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Calculation the Magnetic Dipole Moments of Some Fluorine Nuclei

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Abstract

The magnetic dipole moments and the root mean square radius have been calculated some the Fluorine (A= 17, 19, 20, 21) isotopes based on the *sd*-shell model using universal *sd*-shell interaction A (USDA). All studied isotopes are composed of ^{16}O nucleus that is considered as an inert core and the other valence particles are moving over the *sd*-shell model space within $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbits. The configuration of mixing shell model with limiting number of orbitals in the model space outside the inert core fail to reproduce the measured magnetic dipole moments. Therefore, and for the purpose of enhancing the calculations, the discarded space has been included the core polarization effect through the effective g-factors. The harmonic oscillator potential is used to generate the single particle matrix elements, where the value of the size parameter b is adjusted to get the experimental root mean square of matter radii for each nucleus calculated.

Keywords: Exotic nucleus, proton, neutron and matter density, magnetic dipole moments, occupation numbers.

حساب عزوم ثنائي القطب المغناطيسي لبعض نوى الفلورين

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الخلاصة

تم حساب عزوم ثنائية القطب المغناطيسية وأنصاف الأقطار لنوى الفلورين (A= 17, 19, 20, 21) المستندة على أنموذج القشرة بأستخدام تفاعل تفاعل القشرة– *sd* العالمي A (USDA). أن جميع النظائر المدروسة تتشكل من نواة ^{16}O والتي أفترض الى انها قلب خامل أضافة على جسيمات خارج هذا القلب أفترض على أنها تتحرك ضمن أنموذج فضاء القشرة *sd* وفي المدارات $1d_{5/2}$, $2s_{1/2}$ و $1d_{5/2}$. أن أنموذج القشرة ذي التشكيلات المختلطة والذي يحتوي على عدد محدد من المدارات في أنموذج الفضاء خارج القلب الخامل يفشل في حساب عزوم ثنائية القطب المغناطيسية. لذلك ولغرض تحسين الحسابات، تم تضمين الفضاء المستثنى من خلال تضمين تأثير استقطاب القلب من خلال عوامل– g الفعالة. استخدم جهد المتذبذب التوافقي في توليد عناصر المصفوفة للجسيمة المفردة حيث أفترضت قيمة مناسبة للثابت التوافقي b لكل نوى وذلك لأعادة توليد معدل الجذر التربيعي لأنصاف الأقطار الكتلية.

1.Introduction

The study of exotic nuclei, nuclei far away from the beta-stability line, is one of the major current research areas in nuclear physics. The investigation of such nuclei has become possible only gradually with the increasing intensity of the radioactive beams that can be delivered by accelerators. It has become evident that the shell structure of exotic nuclei may be different from that of stable nuclei [1]. The development of radioactive-ion beams in the mid-80s has enabled the exploration of the nuclear landscape away from stability. This technical breakthrough led to the discovery of exotic nuclear structures such as haloes [2, 3]. The magnetic moment sensitively reflects

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what single particle orbits contribute to the nuclear wave function [4]. The magnetic moment is a good probe in nuclear physics: Theoretically, the shell model has been successful in reproducing the magnetic moments by the spin and orbital *g*-factors for free nucleons. The differences between the single particle *g*-factors and the measured values have been explained by the configuration of mixing: the core nucleus composed of an even number of nucleons is excited into the 1^+ state, which contributes to the magnetic moment of the nucleus [5].

The nuclear properties such as transition rates cannot be reproduced properly by the model-space wave functions if the properties of neutrons and protons are the same as in free space. The shell -model wave functions have to be renormalized in order to include such “core polarization” effects in describing different nuclear properties. Renormalizations of the model -space wave functions can be achieved by introducing effective operators [6]. Garcia et al., [7] measured the optical hyper-fine spectra of the $^{43-51}$Ca isotopes. The ground state magnetic moments of ^{49,51}Ca and quadrupole moments of ^{47,49,51}Ca were measured for the first time, and the ^{51}Ca ground state spin I = 3/2 was determined in a model-independent way. Their results provide a critical test of modern nuclear theories based on shell-model calculations using phenomenological as well as microscopic interactions. The results for the neutron-rich isotopes are in excellent agreement with predictions using interactions derived from chiral effective field theory including three-nucleon forces, while lighter isotopes illustrate the presence of particle-hole excitations of the ^{40}Ca core in their ground state [7].

In the present work, the rms matter radius is reproduced by fixing the size parameter of the single particle wave function of the harmonic oscillator (HO) potential for each isotope in its ground state, where ^{16}O nucleus considered as an inert core. The magnetic dipole moments (μ) of of some Fluorine nuclei $^{17,\ 19,\ 20,\ 21}$F are calculated by using universal *sd*-shell interaction A (USDA) [8]. The proton, neutron and matter density distributions and the corresponding root mean square (rms) radii of the ground states are calculated as well. Calculations are presented with model space and core-polarization (CP) effects by using effective g-factors. The occupation numbers of the valence particles outside ^{16}O core are also calculated for the ground state of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits to identify the contribution of these orbits for the valence particles.

2.Theory

The one-body density matrix elements (OBDM) contains all the information about transitions of given multiplicities, which is imbedded in the model space wave function and is given in second quantization as [9]:

$$OBDM(J_f, J_i, a, b, J, t_Z) = \frac{\langle J_f \| [a^+_{a,t_Z} \otimes \tilde{a}_{b,t_Z}]^J \| J_i \rangle}{\sqrt{2J+1}} \quad \textbf{(1)}$$

where $\langle \| \ \| \rangle$ is the reduced matrix element. J_i , J_f initial and final total angular momentum (spin) respectively. a^+ and $\tilde{a}$ are the creation and annihilation operators of a proton or neutron in the single particle state, and t_z = 1/2 for a proton and -1/2 for a neutron. The initial and final single particle states ($n\ l\ j\ t_z$) are denoted by α and β, respectively. As the nuclear shell wave functions have good isospin, it is appropriate to evaluate the OBDM elements by means of isospin-reduced matrix elements. The relation between tripled-reduced OBDM and the proton or neutron OBDM is given by [10]:

$$\begin{aligned} OBDM\ (J_f, J_i, a, b, J, t_z) = {} & (-1)^{T_f - T_z}\sqrt{2}\begin{pmatrix} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \mathrm{OBDM}\,(\Gamma_f, \Gamma_i, a, b, \Delta T = 0)/2 \\ & + 2t_z(-1)^{T_f - T_z}\sqrt{6}\begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z \end{pmatrix} \mathrm{OBDM}\,(\Gamma_f, \Gamma_i, a, b, \Delta T = 1)/2 \end{aligned} \quad \textbf{(2)}$$

where the bracket $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ denotes the 3j-symbol and $T_Z = \frac{Z-N}{2}$, where Z, N are protons and neutrons numbers, respectively. These triply reduced $OBDM(\Delta T)$ elements are given in terms of the second quantization as:

$$OBDM(\Gamma_f, \Gamma_i, a, b, J, \Delta T) = \frac{\langle \Gamma_f ||| [a_a^+ \otimes \tilde{a}_b]^{(J,\Delta T)} ||| \Gamma_i \rangle}{\sqrt{2J+1}\sqrt{2\Delta T+1}} \quad \textbf{(3)}$$

where $\langle ||| \ \ ||| \rangle$ is matrix element reduced both in angular momentum and isospin space, Greek symbols are utilized to indicate quantum numbers in coordinate space and isospace (i.e., $\Gamma_i \equiv J_i T_i$ and $\Gamma_f \equiv J_f T_f$).

The average occupations number in each subshell j is given by [11]:

$$occ\#(j,t_z) = OBDM(a,b,t_z,J=0)\sqrt{\frac{2j+1}{2J_i+1}} \quad \textbf{(4)}$$

The root mean square radius in terms of occupation numbers for the harmonic oscillator potential with size parameter is given by:

$$\langle r^2 \rangle_m = \frac{1}{A}\sum occ\#(j,t_z)\, b^2\, (N+\frac{3}{2}) \quad \textbf{(5)}$$

where (A) is mass number A=Z+N.

The nuclear shell model calculations were performed using the OXBASH shell model code [12], where the one body density matrix (OBDM) elements given in eq. (2) were obtained.The single particle matrix element of the magnetic transition operator in spin- isospin state is given by [9]:

$$\langle a \| \hat{O}_T(m1) \| b \rangle = \langle n_a l_a | r^{J-1} | n_b l_b \rangle\, f_T^{m1}(a,b) \quad \textbf{(6)}$$

where

$$f_T^{m1}(a,b) = (-1)^{\ell_a}(2J+1)\sqrt{\frac{J(2J-1)\ (2\ell_a+1)\ (2\ell_b+1)\ (2j_a+1)\ (2j_b+1)}{4\pi}}$$
$$\times \begin{pmatrix} \ell_a & J-1 & \ell_b \\ 0 & 0 & 0 \end{pmatrix} \sqrt{2T+1}\,[\frac{g_p^{\ell}+(-1)^T g_n^{\ell}}{J+1}(-1)^{\ell_b+j_b+1/2}\sqrt{2\ell_b(\ell_b+1)(2\ell_b+1)} \quad \textbf{(7)}$$
$$\times \begin{Bmatrix} \ell_a & \ell_b & J \\ j_b & j_a & 1/2 \end{Bmatrix} \begin{Bmatrix} J-1 & 1 & J \\ \ell_b & \ell_a & \ell_b \end{Bmatrix} + \frac{1}{2}\{g_p^s + (-1)^T g_n^s\}\sqrt{3}\begin{Bmatrix} \ell_a & 1/2 & j_a \\ \ell_b & 1/2 & j_b \\ J-1 & 1 & J \end{Bmatrix}]\mu_N$$

For $J=1$ equation (6) become:

$$\langle a \| \hat{O}_T(m1) \| b \rangle = f_T^{m1}(a,b) \quad \textbf{(8)}$$

where $\mu_N = \frac{e\hbar}{2m_p c} = 0.1051 e.fm.$ is the nuclear magneton, with m_p the proton mass.

The orbital and spin free nucleon g-factors are: g_l (p) = 1, g_s (p) = 5.585 for proton and g_l (n) = 0, g_s (n) = -3.826 for neutron [9].

The reduce matrix element of the magnetic transition operator $\hat{O}_k(m1)$ is expressed as the sum of the product of the elements of the one-body density matrix (OBDM) times the single-particle [9]

$$\langle J_i \| \sum_{k=1}^{n} \hat{O}_k(m1) \| J_i \rangle = \sum_{a,b,T} (-1)^{T_i - T_z} \begin{pmatrix} T_i & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} OBDM \quad (a,b,J=1,T) \langle a \| \hat{O}_T(m1) \| b \rangle \quad \textbf{(9)}$$

The magnetic dipole moment μ of a state of total angular momentum J is given by [9]:

$$\mu = \sqrt{\frac{4\pi}{3}} \begin{pmatrix} J_i & 1 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \langle J_i \| \sum_{k=1}^{n} \hat{O}_k(m1) \| J_i \rangle \quad \textbf{(10)}$$

3.Results and Discussion

The radial wave functions for the single-particle matrix elements were calculated with harmonic oscillator (HO) potential with size parameters *b* adjusted for isotopes of Fluorine $^{17, 19, 20, 21}$F to reproduce the measured root mean square matter radius (R_m). The calculations of the proton, neutron, matter rms radii, and magnetic dipole moments μ were carried out using *sd*-shell model space using universal *sd*-shell interaction A (USDA) in OXBASH code [12]. The core polarization (CP) effect was included by using effective g-factors.

The values of calculated matter (R_m), proton (R_p) and neutron (R_n) rms radii and oscillator size parameter *b* of $^{17, 19, 20, 21}$F nuclei are displayed in Table-1.

Table 1- The calculated matter, proton and neutron rms radii and oscillator size parameter (*b*) of $^{17, 19, 20, 21}$F nuclei compared with experimental results.

A	$J^{\pi}T$ $\tau_{1/2}$	*b* (fm)	Rm (fm)		R_p(fm)	R_n(fm)
			Calc.	Exp.		
17	$5/2^+$ 1/2 64.49 s	1.666	2.54	2.54∓0.08 (a)	2.575	2.499
19	$1/2^+$ 1/2 stable	1.668	2.61	2.61∓0.07 (b)	2.578	2.637
20	2^+ 1 11.00 s	1.765	2.79	2.79∓0.03 (c)	2.727	2.840
21	$5/2^+$ 3/2 4.158 s	1.698	2.71	2.71∓0.03 (b)	2.624	2.772

(a) Ref.[13], (b) Ref.[14], (c) Ref.[15]

3.1. ^{17}F nucleus ($J^{\pi}T = 5/2^+$ 1/2, $\tau_{1/2}$ = 64.49 s)

^{17}F nucleus is composed of the core ^{16}O nucleus plus one proton surrounding the core. The outer one proton is considered to move over the *sd*-shell model space.The nuclear halo is among the most peculiar features discovered in unstable nuclei. Considerable effort has been expended to determine the characteristics of this unstable, short-lived nucleus [16]. ^{17}F is the show case of one-proton halo system with much extended matter distribution related to the small energy necessary to remove the proton, S_{1p}= 6.0027(25)keV. The size parameter is fixed at *b*=1.666 fm, which reproduces the rms matter radius value 2.54∓0.08fm [13]. The ground state configurations show that one proton is distributed outside the core over *sd*-shell orbits with 100% over $1d_{5/2}$ orbit.

The magnetic dipole moment with free spin g_s-factors for this transition is $\mu = 4.7928\, n.m$ in comparison with the measured values $\mu_{\text{exp.}} = 4.7213(3)\, n.m$ [17, 18] and $\mu_{\text{exp.}} = 4.7223(12)\, n.m$ [17, 19]. The effective g-factor with g_s(eff)=0.9g_s(free) decreased the magnetic moment value by $\mu = 4.31356\, n.m$. Using the orbital g-factors for proton and for neutron as g_l^p =1.1, g_l^n = 0.1 and the spin g-factors for proton and neutron as g_s(eff)=0.9 g_s(free), the result of μ agrees with that of experimental value by 4.7135n.m. The calculated and experimental values of magnetic dipole moments are tabulated in Table 2. The analysis of the above results shows a strong contribution of $1d_{5/2}$ orbit for valence one proton, and that is so apparent from Figure-1(a).

3.2. ^{19}F nucleus ($J^{\pi}T = 1/2^+$ 1/2, $\tau_{1/2}$ = stable)

Calculations were performed with *sd*-shell model space including core-polarization effects. The configurations $(1s_{1/2})^4$ $(1p_{3/2})^8(1p_{1/2})^4(1d_{5/2})^3$ were used for ^{19}F. The oscillator size parameter (*b*) was taken to be 1.668fm, which gives the rms matter radius equal to 2.61fm in agreement with the measured value 2.61∓0.07fm [14].

The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 2.90388\, n.m$, which is in a good agreement with the measured value $\mu_{\text{exp.}} = 2.628868(8)\, n.m$ [17, 20]. The effective g-factor with g_s(eff)=0.9g_s(free) decreased the magnetic moment value by $\mu = 2.61349\, n.m$.Using the orbital g-factors for proton and neutron as g_l^p =0.9, g_l^n = -0.09 and the spin g-factors for proton and neutron as g_s(eff)=0.9 g_s(free), the result of μ agrees with that of experimental value 2.63141n.m. The calculated and experimental values of magnetic dipole moments are tabulated in Table-2. The major contribution of $1d_{5/2}$ orbit for valence three particles, are shown from Figure-1b, where the occupation number percentages were calculated and presented for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits.

3.3. ^{20}F nucleus ($J^{\pi}T = 2^+$ 1, $\tau_{1/2}$ = 11.00 s)

^{20}F is an exotic neutron-rich nucleus, with half-life time ($\tau_{1/2}$=11.00s [21]) and one-neutron separation energy S_n= 6601.31±5 keV. This isotope is composed of the core ^{16}O nucleus plus one proton and three neutrons distributed over the *sd*-shell model space. The ground state of unstable ^{20}F isotope is $J^{\pi}T = 2^{+}\ 1$. USDA interaction was adopted to generate the OBDM elements. The single-particle wave functions of a harmonic oscillator potential were used with size parameter b=1.765 fm, to reproduce the rms matter radius 2.79±0.03 fm [15].

The calculated magnetic dipole moment of this transition for g_s(free) is $\mu = 2.03528\, n.m$, which underestimates the measured value $\mu_{exp.} = 2.09335(9)\, n.m$ [17, 22] and $\mu_{exp.} = 2.0935(9)\, n.m$ [17, 23].The effective g-factor with $g_s(eff)=0.9g_s(free)$ decreased the magnetic moment value by $\mu = 1.83175\, n.m$.Using the orbital g-factors for proton and neutron as g_l^p =1.164, g_l^n = 0.164 and the spin g-factors for proton and neutron as $g_s(eff)=0.9\ g_s(free)$, the result of μ agrees with that of experimental value 2.093528n.m. The calculated and experimental values of magnetic dipole moments are tabulated in Table-2. The major contribution of $1d_{5/2}$ orbit for valence three particles is shown in Figure-1c, where the occupation number percentages were calculated and presented for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits.

3.4. ^{21}F nucleus ($J^{\pi}T = 5/2^{+}\ 3/2$, $\tau_{1/2} = 4.158$ s)

^{21}F is an exotic neutron-rich nucleus, with half-life time ($\tau_{1/2}$=4.158s [21]) and one-neutron separation energy S_n= 8101.5±18 keV. Calculations were performed with *sd*-shell model space including core-polarization effects. The configurations $(1s_{1/2})^4\ (1p_{3/2})^8 (1p_{1/2})^4 (1d_{5/2})^5$ were used for ^{21}F. The oscillator size parameter was taken to be 1.698fm,which gives the rms matter radius as 2.71fm in agreement with the measured value 2.71∓0.03fm [14].

The calculated magnetic dipole moment for this transition for g_s(free) is $\mu = 3.77289\, n.m$, which is underestimates the measured value $\mu_{exp.} = 3.93(5)\, n.m$ [17, 24]. The effective g-factor with $g_s(eff)=0.9g_s(free)$ decreased the magnetic moment value by $\mu = 3.3956\, n.m$.Using the orbital g-factors for proton and neutron as g_l^p =1.21, g_l^n = 0.21 and the spin g-factors for proton and neutron as $g_s(eff)=0.9\ g_s(free)$, the result for μ agrees with that of experimental value 2.93862n.m. The calculated and experimental values of magnetic dipole moments are tabulated in Table-2. The major contribution of $1d_{5/2}$ orbit for valence three particles, is as shown in Figure-1d, where the occupation number percentages were calculated and presented for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits.

Table 2- Magnetic dipole moments (μ) of $^{17, 19, 20, 21}$F nuclei compared with experimental results

A	$J^{\pi}T$	$\mu_{Calc.}$(n. m)			$\mu_{Exp.}$ (n. m)	Refs.
		g(free)	g(eff.)= 0.9 g(free)	g_l^p g_l^n $\mu_{Calc.}$(n. m)		
17	$3/2^{+}\ 1/2$	4.7928	4.31356	1.1 0.1 4.7135	4.7213(3) 4.7223(12)	[17,18] [17,19]
19	$1/2^{+}\ 1/2$	2.90388	2.61349	0.9 -0.09 2.63141	2.628868(8)	[17, 20]
20	$2^{+}\ 1$	2.03528	1.83175	1.164 0.164 2.09353	2.09335(9) 2.0935(9)	[17, 22] [17, 23]
21	$5/2^{+}\ 3/2$	3.77289	3.3956	1.21 0.21 3.93862	3.93(5)	[17, 24]

Figures-2 represents comparison of the obtained matter, proton and neutron rms radii with the mass number *A* of the considered $^{17,\ 19,\ 20,\ 21}$F nuclei. All curves have the same behaviors but they are different in magnitudes. The rms radii are dependent on occupation numbers for the harmonic oscillator potential and on the size parameter *b*.

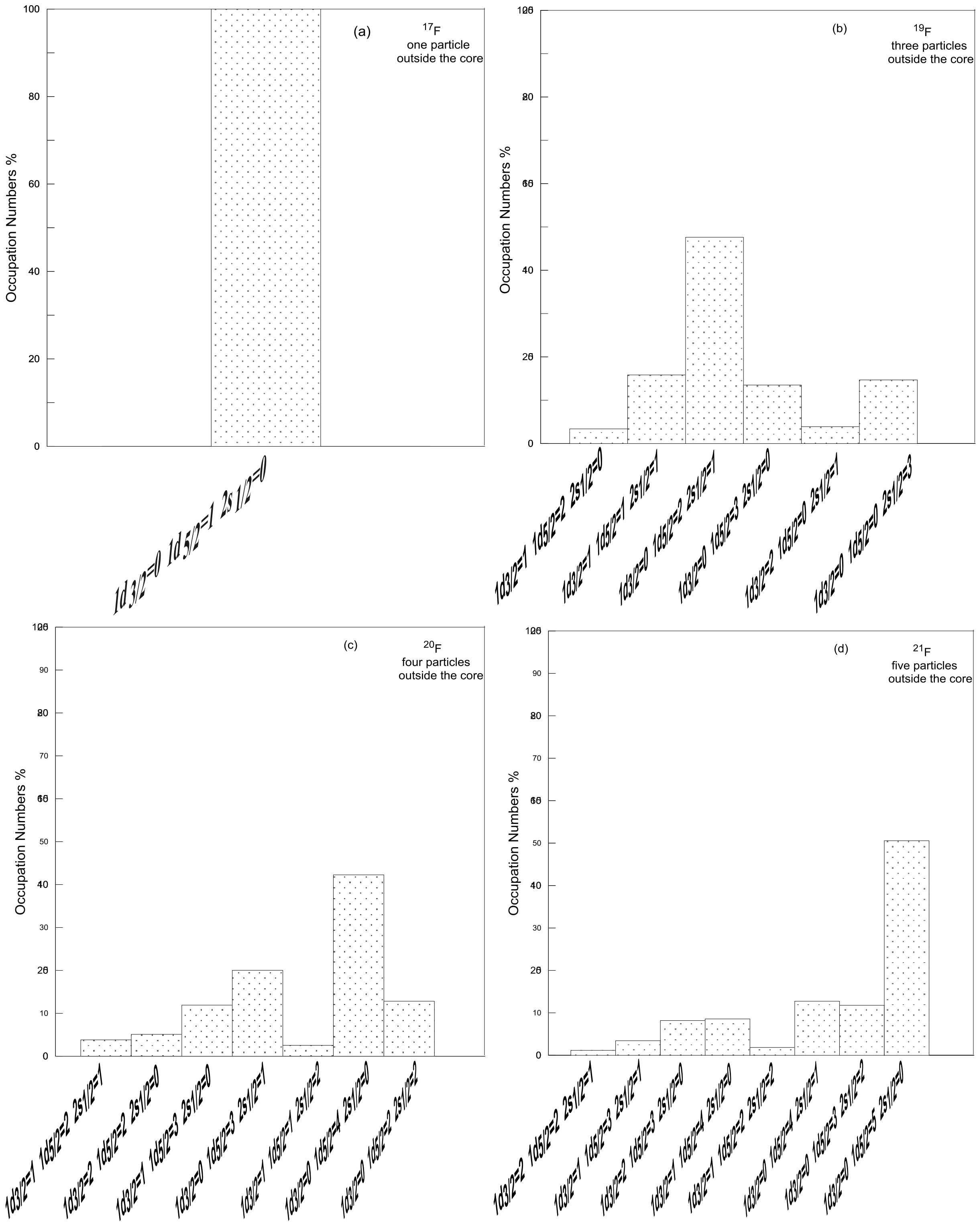

Figure 1- The percent occupation for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits Outside ^{16}O core of the considered $^{17, 19, 20, 21}F$ nuclei.

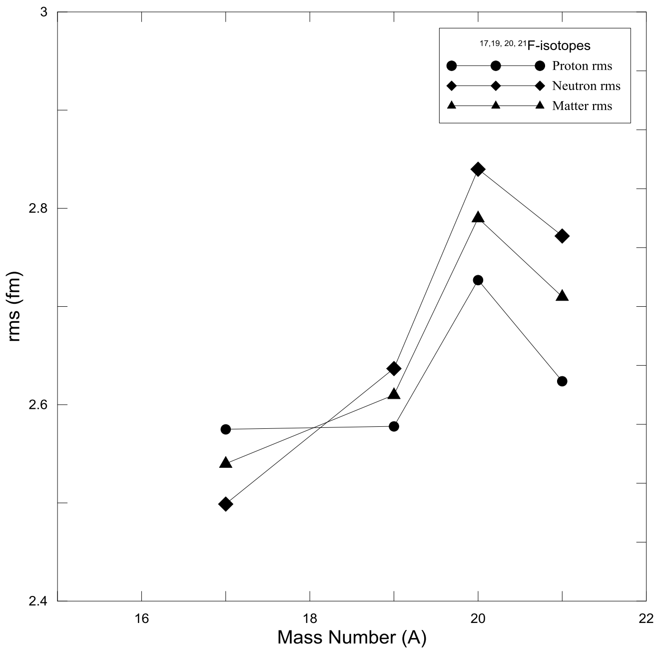

Figure 2- The comparison of the obtained matter, proton and neutron rms radii with the mass number *A* of $^{17, 19, 20, 21}F$ isotopes.

Conclusions

Shell-model calculations are performed for F nuclei including core-polarization effects through first- order perturbation theory. The magnetic dipole moments μ of the $^{17, 19, 20, 21}F$ nuclei depend clearly on assigned configurations and their experimental data are useful to determine the deformations of the ground states of nuclei near the proton and neutron drip line. The inclusion of core polarization (the effective g-factors) is adequate to obtain good agreement between the predicted and measured magnetic dipole moments. The size parameter *b* of the HO potential is fixed to reproduce the measured rms matter radii. The analysis of the calculations for the considered isotopes shows a strong contribution of $1d_{5/2}$ orbit and a clear exotic behavior for the valence nucleons outside ^{16}O core.

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