



# On Intuitionistic Fuzzy bi- Ideal With respect To an Element of a near Ring

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#### Abstract

In this paper we introduce the notions of bi-ideal with respect to an element r denoted by (r-bi- ideal ) of a near ring, and the notion fuzzy bi- ideal with respect to an element of a near ring and the relation between F-r-bi-ideal and r-bi-ideal of the near ring, we studied the image and inverse image of r-bi- ideal under epimomorphism, the intersection of r-bi- ideals and the relation between this ideal and the quasi ideal of a near ring, also we studied the notion intuitionistic fuzzy bi-ideal with respect to an element r of the near ring N, and give some theorem about this ideal .

**Keywords:** Near ring, ideal of a near ring ,bi- ideal, quasi- ideal, P-regular of near ring , intuitionistic fuzzy, intuitionistic fuzzy bi-ideal.

# الحدس الضبابي للمثالية bi بالنسبة لعنصر ما في الحلقة القريبة

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الخلاصة

قدمنا في هذا البحث بعض المفاهيم ومنها المثالية bi-ideal بالنسبة لعنصر ما في الحلقة القريبة N والتي يرمز لها بالرمز (r-bi-ideal)، كما ودرسنا الصور المباشرة ومعكوس الصورة للمثالية تحت التشاكل الشامل كما وأعطينا بعض الخواص التي تتعلق بهذه المثالية وعلاقتها ببعض المثاليات المعروفة وأيضا تطرقنا إلى مفهوم جديد وهو المثالية الضبابية bi-ideal بالنسبة لعنصر ما في الحلقة القريبة N وتم إعطاء يعض المبرهنات والنتائج الخاصة بها .وكما قدمنا مفهوم ideal فيها منا المترافي المتاليات المعروفية بالنسبة لعنصر ما في الحلقة القريبة مع إعطاء بعض المبرهنات والامثلة عليها .

#### Introduction

The notion of near ring is first define by G. Pliz [1] in 1983, the notion of bi- ideal interfused by N.Ganesan [2], in 1986 the notion intutionistis fuzzy sets denoted by K.T. Atanassov [3], in 1987 T.T. Chelvam, N. Ganesan denoted the notion bi-ideals of near-rings, in 1997 the notion Fuzzy Ideal denoted by D.T.K, and Biswas[4], in 2012 the notion P-regular near ring denoted by Aphisit in [5].

## **1.Preliminaries**

In this section we give some concepts that we need.

#### **Definition** (1.1) [1]

A left near ring is a set N together with two binary operations "+" and "." such that

(1) (N,+) is a group (not necessarily abelian )

(2) (N, .) is a semigroup.

**(3)**  $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ 

For all  $n_1, n_2, n_3, \in \mathbb{N}$ ,

#### **Definition** (1.2) [6]:

Let N be a near ring. A normal subgroup I of (N,+) is called a left ideal of N if

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(1)I.N  $\subseteq$  I.

(2).  $\forall$  n, n<sub>1</sub>  $\in$  N and for all i  $\in$  I, (n<sub>1</sub> + *i*)  $\cdot$  n - n<sub>1</sub>  $\in$  I

## **Definition** (1.3) [7]

Let (  $N_1,+,.$ ) and (  $N_2,+',.'$ ) be two near rings. The mapping  $f: N_1 \rightarrow N_2$  is called a near ring homomorphism if for all  $m, n \in N_1$ 

f(m+n) = f(m) + f(n),  $f(m, n) = f(m) \cdot f(n)$ .

Theorem (1.4) [5]

Let  $f: (N_1, +, .) \rightarrow (N_2, +', .')$  be a near ring homomorphism

(1) If I is an ideal of a near ring  $N_1$ , then f(I) is an ideal of a near ring  $N_2$ .

(2) If J is an ideal of a near ring  $N_2$ , then  $f^{-1}(J)$  is an ideal of the near ring  $N_1$ 

## **Definition** (1.5) [8]

A nonempty subset Q of a near ring N is called a quasi ideal of N if

(1) Q together with addition is a subgroup of N.

(2)  $QN \cap NQ \subset Q$ .

## Definition (1.6) [2]

A nonempty subset B of a near ring N is called a bi-ideal of N if

(1) B together with addition is a subgroup of N.

(2)  $BNB \subset B$ .

#### Definition (1.7) [9]

Let N be a near ring with unity and P an ideal of N. Then N is said to be P-regular near ring if for each  $x \in N$  there exists  $y \in N$  such that  $xyx - x \in P$ .

#### **Definition (1.8) [10]**

A near ring N is called a distributive near ring if a(b+c) = ab + ac for all  $a, b, c \in N$ .

#### Theorem(1.9)[11]

Let N be a P-regular near ring . Then for each  $n \in N$  there exists  $n' \in N$  such that  $n'n \in P$ .

## Theorem(1.10) [11]

Let N be a P-regular distributive near ring. Then for every left ideal L and right ideal R of N.

 $(P+R) \cap (P+L) = P + RL$ .

## **Definition** (1.11)[2]

Let N be a non-empty set a mapping  $\mu: N \to [0,1]$  is called a fuzzy subset of N, where [0,1] is a closed interval of real numbers.

## **Definition** (1.12) [4]

Let  $\mu$  be a non-empty fuzzy subset of a near ring N, that is  $\mu(y) \neq 0$  for some  $y \in N$ ) then  $\mu$  is said to be fuzzy ideal of N if it satisfies the following conditions : (1)  $\mu(z - y) \ge \min{\{\mu(z), \mu(y)\}};$ 

(2)  $\mu(z.y) \ge \min\{\mu(z), \mu(y)\};$ 

(3)  $\mu(y+z-y) \ge \mu(z);$ 

(4)  $\mu(z.y) \ge \mu(y)$ ,  $\forall y,z \in \mathbb{N}$ .

When the subset of N satisfies 1, 2 is called fuzzy sub near ring.

## **Definition** (1.13) [3]

An intuitionistic fuzzy set A in a non-empty set X is an object having the form

A={ $(x, \mu_A(x), \lambda_A(x)) | x \in X$ }, where the functions  $\mu_A : X \to [0,1]$  and  $\lambda_A : X \to [0,1]$  denoted the

degree of membership and the degree of non – membership of each element  $x \in X$  to the set A, respectively and  $0 \le \mu_A(x) + \lambda_A(x) \le 1$  for all  $x \in X$ .

## Definition (1.14)[3]

An intuitionistic fuzzy set  $A = (\mu_A, \lambda_A)$  in N is an intuitionistic fuzzy subnear ring of N if for  $x, y \in N$ 

(1)  $\mu_A(x - y) \ge \min \{ \mu_A(x), \mu_A(y) \}$ 

(2)  $\mu_A(xyz) \ge \min \{ \mu_A(x), \mu_A(z) \}$ 

(3)  $\lambda_A(\mathbf{x} - \mathbf{y}) \le \max \{ \lambda_A(\mathbf{x}), \lambda_A(\mathbf{y}) \}$ 

(4)  $\lambda_A(xyz) \le max \{ \lambda_A(x), \lambda_A(z) \}$ 

## **Proposition** (1.15) [1]

Let X be a non- empty set . A mapping  $\mu: N \to [0,1]$  is a fuzzy set in X , the complement of  $\mu$  , denoted by  $\mu^c$ , is the fuzzy set in X given by  $\mu^c(x) = 1 - \mu(x)$  for all  $x \in \mathbb{N}$  for any  $I \subseteq X, X_I$  denote the characteristic function of I.

For any fuzzy set  $\mu$  and  $h \in [0,1]$ , we define two sets,  $\bigcup (\mu_A, h) = \{x \in X | \mu(x) \ge h\}$  and

 $L(\mu_A, h) = \{x \in X | \mu(x) \le h\}$  Which are called upper and lower h-level cut of  $\mu$  respectively, and can be used to the characterization of  $\mu$ .

## 2- bi-ideal with respect to an element of a near ring N

In this section we devoted to study bi-ideal with respect to an element r of a near ring N and give some properties, theorem about this ideal .

## **Definition** (2.1)

A nonempty subset B of a near ring N is called a bi-ideal with respect to an element of N and denoted by r-b-ideal of a near ring if

(1) B together with addition is a subgroup of N.

(2)  $r.BNB \subseteq r.B$ ,  $r \in N$ .

Example (2.2).

Let  $N = \{0,a,b,c\}$  be the near ring defined by caleys

+	0	А	b	с
0	0	А	b	с
a	Α	0	с	b
b	В	С	0	a
с	С	В	а	0

•	0	а	b	с
0	0	0	0	0
А	0	b	0	b
В	0	0	0	0
С	0	b	0	b

Then B={0,a} is b-bi-ideal since  $b.BNB \subseteq b.B$ 

## Remark (2.3)

If  $B_1$  and  $B_2$  be two r-bi-ideal of near ring N, then  $B_1$ .  $B_2$  of N may be not r-bi-ideal

## Example (2.4)

Considers the near ring  $N=\{0,a,b,c\}$  with addition and multiplication defined by the following tables.

ſ	+	0	1	2	3		0	1	2	3
ſ	0	0	1	2	3	0	0	0	0	0
ſ	1	1	0	3	2	1	0	2	0	2
ſ	2	2	3	0	1	2	0	0	0	0
ſ	3	3	2	1	0	3	0	2	0	2

Let  $B_1 = \{0,3\}$  and  $B_2 = \{1,2,3\}$  are two 3-bi-ideal of N but  $B_1 \cap B_2 = \{3\}$  is not two 3-bi-ideal of N. **Permark** (2.5)

## Remark (2.5)

Not all r-bi-ideal of a near ring are bi-ideal of N.

## Example (2.6)

Consider the near ring N in example (2.2) let  $B = \{0,b\}$  be b-bi-ideal but B is not bi-ideal of N since  $BNB \not\subset B$ 

## Theorem (2.7)

Let  $(N_{1,+,.})$  and  $(N_{2,+,+',.})$  be two near rings,  $f: N_1 \to N_2$  be an epimomorphism and B be rbi-ideal of  $N_1$ . Then f(B) is f(r)-bi-ideal of  $N_2$ .

## **Proof**

Let B be r-bi- ideal of  $N_1\;$  , f(B) is a subgroup of  $N_2\;$  to prove f(B) is an r-bi-ideal of  $N_2\;$ 

Let  $r \in N_1$ , such that  $f(r) \in N_2$ , hence

 $r.BNB \subseteq rB$  Since B is r-bi-ideal of N<sub>1</sub>

 $f(r.BN_1B) \subseteq f(r.B)$  f be an epimomorphism

 $f(r).'f(B).'f(N_1).'f(B) \subseteq f(r).'f(B).$ 

 $f(r).'f(B).'N_2.'f(B) \subseteq f(r).'f(B).$ 

f(B) is f(r)- bi- ideal of  $N_2$ 

## **Theorem (2.8)**

Let  $(N_1, +, .)$  and  $(N_2, +', .')$  be two near rings, and  $f: N_1 \rightarrow N_2$  be epiomomorphism and J be a

f(r)-bi-ideal of  $N_2$ . Then  $f^{-1}(J)$  is a r-bi-ideal of  $N_1$ , where y = f(r), ker  $f \subseteq f^{-1}(J)$ .

## **Proof**

Let  $\mathbf{r} \in \mathbf{N}_1$ ,  $\mathbf{f}^1$  (J) is a subgroup of  $\mathbf{N}_1$   $y \ 'JN_2 J \subseteq y \ 'J$   $f^{-1}(y \ JN_2 J) \subseteq f^{-1}(y \ J)$   $f^{-1}(y) \ f^{-1}(J) \ f^{-1}(N_2) \ f^{-1}(J) \subseteq f^{-1}(y) \ f^{-1}(J)$   $r \ f^{-1}(J) \ N_1 f^{-1}(J) \subseteq r \ f^{-1}(J)$  $\rightarrow \mathbf{f}^1(J)$  is a r-bi-ideal of  $\mathbf{N}_1$ .

## **Remark (2.9)**

Not all r-bi-ideals of the near ring N are quasi ideal. **Example (2.10).** 

Let N- J	[0 a b c]	he the	near ring defined by caleys
Let N = 1	10,a,0,0	be the	hear fing defined by caleys

+	0	А	b	c
0	0	А	b	c
a	А	0	с	b
b	В	С	0	a
с	С	В	a	0

	0	а	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
с	0	b	0	b

Let  $B = \{0, a\}$  is c-bi-ideal since  $c.BNB \subseteq c.B$  but is not quasi ideal

## **Theorem (2.11)**

Let N be a P-regular near ring and B a r-bi-ideal of N. Then every  $y \in B$  There exist  $p' \in P$  and  $b' \in B$  such that y = p' + b'.

## **Proof**

Let N be a P-regular near ring and B a r-bi-ideal of N and  $y \in B \subset N$ , there exists  $z \in N$  such that r(yzy) - y = p for some  $p \in P$  thus y = -p + r(yzy) since B is r-bi- ideal of N, we have  $r(yzy) \in rBNB \subseteq rB$  since  $p \in P$  together with addition is a subgroup of N, we have  $-p \in P$  put p' = -p and b' = r(yzy) thus  $y = -p + r(yzy) = p' + b' \in P + B$ .

## **Theorem (2.12)**

Let N be a P-regular distributive near ring and let  $B_1$ ,  $B_2$  are a r-bi-ideals of N. if  $b \in B_1 \cap B_2$  and  $y \in N$ , then the element b can be represented as  $b = p + rb_1y_1b_2$  and  $b_1y_1b_2yP \subseteq P$  for some  $p \in P$ ,  $r, y_i \in N, b_1 \in B_1$  and  $b_2 \in B_2$ .

#### **Proof**

Let  $b \in B_1 \cap B_2$  since N is a P-regular near ring there exists  $y_1 \in N$  such that  $rby_1b - b \in P$ since  $b \in B_1 \cap B_2$  by theorem (1.9) we have  $b = p_1 + b_1$  for some  $p_1 \in P$  and  $b_1 \in B$ ,  $b = p_2 + b_2$  for some  $p_2 \in P$  and  $b_2 \in B$ , since  $rby_1b - b \in P$ , we have  $rby_1b - b = p_3$  for some  $p_3 \in P$ . Thus  $b = -p_3 + rby_1b$ . Hence

 $= -p_3 + r(p_1 + b_1)y_1(p_2 + b_2)$ =  $-p_3 + rp_1y_1p_2 + rp_1y_1b_2 + rb_1y_1p_2 + rb_1y_1b_2$  since P is an ideal of N, t  $-p_3, rp_1y_1p_2, rp_1y_1b_2, rb_1y_1p_2, rb_1y_1b_2 \in P$ Then  $-p_3 + xp_1y_1p_2 + xp_1y_1b_2 + xb_1y_1p_2 = p_4$ 

For some  $p_4 \in P$ . Thus  $b = p_4 + rb_1y_1b_2$ So  $rb_1y_1b_2 = b - p_4$  hence  $rb_1y_1b_2yP = (b - p_4)yP \subseteq byP - p_4yP$  $\subseteq P + P \subseteq P$ .

#### 3-Intuitionistic fuzzy bi- ideal with respect to an element of a near ring

In this section we devoted to study fuzzy bi-ideal with respect to an element of a near ring N, we introduce the notion intuitionist fuzzy bi-ideal with respect to an element of the near ring N, and give some properties, theorem about this ideals.

## **Definition (3.1)**

A fuzzy set  $\mu$  of a near ring is called fuzzy bi- ideal with respect to an element of a near ring N if

(1) 
$$\mu(\mathbf{r}(\mathbf{x} - \mathbf{y})) \ge \min \{ \mu(\mathbf{r}\mathbf{x}), \mu(\mathbf{r}\mathbf{y}) \}$$

$$\forall x, y \in N, r \in N.$$

(2)  $\mu(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \ge \min \{ \mu(\mathbf{r}\mathbf{x}), \mu(\mathbf{r}\mathbf{z}) \}$ 

$$\forall x, y, z \in N, r \in N.$$

It denoted by F-r-bi-ideal of N  $\,$  .

#### Example (3.2)

Consider the near ring  $N = \{0,1,2,3\}$ 

With addition and multiplication defined by the following tables.

+	0	1	2	3		0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	1	0	3	2	1	0	1	2	3
2	2	3	0	1	2	0	0	0	0
3	3	2	1	0	3	0	1	2	3

The fuzzy subset  $\mu$  of N which is defined by

$$\mu(y) = \begin{cases} 0.9 & \text{if } y = 0,1 \\ 0 & \text{otherwise} \end{cases}$$

0 otherwise

# $\mu$ is F-2-bi-ideal of N **Definition (3.3)**

An intuitionistic fuzzy set  $A = (\mu_A, \lambda_A)$  in N is an intuitionistic fuzzy bi-ideal with respect to an element r of N if for all  $x, y, z \in N, r \in N$ ,

(1)  $\mu_A(\mathbf{r}(\mathbf{x} - \mathbf{y})) \ge \min \{ \mu_A(\mathbf{r}\mathbf{x}), \mu_A(\mathbf{r}\mathbf{y}) \}$ (2)  $\mu_A(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \ge \min \{ \mu_A(\mathbf{r}\mathbf{x}), \mu_A(\mathbf{r}\mathbf{z}) \}$ (3)  $\lambda_A(\mathbf{r}(\mathbf{x} - \mathbf{y})) \le \max \{ \lambda_A(\mathbf{r}\mathbf{x}), \lambda_A(\mathbf{r}\mathbf{y}) \}$ (4)  $\lambda_A(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \le \max \{ \lambda_A(\mathbf{r}\mathbf{x}), \lambda_A(\mathbf{r}\mathbf{z}) \}$ 

## Theorem (3.4)

\_An intuitionistic fuzzy set  $A = (\mu_A, \lambda_A)$  in N is an intuitionistic F-r- bi-ideal of N if and only if the fuzzy set  $\mu_A$  and  $\lambda_A^c$  are F-r- bi-ideal of N.

## **Proof**

If  $A = (\mu_A, \lambda_A)$  is an intuitionistic F-r-bi-ideal of N then clearly  $\mu_A$  is a F-r-bi-ideal of N, for all  $x, y \in N$ ,

$$\begin{aligned} \lambda_{A}^{c}\left(\mathbf{r}(\mathbf{x}-\mathbf{y})\right) &= 1 - \lambda_{A}\left(\mathbf{r}(\mathbf{x}-\mathbf{y})\right) \\ &\geq 1 - \max\{\lambda_{A}\left(\mathbf{r}\mathbf{x}\right), \lambda_{A}\left(\mathbf{r}\mathbf{y}\right)\} \\ &= \min\{1 - \lambda_{A}\left(\mathbf{r}\mathbf{x}\right), 1 - \lambda_{A}\left(\mathbf{r}\mathbf{y}\right)\} \\ &= \min\{\lambda_{A}^{c}\left(\mathbf{r}\mathbf{x}\right), \lambda_{A}^{c}\left(\mathbf{r}\mathbf{y}\right)\} \quad , \forall \mathbf{x}, \mathbf{y} \in \mathbf{N}, \mathbf{r} \in \mathbf{N}, \end{aligned}$$

$$\lambda_{A}^{c}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) = 1 - \lambda_{A}^{}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z}))$$

$$\geq 1 - \max\{\lambda_{A}(\mathbf{r}\mathbf{x}), \lambda_{A}(\mathbf{r}\mathbf{z})\}$$

$$= \min\{1 - \lambda_{A}(\mathbf{r}\mathbf{x}), 1 - \lambda_{A}(\mathbf{r}\mathbf{z})\}$$

$$= \min\{\lambda_{A}^{c}(\mathbf{r}\mathbf{x}), \lambda_{A}^{c}(\mathbf{r}\mathbf{z})\}$$

thus  $\lambda_A^c$  is a F-r- bi-ideal of N. conversely that  $\mu_A$  and  $\lambda_A^c$  are F-r- bi-ideal of N, then clearly the conditions 1,2 of definition (3.3) are valid .Now for all  $x, y \in N, r \in N$ .

$$1 - \lambda_{A}(\mathbf{r}(\mathbf{x} - \mathbf{y})) = \lambda_{A}^{c}(\mathbf{r}(\mathbf{x} - \mathbf{y}))$$
$$= \min\{\lambda_{A}^{c}(\mathbf{r}\mathbf{x}), \lambda_{A}^{c}(\mathbf{r}\mathbf{y})\}$$
$$= 1 - \max\{\lambda_{A}(\mathbf{r}\mathbf{x}), \lambda_{A}(\mathbf{r}\mathbf{y})\}$$

ther fore  $\lambda_{A}(\mathbf{r}(\mathbf{x} - \mathbf{y})) \le \max \{\lambda_{A}(\mathbf{r}\mathbf{x}), \lambda_{A}(\mathbf{r}\mathbf{y})\} \quad \forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{N}, \mathbf{r} \in \mathbf{N}, \mathbf{z} \in \mathbf{N}\}$ 

$$1 - \lambda_{A}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) = \lambda_{A}^{c}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z}))$$
$$\geq \min\{\lambda_{A}^{c}(\mathbf{r}\mathbf{x}), \lambda_{A}^{c}(\mathbf{r}\mathbf{z})\}$$
$$= 1 - \max\{\lambda_{A}(\mathbf{r}\mathbf{x}), \lambda_{A}(\mathbf{r}\mathbf{z})\}$$

therfore  $\lambda_{A}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \leq \max \{ \lambda_{A}(\mathbf{r}\mathbf{x}), \lambda_{A}(\mathbf{r}\mathbf{z}) \}$ 

Thus  $A = (\mu_A, \lambda_A)$  is an intuitionistic F-r- bi-ideal of N. **Theorem (3.5)** 

An intuitionistic fuzzy set  $A = (\mu_A, \lambda_A)$  in N, is an intuitionistic F-r-bi-ideal of N if and only if  $A = (\mu_A, \mu_A^c)$  and  $A = (\lambda_A^c, \lambda_A)$  are intuitionistic F-r-bi-ideal of N.

## **Proof**

If  $A = (\mu_A, \lambda_A)$  is an intuitionistic F-r- bi-ideal of N, then  $\mu_A = (\mu_A^c)^c$  and  $\lambda_A^c$  are F-r-bi-ideal of N, from theorem (3.4). Therefore  $A = (\mu_A, \mu_A^c)$  and  $A = (\lambda_A^c, \lambda_A)$  are intuitionistic F-r-bi-ideal of N.

Conversely if  $A = (\mu_A, \mu_A^c)$ ,  $A = (\lambda_A^c, \lambda_A)$  are intuitionistic F-r- bi-ideal of N, then the fuzzy sets  $\mu_A$  and  $\lambda_A^c$  are F-r-bi-ideal of N, therefore  $A = (\mu_A, \lambda_A)$  is intuitionistic F-r- bi-ideal of N.

## **Proposition (3.6)**

An intuitionistic fuzzy set  $A = (\mu_A, \lambda_A)$  in N is a F-r-bi-ideal of N if and only if all non –empty set  $\bigcup (\mu_A, h)$  and  $\bigcup (\lambda_A, t)$  are r-bi-ideal of N for all of N  $h \in \text{Im}(\mu_A)$  and  $t \in \text{Im}(\lambda_A)$  respectively. **Proof** 

Suppose that  $A = (\mu_A, \lambda_A)$  is an intuitionistic F-r-bi-ideal of N, for  $x, y \in \bigcup(\mu_A, h)$ , we have  $\mu_A(\mathbf{r}(x - y)) \ge \min\{\mu_A(\mathbf{r}x), \mu_A(\mathbf{r}y)\} \ge h$ 

Therefore  $r(x-y) \in \bigcup(\mu_A, h)$  let  $x, z \in \bigcup(\mu_A, h)$  and  $y \in N$ . then

$$\mu_{A}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \ge \min\{\mu_{A}(\mathbf{r}\mathbf{x}), \mu_{A}(\mathbf{r}\mathbf{z})\} \ge h \text{ and so } \mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z}) \in \bigcup(\mu_{A}, h) \text{ hence } \bigcup(\mu_{A}, h) \text{ is }$$

a r-bi-ideal of N.

For all  $h \in \text{Im}(\mu_A)$  similarly we can show that  $\bigcup (\lambda_A, t)$  is also a r- bi-ideal of N for all  $t \in \text{Im}(\lambda_A)$ . Conversely suppose that  $\bigcup (\mu_A, h)$  and  $\bigcup (\lambda_A, t)$  are r- bi-ideal of N for all  $h \in \text{Im}(\mu_A)$  and  $t \in \text{Im}(\lambda_A)$  respectively .Suppose that  $x, y \in N$  and  $\mu_A^{(r(x-y)) \le \min\{\mu_A^{(rx)}, \mu_A^{(ry)}\}}$ 

Choose h such that  $\mu_{A}(\mathbf{r}(\mathbf{x} - \mathbf{y})) \prec \mathbf{h} \prec \min\{\mu_{A}(\mathbf{rx}), \mu_{A}(\mathbf{ry})\}$  Then we get  $x, y \in \bigcup(\mu_{A}, h)$  but

 $r(x-y) \notin \bigcup(\mu_A, h)$  a contradiction. Hence  $\mu_A(r(x-y)) \ge \min\{\mu_A(rx), \mu_A(ry)\}$ .

A similar argument shows that  $\mu_{\mu}(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \ge \min\{\mu_{\mu}(\mathbf{r}\mathbf{x}), \mu_{\mu}(\mathbf{r}\mathbf{z})\}\$ 

For all  $x, y, z \in N$ . likewise we can show that

 $\lambda_A(\mathbf{r}(\mathbf{x} - \mathbf{y})) \le \max\{\lambda_A(\mathbf{r}\mathbf{x}), \lambda_A(\mathbf{r}\mathbf{y})\}$ 

 $\lambda_A(\mathbf{r}(\mathbf{x}\mathbf{y}\mathbf{z})) \le \max \{ \lambda_A(\mathbf{r}\mathbf{x}), \lambda_A(\mathbf{r}\mathbf{z}) \}$ 

Hence  $A = (\mu_A, \lambda_A)$  is an intuitionistic F-r-bi-ideal of N.

## Theorem (3.7)

A non empty set B of N is r-bi-ideal of N if and only if  $A = (\chi_B, \chi_B^c)$  is an intuitionistic F-r-bi-ideal of N.

Proof

Straight forward.

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