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# Charge density distributions and electron scattering form factors of $^{19}$F, $^{22}$Ne and $^{26}$Mg nuclei

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**Abstract**

An effective two-body density operator for point nucleon system folded with the tenser force correlations ( TC's), is produced and used to derive an explicit form for ground state two-body charge density distributions (2BCDD's) applicable for $^{19}$F,$^{22}$Ne and $^{26}$Mg nuclei. It is found that the inclusion of the two-body TC's has the feature of increasing the central part of the 2BCDD's significantly and reducing the tail part of them slightly, i.e. it tends to increase the probability of transferring the protons from the surface of the nucleus towards its centeral region and consequently makes the nucleus to be more rigid than the case when there is no TC's and also leads to decrease the $\langle r^2 \rangle^{1/2}$ of the nucleus. It is also found that the effect of the TC's and the effect of increasing the values of $\hbar\omega$ on the 2BCDD's, elastic electron scattering form factors and $\langle r^2 \rangle^{1/2}$ are in the same direction for all considered nuclei.

**Keywords:** elastic electron scattering ,Charge density ,root mean square chargeradii.

# توزيعات كثافة الشحنة وعوامل التشكل للأستطارة الألكترونية للنوى $^{19}$F,$^{22}$Ne و $^{26}$Mg

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**الخلاصة**

تم توليد مؤثر الكثافة النووية الفعال ذو صيغة الجسيمين لنظام النوية النقطية الذي يتعامل مع نويات النواة بمثابة جسيمات نقطية ليس لها حجم. تم ارتباط هذا المؤثر بدالة ارتباط الجسيمين التي تأخذ بنظر الأعتبار تأثير قوة tensor التجاذبية الطويلة المدى ( TC) للنوى $^{19}$F,$^{22}$Ne و $^{26}$Mg . اظهرت هذه الدراسة بان ادخال دالة TC في الحسابات يؤدي إلى زيادة احتمالية انتقال البروتونات من منطقة السطح الى مركز النواة يرافقه زيادة واضح في الجزء المركزي و نقصان طفيفة في الجزء ألذيلي من توزيعات كثافة الشحنة النووية 2BCDD's وبالتالي يؤدي ذلك إلى نقصان القيم المحسوبة لجذر معدل مربع نصف القطر $\langle r^2 \rangle^{1/2}$ . لقد وجد بان هنالك تأثيرا متشابه لكل من زيادة قيم ( $\hbar\omega$ ) وإدخال دالة TC على حسابات 2BCDD's, عوامل التشكل للاستطارة الاكترونية المرنة و $\langle r^2 \rangle^{1/2}$ ولجميع النوى قيد الدراسة.

## Introduction

Electron scattering is an excellent tool for studying the nuclear structure because of many reasons. Since the interaction between the electron and the target nucleus is relatively weak and known where

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the electron interacts electromagnetically with the local charge, current and magnetization densities of nucleus. Besides , the measurements can be obtained without greatly disturbing the structure of the target[1, 2]. Sugimoto et al.[3] studied the effect of the tensor correlation on the alpha-alpha interaction in $^{8}Be$ using an alpha cluster model. They had used the wave function of the alpha particle calculated by the projected Hartree-Fock method, which could treat the effect of the tensor correlation. Radhi et al.[4] studied the elastic longitudinal electron scattering form factors for $^{9}$Be in the frame work of 1p-shell model, which considered as the core of $^{4}$He with five neucleons disturbuted out of the core. Their results are in good agreement with the experimental data for both models considered. The reduced transition probabilities B(C2) calculated for the two kinds of model space and for the effective charges which are used in this work . Hamoudi et al. [5] studied the Nucleon Momentum Distributions (*NMD*) and elastic electron scattering form factors of the ground state for 1p-shell nuclei with Z=N (such as $^{6}$Li, $^{10}$B, $^{12}$C and $^{14}$N nuclei ) in the framework of the Coherent Density Fluctuation Model (*CDFM*) and expressed in terms of the weight function $f\,|x|^2$. Hamoudi et al. [6] studied the NMD for the ground state and elastic electron scattering form factors in the framework of the coherent fluctuation model and expressed in terms of the weight function (fluctuation function). The inclusion of tensor correlation effects is rather a complicated problem especially for the microscopic theory of nuclear structure. Several methods were proposed to treat complex tensor forces and to describe their effects on the nuclear ground state [7-9].

The aim of the present work is to derive an expression for the ground state 2BCDD (two body charge density distrbutions), based on the use of the two - body wave functions of the harmonic oscillator in order to employ it for studying of the effects of the TC's and oscillator parameter on the root mean square charge radii $\langle r^2\rangle^{1/2}$, 2BCDD, elastic electron scattering form factors for $^{19}$F, $^{22}$Ne and $^{26}$Mg nuclei.

**Theory:**

The one body density operator of Eq. (1) could be transformated into a two-body density form by the following transformation [10].

$$\hat{\rho}^{(1)}(\vec{r}) = \sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i) \qquad \textbf{(1)}$$

$$\hat{\rho}^{(1)}(\vec{r}) \Rightarrow \hat{\rho}^{(2)}(\vec{r})$$

i.e.

$$\sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i) \equiv \frac{1}{2(A-1)}\sum_{i\neq j}\left\{\delta(\vec{r}-\vec{r}_i)+\delta(\vec{r}-\vec{r}_j)\right\} \qquad \textbf{(2)}$$

Where $\delta(\vec{r}-\vec{r}_i)$ : Dirac delta function , A is the nucleon number

In fact, a further useful transformation can be made which is that of the coordinates of the two – particles, $\vec{r}_i$ and $\vec{r}_j$, to be in terms of that relative $\vec{r}_{ij}$ and center – of – mass $\vec{R}_{ij}$ coordinates [11], i.e.

$$\vec{r}_{ij} = \frac{1}{\sqrt{2}}(\vec{r}_i - \vec{r}_j) \qquad \textbf{(3-a)}$$

$$\vec{R}_{ij} = \frac{1}{\sqrt{2}}(\vec{r}_i + \vec{r}_j) \qquad \textbf{(3-b)}$$

Subtracting and adding (3-a) and (3-b) we obtain

$$\vec{r}_i = \frac{1}{\sqrt{2}}(\vec{R}_{ij} + \vec{r}_{ij}) \qquad \textbf{(3-c)}$$

$$\vec{r}_j = \frac{1}{\sqrt{2}}(\vec{R}_{ij} - \vec{r}_{ij})$$

(3-d) Introducing Eqs.(3-c) and (3-d) into Eq. (2) yields

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{1}{2(A-1)}\sum_{i\neq j}\left\{\delta\left[\vec{r} - \frac{1}{\sqrt{2}}(\vec{R}_{ij} + \vec{r}_{ij})\right] + \delta\left[\vec{r} - \frac{1}{\sqrt{2}}(\vec{R}_{ij} - \vec{r}_{ij})\right]\right\} \quad \textbf{(4)}$$

Eq. (4) may be written as

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{1}{2(A-1)}\sum_{i\neq j}\left\{\delta\left[\frac{1}{\sqrt{2}}(\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij})\right] + \delta\left[\frac{1}{\sqrt{2}}(\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij})\right]\right\}$$

$$= \frac{2\sqrt{2}}{2(A-1)}\sum_{i\neq j}\left\{\delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}\right]\right\} \quad \textbf{(5)}$$

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{(A-1)}\sum_{i\neq j}\left\{\delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}\right]\right\} \quad \textbf{(6)}$$

where the following identities [12] have been used

$$\delta(a\,x) = \frac{1}{|a|}\delta(x) \qquad \text{(for one – dimension )}$$

$$\delta(a\,\vec{r}) = \frac{1}{|a^3|}\delta(\vec{r}) \qquad \text{(for three – dimension )}$$

For closed shell nuclei with $N=Z$, the two – body charge density operator can be deduced from Eq.(6) as

$$\hat{\rho}^{(2)}_{ch}(\vec{r}) = \frac{1}{2}\hat{\rho}^{(2)}(\vec{r})$$

i.e.

$$\hat{\rho}^{(2)}_{ch}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\left\{\delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}\right]\right\} \quad \textbf{(7)}$$

Finally, an effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the operator of Eq.(7) with the two-body correlation functions $\tilde{f}_{ij}$ as

$$\hat{\rho}^{(2)}_{eff}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\tilde{f}_{ij}\left\{\delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}\right]\right\}\tilde{f}_{ij} \quad \textbf{(8)}$$

In the present work, a simple model form of the two-body full correlation operators of ref. [13] will be adopted, i.e.

$$\tilde{f}_{ij} = \left\{1 + \alpha(\mathrm{A})\,S_{ij}\right\}\Delta_2 \quad \textbf{(9)}$$

TC's presented in the Eq.(9) are induced by the strong tensor component in the nucleon- nucleon force and they are of longer range. Here $\Delta_2$ is a projection operator onto the $^3S_1$ and $^3D_1$ states only. However, Eq. (9) can be rewritten as

$$\tilde{f}_{ij} = \sum_{\gamma}\left\{1 + \alpha_{\gamma}(\mathrm{A})\,S_{ij}\right\}\Delta_{\gamma} \quad \textbf{(10)}$$

where the sum $\gamma$, in Eq.(10), is over all reaction channels, $S_{ij}$ is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i . \vec{r}_{ij})(\vec{\sigma}_j . \vec{r}_{ij}) - \vec{\sigma}_i . \vec{\sigma}_j \quad \textbf{(11)}$$

while $\alpha_\gamma(\mathrm{A})$ is the strength of tensor correlations and it is non zero only in the ${}^3S_1 - {}^3D_1$ channels.
Elastic electron scattering form factor from spin zero nuclei ($J = 0$), can be determined by the ground – state charge density distributions (CDD). In the Plane Wave Born Approximation (PWBA), the incident and scattered electron waves are considered as plane waves and the CDD is real and spherical symmetric, therefore the form factor is simply the Fourier transform of the CDD. Thus [1,2]

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_o(\mathrm{r})\, j_0(q\mathrm{r})\,\mathrm{r}^2 d\mathrm{r} \quad \textbf{(12)}$$

where $\rho_o(\mathrm{r})$ is the ground state 2BCDD of Eq. (12) .
$j_0(q\mathrm{r}) = \sin(q\mathrm{r})/(q\mathrm{r})$ is the zeroth order of the spherical Bessel function and $q$ is the momentum transfer from the incident electron to the target nucleus. Eq. (12) may be expressed as

$$F(q) = \frac{4\pi}{qZ}\int_0^\infty \rho_o(\mathrm{r})\sin(q\mathrm{r})\,\mathrm{r}\ d\mathrm{r} \quad \textbf{(13)}$$

Inclusion of the finite nucleon size correction $F_{fs}(q)$ and the center of mass correction $F_{cm}(q)$ in our calculations requires multiplying the form factor of Eq.(13) by these corrections. $F_{fs}(q)$ is considered as free nucleon form factor and assumed to be the same for protons and neutrons. This correction takes the form [14].

$$F_{fs}(q) = e^{-0.43q^2/4} \quad \textbf{(14)}$$

The correction $F_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [14] .

$$F_{cm}(q) = e^{q^2b^2/4A} \quad \textbf{(15)}$$

where *A* is the nuclear mass number. Introducing these corrections into Eq.(13), we obtain

$$F(q) = \frac{4\pi}{qZ}\int_0^\infty \rho_o(\mathrm{r})\, Sin(q\mathrm{r})\,\mathrm{r}\ d\mathrm{r}\, F_{fs}(q) F_{cm}(q) \quad \textbf{(16)}$$

In the limit of $q \to 0$, the target will be considered as a point particle, and from Eq.(16), the form factor of this target nucleus is equal to unity, i.e. $F(q \to 0)=1$. The elastic longitudinal electron scattering form factor with the inclusion of the effect of the two-body TC's in light nuclei can now be obtained by introducing the ground state 2BCDD of Eq.(8) into Eq.(16).
We also wish to mention that we have written all computer programs needed in this study using Fortran languages.

**Results and discussion**

The dependence of the ground state 2BCDD's ( in fm$^{-3}$ ) on r ( in fm) for $^{19}$F,$^{22}$Ne and $^{26}$Mg nuclei are displayed in Figures-1 (a, b and c) respectively, where all parameter requried to the calculation represented buy Table-1.

In Figures-1 (a, b and c) the calculated 2BCDD's without TC's (the dashed curves with $\alpha = 0$) and with TC's (the solid curves with $\alpha \neq 0$) are compared with experimental results (the dotted symbols) [15]. From these figures, the dashed curves deviate slightly from the solid curve especially at small r. Introducing the effect of TC's in the calculations tends to remove these deviations from the region of small r as seen in the solid curves. It is evident from these figures that the calculated 2BCDD's represented by the solid curves are in excellent accordance with those of experimental data hence they coincide with each other throughout the whole range of r.

**Table1-**Parameters which have been used in the calculations of the present work for the 2BCDD's, $\langle r^2\rangle^{1/2}$and elastic longitudinal *F(q)*'s of all nuclei under study.

| Nucleus | ħω | $\alpha$ | $\langle r^2\rangle^{1/2}_{\alpha=0}$ | $\langle r^2\rangle^{1/2}_{\alpha\neq 0}$ | $\langle r^2\rangle^{1/2}_{exp.}$ [15] | $\langle r^2\rangle_{TC's}$ |
|---|---|---|---|---|---|---|
| $^{19}F$ | 11.7 | 0.2 | 2.821390 | 2.791368 | 2.901144 | -0.030022 |
| $^{22}Ne$ | 11.25 | 0.198 | 2.956032 | 2.791368 | 2.901144 | -0.030022 |
| $^{26}Mg$ | 11.25 | 0.196 | 3.111177 | 3.072351 | 3.059367 | -0.038826 |

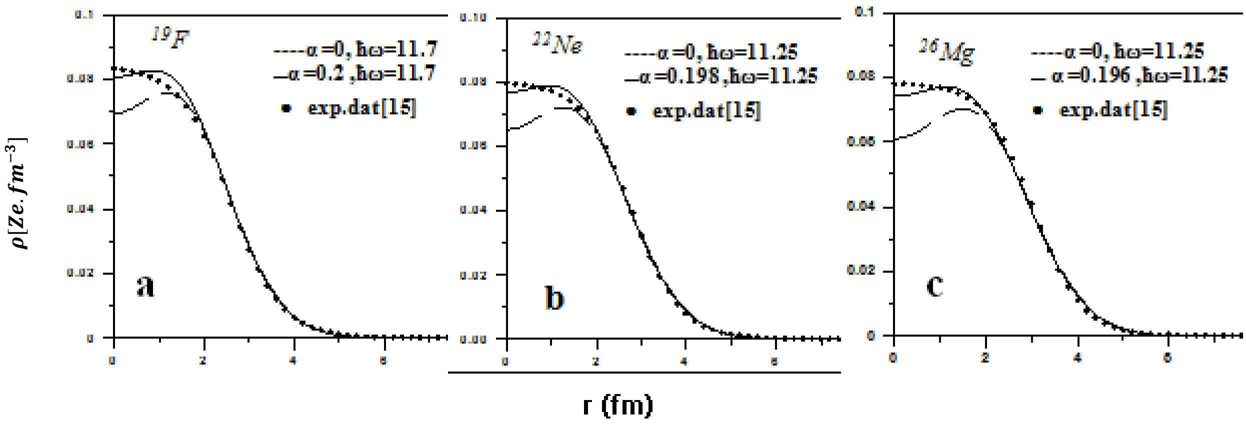


**Figure 1**-(a, b and c): Dependence of the 2BCDD onr for $^{19}$F,$^{22}$Ne and $^{26}$Mg nuclei respectively.The dotted symbols are the experimental data of ref [15].

The elastic longitudinal form factors *F(q)*'s of $^{19}F$ nucleus is presented in Figure-2 (a). As it is evident from this figure that the dashed curve is good agreement with the experimental data up to momentum transfer of $q$ =2.5 fm$^{-1}$ and it underestimates clearly these data at the region of $q$ >2.5fm$^{-1}$. It is so clear that the location of the observed diffraction minimum is not reproduced in the correct place by the dashed and solid curves. It is noticed that the first diffraction minimum at the region of $q$ =1.6 fm$^{-1}$ .It is clear from the Figure-2(a),the location of second diffraction minimum was shifted to the region of $q$ =3.4fm$^{-1}$ . However, an improvement results for *F(q)*'s at the region of higher momentum transfer of $q$ > 2.6fm$^{-1}$ is obtained by the solid curve, where the introduction of the TC's in the calculations of the solid curve tends to push the location of the calculated first diffraction minimum into the region with high momentum transfer and closer to that of the observed one. The elastic form factors of $^{22}$Ne nucleus is presented in Figure-2(b). It is noticed from this figure that both of the magnitude and the behavior of the calculated form factors (the dashed and solid curves) are in reasonable agreement with those of experimental data [15] throughout the range of momentum transfer $q\leq 2.6$. Besides, the locations of the calculated of first diffraction minima in solid curve are reproduced in the correct place. It is noticed that the first diffraction minimum at the region of $q$=1.4fm$^{-1}$. It is clear from the Figure-2(b) ,the location of second diffraction minimum was shifted to the region of $q$=3.4fm$^{-1}$.The form factor of $^{26}Mg$ nucleus is displayed in Figure-2(c).As we can see from this figure that the available data of $^{26}Mg$ nucleus are restricted for a small region of momentum transfer $q\leq$ 2.3fm$^{-1}$. It is noticed that the first diffraction minimum at the region of $q$ =1.4fm$^{-1}$ .It is clear from the Figure-2(c),the location of second diffraction minimum was shifted to the region of $q$ =2.7fm$^{-1}$ and the location of third diffraction minimum was shifted to the region of $q$ =3.2fm$^{-1}$ . It is evident from this figure that the calculated results obtained in both of the dashed and solid curves are in excellent agreement with those of experimental data [15]. up to q=2.2 it is also noted that the effect of considering the TC's becomes more effective at higher momentum transfer of $q$>2.5fm$^{-1}$ and even it becomes progressively larger with increasing $q$. Besides, this effect enhances the calculated *F(q)*'s at $q$>2.5fm$^{-1}$ as seen in the solid curve of this figure .

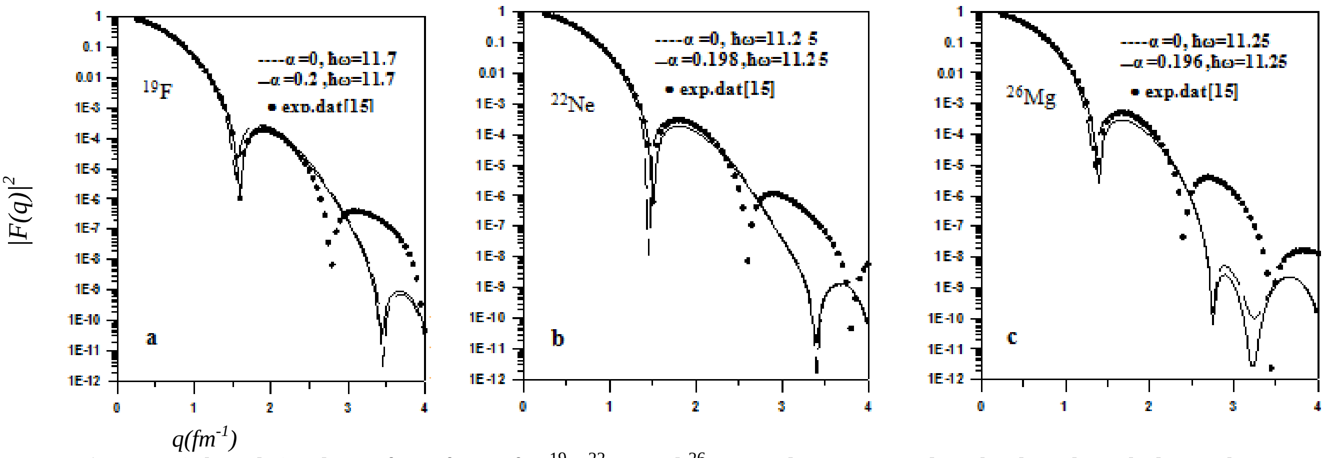


**Figure 2**-(a,b and c) :Elastic form factor for $^{19}$F,$^{22}$Ne and $^{26}$Mg nuclei respectively. The dotted symbols are the experimental data of ref [15].

## Conclusions

The two-body TC's exhibits a mass dependence due to the strength parameter $\alpha$ (A) and the two-body TC's have the feature of increasing the central part of the 2BCDD's significantly and reducing the tail part of them slightly, i.e. it tends to increase the probability of transferring the protons from the surface of the nucleus towards its central region (the central region of the nucleus towards its surface) and consequently makes the nucleus to be more rigid than the case when there is no TC's and also leads to decrease the $\langle r^2 \rangle^{1/2}$of the nucleus.

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