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Determination local geoid Heights Using RTK-DGPS/Leveling and transformation methods

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Abstract

Geodesy is concerned with the relative positioning of points and the gravity field of the earth. For this task, a well-defined coordinate system is needed on which measurements are normally tied to a set of reference points called a geodetic datum (geoid or ellipsoid). The Global Positioning System GPS gives accurately the three-dimensional position of a point (latitude, longitude, and ellipsoidal height) and can measure under all weather conditions. The coordinates of the GPS reference to the World Geodetic System1984 (WGS 84), a global ellipsoid having its origin as the mass center of the earth, and height, referenced to the surface of the ellipsoid . In this research , using RTK-DGPS technique Data collection for study local and leveling, and Earth Gravitational Models (EGM2008,EGM96) for determined to geoid undulation utilized 2D polynomial models , and then Using a surface interpolation (kriging) approach, the coordinate and the computed geoidal heights of some well selected points .

Keywords: Geoid height, Real Time Kinematic, Differential Global Position System, World Geodetic, System1984

تحديد الارتفاعات الجيوديسية المحلية باستخدام (الرصد المتحرك اللحظي – بجهاز التموضع التفاضلي العالمي) للمناسيب و طرق التحويل

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الخلاصة

علم الجيوديسيا يختص بتحديد المواقع النسبية للنقاط بالنسبة الى مجال الجاذبية للارض .لهذه المهمة،نظام الاحداثيات الدقيق يحتاج الى قياسات ترتبط عادةً الى مجموعة من النقاط المرجعية تسمى (الجيود او الاهليلجي) ، نظام تحديد المواقع العالمي يعطي بدقة موقع ثلاثي الابعاد للنقطة (خطوط الطول والعرض ، والارتفاع الاهليلجي) ،ويمكن القياس تحت جميع الظروف الجوية ، احداثيات لنظام تحديد المواقع العالمي مرجع لنظام الجيوديسي العالمي 1984 ، وجود الاهليلجي العالمي اصله من مركز كتلة الارض و مرجع الارتفاع الى سطح الاهليلجي . في هذا البحث استخدمت تقنية الجي بي اس التفاضل – الرصد المتحرك اللحظي لجمع البيانات للمنطقة واستخدام ميزان التسوية ونماذج الجاذبية الارضية EGM(2008,1996) لتحديد حيود الجيود باستخدام نماذج متعددة الحدود ثنائية الابعاد ، وبعدها تم استخدام طريقة (kriging) لاستيفاء السطوح لارتفاعات واحداثيات الجيود المحسوبة لبعض النقاط المختارة.

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Introduction

One of the basic goals of geodesy is the determination of the geoid which is the equipotential surface of the earth gravity field and which coincides on the average with the mean sea level [1].The geoid surface is more irregular than the ellipsoid of revolution often used to approximate the shape of the physical Earth, but considerably smoother than the Earth's physical surface. The transformation of ellipsoidal heights to orthometric heights therefore requires that the geoid height refer to the same reference ellipsoid In the case of GPS-derived ellipsoidal heights the geocentric WGS84 ellipsoid are used..by using the equation(1)[2].

$$\mathbf{H = h - N} \quad (1)$$

Orthometric heights (H) are defined as the geometric distance, measured along the plumb line, from the geoid to a point of interest above. Ellipsoid heights (h) are defined as the geometric distance, measured along a normal straight line to a reference ellipsoid, from the reference ellipsoid to a point of interest above. The geoid height (or geoid undulation, N) is defined as the geometric distance, measured along a normal straight line to a reference ellipsoid, from the reference ellipsoid to a point on the geoid above. Equations (1) has been used for the determination of orthometric heights from ellipsoidal heights and a geoid model, this is called GPS/leveling [3] The relationships between orthometric, ellipsoidal and geoid heights are shown in Figure-1.

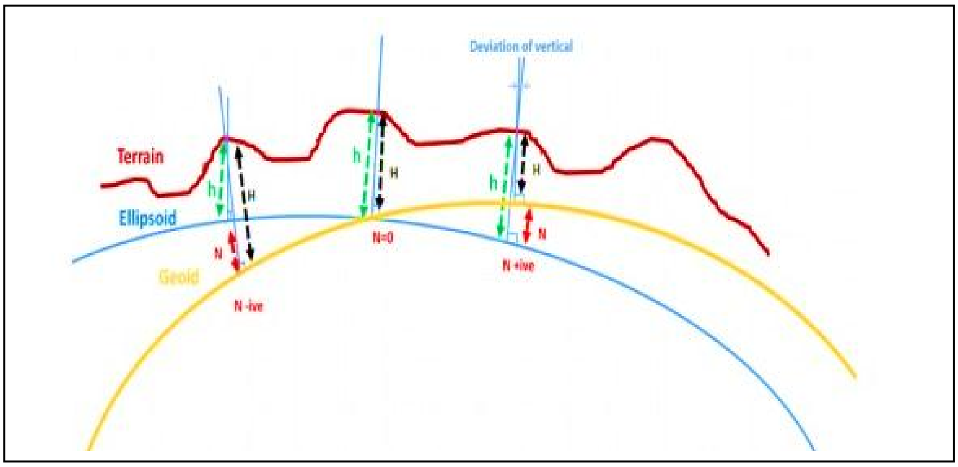

Figure 1 - Illustration of the relationships between orthometric, ellipsoidal and geoid heights.[4]

Global Geopotential Models (GGM)

Global Geopotential models (GGMs) describe the Earth's gravitational potential in terms of an infinite series of spherical harmonics outside the Earth attracting masses. They are determined by a combination of satellite and terrestrial observations and used as reference fields in the determination of local and regional geoids. The geopotential is usually given as a truncated set of harmonic coefficients, obtained when solving a Laplace equation in spherical coordinates described [5] .

$$V(r,\theta,\lambda) = \frac{GM}{r}\left\{1 + \sum_{n=1}^{N_{max}} \sum_{m=0}^{l} \left(\frac{\alpha}{r}\right)^{l} \left[\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm}(\sin\theta)\right\} \quad (2)$$

Where:

GM Earth's gravity constant

r magnitude of radius vector

n, m degree and order of spherical harmonics

$\overline{P}_{nm}$ Legendre functions

$\overline{C}_{nm}, \overline{S}_{nm}$ coefficients of spherical harmonics

θ Latitude

λ Longtude

The disturbing potential *T* at a point $V(r, \theta, \lambda)$ is the differences between the actual gravity potential of the Earth and the normal potential associated with the a rotating equipotential ellipsoid at *V*. Based on equation (2) the spherical harmonic representation of T is :

$$T(r,\theta,\lambda) = \frac{GM}{r} \sum_{n=1}^{N_{max}} \sum_{m=0}^{l} \left(\frac{\alpha}{r}\right)^{l} \left[\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm}(\sin\theta) \quad (3)$$

Equation (3) have been expanded for several numerous processes to get the element of the Earth's gravity field such as gravity anomalies (Δg) and geoid height (N). The relationship between the coefficient of spherical harmonic with gravity anomalies (Δg_{GM}) and geoidal height (N_{GM}) is given by the following formula, respectively:[6]

$$\Delta g_{GM} = \frac{GM}{r^2}\sum_{n-2}^{N_{max}}(n-1)\sum_{m=0}^{l}\left(\frac{\alpha}{r}\right)^{l}[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda]\bar{P}_{nm}(\sin\theta) \quad (4)$$

$$N_{GM} = \frac{GM}{r\gamma}\sum_{n-2}^{N_{max}}(n-1)\sum_{m=0}^{l}\left(\frac{\alpha}{r}\right)^{l}[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda]\bar{P}_{nm}(\sin\theta) \quad (5)$$

The development of accurate potential coefficient models is dependent on accurate analyses of the perturbations of the orbits of artificial satellites (e.g. GPS) and from the combination of such information with surface gravity data, and relatively recently with satellite altimeter data [6].

In this study, used two models (EGM96, EGM2008). Earth Gravitational Model 1996 EGM96 is a geopotential model of the Earth consisting of spherical harmonic coefficients complete to degree and order 360, This model was developed by Goddard Space Flight Center, The National Imagery and Mapping Agency (NIMA) and the Ohio State University (OSU) .The model gives the geoidal separation from WGS84 ellipsoid and utilized worldwide for converting ellipsoidal elevation values to Orthometric heights. [7]. Earth Gravitational Model 2008 (EGM2008) is a spherical harmonic model of the Earth's gravitational potential developed by the Department of Defense. However, National Geospatial-Intelligence Agency (NGA) is developing a comprehensive Earth Gravitational Model Implementation Plan for use within the National System for Geospatial-Intelligence. The Implementation Plan will include specific data, format, file and algorithm guidance for system developers on EGM2008 and future geoid releases.[8] Together with other satellite-only and combined GGMs that have been available after the GRACE and CHAMP missions, the EGM08 model represents the state-of-art in global gravity field mapping and it contributes significantly to the continuing efforts of the geodetic community for a highly accurate reference model of Earth's gravity field.[8] EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/ Leveling values are on the order of 5 to 10 cm.[8]

Real Time Kinematic DGPS

RTK surveying is a carrier phase based relative positioning technique that, like the previous methods, employs two or more receivers simultaneously tracking the same satellites . This method is suitable when: (1) the survey involves a large number of unknown points located in the vicinity (i.e., within up to about 10.15 km) of a known point; (2) the coordinates of the unknown points are required in real time; and (3) the line of sight, the propagation path, is relatively unobstructed [10]. The RTK approach is a differential positioning technique that uses known coordinates of a reference station occupied by one receiver to determine coordinates of unknown points visited by a rover receiver [11].Because of its ease of use as well as its capability to determine the coordinates in real time, this method is the preferred method by many users. The typical nominal accuracy for these dual-frequency systems is 1 centimeter ± 2 parts-permillion (ppm) horizontally and 2 centimeters ± 2 ppm vertically.

Modeling local GPS/levelling geoid technique

The GPS/levelling technique simply involve the use of ellipsoidal heights derived from the Global Positioning System and orthometric heights obtained via the levelling process, to determine the geoid height and subsequently the geoid model . In modelling local GPS/levelling geoid with geometric approach, a geoid reference benchmarks network having coverage of entire area is constituted. One of the limitations of local geoid determination is datum inconsistency problem. But, in this study, this problem is not going to be considered because the focus is on testing surface fitting algorithms as a geometrical approach for modeling a local geoid using GPS and Leveling data. [12]. The geoid reference benchmarks are generally selected from the common points of order GPS benchmarks and the 1st , the 2^{nd} and 3^{rd} orders levelling network points, With existing *n* reference benchmarks having GPS ellipsoidal and levelling heights (and hence with known geoid heights: $N_{GPS/lev.} = h_{GPS} - H_{levelling}$)

in a local area, the general equation of polynomial interpolation to estimate GPS/levelling geoid heights at unknown points in the area can be given as(equation 6):

$$\boldsymbol{N(x,y) = \sum_{m=0}^{L} \sum_{n=0}^{M} a_{mn} x^m y^n} \tag{6}$$

where x and y represent the position coordinates, a_{mn} symbolize the polynomial coefficients, and L is the degree of the polynomial. The position coordinates can be constituted in various ways, and in this study they are obtained from the ellipsoidal geographical coordinates as following:

$x=(\varphi - \varphi_0)$, $y=(\lambda - \lambda_0)$

where φ_0 and λ_0 are the arithmetic averages of the latitudes and longitudes of the data set. For modeling the geoid of this area, a 1st ,2nd and 3rd degrees polynomials were used as a trend functions (shows equations 7,8 and 9).

$$\boldsymbol{N(x,y) = a_0 + a_1 x + a_2 y} \tag{7}$$

$$\boldsymbol{N(x,y) = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2} \tag{8}$$

$$\boldsymbol{N(x,y) = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 + a_6 xy^2 + a_7 yx^2 + a_8 x^3 + a_9 y^3} \tag{9}$$

The (a_0 , a_1 , a_3,…,a_9) polynomial coefficients , determined according to Least Squares Adjustment (LSA) method.

The Study Area

Baghdad University Compass **region(The** Baghdad **government)**, which covers about (2.903225 Square Kilometers). The survey for this area were accomplished using Real Time Kinematic - Differential Global Position System RTK-DGPS, type Topcon Hiper-II. The study region located in the middle of the Iraqi country, Latitude (33º 16' 32.1") to (33º 16' 2.9") N, Longitude (44º 22' 10.1") to (44º 23' 18.5") E. A total of 34 points (bench marks) were observed using both GPS and leveling . One of the best available data is QuickBird satellite image of 0.6 spatial resolutions, shown Figure-2.

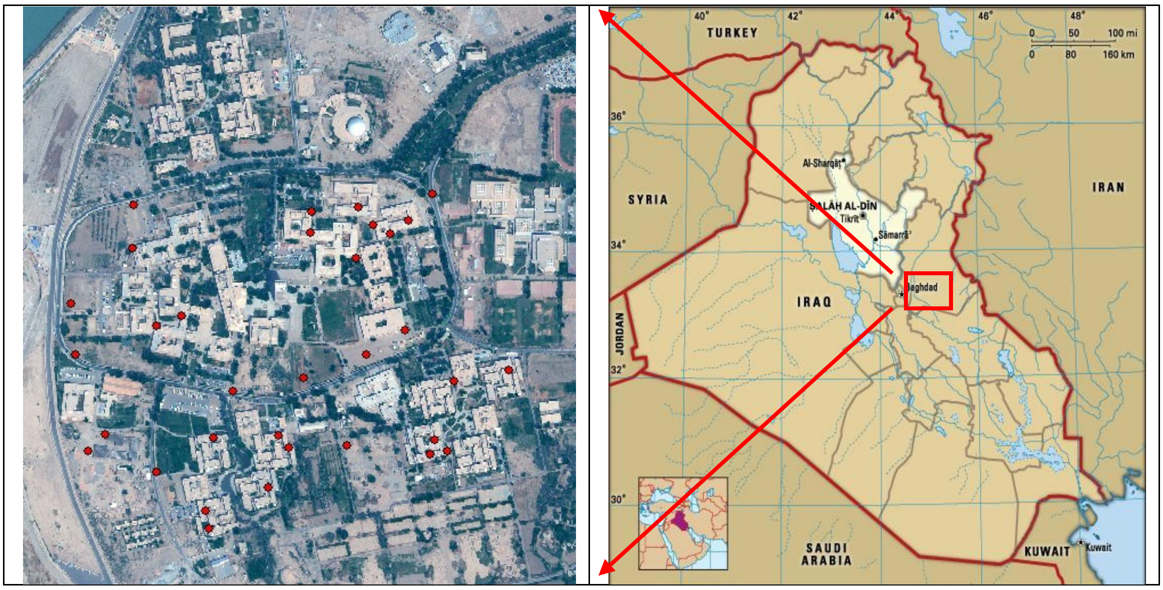

Figure 2 - Satellite image QuickBird, 0.6m sensors show study area (Baghdad University)

Data Acquisition

The Easting and Northing of 34 points located on the studied area have been navigated using GPS survey, with the Universal Transverse Mercator UTM projection and world geodetic system 1984 (WGS-84).A total of points (bench marks) were observed using both GPS and leveling, Figure-2 shows the distribution of these GPS/leveling . The survey for this study were accomplished using Differential Global Position System (DGPS), type Topcon Hiper-II.

Results and Discussion

These methods is based on the direct determination of the orthometric (H) and ellipsoidal (h) heights , for the purpose of calculating the undulation height values N of the geoid of these points. And also determine the geoid height using of the Earth Gravitational Model 2008 (EGM2008) and EGM96 using matlab .The Table-1 Illustrate the results for all modeling mathematical using in this research.

Table 1 - Differences between Geoid Heights by each modeling 1st ,2nd , 3rd and EGM (2008,96)

Point ID	Easting (m) meter	Northing meter	N observer (h-H)meter	N 1st order meter	N2nd order meter	N3rd order meter	N(EGM2008) meter	N(EGM96) meter
1	442096.461	3681963.586	-1.57	-1.5011	-1.4934	-1.5061	-1.6	-2.23
2	442201.863	3681901.42	-1.53	-1.5001	-1.5056	-1.5235	-1.61	-2.23
3	442098.61	3682011.521	-1.53	-1.5016	-1.4904	-1.4959	-1.6	-2.23
4	442316.348	3681728.922	-1.51	-1.4979	-1.5097	-1.5277	-1.62	-2.24
5	442241.894	3681980.964	-1.56	-1.5008	-1.5084	-1.5139	-1.61	-2.23
6	442207.086	3682021.943	-1.48	-1.5013	-1.5026	-1.5011	-1.61	-2.23
7	441798.015	3681763.733	-1.43	-1.5002	-1.4951	-1.4931	-1.61	-2.23
8	442022.106	3681478.486	-1.51	-1.4965	-1.4915	-1.4787	-1.61	-2.23
9	441620.367	3681479.598	-1.57	-1.498	-1.5256	-1.5235	-1.6	-2.22
10	441872.341	3681473.405	-1.53	-1.497	-1.5011	-1.4851	-1.61	-2.24
11	441853.324	3681299.472	-1.55	-1.4953	-1.4954	-1.5044	-1.62	-2.25
12	441997.676	3681353.789	-1.44	-1.4953	-1.4838	-1.4758	-1.6	-2.22
13	442374.584	3681433.055	-1.52	-1.4947	-1.4756	-1.4731	-1.61	-2.23
14	442384.395	3681468.698	-1.46	-1.495	-1.4816	-1.4808	-1.61	-2.23
15	442413.724	3681441.436	-1.44	-1.495	-1.4769	-1.4747	-1.6242	-2.2541
16	441861.491	3681256.961	-1.48	-1.4948	-1.4912	-1.5126	-1.6115	-2.2412
17	442430.342	3681607.564	-1.59	-1.4963	-1.5037	-1.5124	-1.6213	-2.251
18	442044.984	3681448.964	-1.53	-1.4961	-1.4883	-1.4758	-1.6131	-2.2427
19	441738.001	3681740.837	-1.51	-1.5002	-1.4982	-1.4927	-1.5979	-2.227
20	441682.67	3681926.304	-1.41	-1.5023	-1.4745	-1.4773	-1.5925	-2.2214
21	442324.741	3681992.562	-1.57	-1.5005	-1.5191	-1.5143	-1.6102	-2.2395
22	442280.752	3681959.276	-1.48	-1.5004	-1.5135	-1.5206	-1.6096	-2.2389
23	442374.584	3681433.055	-1.46	-1.4947	-1.4756	-1.4731	-1.6232	-2.253
24	442555.524	3681633.383	-1.52	-1.4961	-1.5155	-1.5171	-1.6244	-2.2542
25	441685.92	3682028.417	-1.45	-1.5033	-1.4564	-1.4551	-1.5905	-2.2193
26	441540.399	3681792.937	-1.57	-1.5015	-1.4983	-1.4988	-1.5909	-2.2199
27	441550.983	3681671.229	-1.52	-1.5002	-1.5153	-1.5115	-1.5937	-2.2228
28	441580.087	3681441.041	-1.44	-1.4977	-1.5326	-1.543	-1.5993	-2.2286
29	441738.837	3681390.77	-1.56	-1.4966	-1.5133	-1.5118	-1.6051	-2.2346
30	441916.108	3681583.916	-1.42	-1.4979	-1.4997	-1.4877	-1.6065	-2.2358
31	442080.15	3681615.666	-1.53	-1.4976	-1.4966	-1.4962	-1.6107	-2.2402
32	442225.671	3681672.508	-1.41	-1.4976	-1.5016	-1.5141	-1.6139	-2.2434
33	442180.692	3681454.27	-1.43	-1.4956	-1.4826	-1.4756	-1.617	-2.2467
34	442379.13	3682054.875	-1.43	-1.5009	-1.5269	-1.4924	-1.6105	-2.2398

For all measuring modeling described in this paper, the average, min.,max., and root mean square error were calculated. The results are shown in Table-2.

Table 2 - Statistical analysis for all modeling

Geoid model	Min. (m)	Max. (m)	Average (m)	RMSE (m)
N observed	-1.59	-1.41	-1.4982	0.0682
N (1st order)	-1.5033	-1.4946	-1.4982	0.0028
N (2nd order)	-1.5326	-1.4564	-1.4982	0.0287
N (3rd order)	-1.543	-1.455	-1.4982	0.0085
N (EGM 2008)	-1.6244	-1.5905	-1.6081	0.0024
N (EGM96)	-2.2542	-2.2193	-2.2351	0.0047

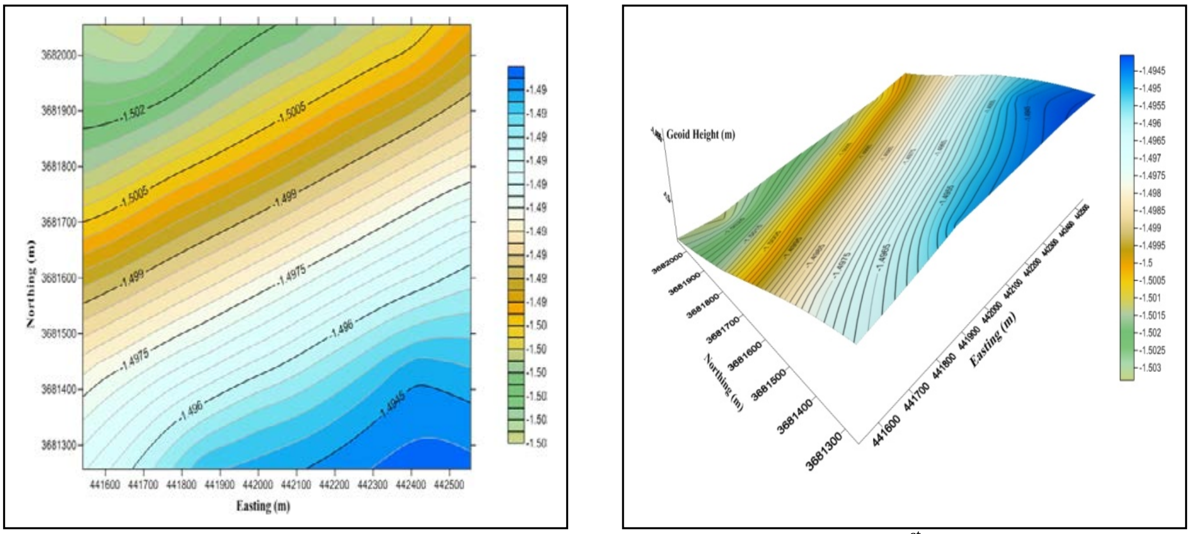

Figure 3- Modelling Surface and Digital Terrain Model using 1st order

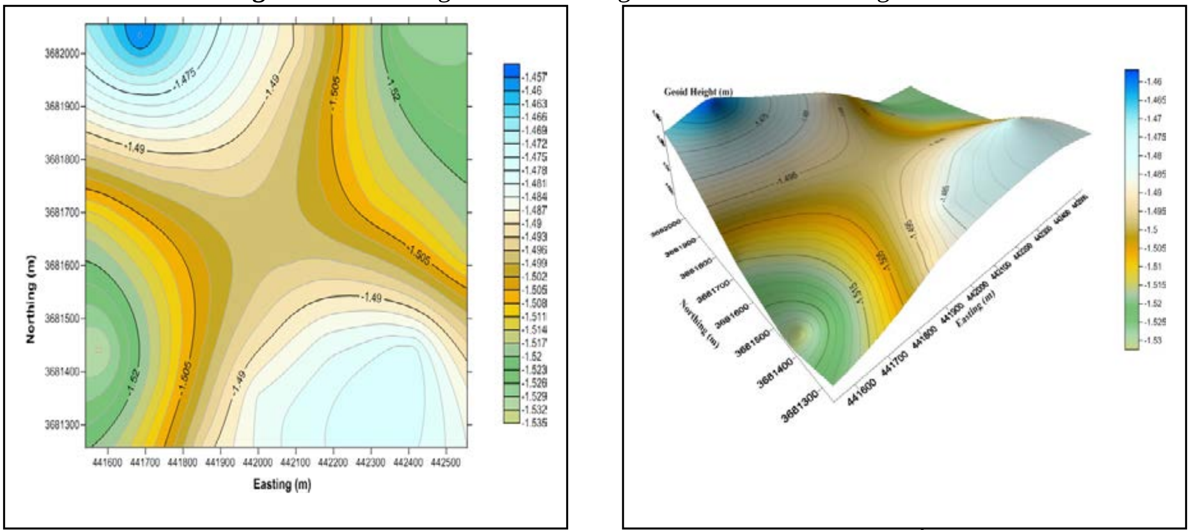

Figure 4- Modelling Surface and Digital Terrain Model using 2nd order

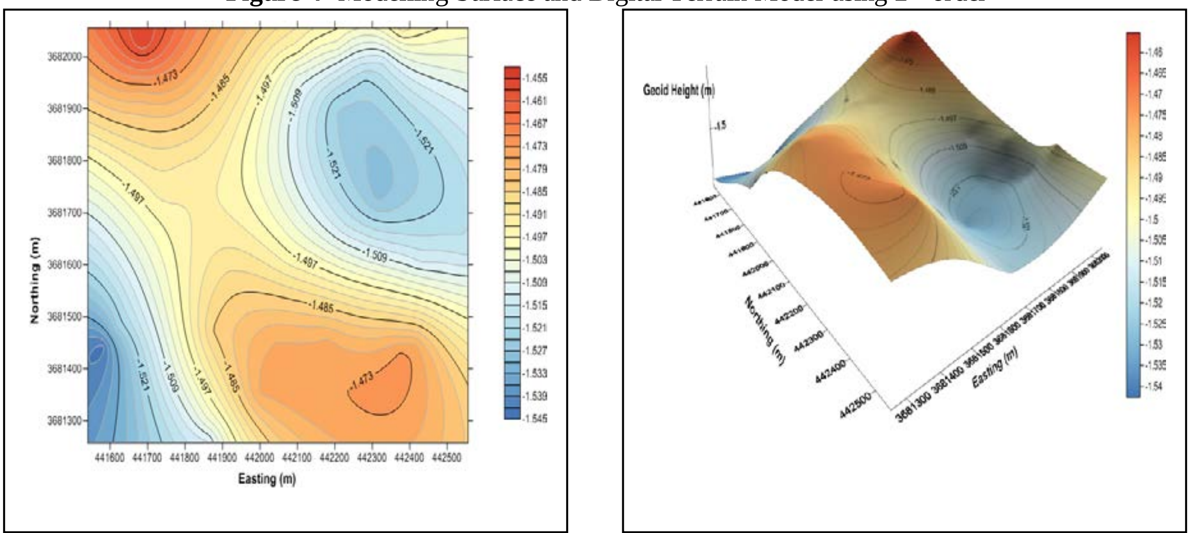

Figure 5- Modelling Surface and Digital Terrain Model using 3rd order

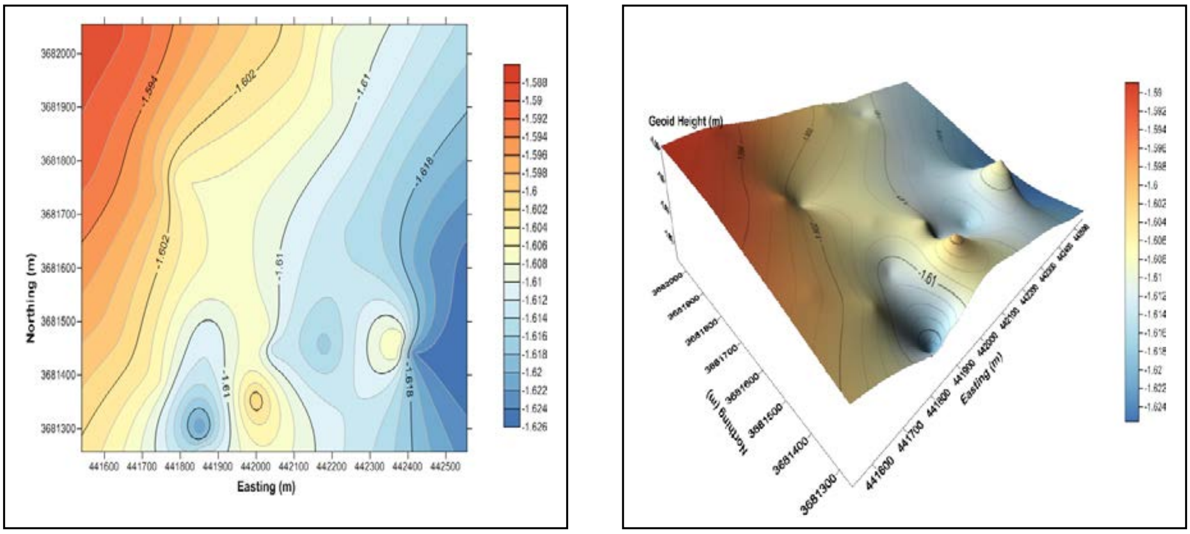

Figure 6- Modelling Surface and Digital Terrain Model using EGM 2008

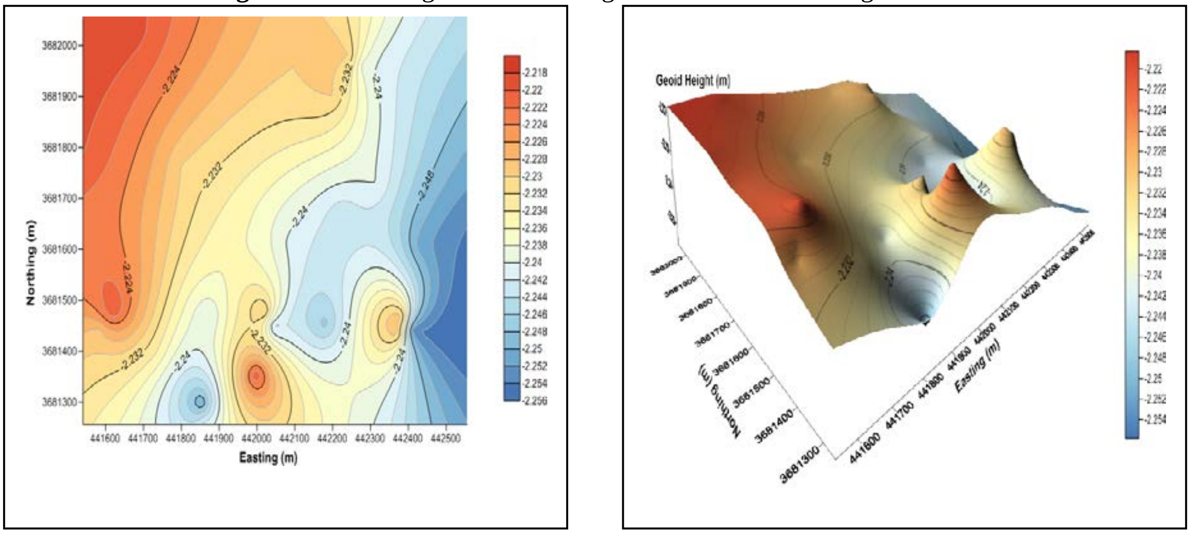

Figure 7- Modelling Surface and Digital Terrain Model using EGM96

Conclusions:

Despite the fact that the GPS heighting is fairly muddled methodology since it is gathers a few strategies for situating and gravity field determination GPS, leveling and geoid modeling Geoid modeling utilizing geometrical interpolation procedure has been discussed; the likelihood of utilizing a lower request polynomial for modeling geoid. This is using high degree polynomial as depending on well distributed and sufficient number of GPS/Leveling points that are called as benchmarks and by the way expressing the surface of geoid as an analytical surface is one of the ways of modeling. This polynomial that means to make it more accurate with some techniques is possible. Applying weighted corrections from the reference points to the interpolation points or according to the least squares collocation method can be considered in to these techniques.

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