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Coefficients Estimates of New Subclasses for Univalent Functions Related to Complex Order

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Abstract

We introduce the class of analytic and univalent functions of the form $f(z)=z+\sum_{j=2}^{\infty}a_j z^j$ for $z\in\triangle:=\{z:|z|<1\}$ that satisfies $1+\frac{1}{\mu}\{(S_\lambda^\tau f(z))'-1\}\prec\frac{(1+Lz)}{(1+Mz)}$, $z\in\triangle$, such that L and M are fixed numbers where satisfy $-1\leq M<L\leq 1$, $\tau\geq 0,\lambda>-1$ and μ is arbitrary complex number. We denote this subclass by $Q^\mu(\tau,\lambda,L,M)$. In this paper, we determine coefficients estimates, distortion theorem and maximization theorem for the new subclass $Q^\mu(\tau,\lambda,L,M)$.

Keywords: Analytic function, Univalent, Sharp coefficient estimates, Distortion theorem, Maximization theorem.

1. Introduction

Suppose p be the class of functions of the form

$$p(z)=\sum_{j=1}^{\infty}c_j z^j\,, \tag{1.1}$$

which are analytic in the unit disk $\triangle=\{z:|z|<1\}$ and satisfying the conditions $p(0)=0$ and $|p(z)|<1$.

Our aim is to introduce new subclasses of univalent and analytic functions and study their coefficients estimates and other geometric properties such as, distortion theorems and maximization theorem.

Now, let the new subclass $Q^\mu(\tau,\lambda,L,M)$ of functions of the form

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$$f(z) = z + \sum_{j=2}^{\infty} a_j \, z^j, \tag{1.2}$$

which is analytic and univalent in △ and satisfying the condition

$$1 + \frac{1}{\mu}\{(S_\lambda^\tau f(z))' - 1\} \prec \frac{(1+Lz)}{(1+Mz)}, \ z \in \triangle \tag{1.3}$$

where L and M are fixed numbers such that, $-1 \le M < L \le 1$ and $\mu \neq 0$ is arbitrary complex number or, equivalently (1.3) can be rewritten as

$$\left| \frac{(S_\lambda^\tau f(z))' - 1}{\mu(L - M) - M\left\{\left(S_\lambda^\tau f(z)\right)' - 1\right\}} \right| < 1, \qquad z \in \triangle, \tag{1.4}$$

where S_λ^τ is the generalized Jung-Kim-Srivastava integral operator [6] defined by

$$S_\lambda^\tau f(z) = \frac{\Gamma(\lambda+\tau+1)}{z^\lambda \, \Gamma(\tau)\Gamma(\lambda+1)} \int_0^z t^{\lambda-1}\left(1 - \frac{t}{z}\right)^{\tau-1} f(t)dt,$$

$$= z + \frac{\Gamma(\lambda+\tau+1)}{\Gamma(\lambda+1)} \sum_{j=2}^{\infty} \frac{\Gamma(\lambda+j)}{\Gamma(\lambda+\tau+j)} \, a_j \, z^j \, , \tag{1.5}$$

for $\tau \ge 0, \lambda > -1$, we observe that for $\tau = 0$, we have $S_\lambda^0 f(z) = f(z)$.
By giving specific values to μ, τ, L and M in (1.4), we obtain some subclasses that are previously studied by several authors as follows;

(i) for $\tau = 0$, we obtain the subclass of functions $f(z)$ satisfying the condition

$$\left| \frac{f'(z) - 1}{\mu(L - M) - M\{f'(z) - 1\}} \right| < 1, \quad z \in \triangle$$

studied by Dixit and Pal in [3].

(ii) for $\mu = e^{-i\eta}\cos\eta$ and $\tau = 0$, we obtain the subclass of functions $f(z)$ satisfying the condition

$$\left| \frac{e^{i\eta}\{f'(z) - 1\}}{Me^{i\eta} f'(z) - (L\cos\eta + iM\sin\eta)} \right| < 1, \quad z \in \triangle$$

studied by Dashrath in [2].

(iii) for $\mu = 1, L = \rho, M = -\rho$ and $\tau = 0$, we obtain the subclass of functions $f(z)$ satisfying the condition

$$\left| \frac{f'(z) - 1}{f'(z) + 1} \right| < \rho, \quad z \in \triangle,$$

where $0 \le \rho < 1$, studied by Caplinger and Causey in [1] and Padmanabban in [8].

(iv) for $\mu = 1, L = (1 - 2\rho)\theta, M = -\theta$ and $\tau = 0$, we obtain the subclass of functions $f(z)$ satisfying the condition

$$\left| \frac{f'(z) - 1}{f'(z) + 1 - 2\rho} \right| < \theta, \quad z \in \triangle,$$

where $0 \le \rho < 1, 0 < \theta \le 1$, studied by Juneja and Mogra in [5].

(v) For $\mu = 1$ and $\tau = 0$, we obtain the subclass of functions $f(z)$ satisfying the condition

$$\left|\frac{f'(z)-1}{Mf'(z)-\mathrm{L}}\right|<1, \quad \mathrm{z} \in \triangle$$

studied by Goel and Mehrok in [4].
We refer the interested readers to [5,9,11] concerning study of certain subclasses of analytic functions.
We state the following lemma that needed in our results.

Lemma 1.1 *Keogh and Merkes* [7]
Let $p(z)=\sum_{j=1}^{\infty} c_j z^j$ be analytic with $|\mathrm{p(z)}|<1$ in $\triangle$. Then for any complex number *t*

$$|c_2-tc_1^2| \leq max(1,|t|).$$

The equality is obtained with the functions $p(z)=z^2$ and $\mathrm{p(z)=z}$.

2. Coefficient Estimates

Theorem 2.1 Let the function $f(z)$ given with *(1.2)* be in the class $Q^{\mu}(\tau,\lambda,L,M)$, then

$$\left|a_j\right| \leq \frac{(\Gamma(\lambda+1)\Gamma(\tau+\lambda+j))}{j(\Gamma(\tau+\lambda+1)\Gamma(\lambda+j))}(L-M)|\mu|,$$

the estimates are sharp.
Proof. Let $f \in Q^{\mu}(\tau,\lambda,L,M)$, then for an analytic function $p(z)$ with $p(0)=0$ and $|p(z)|<1$ for all $z \in \triangle$, we have

$$1+\frac{1}{\mu}\{(S_{\lambda}^{\tau}f(z))'-1\}=\frac{1+Lp(z)}{1+Mp(z)}, \ z \in \triangle, \tag{2.1}$$

from (2.1) we obtain

$$\left[(M-L)+\left(\frac{M}{\mu}\right)\sum_{j=2}^{\infty}\frac{j\Gamma(\tau+\lambda+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\tau+\lambda+j)}a_j z^{j-1}\right]p(z)=-\frac{1}{\mu}\sum_{j=2}^{\infty}\frac{j\Gamma(\tau+\lambda+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\tau+\lambda+j)}a_j z^{j-1},$$

which can be written as

$$\left[(L-M)-\left(\frac{M}{\mu}\right)\sum_{j=2}^{\infty}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}\right]\left[\sum_{j=1}^{\infty}c_j z^j\right]$$
$$=\frac{1}{\mu}\sum_{j=2}^{\infty}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}. \tag{2.2}$$

By equating coefficients in (2.2). Then for $j \geq 2$, (2.2) can be expressed as

$$\left[(L-M)-(M/\mu)\sum_{j=2}^{n-1}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}\right][p(z)]$$
$$=\frac{1}{\mu}\sum_{j=2}^{n}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}+\sum_{j=n+1}^{\infty} w_j z^{j-1}, \tag{2.3}$$

this leads to

$$\left|(L-M)-(M/\mu)\sum_{j=2}^{n-1}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}\right|$$
$$\geq\left|\frac{1}{\mu}\sum_{j=2}^{n}\frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_j z^{j-1}+\sum_{j=n+1}^{\infty} w_j z^{j-1}\right|, \tag{2.4}$$

squaring both sides of (2.4) and integrating round $|z|=r$ for $0<r<1$, we have

$$(L-M)^2+\frac{M^2}{|\mu|^2}\sum_{j=2}^{n-1}\frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2 r^{2j-2}$$
$$\geq\frac{1}{|\mu|^2}\sum_{j=2}^{n}\frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2 r^{2j-2}+\sum_{j=n+1}^{\infty}|w_j|^2 r^{2j-2},$$

we assume $r \to 1$, then we obtain

$$(L-M)^2+\frac{M^2}{|\mu|^2}\sum_{j=2}^{n-1}\frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2$$

$$\geq \frac{1}{|\mu|^2}\sum_{j=2}^{n} \frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2$$

or

$$(1-M)^2\sum_{j=2}^{n-1} \frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2 + \frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2 \leq (L-M)^2|\mu|^2, \qquad (2.5)$$

since $-1 \leq M < 1$, then (2.5) can be expressed as;

$$\frac{j^2(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))^2}{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))^2}|a_j|^2 \leq (L-M)^2|\mu|^2,$$

this leads to

$$|a_j| \leq \frac{(\Gamma(\lambda+1)\Gamma(\lambda+\tau+j))}{j(\Gamma(\lambda+\tau+1)\Gamma(\lambda+j))}(L-M)|\mu|, \qquad j = 2,3,\ldots \ .$$

The sharpness of the theorem “the equality of coefficients $|a_j|$" is obtained for the function

$$f(z) = \int_0^z \left\{1 + \frac{\frac{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}{\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}(L-M)\mu\, z^{j-1}}{1 + M\, z^{j-1}}\right\}dz$$

$$= \quad z + \frac{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}{\mathrm{j}\,\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}\,(L-M)\mu\, z^j + \cdots \qquad (2.6)$$

with $j \geq 2$ and $z \in\triangle$.

Theorem 2.2 *Suppose the analytic function $f(z)$ given as in (1.2). Then $f \in Q^{\mu}(\tau,\lambda,L,M)$ if*

$$\sum_{j=2}^{\infty} \frac{\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}(1+|M|)j|a_j| \leq (L-M)|\mu|, \qquad (2.7)$$

where $-1 \leq M < L \leq 1$. The result is sharp.

Proof. Let (2.7) is given, then we obtain for $|z| < 1$

$$|S_\lambda^\tau f(z))' - 1| - |\mu(L-M) - M\{(S_\lambda^\tau f(z))' - 1\}|$$

$$= \left|\sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)} a_j z^{j-1}\right| - \left|\mu(L-M) - \ M\sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)} a_j z^{j-1}\right|$$

$$\leq \sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}|a_j| r^{j-1} - |\mu|(L-M)$$

$$+ |M|\sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}|a_j| r^{j-1},$$

$$< \sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}|a_j| - |\mu|(L-M) + |M|\sum_{j=2}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}|a_j|,$$

$$= \sum_{n=j}^{\infty} \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}(1+|M|)|a_j| - (L-M)|\mu| \leq 0, \quad by \quad (2.7).$$

Then we have

$$\left|\frac{S_\lambda^\tau f(z))' - 1}{\mu(L-M) - M\{(S_\lambda^\tau f(z))' - 1\}}\right| < 1, z \in\triangle.$$

Then we deduced that $f \in Q^{\mu}(\tau,\lambda,L,M)$.

The sharpness is obtained for the function

$$f(z) = z + \frac{(L-M)\mu\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}{j(1+|M|)\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)} z^j, \ j \geq 2 \ \ and \ \ z \in\triangle.$$

3. Distortion Theorem

Theorem 3.1 *If $f(z)$ given as in (1.2) be in the class $Q^{\mu}(\tau,\lambda,L,M)$, then*

$$Re\{(S_\lambda^\tau f(z))'\} \geq \frac{1-LMr^2Re(\mu)-M^2r^2Re(1-\mu)-(L-M)|\mu|r}{1-M^2r^2}$$

and

$$Re\{(S_\lambda^\tau f(z))'\} \leq \frac{1-LMr^2Re(\mu)-M^2r^2Re(1-\mu)+(L-M)|\mu|r}{1-M^2r^2}.$$

The sharpness is obtained.

Proof. As $f \in Q^\mu(\tau,\lambda,L,M)$, we obtain for an analytic function $p(z)$ with $|p(z)| < 1$ and $|z| < 1$

$$1+\frac{1}{\mu}\{(S_\lambda^\tau f(z))'-1\} = \frac{1+Lp(z)}{1+Mp(z)}. \tag{3.1}$$

by setting

$$v(z) = \frac{1+Lp(z)}{1+Mp(z)}$$

which transforms the circle $|p(z)| \leq r < 1$ to the circle

$$\left|v(z)-\frac{1-LMr^2}{1-M^2r^2}\right| \leq \frac{(L-M)r}{1-M^2r^2}, \tag{3.2}$$

then (3.1) became

$$1+\frac{1}{\mu}\{(S_\lambda^\tau f(z))'-1\} = v(z), \tag{3.3}$$

then by equating (3.3) and (3.2) we have

$$|(S_\lambda^\tau f(z))'-\frac{1-LM\mu r^2+M^2r^2(\mu-1)}{1-M^2r^2}| \leq \frac{(L-M)|\mu|r}{1-M^2r^2}.$$

Then

$$Re\{(S_\lambda^\tau f(z))'\} \geq \frac{1-LMr^2Re(\mu)-M^2r^2Re(1-\mu)-(L-M)|\mu|r}{1-M^2r^2}$$

and

$$Re\{(S_\lambda^\tau f(z))'\} \leq \frac{1-LMr^2Re(\mu)-M^2r^2Re(1-\mu)+(L-M)|\mu|r}{1-M^2r^2}.$$

The function

$$S_\lambda^\tau f(z) = \frac{M+(L-M)\mu}{M}z-\frac{(L-M)\mu}{M^2e^{i\theta}}\log(1+Mze^{i\theta})$$

where

$$e^{i\theta} = \frac{|\mu|-Mz\mu}{\mu-Mz|\mu|}.$$

satisfies the sharpness of this theorem.

Theorem 3.2 *Let $f(z)$ given as in (1.2) be in the class $Q^\mu(\tau,\lambda,L,M)$, then for any complex number σ*

$$|a_3-\sigma a_2^2| \leq \frac{|\mu|(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\lambda+1)(\lambda+2)}\max\left\{1,\frac{|4M(\lambda+1)(\lambda+\tau+2)+3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)|}{4(\lambda+1)(\lambda+\tau+2)}\right\}. \tag{3.4}$$

The sharpness is obtained.

Proof. Let $f \in Q^\mu(\tau,\lambda,L,M)$, then we obtain

$$1+\frac{1}{\mu}\{(S_\lambda^\tau f(z))'-1\} = \frac{1+Lp(z)}{1+Mp(z)} \tag{3.5}$$

for an analytic function $p(z)$ given with $p(z) = \sum_{j=1}^\infty c_jz^j$ and defined with $p(0) = 0$, $|p(z)| < 1$ for $z \in \triangle$. Hence from (3.5) we obtain

$$\begin{aligned} p(z) &= \frac{(S_\lambda^\tau f(z))'-1}{\mu(L-M)-M\{(S_\lambda^\tau f(z))'-1\}} \\ &= \frac{\sum_{j=2}^\infty \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_jz^{j-1}}{\mu(L-M)-M\sum_{j=2}^\infty \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_jz^{j-1}} \\ &= \frac{\sum_{j=2}^\infty \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_jz^{j-1}}{\mu(L-M)}\left[1+\frac{M}{\mu(L-M)}\sum_{j=2}^\infty \frac{j\Gamma(\lambda+\tau+1)\Gamma(\lambda+j)}{\Gamma(\lambda+1)\Gamma(\lambda+\tau+j)}a_jz^{j-1}+\ldots\right] \end{aligned}$$

by equating coefficients of z and z^2 on both sides, we obtain

$$c_1 = \frac{2(\lambda+1)}{\mu(L-M)(\lambda+\tau+1)}a_2,$$

$$c_2 = \frac{3(\lambda+1)(\lambda+2)}{\mu(L-M)(\lambda+\tau+1)(\lambda+\tau+2)} a_3 + \frac{4M(\lambda+1)^2}{\mu^2(L-M)^2(\lambda+\tau+1)^2} a_2^2.$$

Hence

$$a_2 = \frac{\mu(L-M)(\lambda+\tau+1)}{2(\lambda+1)} c_1$$

and

$$a_3 = \frac{\mu(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\lambda+1)(\lambda+2)} [c_2 - Mc_1^2].$$

Then

$$a_3 - \sigma a_2^2 = \frac{\mu(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\lambda+1)(\lambda+2)} [c_2 - Mc_1^2] - \frac{\mu^2(L-M)^2(\lambda+\tau+1)^2}{4(\lambda+1)^2} \sigma c_1^2$$
$$= \frac{\mu(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\tau+1)(\tau+2)} \left[c_2 - \left\{M + \frac{3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)}{4(\lambda+1)(\lambda+\tau+2)}\right\} c_1^2\right],$$

then,

$$|a_3 - \sigma a_2^2| = \frac{|\mu|(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\lambda+1)(\lambda+2)} \left|c_2 - \left\{M + \frac{3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)}{4(\lambda+1)(\lambda+\tau+2)}\right\} c_1^2\right|. \quad (3.6)$$

Using Lemma 1.1 in (3.6) we have,

$$|a_3 - \sigma a_2^2| \leq \frac{|\mu|(L-M)(\lambda+\tau+1)(\lambda+\tau+2)}{3(\lambda+1)(\lambda+2)} \max\left\{1, \frac{|4M(\lambda+\tau+2)(\lambda+1)+3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)|}{4(\lambda+1)(\lambda+\tau+2)}\right\}.$$

which is (3.4).

We found that the sharpness is obtained for the function

$$f(z) = z + \mu(L-M) \sum_{j=2}^{\infty} \frac{(-1)^j M^{j-2}(\lambda+\tau+j-1)!\lambda!}{j(\lambda+j-1)!(\lambda+\tau)!} z^j,$$

where $\frac{|4M(\lambda+\tau+2)(\lambda+1)+3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)|}{4(\lambda+1)(\lambda+\tau+2)} > 1.$

Also, we found that the sharpness is obtained for the function

$$f(z) = \frac{(\lambda+1)(\lambda+2)+(\lambda+\tau+1)(\lambda+\tau+2)}{(\lambda+1)(\lambda+2)} z - \frac{(\lambda+\tau+1)(\lambda+\tau+2)}{(\lambda+1)(\lambda+2)} \int_0^z \frac{dt}{1+\mu(L-M)t^2},$$

where $\frac{|4M(\lambda+\tau+2)(\lambda+1)+3\sigma\mu(L-M)(\lambda+\tau+1)(\lambda+2)|}{4(\lambda+1)(\lambda+\tau+2)} < 1.$

4. Conclusion

We determined the coefficients estimates for the new subclass $Q^{\mu}(\tau,\lambda,L,M)$. Also we obtained the distortion theorem and maximization theorem for this new class. This study will aid authors later to expand other analytic properties for this class.

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