mu = G(M_E + m_s)$, r is the distance from canter of earth to satellite. $\vec{a}_p$ is the resultant vector of all the perturbing accelerations.

$$\vec{a}_p = \vec{a}_{non-spherical} + \vec{a}_{drag} + \vec{a}_{third\ body} + \vec{a}_{SRP} + \vec{a}_{others}$$

In the solar system, the magnitude of the a_p for all the satellite orbits is at least 10 times smaller than the central force or two-body accelerations, or $a_p << \mu r/r^3$.

2. Theory

The mean motion (n) can be written as in the following equation: [5]

$$n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}} \tag{2}$$

Where T is orbital period of the elliptical orbit, then:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{3}$$

At any time the mean anomaly is used to describe the location of the satellite in an orbit.

$$M = n(t - \tau) \tag{4}$$

Where τ is time of perigee passage. The eccentric anomaly (E) for the orbit calculated as: [5]

$$E_{i+1} = M + e \sin E_i \tag{5}$$

Where e is the orbital eccentricity of the elliptical orbit

There are many methods to calculate (E), One of the analytical methods is used to solve Kepler's equation by iterative methods only. A common way is to start with an approximation of $E_0 = M$ or and employ Newton's method to calculate successive refinements E_i until the result changes by less than a specified amount from one iteration to the next. [6]

Then, the value of eccentric anomaly used to calculate the true anomaly(f) :

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \tag{6}$$

The general equation of motion can be written by using Newton's laws in motion with general gravitational law given as:[2- 4]

$$\ddot{\vec{r}}_i + \frac{\mu}{r^3}\vec{r} = 0 \tag{7}$$

The solution of the equation of motion written as a polar equation of a conic section which can be expressed by the following equation: [7]

$$r = \frac{h^2/\mu}{1 + e\cos f} \tag{8}$$

Where (h) is the angular momentum which is twice the rate of description of area by the radius vector, the relation can write as:

$$h = \frac{2\pi ab}{T} \quad (9)$$

Where b is semi minor axis . the velocity is: [7]

$$v = \left(\mu\left(\frac{2}{r} - \frac{1}{a}\right)\right)^{1/2} \quad (10)$$

To find the Cartesian coordinate (x_w, y_w, z_w) for the satellite in its orbit the following equations will be used: [8]

$$\left.\begin{aligned} x_w &= a(\cos E - e) \\ y_w &= a\sqrt{1-e^2}\sin E \\ z_w &= 0 \end{aligned}\right\} \quad (11)$$

By differentiation for (x_w, y_w, z_w) for a satellite in its orbit, and use $p=a(1-e^2)$, we get: [8]

$$\left.\begin{aligned} \dot{x}_w &= -\sqrt{\mu/p}\ \sin f \\ \dot{y}_w &= \sqrt{\mu/p}\ (e + \cos f) \\ \dot{z}_w &= 0 \end{aligned}\right\} \quad (12)$$

The velocity $\dot{r}$ and a momentum can be written as:

$$\dot{r} = \frac{\sqrt{\mu a}}{r}\ e \sin E \quad (13)$$

The momentum vector $\vec{h}$ can be analysed of components, where

$$\vec{h} = \vec{r} \times \vec{v} \quad (14)$$

$$\vec{h} = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} i & j & k \\ x & y & z \\ \dot{x} & \dot{y} & \dot{z} \end{bmatrix} \quad (15)$$

Euler angles(i , Ω , ω) are used in Gauss Matrix to convert the coordinates from satellite orbit plane to equatorial plane: [1,2,8]

$$\begin{pmatrix} \sin i \sin \Omega \\ \sin i \cos \Omega \\ \cos i \end{pmatrix} = \begin{pmatrix} +h_x/h \\ -h_y/h \\ +h_z/h \end{pmatrix} = \begin{pmatrix} +W_x \\ -W_Y \\ +W_Z \end{pmatrix} \quad (16)$$

3. Calculation the Position of the Sun and Moon:

The Mean longitude (L) and the mean anomaly (M_s) of the sun are given by:[9]

$$L = 280°.46 + 0°.985647359\, n \quad (17)$$

$$M_s = 357°.528 + 0°.985600281\, n \quad (18)$$

The longitude of the Sun can be calculated by using the formula:

$$\lambda_s = L + 1.915262268° \sin M_s + 0.020008486° \sin 2M_s + 0.000289389° \sin 3M_s \quad (19)$$

Where *(n)* is the number of days since the epoch J2000.0 which can be obtained by:

$$n = JD - 2451545.0 \quad (20)$$

The right ascension of the sun (α_s) and declination (δ_s) can be calculated from:

$$\mathrm{Tan}\ \alpha_s = \frac{\cos\varepsilon \sin\lambda_s}{\cos\lambda_s} \quad (21)$$

$$Sin\ \delta_s = sin\ \varepsilon\ sin\ \lambda_s \quad (22)$$

Where ε, the obliquity of the ecliptic , is given by:[9]

$$\epsilon = 23°.452\,294 - 0°.013\,0125\, T_1 - 0°.000\,00164\, T_1^2 - 0°.000\,000\,503\, T_1^3 \quad (23)$$

Where (T_1) is the number of Julian centuries beginning of 1st January 1900 can be calculated as : [9]

$$T_1 = \frac{JD - 2415020}{36525} \quad (24)$$

While the number of Julian centuries beginning of 1st January 2000 can be calculated as :

$$T_2 = \frac{JD - 2451545.5}{36525} \quad (25)$$

The Sun radial distance is :

$$r_s = \frac{a_e(1 - {e_e}^2)}{1 + e_e \cos(\lambda_s - 282.596403°)} \quad (26)$$

where (r_s) is the heliocentric distance of planet, (a_e) is the semi major axis of Earth, (e_e) is the eccentricity of Earth orbit. The position components distance in Cartesian coordinate can be calculated using equatorial coordinate as the following: [2, 9]

$$\left.\begin{aligned} x_s &= r_s \cos\delta_s \cos\alpha_s \\ y_s &= r_s \cos\delta_s \sin\alpha_s \\ z_s &= r_s \sin\delta_s \end{aligned}\right\} \tag{27}$$

The distance of the Sun from the Earth centre can be also found as: [8]

$$R_s = \sqrt{x_s^2 + y_s^2 + z_s^2} \tag{28}$$

The Moon's longitude is given by: [10]

λ_l = 218.32 + 481267.883T_2 + 6.29 sin (134.9 + 477198.85T_2) -1.27 sin (259.2 – 413335.38 T_2) +0.66 sin (235.7+ 890534.23 T_2) + 0.21 sin (269.9 + 954397.7 T_2) – 0.19 sin (357.5 + 35999.05 T_2) – 0.11 sin (186.6 + 966404.05 T_2) (29)

The Moon's latitude is given by:

$$\beta_l = 5.13\sin(93.3 + 483202.03\,T_2) + 0.280606\,\sin(228.2 + 960400.87\,T_2) - 0.28\sin(318.3 + 6003.18\,T_2) - 0.17\sin(217.6 - 407332.2T_2) \tag{30}$$

The Moon's distance from the centre of the Earth can be calculated as the following:

r_l = 385000 – 20905 cos M_l – 3699 cos (2D_{sl} – M_l) – 2956 cos (2 D_{sl}) – 570 cos (2 M_l) + 246 cos (2 M_l – 2 D_{sl}) – 152 cos (M_l + M_S – 2 D_{sl}) (31)

Where M_l is mean anomaly of the Moon, M_S is mean anomaly of the Sun and D_{sl} is the difference between the mean longitudes of the Sun and the Moon, which are functions of Julian centuries (T_{2000}) and calculated as: [9]

$$\left.\begin{aligned} M_l &= 134°.96292 + 477198°.86753\,T_2 \\ M_S &= 357°.52543 + 35999°.04944\,T_2 \\ D_{sl} &= 297°.85027 + 445267°.11135\,T_2 \end{aligned}\right\} \tag{32}$$

Used the following formulae to transformation elliptical coordinates to equatorial coordinates (α_l, δ_l),.

$$\left.\begin{aligned} \tan\alpha_l &= (\sin\lambda_l \cos\varepsilon - \tan\beta_l \sin\varepsilon) \,/\, \cos\lambda_l \\ \sin\delta_l &= \sin\beta_l \cos\varepsilon + \cos\beta_l \sin\varepsilon \sin\lambda_l \end{aligned}\right\} \tag{33}$$

The position components distance in Cartesian coordinate can be calculated using equatorial coordinate as the following :[2]

$$\left.\begin{aligned} x_l &= r_l \cos\delta_l \cos\alpha_l \\ y_l &= r_l \cos\delta_l \sin\alpha_l \\ z_l &= r_l \sin\delta_l \end{aligned}\right\} \tag{34}$$

The distance of the Moon from the Earth centre can be also found as:

$$R_l = \sqrt{x_l^2 + y_l^2 + z_l^2} \tag{35}$$

4. The Non-spherical Earth Perturbation:

The Earth is flattened somewhat at the poles and bulges correspondingly at the equator. Such a shape is called an oblate spheroid. The difference between the equatorial and polar radii is (21.4) kilometers. Another surface commonly used in geodesy is the geoid, which is the equipotential surface that coincides on the average with mean sea level in the oceans. The surface gravitation was representing by the series of spherical harmonics represented by the symbol (J) . The zonal harmonic (J_0) expresses the overall size of the geoid, while (J_1), the first degree harmonic determines the center point of the geoid in the north-south direction. The other harmonics represent deviations from the spherical shape. A harmonics locational factors values are available in the literature for the basic models of the form of the earth by satellites records as: [11,12]

$J_2 = 1082.63 \times 10^{-6}$, $J_3 = -2.54 \times 10^{-6}$, $J_4 = -1.62 \times 10^{-6}$, $J_5 = -0.23 \times 10^{-6}$, $J_6 = 0.25 \times 10^{-6}$

Mathematical expressions for Cartesian coordinates and use accelerating instead of potential as follow equation: [12]

$$\vec{a}_{non-spherical} = \ddot{r} = \frac{dU}{dX}X + \frac{dU}{dY}Y + \frac{dU}{dZ}Z \tag{36}$$

Where:

$$\ddot{X} = \frac{dU}{dX} = \frac{\mu X}{r^3}\{J_2 \frac{3}{2}\left(\frac{a_e}{r}\right)^2\left(5\frac{z^2}{r^2} - 1\right) + J_3\frac{5}{2}\left(\frac{a_2}{r}\right)^3\left(3\frac{z}{r} - 7\frac{z^3}{r^3}\right) - J_4\frac{5}{8}\left(\frac{a_e}{r}\right)^4\left(3 - 42\frac{z^2}{r^2} + 63\frac{z^4}{r^4}\right) - J_5\frac{3}{8}\left(\frac{a_e}{r}\right)^5\left(35\frac{z}{r} - 210\frac{z^3}{r^3} + 231\frac{z^5}{r^5}\right)J_6\frac{1}{16}\left(\frac{a_e}{r}\right)^6\left(35 - 945\frac{z^2}{r^2} + 3465\frac{z^4}{r^4}3003\frac{z^6}{r^6}\right)\} \tag{37.a}$$

$$\ddot{Y} = \frac{dU}{dY} = \frac{Y}{X}\ddot{X} \tag{37.b}$$

$$\ddot{Z} = \frac{dU}{dZ} = -\frac{\mu Z}{r^3}\left\{J_2\frac{3}{2}\left(\frac{a_e}{r}\right)^2\left(3 - 5\frac{Z^2}{r^2}\right) + J_3\frac{3}{2}\left(\frac{a_e}{r}\right)^3\left(10\frac{Z}{r} - \frac{35}{3}\frac{z^3}{r^3} - \frac{r}{Z}\right) - J_4\frac{5}{8}\left(\frac{a_e}{r}\right)^4\left(15 - 70\frac{Z^2}{r^2} + 63\frac{Z^4}{r^4}\right) - J_5\frac{3}{8}\left(\frac{a_e}{r}\right)^5 + J_2\frac{1}{16}\left(\frac{a_e}{r}\right)^6\left(3003\frac{Z^6}{r^6} - 4851\frac{Z^4}{r^4} + 2205\frac{Z^2}{r^2} - 245\right)\right\} \tag{37.c}$$

5. Atmospheric Drag Perturbation:

Drag is a resistance offered by the atmosphere to the satellite. it is caused by frequent collisions of gas molecules with the satellite. Drag force acts in a direction opposite to the direction of its motion. It is the major cause of orbital decay for satellites in LEO. This drag is the greatest during launch and reentry. the action of drag on a satellite will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within (120 to 160 km) of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about (80 km). Atmospheric drag resulting in satellite re-entry can be described by the following sequence: [13,14]

lower altitude → denser atmosphere → increased drag → increased heat → usually burns on re-entry.

The drag caused by the Earth's atmosphere also causes satellites to spiral downward. Together with the oblateness of the Earth, The relation that describes the acceleration due to the atmospheric drag ($\vec{a}_{\text{Drag}}$) is : [15]

$$\vec{a}_{\text{Drag}} = -\frac{1}{2}\rho\frac{C_D A}{m}v_{rel}^2\frac{\vec{v}_{rel}}{\|\vec{v}_{rel}\|} \tag{38}$$

Where: ρ is the atmospheric density value, (C_D =2.2) is the drag coefficient, A is the projected area in the direction of the velocity vector relative to the atmosphere, m is a total mass of the satellite and $\vec{v}_{rel}$ is the velocity of the satellite relative to the atmosphere.

6. Solar Radiation Pressure (SRP):

Solar radiation pressure (SRP) is force acting on the satellite's surface caused by sunlight. The force acting directly on the satellite is proportional to the effective satellite surface, to the reflectivity of surface and to the solar flux; it is inversely proportional to the velocity of light. The acceleration result from solar radiation pressure is:[16]

$$a_{SPR} = -\mu P_s C_R \frac{A}{m} \tag{39}$$

Where: C_R is reflectivity coefficient, A is cross section area of the satellite, m is mass of satellite, μ is shadow function, (is equal 1 for complete sunlight, 0 for umbra phase and $0 < \mu < 1$ for penumbra phase). P_s is solar radiation pressure for sunlight (equal $\frac{E}{C}$) , where E is solar constant (1358 w/m^2), C is vacuum speed of light.

7. Third Body problem:

The term (third body) refers to any other body in space beside the Earth which could have a gravitational influence on the satellite such as Moon and Sun, the Sun is extremely massive, while the Moon is very close. The third body perturbation becomes more relevant for high altitude orbits that are when the atmospheric drag effect begins to diminish. the third body force is perturbation proportion to Earth’s gravitational force, therefore, the greater effect is happening on the high altitude orbits. The solar gravity perturbations can be modeled as a third body effect in inertial geocentric axes using the equation: [17,18]

$$\ddot{\boldsymbol{r}}_s = -\mu_s\left(\frac{\vec{\boldsymbol{r}}_s - \vec{\boldsymbol{r}}}{|r_s - r|^3} - \frac{\vec{\boldsymbol{r}}_s}{{r_s}^3}\right) \tag{40}$$

Where: ($\mu_s = GM_s$ = 1.327124 ×10^{11} (N. km^2 / kg)), (r_s) radial distance between center of Sun and center of Earth, which vary between (147 x 10^6 km and 152 x 10^6 km) along a year, (r) radial distance between the satellite and the center of Earth.

The effects of the Moon will be treated as a third body acting on the satellite. Although the mass of the Moon is much lower than that of the Sun the reduced distance between perturbing body and satellite makes the Lunar perturbation about equal to the Solar. Re-writing equation (28), replacing the Solar variables by the equivalent Lunar ones:[18,19]

$$\ddot{\boldsymbol{r}}_L = -\mu_L\left(\frac{\vec{\boldsymbol{r}}_L - \vec{\boldsymbol{r}}}{|r_L - r|^3} - \frac{\vec{\boldsymbol{r}}_L}{{r_L}^3}\right) \tag{41}$$

Where ($\mu_L = GM_L$ = 4.902794 × 10^3 (N. km^2 / Kg)), (r_L) radial distance between center of Moon and center of Earth which must be calculated.

8. The solution of the perturbed equation:

There are many methods to solve the equations of perturbation, Runge Kutta method is used to solve the equations of motion of a satellite which are described by the following system of six ordinary linear differential equations to obtain satellites position and velocity vector in time. [3,4,18]

$$\frac{dx}{dt} = \dot{x}\ ,\qquad \frac{dy}{dt} = \dot{y}\ ,\qquad \frac{dz}{dt} = \dot{z} \tag{42}$$

$$\left.\begin{aligned}
\frac{d\dot{x}}{dt} &= \ddot{x} = \textstyle\sum_k \frac{F_{x,k}}{m} = \sum_k a_{x,k} = f(x,y,z)\\
\frac{d\dot{y}}{dt} &= \ddot{y} = \textstyle\sum_k \frac{F_{y,k}}{m} = \sum_k a_{y,k} = f(x,y,z)\\
\frac{d\dot{z}}{dt} &= \ddot{z} = \textstyle\sum_k \frac{F_{z,k}}{m} = \sum_k a_{z,k} = f(x,y,z)
\end{aligned}\right\} \tag{43}$$

The classical Runge-Kutta algorithm is of 4^{th} order and has 4 stages. The stage number indicates how often the right hand function *f* (x, *t*) has to be evaluated. The new state vector can be obtained by:[4,18]

$$x_{i+1} = x_i + \frac{h_{step}}{6} * (k_1 + 2 * k_2 + 2 * k_3 + k_4) \tag{44}$$

h_{step} is stepped width in time. Where the four derivatives k_1 through k_4 are computed as follows:

$$\left.\begin{aligned}
k_1 &= f(x_i) \qquad k_2 = f\left(x_i + \tfrac{h_{step}}{2} \,.k_1\right)\\
k_3 &= f\left(x_i + \tfrac{h_{step}}{2} \,.k_2\right)\ k_4 = f\left(x_i + h_{step} \,.k_3\right)
\end{aligned}\right\} \tag{45}$$

9. The algorithm : The programs were design to solve kepler equation and calculate the following:

a. Calculate the Julian date from(2018:01:01:12:0:0) and beyond then calculate the delta Julian date.

b. Calculating the period with assume semi-major axis and assume step=period/1000 , where the period is not constant when the perturbation included.

c. Calculate the position and velocity for satellite and its coordinates (x, y, z, v_x, v_y, v_z), for one period without perturbation.

d. Calculate the variation of the position of sun and moon from earth and satellite during one year, and calculate the sun and moon attraction perturbations at all step of time, and using the acceleration at the end of periods to plot the figures.

e. Calculate the atmospheric drag acceleration for heights (100,200,---1000 km).

f. Calculate the position, velocity and the angular momentum for satellite, and its components(x, y, z, v_x, v_y, v_z), for one period with perturbation.

g. Compare the results with and without perturbation.

10. Results and Discussion:

Adoption of The orbital elements of satellite with (h =200 km, e = 0.01, Ω = 40˚, ω = 30˚, i = 116.5˚). and calculate of the coordinates of the position, velocity and its component with the time and mean anomaly with and without perturbations. the results were as follows:

First: calculate the state vectors of satellite without perturbation:

1- Figure-1 represents the change of the distance of satellite position with the mean anomaly within one day (16 periods) and without perturbations. It shows from the figure that the value of the distance varies between (6578,165039 - 6711,057259) km, and the maximum and minimum value are fixed as well as the period as (1.49 hours).

2- Figure-2: represents the change of the velocity with the mean anomaly within one day (16 periods) without perturbations, where the change of velocity values are constant between (7, 82307 - 7,668157) km /sec.

3- Figure-3: shows the change of position compounds (x, y, z) with the mean anomaly during one period without perturbations, where the compounds change regular periodically between positive and negative values, indicating the validity of the results.

4- Figure-4: shows the change of velocity compounds (vx, vy, vz) with the mean anomaly during one period without perturbations. It shows that values change regularly between negative and positive values. As shown from the figure, the values of (v_z) change more than the values of (v_x, v_y) due to the high inclination orbit (116.5˚).

5- Figures-(5, 6): shows that the values of distance (r) and velocity (v) are changed with mean anomaly during one period without perturbations, it shows that the values of distance (r) varies between (6578,165039-6711,067259) km, and the values of velocity (v) varies between (7,82307- 7,668,157) km /sec. The values of r_{min} correspond to v_{max} values and vice versa.

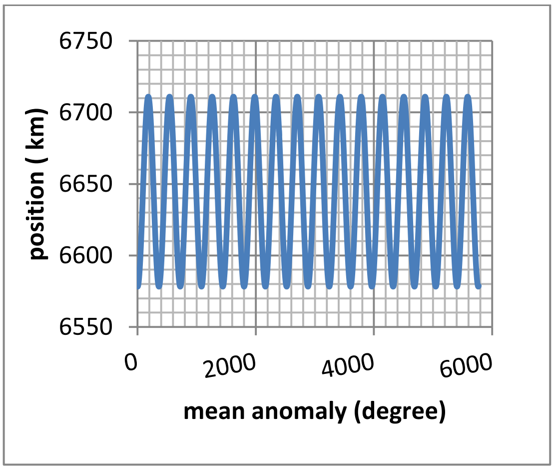

Figure 1-variation of position due to mean anomaly at one day without perturbation

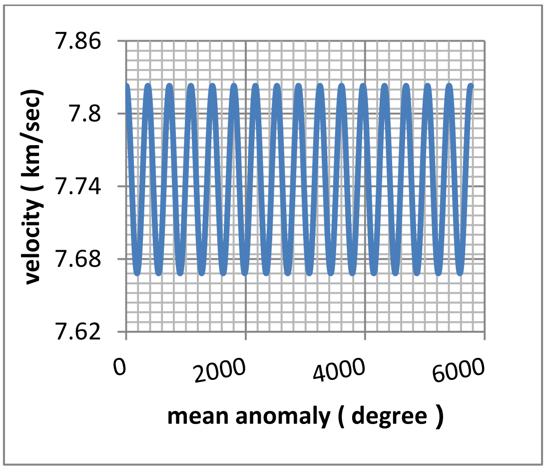

Figure 2-variation of velocity due to mean anomaly at one day without perturbation

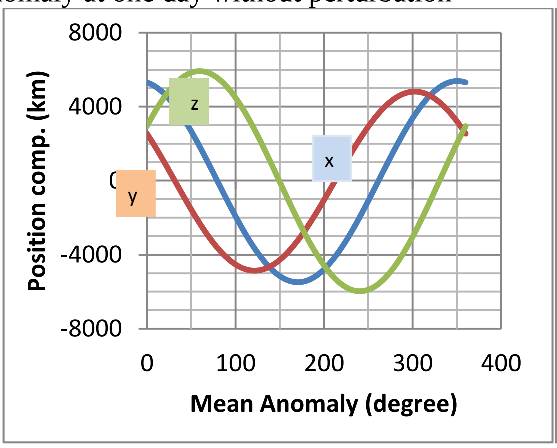

Figure 3-ariation of x,y,z Position component with Mean Anomaly at one period without perturbation

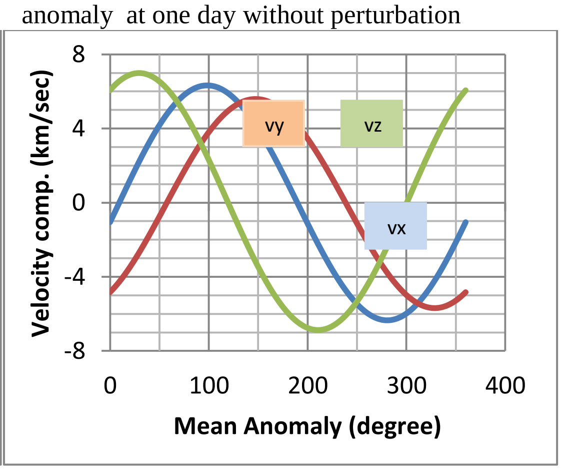

Figure 4- variation of v_x, v_y, v_z velocity component with Mean Anomaly at one period without perturbation

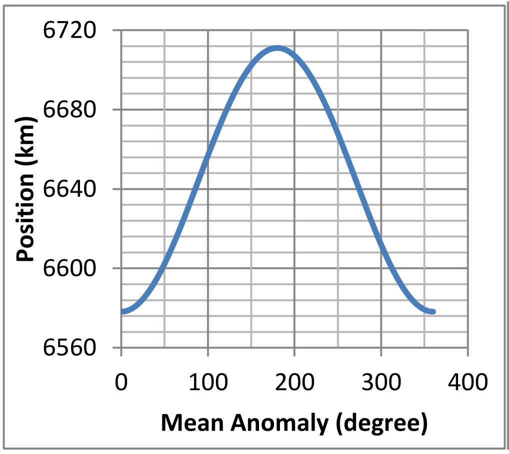

Figure 5-variation of Position with Mean Anomaly at one period without perturbation

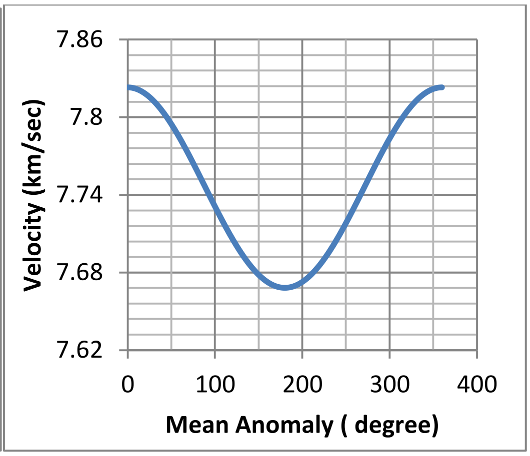

Figure 6- variation of velocity with Mean Anomaly at one period without perturbation

Second: calculation of the moon and the sun equatorial coordinates and distances from the earth

1- Figure-7: shows the change in the distance of the moon from the center of the earth within one year. It shows that the values of the distance during the anomalistic month (27.55455) day vary between the value of maximum and minimum, which is not fixed from month to month. It is also view that the values of ($r_{perigee}$) change more than (r_{apogee}), because of the change in the effect of the sun's attraction to the moon resulting from the rotation of the earth and the moon around the sun.

2- Figure-8: shows the change of the right ascension of the Moon during one year between (0-360˚). We found that the number of lunar periods during one year is (13.369) periods.

3- Figure-9: shows the change in the declination of the moon during one year between the maximum value and the minimum value (13,369) times, and the minimum values change during 2018 between (-19.89˚ , -21.649˚), while the maximum values vary between (20.111˚ , 21.494˚).

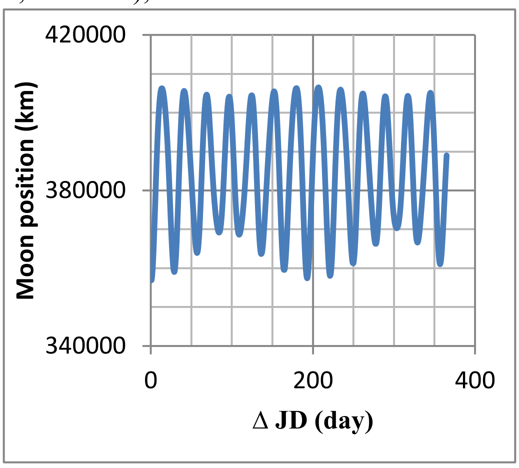

Figure 7-variation of Moon position with Δ Julian date at year (2018)

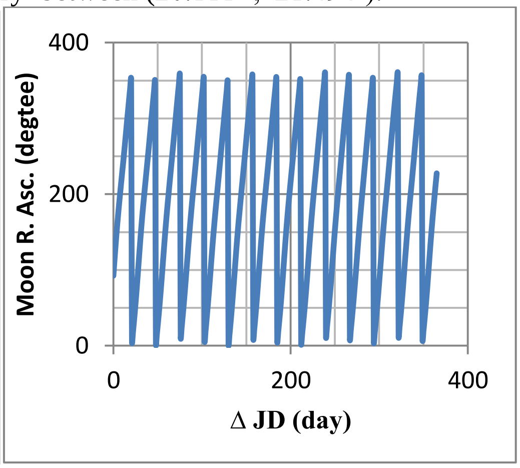

Figure 8- variation of Moon Right Ascension with Δ JD at year (2018)

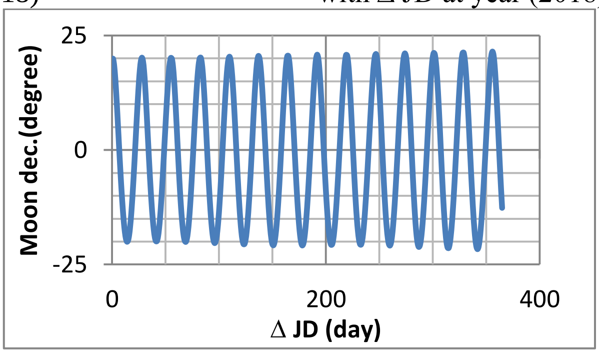

Figure 9-variation of Moon Declination with Δ JD at year (2018)

Third: calculation of acceleration attraction of the sun and the moon on the low earth orbit satellite.

1- Figure-10 shows the change in the lunar attraction perturbation with the moon- satellite distance during one period, It is clear that the acceleration varies between (7,120210861 * 10^{-10} - 11,64525787 * 10^{-10}) km /sec , and The minimum value of acceleration when the moon at the distance (356731,2942)km.
2- Figure-11 shows that the acceleration of the sun's attraction perturbation varies between (2,757376704*10^{-10} - 4,006389882*10^{-10}) km ^{2}/sec.
3- Figures-(12,13) shows the change of acceleration of the moon and the sun attraction perturbation with time in one period at the same values in paragraphs (1,2) above.

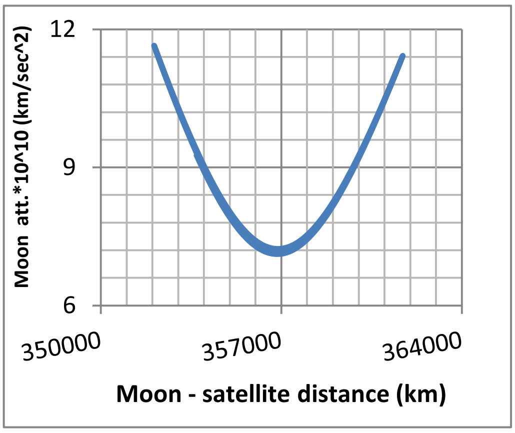

Figure 10- variation of Moon attraction per with Moon-Satellite distance for one Period at 2018:01:01:12:00:00

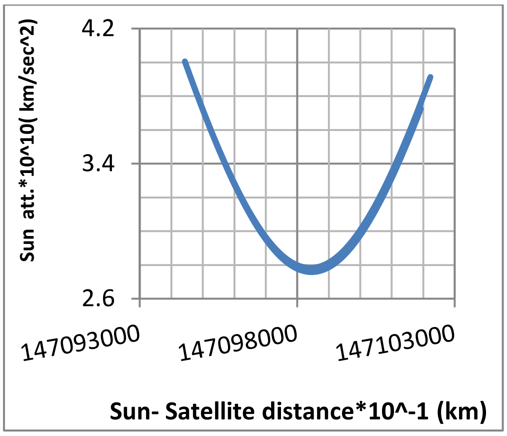

Figure 11-variation of Sun attraction per with Sun-Satellite distance for one Period at 2018:01:01:12:00:00

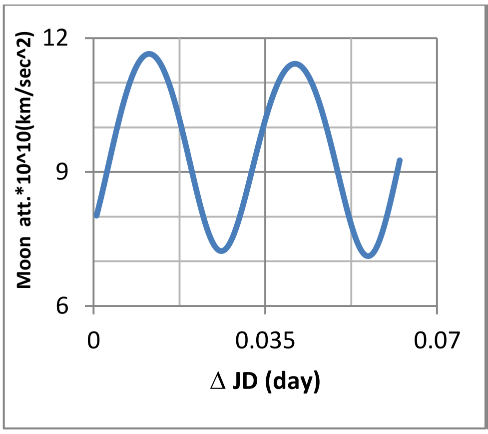

Figure 12-variation of Moon attraction per with Δ Julian date for one Period at 2018:01:01:12:00:00

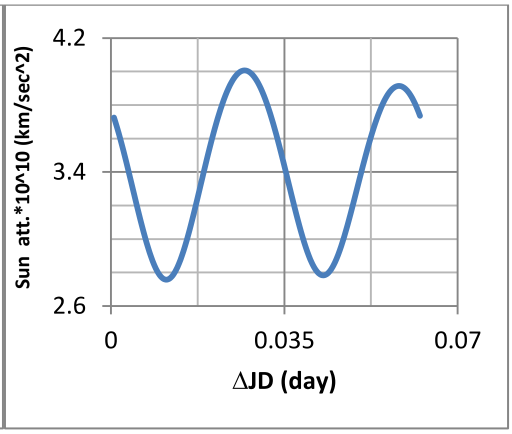

Figure 13-variation of Sun attraction per with Δ Julian date for one Period at 2018:01:01:12:00:00

Fourth: the other perturbation calculation
1- Figure-14 shows the change in atmospheric drag perturbation with the high of perigee point of the orbit, it is clear that the drag perturbation decreases significantly with the increase in height, this change can be described by the relation (a_{drag} α e^{-hp}). The drag values appeared higher than usual values, because the orbit is retrograde, where the relative velocity between the atmosphere and the satellite is higher.
2- Figure-15 represents the change of solar radiation pressure perturbation with the distance of the satellite from the sun; from the figure we show that the relationship between the change of distance and perturbation is linear.
3- Figure-16 represents the change of the solar radiation pressure perturbation with the mean anomaly angle, we find that the SRP is periodic rotation with mean anomaly and have a minimum value at (M = 180˚) where the satellite on the other side of the sun.

4- Figure-17: shows the earth oblateness acceleration (aj_2) has a periodic variation with the satellite distance from the earth center. That is clear view two big peaks nested with a small peak. The same behavior sees for (aj_2) with the mean anomaly but in a different phase as in Figure-18, because the distance depends on the mean anomaly where the orbit is an ellipse.

5- Figure-19 shows the variation of the satellite distance with delta Julian date in a unit (day) under all perturbations for one period. The perturbations are very small; their effect does not appear clearly through one period. The position variation between (6578.165039 – 6711.057312) km and (Δr = 132.892273) km. this variation is not the same for all periods. As well as the variation of the velocity with delta Julian date as shown in Figure-20, the minimum velocity appears at the same time of perigee distance and vice versa.

6- Figures (21, 22) show the angular momentum of the satellite variation with delta Julian date through 10 days (160 periods). it's clear that the angular momentum is constant without perturbation along the orbit and along the time, but with perturbation the angular momentum has a secular curvature increased with a small magnitude depend on the perturbations effects.

7- Figure-23: shows that the perigee distance is constant with time for Keplerian orbit without perturbations, but it is decreased linearity with time under all perturbations as the Figure-24. The slop = 0.00458/10 = 0.458 meter per day, that mean the low orbit at (hp = 200 km) will be decreases with time under atmospheric drag perturbation.

8- Figure-25: shows the apogee distance is decreased as a concave curve with all perturbations through 10 days with magnitude (0.598 meter per day). From Figures-(24, 25) we can find that the low orbit close circle under the atmospheric drag perturbation.

9- Figure-26: shows the minimum velocity (v_{apogee}) is slowly increased with time under perturbations as a concave curve with magnitude (Δv_{apogee} = 0.00457 m/s per day. The strange thing that the velocity at perigee ($v_{maximum}$) is decrease with time even if the $r_{perigee}$ is decrease also as the Figure-27. The decreasing value is (Δv_{apogee}= 0.00156 m/s per day).

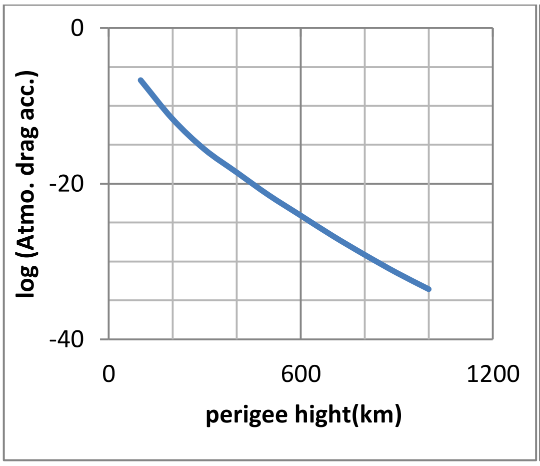

Figure 14-variation of log (atmospheric drag acceleration) height of perigee

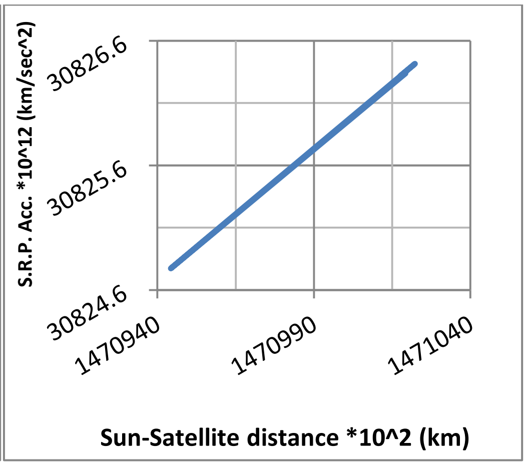

Figure 15-variation of Solar R. P. Acc.. with with Sun-Sat. distance at one Period

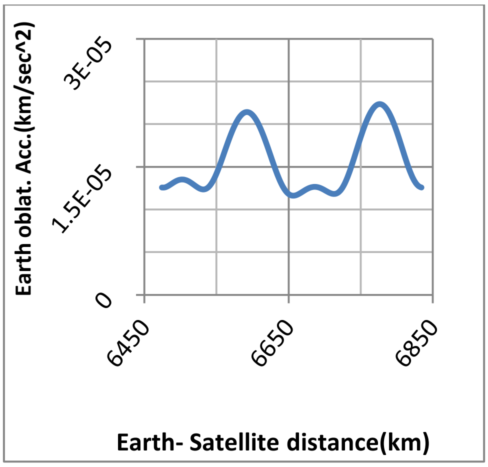

Figure 16-variation of Solar R. P. Acc. with Mean Anomaly at one Period

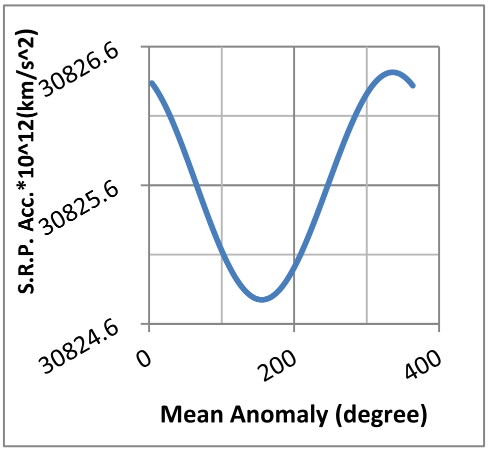

Figure 17- variation of Earth Oblatness Acc. with Earth-Satellite distance at one Period

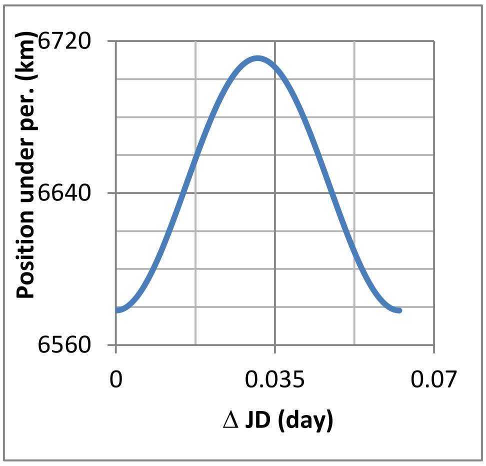

Figure 18-variation of Earth Oblatness Acc. with Mean Anomaly at one Period

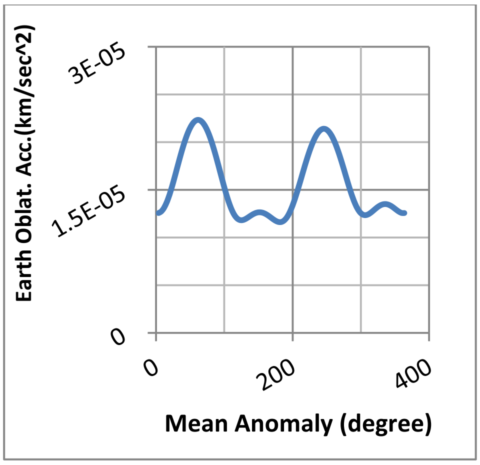

Figure 19-variation of Position under all perturbations with Julian date for one Period

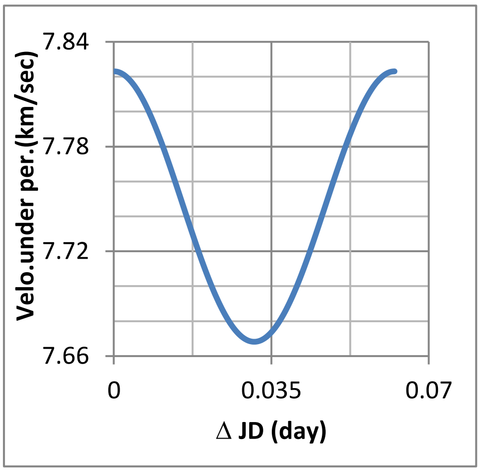

Figure 20- variation of Velocity under all perturbations with Julian date for one Period

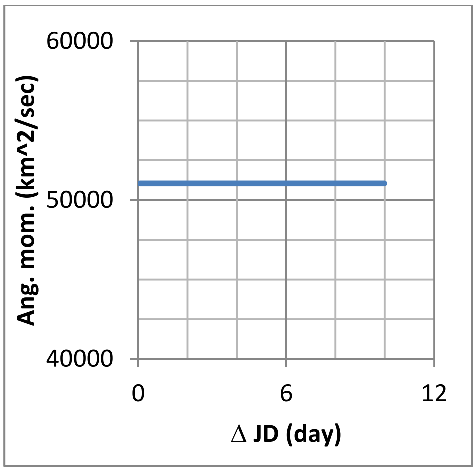

Figure 21- variation of Angular momentum (without perturb.) with Julian date for ten days (160 Periods)

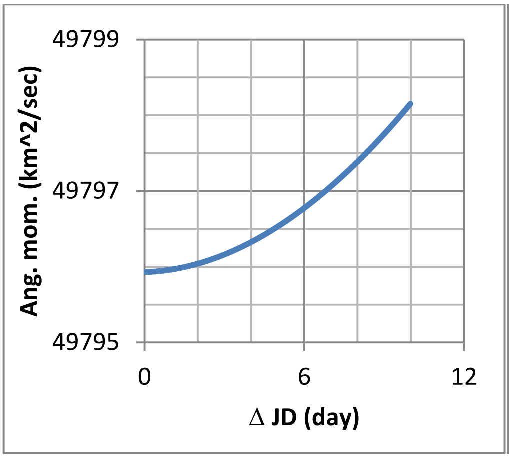

Figure 22-variation of Angular momentum under perturbation with Julian date for ten days (160 Periods)

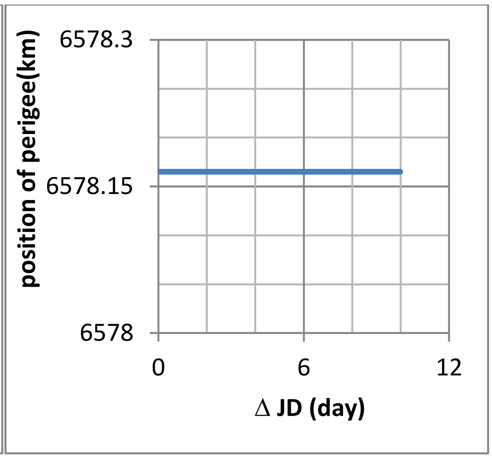

Figure 23 -variation of position at perigee with delta Julian date during 10 days without perturbations

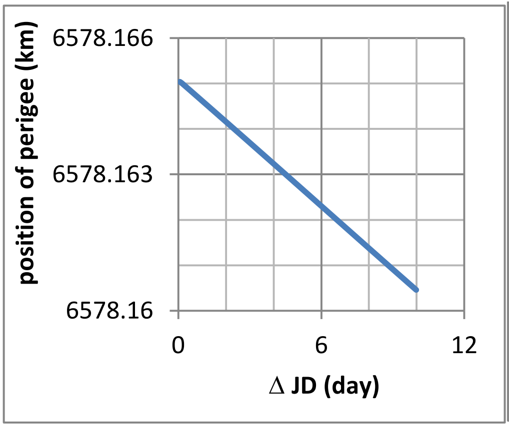

Figure 24-variation of position at perigee under all perturbations with delta Julian date during 10 days

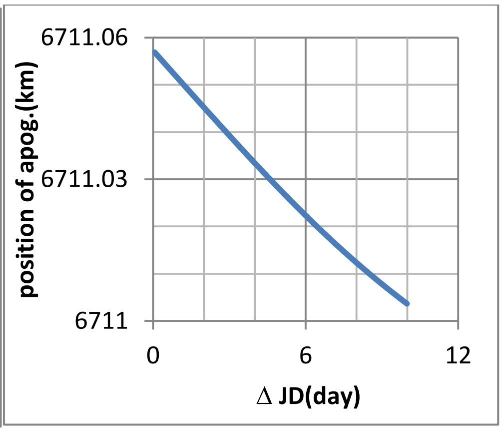

Figure 25-variation of position at apogee under all perturbations with delta Julian date during 10 days

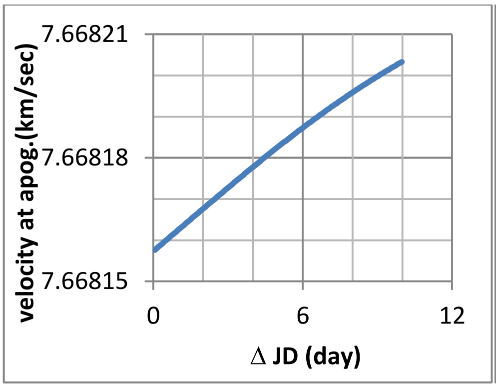

Figure 26- variation of velocity at apogee under all perturbations with delta Julian date during 10 days date

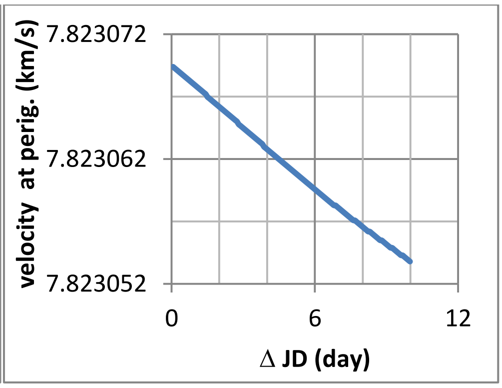

Figure 27-variation of velocity at perigee under all perturbations with delta Julian during 10 days

11. Conclusions

1- All values indicate that the perturbation acceleration effect on the distances and velocity are very small through the period, but the effect is more important after many hundreds of periods.
2- The drag acceleration perturbation is inversely proportional with high of the satellite.
3- Angular momentum is increased under the effect of all perturbations for low orbits that causes a variation on orbital elements after several periods.

4- The effect of attraction of the sun and moon is as a minimum value when the satellite in orbit is perpendicular to the sun or moon.
5- The acceleration of sun and moon attraction do not depend on the distance of the satellite from the sun or moon directly but depends on the angle (sun - Earth - satellite) and (moon - Earth - satellite).
6- The earth oblateness perturbation depends on the position of the satellite in orbit and orbital inclination, more than the distance of the satellite from the earth and this effect is periodic or not accumulated with the date.
7- The atmospheric drag perturbation beyond height (1000 km) is neglected.

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