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The Minimum Cost for the Vascular Network Using Linear Programming Based Its Path Graph

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Abstract
As of late, humankind has experienced radiation issues either computerized tomography (CT) or X-rays. In this investigation, we endeavor to limit the effect of examination hardware. To do this the medical image is cropping (cut and zoom) then represented the vascular network as a graph such that each contraction as the vertices and the vessel represented as an edges, the area of the coagulation was processed already, in the current search the shortest distance to reach to the place of the blood vessel clot is computed

Keywords: vascular network, connected graph, algorithm.

التكلفة الدنيا لشبكة الأوعية الدموية باستخدام البرمجة الخطية

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الخلاصة
في الآونة الأخيرة ، شهدت البشرية مشاكل الإشعاع إماعن طريق التصوير المقطعي المحوسب (CT) أو الأشعة السينية. نسعى في هذا البحث إلى الحد من تأثير أجهزة الفحص عن طريق القيام بعمليه الاقتصاص (القطع والتكبير) للصورة الطبية تم تمثيل الشبكة الوعائية كرسومات بيانية بحيث يمثل كل تقاطع القمم وتمثل الاوعيه الدمويه كحواف في البيان وثم تحديد منطقة التخثر, كذلك تم حساب أقصر مسافة إلى مكان التجلط.

1. Introduction:

Graph theory includes several applications and has evidenced to be a particularly great tool in analyzing numerous sensible issues. For example, graphs can be utilized to represent telecommunications networks or electrical, passing systems, flow charts, and pipelines. Graph theory owns too found to be useful while working with chemical compounds, biological evolutionary trees, computer data structures, organizational charts etc.

A flag of analyzing medical issues supported completely different imaging modalities and digital image analysis techniques are called the medical image analysis. The process of partitioning a picture into a group of non-intersecting regions specified every region is homogeneous and therefore the transition from one region to a different is sharp is said to be Segmentation [1]

The medical image is a standout amongst the most dynamic research points in a picture examination. Dissecting and portioning medical image in a clinical setting remains a testing undertaking because of the variety of imaging modalities and the inconstancy of the patient's attributes and pathologies.

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Instead of the past strategies in which the base cost associated sub diagram (MCCS) showed up in numerous medicinal picture investigation, noticeably to segment medical image. In our strategy, the MCCS is utilized for figuring the most limited way to go after the area of the coagulation. The area of the coagulation is accomplished by processing a coordinated graph G_I, which is an overcomplete division of the vasculature by setting vertices and edges; this sort of division is additionally called vascular recreation thus, minimal way to the area of the coagulation is ascertained by applying calculations to overcomplete chart G_I.

The minimum cost connected subgraph (MCCS) optimization problem seems in numerous medical image analysis tasks, the foremost obvious is for segmenting neural structures [2], reconstructing tube networks [3], variations of this optimization problem have been proposed for anatomical labeling of vasculature [4], artery-vein separation [5], an attempt of minimization the impact of medical examination equipment. This is done by represented the blood vessel network as a graph and the weight of the edge represent the length of the vessel, the shortest distance to the place of the clot is computed [6]. In [7] the place of the clot is found. This can be done by representing the weight of each edge as the amount of the blood in the vessel. If the amount is less than the normal flux then a sign is putting which represents the place of thrombosis.

2. Basic concepts

Some basic concepts that relate to the graph theory

Definition (1), [8]

Let (V (G), E (G)) be a ***graph (G)*** where V (G) may be a nonempty set whose parts are referred to as points (vertices). E (G) may be nonempty **set** of unordered pairs of elements of V (G). The elements of E (G) are named lines (edges) of the graph G. A graph G= (V, E) with n vertices and m edges is termed a (n, m)-graph.

Definition (2), [9]

The quantity of vertices during a graph is alleged to be the **order** of the graph, indicate by |V (G) |. The number of edges in a very given graph is claimed to be the **size** of the graph, indicate by |E (G)|.

Definition (3), [10]

A simple graph could be a graph that has neither loop nor multiple edges.

Definition (4), [10]

A trivial graph is a graph of order one**.** Else it is said to be ***nontrivial graph***.

Definition (5), [11]

The degree of a vertex *V* of any graph G is defined as the number of edges incident on *V*. It is denoted by d(V) or d(V).

Definition (6), [11]

An ***isolated vertex*** is a vertex of degree zero.

A vertex of degree one is also said to be an ***end-vertex,*** **leaf** *or* ***Pendent vertex***.

Definition (7), [12]

A graph H may be a ***graph*** of G if each vertex of H may be a vertex of G, and each edge of H is an edge of G. The sub graph H may be ***a spanning sub graph*** of G if H include all vertices of G.

Definition (8), [8]

The ***complement graph*** ($\overline{G}$) of a simple graph *G* with vertex set *V (G)* is the graph in which two vertices are adjacent if and on if they're not adjacent in G.

Definition (9), [8]

A ***path graph*** **(P_n)** is a sequence of vertices without repeated, of order n, n ≥ 1 and size n-1, the vertices are labeled $V_1, V_2, \dots, V_{m-1}, V_m$ and the edges are $V_1V_2, V_2V_3, \dots, V_{m-1}V_m$.

Definition (10), [8]

The closed path is said to a ***cycle graph (*C_m*)*** of order and size m, $m \geq 3$.

Definition (11), [12]

A tree could be a connected graph of order n, n ≥ 1 and size n-1 without the cycle.

Definition (12), [9]

The shortest path joining between two vertices v and u is called ***distance d*** $(\boldsymbol{v}, \boldsymbol{u})$

Definition (13), [9]

The maximum distance between all pairs of vertices in G is named ***diameter*** of G denoted (**diam** (**G**)).

Definition (14), [13]

A digraph (or directed graph) D is a finite nonempty set V of objects called vertices together with a set E of ordered pairs of distinct vertices. The elements of E are called directed edges or arcs. If (u, v) is a directed edge, then we indicate this in a diagram representing D by drawing a directed line segment or curve from u to v.

Definition (15), [13]

Cyclic is a directed graph with at least one directed circuit**.**

Definition (16), [14**]**

acyclic graph is a graph that contains no cycles.

Definition (17), [14**]**

An acyclic graph $G = (V, E)$ if G has more than one component, and then G is said to be a ***forest***. If G has one component, then G is a tree.

Definition (18), [14**]**

Let G= (V, E) be a graph if T=$(\acute{V}, \acute{E})$ is an acyclic sub graph of G such that V=$\acute{V}$ then T is said to be a ***spanning forest*** of G. If T has precisely one ingredient, then F is said to be a ***spanning tree***.

Definition (19), [12]

A spanning tree of G whose weight is minimum among all spanning trees of such a spanning tree is said to be a ***minimum spanning tree***.

The problem of finding a minimum spanning tree in a very connected weighted graph is termed the ***Minimum Spanning Tree Problem.***

Definition (20), [14**]**

A pair (G, w) where $G = (V, E)$ is said to be a weighted graph and $w: E \to R$ is a weight function of the path which is equal to:

$$\sum_{i=1}^{n} w(e_i)$$

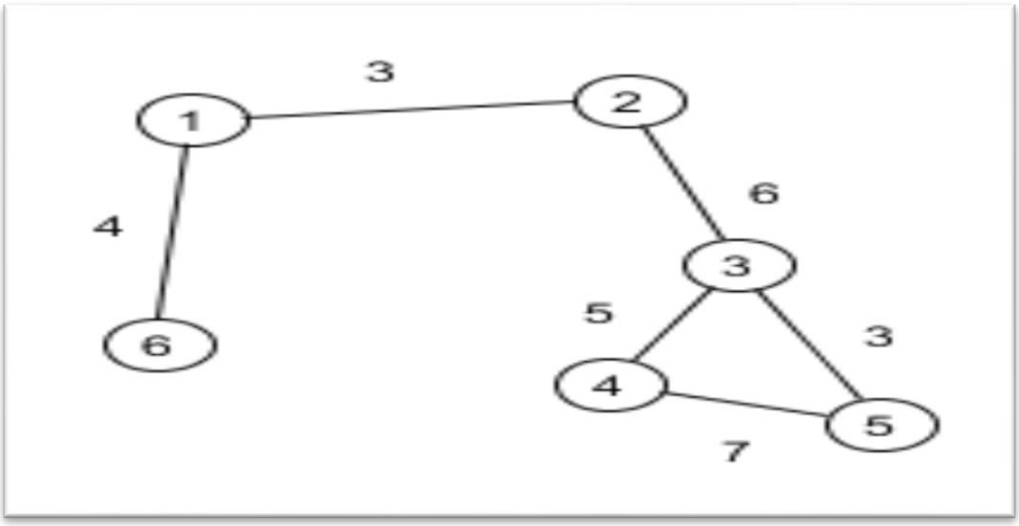


Figure 1-A weighted graph

Thus in a weighted graph, the distance between two vertices $V_1$and $V_2$is the weight.

Definition (21), [14**]**

Let (G, w) be a weighted graph with $G = (V, E)$ if $H = (\acute{V}, \acute{E})$ is sub graph of G is called ***Sub graph weight***, then weight of H is:

$$w(H) = \sum_{e \in \acute{E}} w(e)$$

3- The shortest distance to the clotted vessel

The minimum distance (path) of a connected subgraph is found associate degreed it's an accomplished over a diversity of network optimization algorithms that are

1- Linear programming formulation of the shortest path problem[15]
2- Shortest path algorithm.[15]

Now, the discussion of each algorithm is introduced

4.1 Find formula for solving the shortest path problem by Linear programming (LP).

Such can be utilized to reveal the shortest path between any two vertices in the network. Which include n nodes, The LP suppose that one unit of flux enters the network at node s and leaves at node t.

To detect the shortest path of node s to node t: we follow the following steps:

1. Let for each arc z_{ij}represented the flow on arc (i,j).
2. Flow balance in each node.
3. It arc(i,j) is in the shortest path then $z_{ij} = 1$ otherwise $z_{ij} = 0$
4. The linear programming formulation (assume x_{ij}≥ 0):

$$z_{ij} = amount\ of\ flow\ in\ arc(i,j)$$

$$=\begin{cases} 1, if\ arc(i,j) is\ on\ the\ shortest\ route \\ o, otherwise \end{cases}$$

$$x_{ij} = length\ of\ arc(i,j)$$

Thence, the objective function of the linear program becomes

$$Min\ z = \sum_{\substack{all\ defined \\ arcs(i,j)}} x_{ij}\, z_{ij}$$

And the constraints represent as

Subject to

$$\sum_{i:(i,k)\in arcs(i,j)} z_{ik} - \sum_{j:(k,j)\in arcs(i,j)} z_{kj} = \begin{cases} -1, k = s \\ 1, k = t \\ 0, k \in arcs(i,j)\backslash\{s,t\} \end{cases}$$

$z_{ij} \geq 0$, (i,j)∈ $arcs(i,j)$

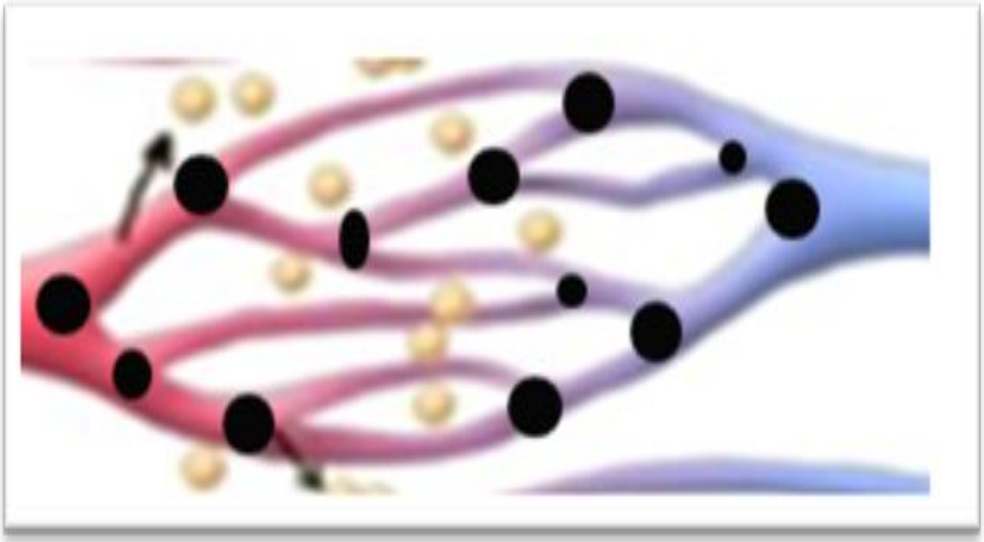
The linear programming dual is

Max $y_t - y_s$,

Subject to

$$y_j - y_i \leq x_{ij}, (i,j) \in arcs(i,j)$$

Example1.To finds the shortest distance for reaching the clotted vessel using linear programming: Figure-2.



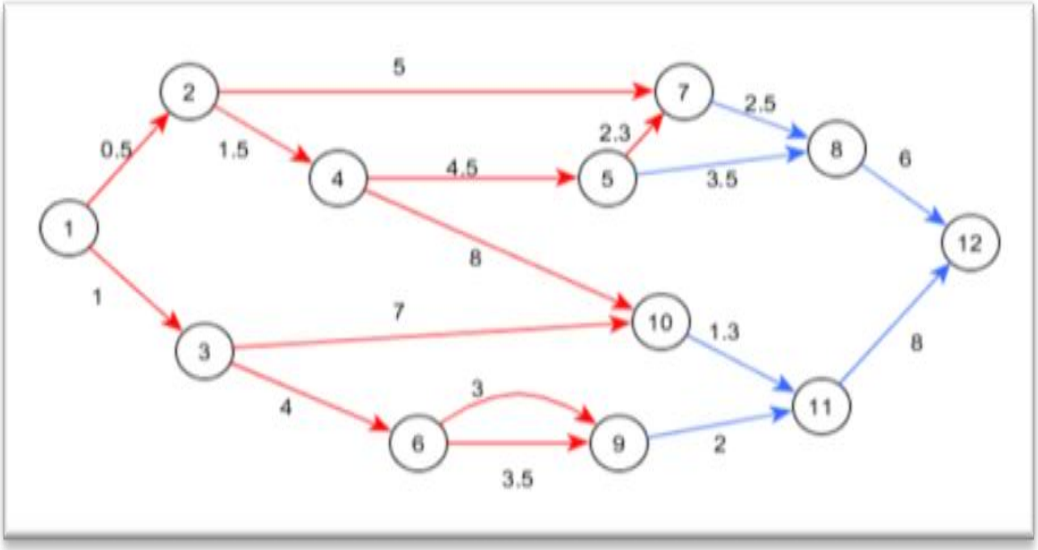


Figure 2-Constructing the vessels of the Overcomplete Graph GI Assume that the blood vessel (edge) that connects the vertex 10 with the rest of the blood vessels contains a clot, to get to the vertex 10 at the shortest distance as following:

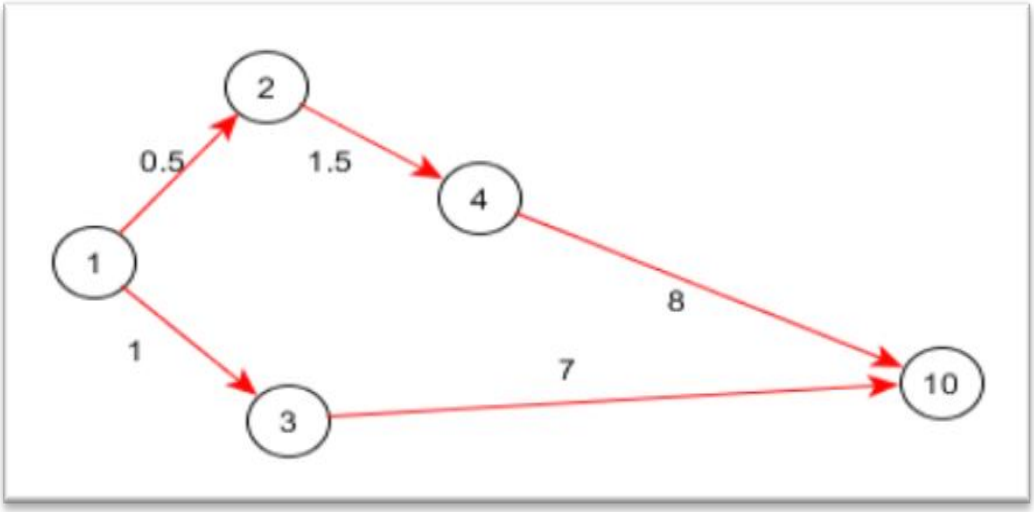


Figure 3-subgraph of the Overcomplete Graph GI.

Min z $= o.5z_{1,2} + z_{1,3} + 1.5z_{2,4} + 7z_{3,10} + 8z_{4,10}$

$$z_{1,2} + z_{1,3} = 1$$
$$z_{4,10} - z_{2,4} = 0$$
$$z_{3,10} - z_{1,3} = 0$$
$$-z_{4,10} - z_{3,10} = -1$$

Max z $= z_{10} - z_1$

$$z_2 - z_1 \leq 0.5$$
$$z_3 - z_1 \leq 1$$
$$z_4 - z_2 \leq 1.5$$
$$z_{10} - z_3 \leq 7$$
$$z_{10} - z_4 \leq 8$$

Max z$= z_{10} - z_1 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5$

Max z $-z_{10} + z_1 - 0s_1 - 0s_2 - 0s_3 - 0s_4 - 0s_5 = 0$

$$z_2 - z_1 + s_1 = 0.5$$
$$z_3 - z_1 + s_2 = 1$$

$$z_4 - z_2 + s_3 = 1.5$$
$$z_{10} - z_3 + s_4 = 7$$
$$z_{10} - z_4 + s_5 = 8$$

By using the simplex method we get the following results

B.V	z_1	z_2	z_3	z_4	z_{10}	s_1	s_2	s_3	s_4	s_5	R.H.S
Z	1	0	0	0	-1	0	0	0	0	0	0
S_1	-1	1	0	0	0	1	0	0	0	0	0.5
S_2	-1	0	1	0	0	0	1	0	0	0	1
S_3	0	-1	0	1	0	0	0	1	0	0	1.5
S_4	0	0	-1	0	1	0	0	0	1	0	7
S_5	0	0	0	-1	1	0	0	0	0	1	8
B.V	z_1	z_2	z_3	z_4	z_{10}	s_1	s_2	s_3	s_4	s_5	R.H.S
Z	1	0	-1	-1	0	0	0	0	1	0	7
s_1	-1	1	0	0	0	1	0	0	0	0	0.5
s_2	-1	0	1	1	0	0	1	0	0	0	1
s_3	0	-1	0	0	0	0	0	1	0	0	1.5
z_{10}	0	0	-1	-1	1	0	0	0	1	0	7
s_5	0	0	1	1	0	0	0	0	-1	1	1
B.V	z_1	z_2	z_3	z_4	z_{10}	s_1	s_2	s_3	s_4	s_5	R.H.S
Z	0	0	0	0	0	0	1	0	1	0	8
s_1	-1	1	0	0	0	1	0	0	0	0	0.5
z_3	-1	0	1	0	0	0	1	0	0	0	1
s_3	0	-1	0	1	0	0	0	1	0	0	1.5
z_{10}	-1	0	0	0	1	0	1	0	1	0	8
s_5	-1	0	0	-1	0	0	1	0	-1	1	0

Max z $= z_{10} - z_1 = 8 - 0 = 8$

$$z_{1,3} = z_{3,10} = 1, z_{1,2} = z_{2,4} = z_{4,10} = 0$$

Min z =8 shortest rout

$$z_1 \rightarrow z_3 \rightarrow z_{10}$$

Therefore the shortest distance is 8.

4.2 Shortest-path algorithms:

To find the shortest path between an origin and destination in a transport network can be used Dijkstra's algorithm, this is often used the formula for resolution each acyclic networks and cyclic (i.e., containing loops).

In this work must impose the connection on both very dense and large grid graph must be imposed; this is because blood vessels are connected.

The following offers the Dijkstra's algorithm that is let u_i be the shortest distance from origin vertex one to node i, and define $d_{ij} (\geq 0)$ as the length of edge the (i,j) .

$$[u_j, i] = [u_i + d_{ij}, i], d_{ij} \geq 0$$

The name at the first vertex is [0,-], referred that the vertex has no precursor.

Vertex labels in Dijkstra's algorithm are 2types: temporary and permanent. A temporary label is changed of a shorter path to a vertex can be found. If no better path ability is found, the case of the temporary label is modified to permanent.

Step1. The origin vertex is Classified (vertex 1) with the permanent class. Set i=1.

Step 2. (a) the temporary labels is computed $[u_i + d_{ij}, i]$ for every vertex j that may be connected from vertex I, provided j is not permanently labeled.

If vertex j is already labeled with $[u_j, k]$ through another node k and if$[u_i + d_{ij} < u_i]$, replace $[u_j, k]$ with $[u_i + d_{ij}, i]$.

(b) If every the vertex have permanent labels, the solution ends. Otherwise, the label is chosen $[u_r, s]$ having the shortest distance ($=u_r$) among all temporary labels (break ties arbitrarily) .set i=r and as will as iterate step i.

Example2 : To find shortest distance to reach the clotted vessel, which is given in Figure-4

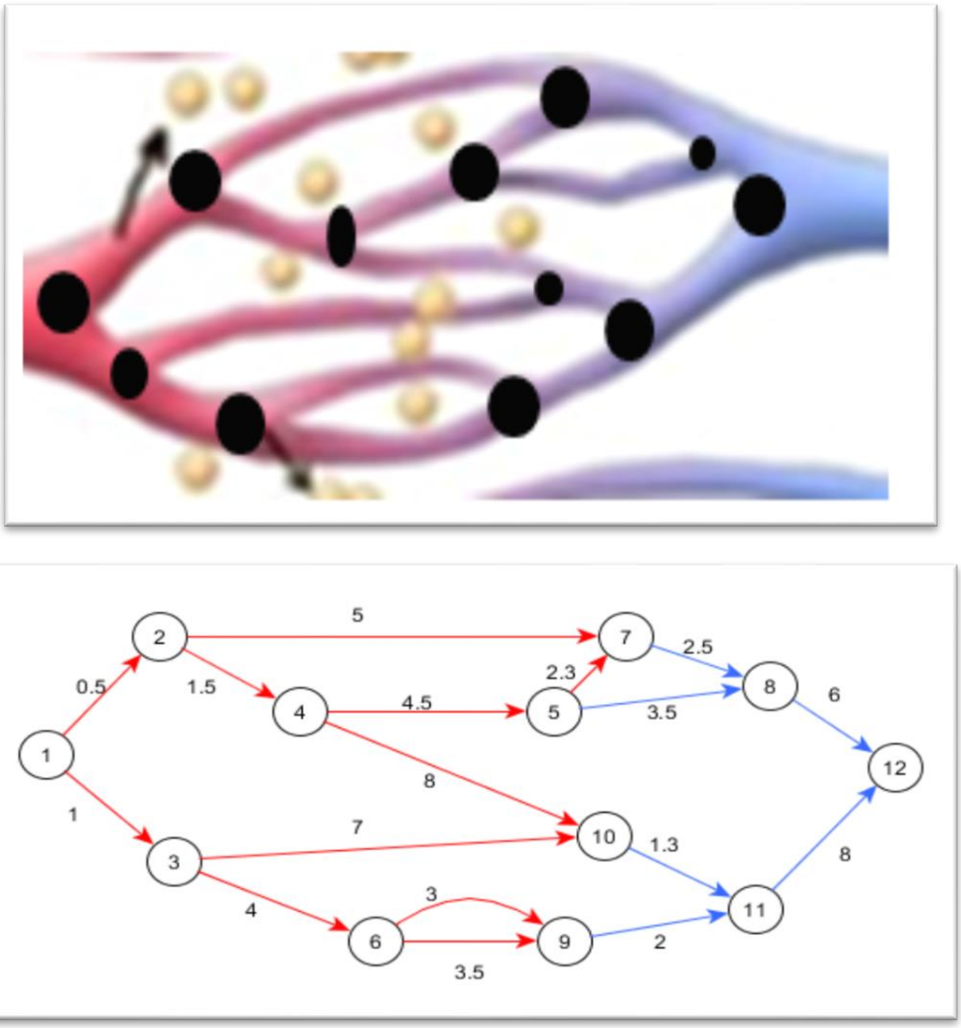


Figure 4-To constructing the vessels of the Overcomplete Graph G_ILet edge (6, 9) a clotted blood vessel, to get to the vertex 6 at the shortest distance as following:

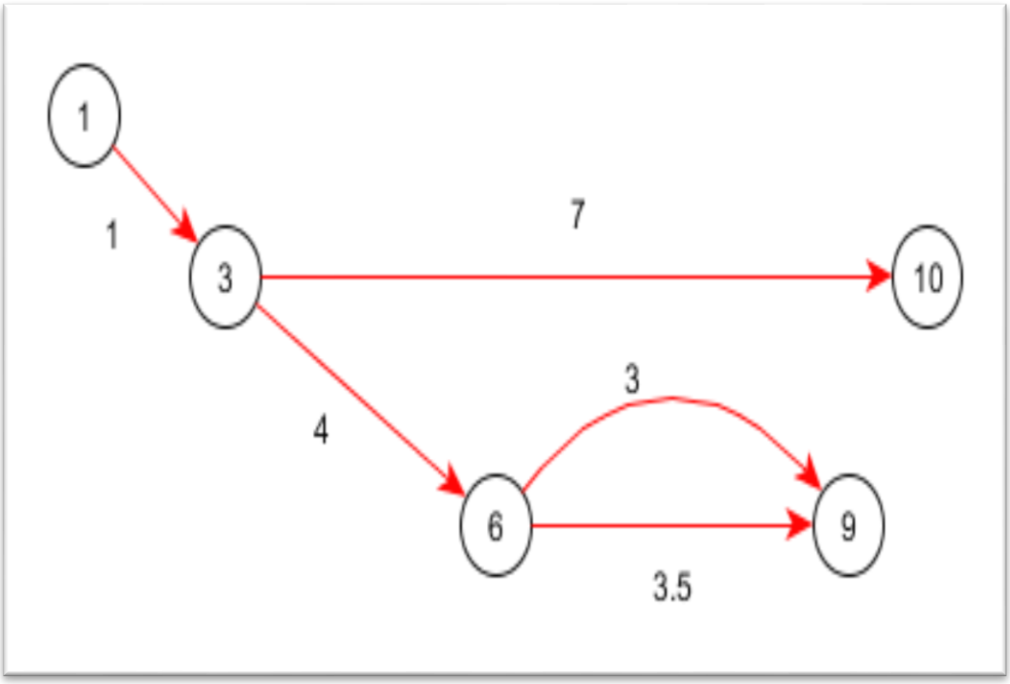


Figure 5-subgraph of the Overcomplete Graph GI

Iteration0. The permanent label [0,-] to vertex one.

Iteration1. Vertex three are often reached from (the last permanently labeled) vertex one.

Iteration 2.vertex ten and six are often reached from (the other permanently labeled) vertex three. s, the table of labeled vertices (temporary and permanent) becomes.

Vertex	Label	Status
1	[0,-]	permanent
3	[0+1,1]=[1,1]	permanent
10	[1+7,3]=[8,3]	temporary
6	[1+4,3]=[5,3]	temporary

For the two temporary labels [8, 3] and [5, 3], vertex 6 yields the smaller distance ($u_6 = 5$). Thus, the standing of vertex 6 is modified to permanent.

Iteration 3. Vertex nine is can be reached from vertex six.

Vertex	Label	Status
1	[0,-]	permanent
3	[1,1]	permanent
10	[8,3]	temporary
6	[5,3]	permanent
9	[5+3,6]=[8,6]	temporary

Vertex 9 It has two values [8, 6] and [8.5, 6] the choose a shorter distance.

Iteration 4. This vertex 9 and 10 as the only temporary label. Because vertex 9 and 10 do not drive to other vertices, its event is converted to permanent, and also the method ends.

Vertex	Label	Status
1	[0,-]	permanent
3	[1,1]	permanent
10	[8,3]	permanent
6	[5,3]	permanent
9	[8,6]	permanent

The shortest distance between $V_1$and V_6in the network is specified by beginning at the required goal vertex and backtracking out of the vertices utilize the data given by permanent tags.

Shortest distance between vertex 1 to vertex 6

(6) → [5, 3] → (3) → [1, 1] → (1)

1 → 3 → 6
1+4=5 therefore the shortest distance is 5.

Conclusion

we employed mathematics to give the connection between the vascular network and the graph by through algorithms used to reach the clot that injury of an artery or vein through minim path, where the vertices of the graph represent as the vascular endpoints and the edges represent as the vessel and the weights represent the length of it.

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