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Verification of Phase Space Inversions Based on The Initial Conditions of the Chaotic Chen System

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Abstract

Theoretically, an eight-term chaos system is presented. The effect of changing the initial conditions values on behavior Chen system was studied. The basic dynamical properties of system are analyzed like time series, attractor, FFT spectrum, and bifurcation. Where the system appears steady state behavior at initial condition x_i, y_i, z_i equal $(0, 0, 0)$ respectively and it convert to quasi-chaotic at x_i, y_i, z_i equal $(-0.1, 0.5, -0.6)$. Finally, the system become hyper chaotic at x_i, y_i, z_i equal $(-0.5, 0.5, -0.6)$ that can used it in many applications like secure communication.

Keywords: Chaos, Chen model, time series, attractor, bifurcation

التحقق من الانقلابات الفضائية الطورية بناءً على الشروط الابتدائية لنظام تشن الفوضوي

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الخلاصة

من الناحية النظرية، يتم تقديم نظام فوضوي من ثماني مقادير. تمت دراسة تأثير تغيير قيم الشروط الابتدائية على سلوك نظام تشن. يتم تحليل الخصائص الديناميكية الأساسية للنظام مثل السلاسل الزمنية والجاذب وطيف تحويلات فورييه السريع والتشعب. حيث يظهر النظام سلوكًا ثابتًا في الحالة الأولية x_i, y_i, z_i يساوي $(0, 0, 0)$ على التوالي ويتحول إلى شبه فوضوية عند x_i, y_i, z_i متساوية $(-0.1, 0.5, -0.6)$. أخيرًا، يصبح النظام شديد الفوضى عند x_i, y_i, z_i يساوي $(-0.5, 0.5, -0.6)$ ويمكن استخدامه في العديد من التطبيقات مثل الاتصال الآمن.

1. Introduction

Chaotic dynamics are non-periodic, broadband systems that are similar to noise but are governed by a set of equations and are highly sensitive to initial conditions. Therefore, these systems have helped to understand many systems that show random behavior. There are many chaotic nonlinear systems which are becoming of great and increasing importance in many fields. The first scientist to study chaos was Edward Lorenz in 1963 [1]. Then great and rapid progress was made in the science of chaos with the help of a group of researchers such as Rossler [2] and Chua et al. [3] who presented a set of models to understand more about this strange system. Chaotic order appears in many fields, including physics and mathematics, and

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continues to grow in a variety of other areas of life, such as weather forecasting and wind movement, and in the sciences of population spread, and in the biological field such as the survival of single-celled organisms, the study of the nervous system and the signals sent from the brain to the organs of the organism, and also the study of fluid movement inside the organism [4-14]. Chaotic systems and their dynamic equations can be represented through electronic circuits, and this was actually done by the scientist Chua, who built the first electronic circuit that shows chaotic behavior in the simplest representation [14]. The simple Chua circuit has been used in various fields, mostly in the field of secure communications. Subsequently, many developed electronic circuits appeared, such as RLC circuits, which show complex chaotic behavior [15-18], electronic oscillators [19], energy storage circuits [20-21], digital filters [22-23] and capacitor circuits. Recently, new chaotic systems have emerged that show a different chaotic behavior than the previous systems that have been used in many fields [24-27]. Currently, the details of the chaotic Lu-Chen model system are presented and the effect of initial conditions on the behavior of this system is studied.

2. Theory

The Chen system is introduced by Guanrong Chen, where described by three first-order nonlinear differential equations, which resembles some familiar behavior from the Rossler and Lorenz attractors. The Chen system equation as follow [16]:

$$\dot{x} = a(y - x) \quad (1)$$

$$\dot{y} = (c - a)x - xz + cy \quad (2)$$

$$\dot{z} = xy - bz \quad (3)$$

Where the typical parameters a , b , and c are real numbers that equal 40, 3 and 28 respectively at chaotic case. Likewise, for other parameter sets of $\{a, b, c\}$, system may not be chaotic. The system becomes not chaotically if this set of parameters is changed. The advantage of this system is that it has three ordinary differential equations and has two types of quadratic nonlinearities in (xy) and (xz) . Until the mathematicians did not agree on a specific formula for defining chaos, but they agreed that the chaotic system is sensitive to the initial conditions and that the presence of double-period cycles leads to the emergence of chaos. Figure 1 shows the flowchart of Chen model.

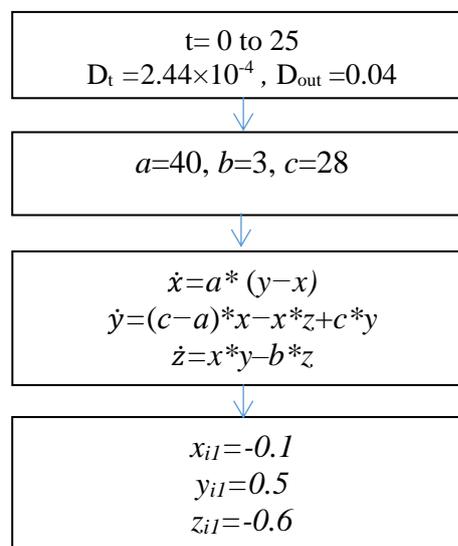


Figure 1: Flow chart of Chen model

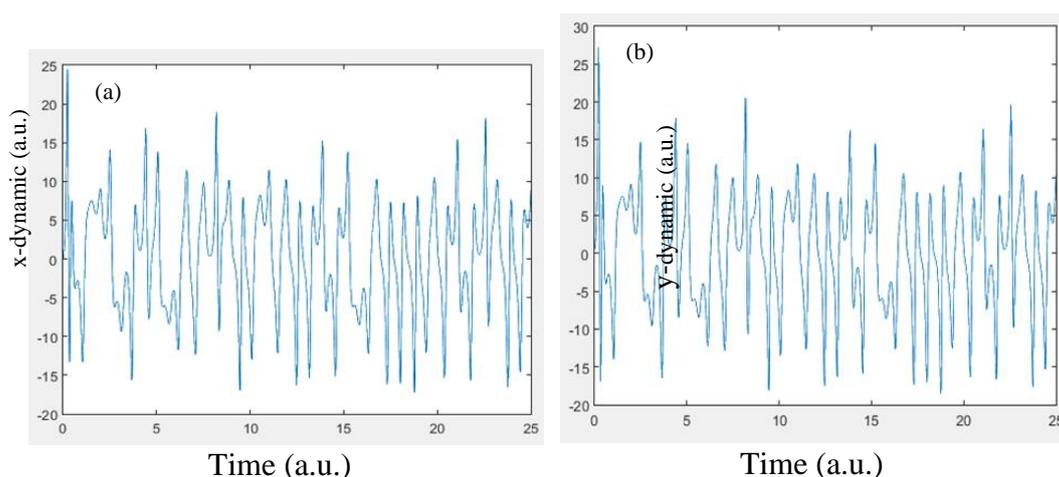
The Chen model is shown in Figure 1, where used MATLAB program to solve the three first-order nonlinear differential equations. The first step of the program is to determine the value of the time period for the chaotic system, which starts from 0 a.u to 25 a.u., and these values can be changed to any required extent, where D_t is the time interval that equal to 2.44×10^{-4} a.u., and D_{out} equal 0.04 a.u. The second step in the program is to determine the parameter values of the chaotic system. The third step represents the three nonlinear differential equations of the chaotic Chen system. The last step in the program is to determine the values of the initial conditions of the Chen system, and these values are responsible for determining the behavior of the system, whether it is chaotic, periodic, or stable (steady state). The values of these conditions were determined as follows [$x_{i1}=-0.1$, $y_{i1}=0.5$, and $z_{i1}=-0.6$]. The simulation time of total program execution for any chaotic system depends on several parameters, such as the time period D_t , the time interval D_{out} and the values of the initial conditions.

3. Results

To analyze and study the effect of changing the initial conditions (x_i , y_i , z_i) on the behavior of the chaotic system, two different conditions were taken, which are as follows: (-0.1, 0.5, -0.6) and (-0.5, 0.5, -0.6). The MATLAB program was used to solve the differential equations using Rang- Kuta 4th, where the time scale taken is 25 a.u. The code results of Chen system as follow: The time series of all dynamics for Chen system at initial condition (-0.1, 0.5, -0.6) are given in Figure 2, where the Table 1 shows the amplitude values of all dynamics (peck to peck). Through the Figure 2 notice, there is a similar in the behavior of the x and y dynamics approximately (but difference in amplitude) with a difference behavior in z dynamic.

The strange attractant for the three dynamics x, y, and z are given in Figure 3. It is shaped like a butterfly (2-scrolls attractor) with complex topological structure, So Chen system is a specific case of Lorenz [17] according to this. Figure (3-d) shows the strange attractor in three dimensions (3D).

To analysis the bandwidth of Chen system, Figure 4 shows the Fast Fourier Transformation (FFT) spectrum of it.



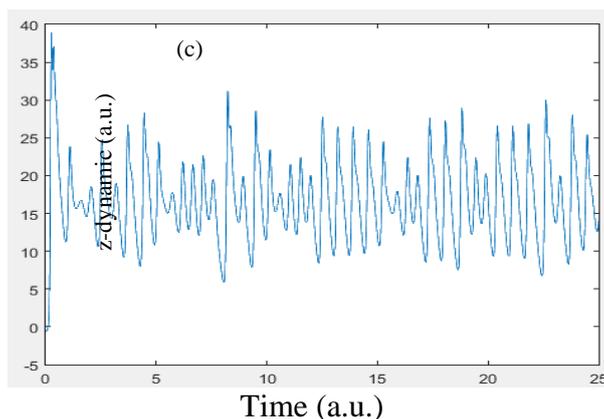


Figure 2: Time series of Chen system, where (a):x-dynamic , (b): y-dynamic, (c):z-dynamic at initial conditions values x_i , y_i , and z_i equals -0.1, 0.5, and -0.6 respectively.

Table 1: The amplitude values of time series of x, y, and z-dynamic

Dynamics	$x_{p,p}$ (a.u)	$y_{p,p}$ (a.u)	$z_{p,p}$ (a.u)
Amplitude	24.8:-17.5	27:-18	39:-0.6

Chen system is showing exponential decay behavior and broadband system and Table 2 shows the values of maximum amplitudes, frequency bandwidth, and Full width at half maximum (FWHM). The Lyapunov exponent can quantitatively reflect the chaotic performance of a system. A Hyper-chaotic system can be described as containing more than one positive Lyapunov exponent, which indicates that these systems expand in many directions and this leads to the emergence of a very complex attractor.

One of the important tools for studying chaotic behavior and studying the effect of changing system parameters is the bifurcation diagram shown in Figure 5. When changing parameter (c), we notice that the system shows different behaviors over time, starting from fixed points and ending with excessive chaos and passing through different periodic states. The system can return to the periodic state at certain values of parameter (c) at range (6.5-10) and range (20.5-21), and the system shows hyper chaotic state at range (25-29).

To verify that the behavior of the Chua system changes by changing its initial conditions, the new value of the initial conditions in x-dynamic is taken as (-0.5) as shown in Figure 6. Table 3 shows the amplitude values that occurred as a result of this changing, where noticed that when changing the initial condition of x_i , the values of amplitudes of x and y have increased compared to the first case with remain the amplitude value in z-dynamic, as well as the general behavior of the Chen system has remained the same but different in the positions of pecks in time series of three dynamics, this result is in agreement with [27-28].

By comparing between two Figures 7 and 3 of strange attractor, where it seems the Figure 7 is lower dynes (trajectories) than Figure 3, when changing the initial condition in x-dynamic. So not all initial conditions values will be investigate the hyper chaotic system

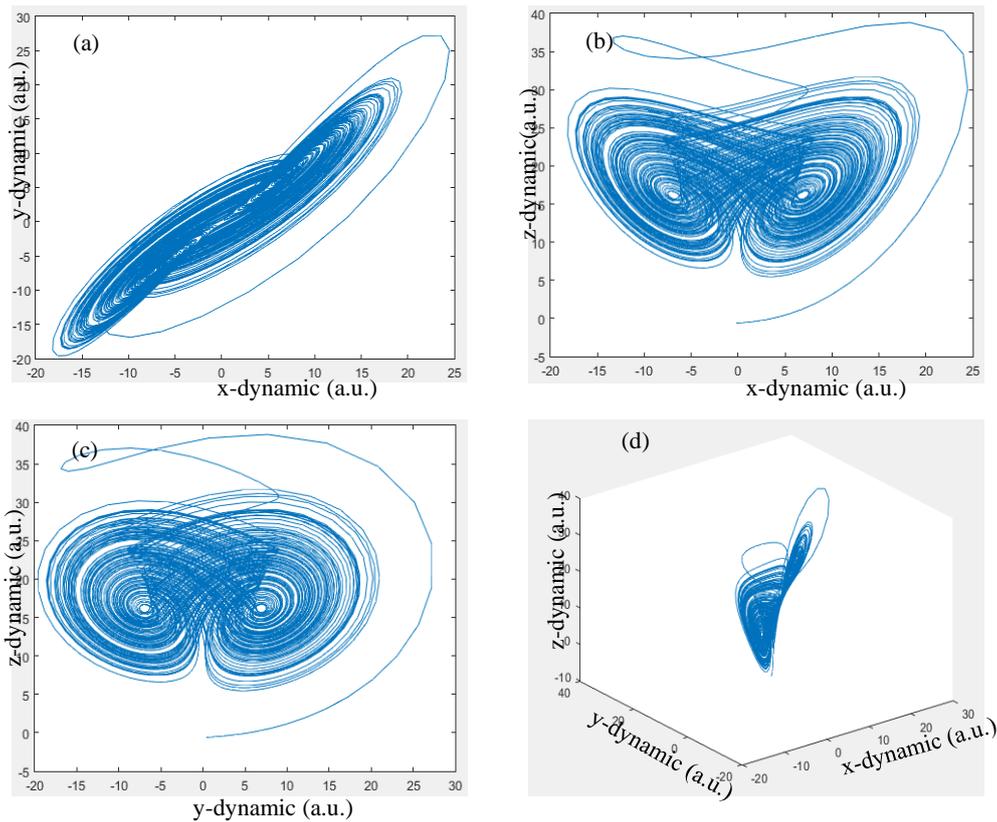


Figure 3: Strange attractor of Chen system (a):x-y plane, (b) x-z plane , (c) y-z plane and (d) 3D (x-y-z) attractor , at initial conditions values x_i , y_i , and z_i equals -0.1, 0.5, and -0.6 respectively.

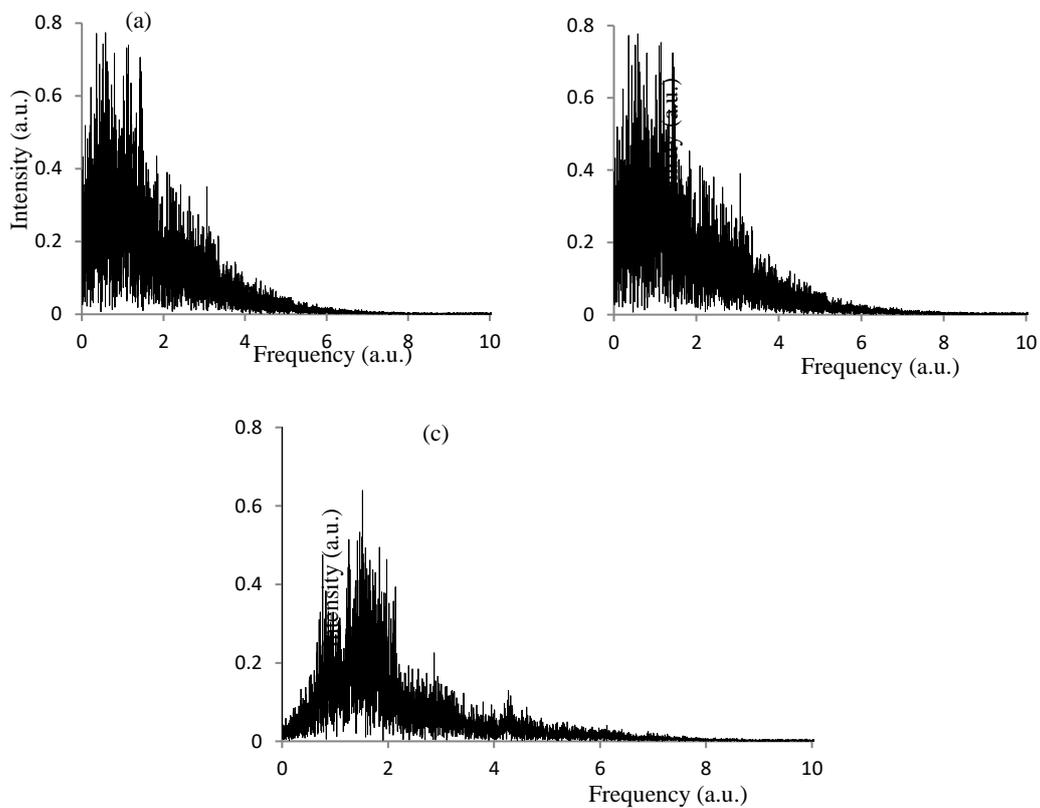
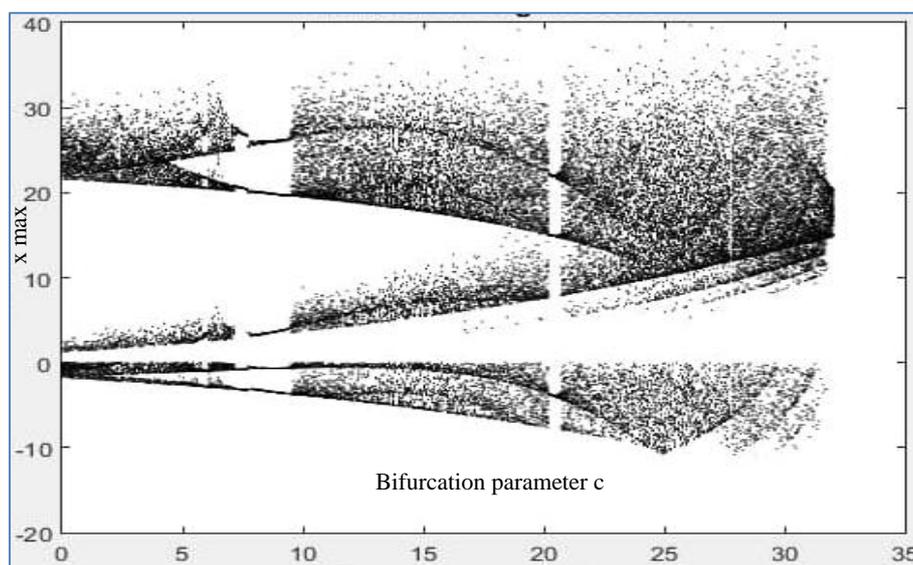


Figure 4: FFT spectrum of Chen system,(a):x-dynamic, (b):y-dynamic, and (c): z-dynamic , at initial conditions values x_i , y_i , and z_i equals -0.1, 0.5, and -0.6 respectively.

Table 2: The important values of FFT spectrum of x, y, and z-dynamic

Dynamics	x-dynamic	y-dynamic	z-dynamic
Max amplitude (a.u.)	0.8	0.8	0.8
Freq. Bandwidth (a.u.)	8	10	8
FWHM (a.u.)	1.8	1.8	1.4

**Figure 5:** Bifurcation diagram of Chen system, when changing c parameter at initial conditions values x_i , y_i , and z_i equals -0.1, 0.5, and -0.6 respectively.

Sometimes, the system is converted to quasi-chaotic that is not advantageous in our applications especially in secure communication. Indeed, this hyper chaotic (Figure 3) system has wide range of frequencies.

To certain the effect of the initial condition values for changing the system behavior Figure 8 shows fast Fourier transformation after changing the initial condition in x, so the bandwidth frequency have less than the first condition, Table 4 shows these changes.

The bifurcation diagram in Figure 9 shows less chaotic behavior in the (25-28) range, with the transition states of the system remaining unchanged. This means that this value chosen from the initial conditions does not affect the behavior and states of the system.

For comprehensive study, Figure 10 shows the time series in x-dynamic when changing a set of initial conditions as the Chen system appears a change from hyper-chaotic to steady state and Table 5 shows some changes in the initial state of the system along with its behavior. Through this study, it was noted that the selection of initial conditions is important in obtaining an excessively chaotic system, especially when applied in secure communications.

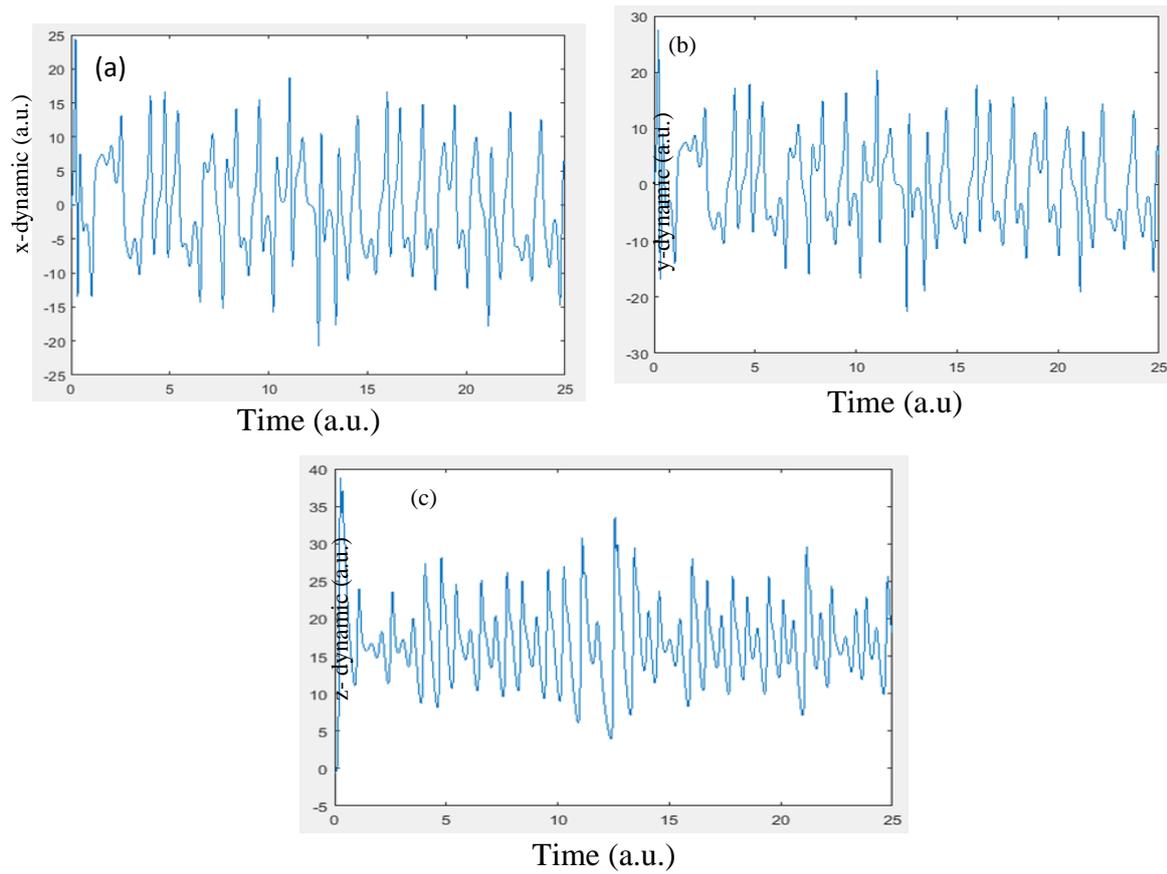
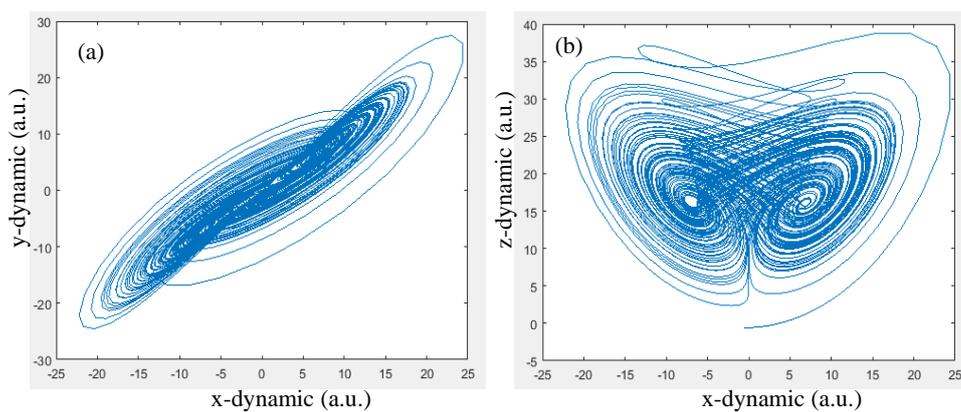


Figure 6: Time series of Chen system, where (a):x-dynamic , (b): y-dynamic, (c):z-dynamic at initial conditions values x_i , y_i , and z_i equals -0.5, 0.5, and -0.6 respectively

Table 3: The amplitude values with changing in initial condition

Dynamics	$x_{p,p}$ (a.u)	$y_{p,p}$ (a.u)	$z_{p,p}$ (a.u)
Amplitude	24.8:-21	28:-21	39:-0.6



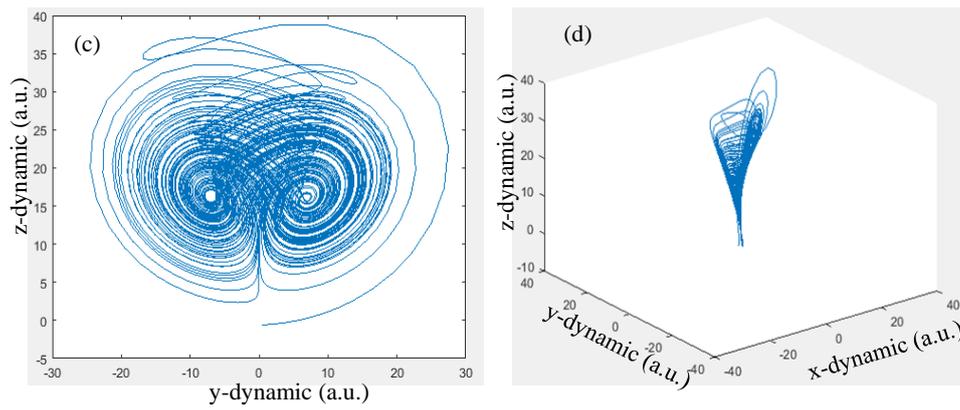


Figure 7: Strange attractor of Chen system,(a): x-y plane, (b): x-z plane, (c): y-z plane and (d): 3D (x-y-z) attractor at initial conditions values $x_i, y_i,$ and z_i equals -0.5, 0.5, and -0.6

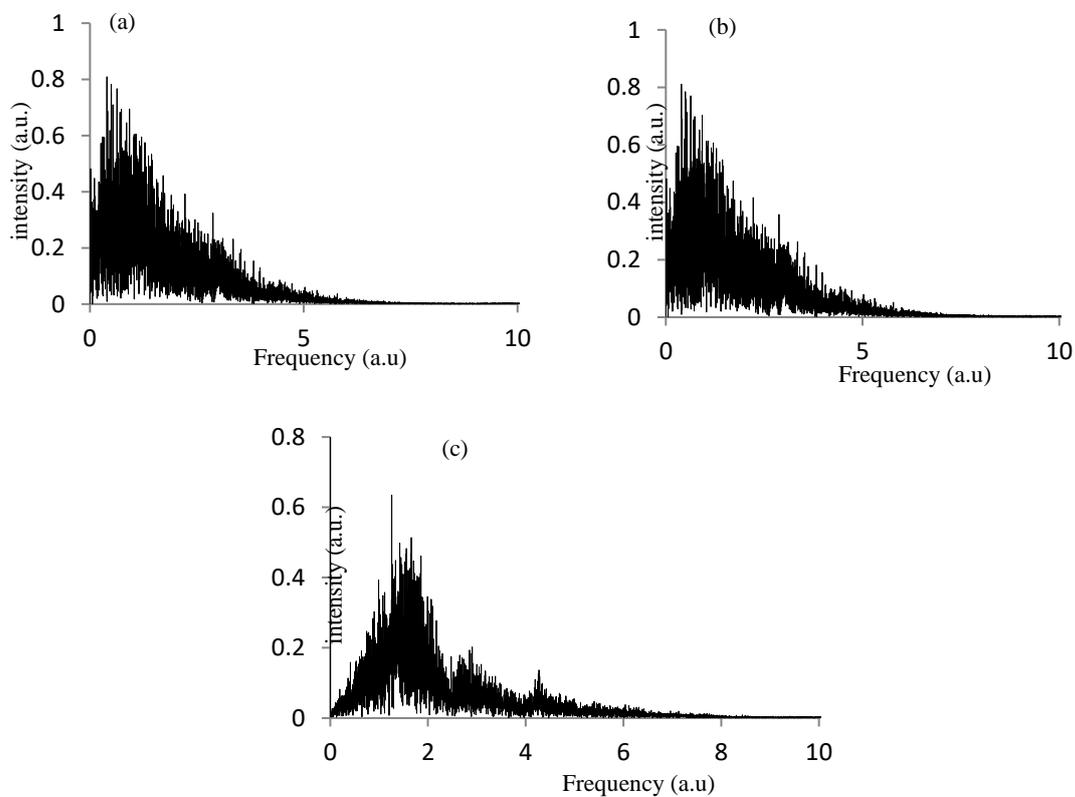


Figure 8: FFT spectra of Chen system,(a):x-dynamic , (b): y-dynamic, (c):z-dynamic at initial conditions values $x_i, y_i,$ and z_i equals -0.5, 0.5, and -0.6 respectively.

Table 4: Characteristics of the x, y, and z-dynamic FFT spectra

Dynamics	x-dynamic	y-dynamic	z-dynamic
Max amplitude	0.7	0.7	0.45
Freq. Bandwidth	7	8	8.25
FWHM	2	2	1.27

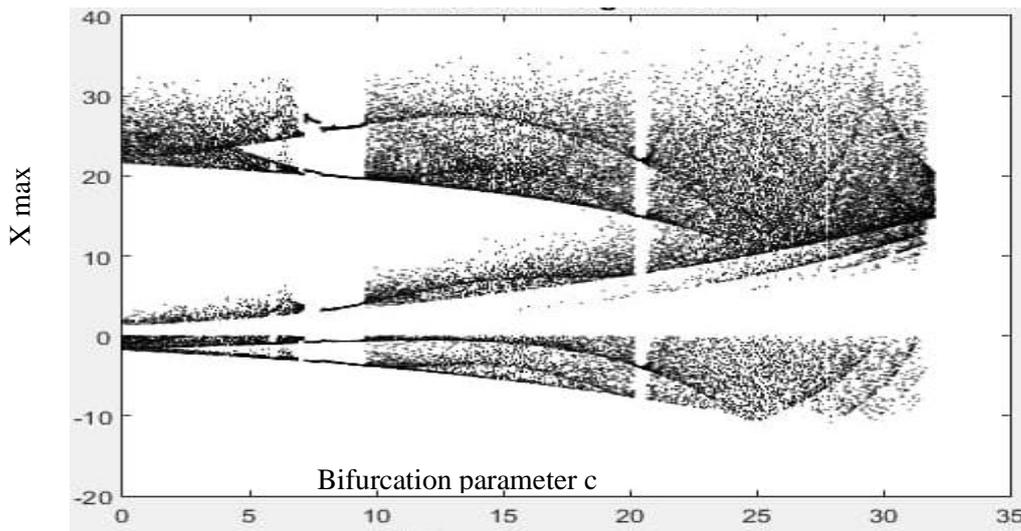


Figure 9: Bifurcation of Chen system, when changing c parameter at initial conditions values $x_i, y_i,$ and z_i equals -0.5, 0.5, and -0.6 respectively.

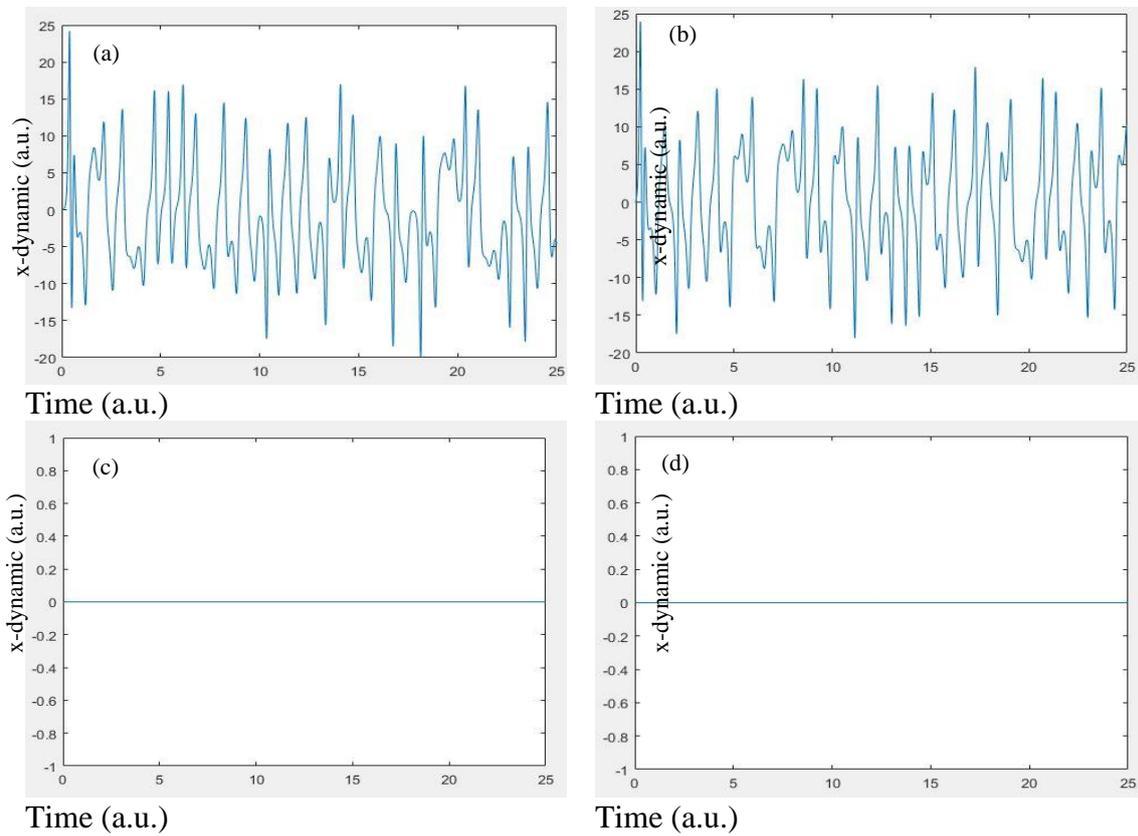


Figure 10: shows the time series in x-dynamic of observably state.

Table 5: Shows some changing in initial condition of system with its behavior

Dynamics state	x-dynamic	y-dynamic	z-dynamic	State dynamic
(a)	-0.1	0	-0.6	Chaotic
(b)	-0.1	0.5	0	Chaotic
(c)	0	0	-0.6	Steady state
(d)	0	0	0	Steady state

4. Conclusions

In this study, the Chen system showed a sensitive dependence on specific values of the initial conditions and not all the values, as this specific value made the behavior system very chaotic (increasing complexity) of the Chen system, which will help us choose suitable values without the other to be used in many applications, including in secure communications.

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