



Elastic Form Factors and Proton Momentum Distributions for Some fp-Shell Nuclei Using the Coherent Density Fluctuation Model

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Abstract

The ground state proton momentum distributions (PMD) and elastic charge form factors for some odd 1f - 2p shell nuclei, such as ⁵⁹Co, ⁶³Cu, and ⁶⁵Cu have been studied using the Coherent Density Fluctuation Model and formulated by means of the fluctuation function (weight function) $|f(x)|^2$. The fluctuation function has been

connected to the charge density distribution of the nuclei and determined from the theory and experiment result. The feature of the long-tail behavior at high momentum region of the PMD has been calculated by both the theoretical and experimental fluctuation functions. It is found that the inclusion of the quadrupole form factors $F_{C2}(q)$ in all nuclei under study, which are described by the undeformed 1f - 2p shell model, is necessary for obtaining a notable accord between the theoretical and experimental form factors.

Keywords: density distributions; elastic electron scattering, quadrupole form factors; momentum distributions; 2s - 1d shell nuclei; root mean square radii.

عوامل التشكل المرنة و توزيع زخم البروتون لبعض نوى القشرة fp باستخدام انموذج التموج للكثافة المتشكل المرنة و

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الخلاصة:

Introduction

The most accurate determination of the charge distributions in nuclei can be obtained from electron-nucleus scattering. The interest in charge densities results from the very important fact is reflected the behavior of wave functions of protons in nuclei, where the charge density distribution is the sum of the proton wave functions squared. Charge density distributions for stable nuclei have been well studied by [1-3]. For the case of the unstable exotic nuclei the corresponding charge distributions

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are planned to be studied by colliding electrons with these nuclei in storage rings, the GSI physics program [4] and the plan of RIKEN [5]. A number of interesting issues can be analyzed by the electron experiments. One of them is to study how the charge distribution evolves with increasing neutron number at fixed proton number or to what extent the neutron halo or skin may trigger sizable changes of the charge root-mean-squared radius and the diffuseness in the peripheral region of the charge distribution. The measurements of the charge form factor F_{ch} of various nuclei in a range of momentum transfers has stimulated extensive theoretical work for the calculation of this quantity. In [6] the calculations of the charge form factors of exotic nuclei were extended from light (He, Li) to medium and heavy nuclei (Ni, Kr and Sn). For the He and Li isotopes the proton and neutron densities obtained in the large-scale shell-model (LSSM) method have been used, while for Ni, Kr and Sn isotopes the densities have been obtained in the deformed self-consistent mean-field Skyrme-Hartree-Fock (HF)+BCS method [7]. In [6] the charge form factors calculated not only within the PWBA but

also in DWBA by the numerical solution of the Dirac equation [8] for electron scattering in the

coulomb potential on the charge density of a given nucleus. Another important characteristic of the nuclear ground state is the nucleon momentum distribution (NMD). In [9] the neutron and proton momentum distributions in some stable nuclei (${}^{12}C$, ${}^{16}O$, ${}^{40}Ca$, ${}^{56}Fe$ and ${}^{208}Pb$) were calculated along with those of light neutron-rich isotopes of Li , Be, B and C using the natural –orbital representation (NOR) on the basis of the empirical data for n(k) in ${}^{4}He$. It is importance to study the NMD not only in stable but also in exotic nuclei. In [10], the NMD of even-even isotopes of Ni, Kr and Sn have been calculated in the framework of deformed self-consistent mean-field Skyrme (HF)+BCS method, as well as of theoretical correlation methods based on light-front dynamics and local density approximation. The isotopic sensitivities of the calculated neutron and proton momentum distributions are investigated together with the effect of pairing and nucleon-nucleon correlations. Al-Rahmani and Hussein [11] have studied the charge density distributions CDD and elastic electron scattering form factors of some 2s - 1d shell nuclei utilizing the PWBA and illustrated that the inclusion of the higher 1f - 2p shell in the calculations leads to produce a good results in comparison with those of the experimental data.

In the coherent density fluctuation model (CDFM), which is characterized by the work of Antonov et al. [12, 13], the local nucleon density distribution (NDD) and the NMD are simply linked and specified by an experimentally obtainable fluctuation function weight function $|f(x)|^2$. Also they studied the NMD of (⁴He and ¹⁶O), ¹²C and (³⁹K, ⁴⁰Ca and ⁴⁸Ca) nuclei employing weight functions $|f(x)|^2$ specified by the two parameter Fermi (2PF) NDD [14], the data of Reuter et al. [15] and the model independent of the NDD [14], respectively. It is significant to remark that all above studies, employed the framework of the CDFM, proved a high momentum tail in the NMD. Elastic electron scattering from ⁴⁰Ca nucleus was also studied in Ref. [12], where the calculated elastic differential cross sections ($d\sigma/d\Omega$) were found to be in good agreement with those of 2PF [14].

Nearly all the CDFM investigations are based on the use of weight functions originated in terms of the experimental NDD, are employed in the present study. The CDFM with weight functions originated in terms of theoretical CDD. In the present study, at first a theoretical form for the CDD is derived, which is applicable through out the lower region of the 1f-2p shell nuclei with $Z \ge 27$, based on the use of the single particle harmonic oscillator wave function and the occupation numbers of the states. The derived form of the CDD is employed in determining the theoretical weight function $|f(x)|^2$, which is then used in the CDFM to study the PMD and elastic electron scattering charge form factors for ⁵⁹Co, ⁶³Cu and ⁶⁵Cu nuclei. The effect of considering the quadrupole form factor in these nuclei is also studied. It is found that the theoretical weight function $|f(x)|^2$ based on the derived CDD is capable to give information about the PMD and elastic charge form factors as do those of the experimental data.

Theory

The charge density distribution CDD of the shell nuclei can be evaluated by means of the radial part of the wave functions of a harmonic oscillator, since [16]

$$\rho_{c}(r) = \frac{1}{4\pi} \sum_{nl} \zeta_{nl} 2(2l+1) |R_{nl}(r)|^{2}$$
(1)

where $\rho_c(r)$ is the CDD of nuclei, ζ_{nl} is the proton occupation probability of the state nl ($\zeta_{nl} = 0$ or 1 for closed shell nuclei and $0 < \zeta_{nl} < 1$ for open shell nuclei) and $R_{nl}(r)$ is the radial part of the single-particle harmonic oscillator wave function. To derive an explicit form for the CDD of 1f - 2p shell nuclei, it is supposed that there is a core of filled 1s and 1p and 1d shells and the proton occupation numbers in 2s, 1f and 2p shells are equal to $(2 - \gamma_1)$, γ_2 and $(Z - 20 - \gamma_2 + \gamma_1)$, respectively, instead of 2, (Z - 20) and 0 as in the simple shell model. Using this assumption in eq.(1),one can get:-

$$\rho_{c}(r) = \frac{1}{4\pi} \left\{ 2 |R_{10}(r)|^{2} + 6 |R_{11}(r)|^{2} + 10 |R_{12}(r)|^{2} + (2 - \gamma_{1}) |R_{20}|^{2} + \gamma_{2} |R_{13}|^{2} + (Z - 20 - \gamma_{2} + \gamma_{1}) |R_{21}|^{2} \right\}$$
(2)

where Z is the atomic number of nuclei, the parameter γ_1 characterizes the deviation of the proton occupation numbers from the prediction of the simple shell model ($\gamma_1 = 0$), the parameter γ_2 is assumed as a free parameter to be adjusted in order to obtain the agreement with the experimental (CDD). After introducing the form of $R_{nl}(r)$ with a harmonic oscillator size parameter b in Eq.(2), an analytical form for the ground state CDD of the 1f - 2p shell nuclei is expressed as

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3} \{ (5 - \frac{3}{2}\gamma_1) + [\frac{11}{3}\gamma_1 + \frac{5}{3}(Z - 20 - \gamma_2)](\frac{r}{b})^2 + [4 - 2\gamma_1 - \frac{4}{3}(Z - 20 - \gamma_2)](\frac{r}{b})^4 + [\frac{8}{105}\gamma_2 + \frac{4}{15}(Z - 20 - \gamma_2) + \frac{4}{15}\gamma_1](\frac{r}{b})^6 \},$$
(3)

The mean square charge radius (MSR) can be determined according to the following equation [12,13]

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_c(r) r^4 dr, \qquad (4)$$

where the normalization condition of the $\rho_c(r)$ is given by [12,13]

$$Z = 4\pi \int_{0}^{\infty} \rho_c(r) r^2 dr , \qquad (5)$$

And the corresponding MSR is

$$\langle r^2 \rangle = b^2 \{ \frac{9}{2} - \frac{30}{Z} + \frac{\gamma_1}{Z} \}$$
 (6)

The central $\rho_c(r=0)$ is obtained from eq. (3) as

$$\rho_c(0) = \frac{1}{\pi^{3/2} b^3} \left\{ 5 - \frac{3}{2} \gamma_1 \right\},\tag{7}$$

The parameter γ_1 is determined from the central CDD of eq. (6) as

$$\gamma_1 = \frac{2}{3} \left\{ 5 - \pi^{3/2} b^3 \rho_c(0) \right\}$$
(8)

In eq. (8), the values of the central density, $\rho_c(0)$, are taken from the experiments whereas the harmonic oscillator size parameter b is chosen in such away so as to reproduce the experimental root mean square charge radii $\langle r^2 \rangle_{exp}^{1/2}$ of the considered nuclei.

The PMD, n(k), for the 1f - 2p shell nuclei is studied using two distinct methods. In the first method, it is determined by the shell model using the single-particle harmonic oscillator wave functions in momentum representation and expressed as

$$n(k) = \frac{b^{3}e^{-b^{2}k^{2}}}{\pi^{3/2}} \begin{vmatrix} 5 - \frac{3}{2}\gamma_{1} + \{\frac{11}{3}\gamma_{1} + \frac{5}{3}(Z - 20 - \gamma_{2})\}(bk)^{2} \\ + \{4 - 2\gamma_{1} - \frac{4}{3}(Z - 20 - \gamma_{2})\}(bk)^{4} \\ + \frac{8}{105}\gamma_{2} + \frac{4}{15}(Z - 20 - \gamma_{2}) + \frac{4}{15}\gamma_{1}\}(bk)^{6} \end{vmatrix},$$
(9)

whereas in the second method, the n(k) is determined by the CDFM, where the mixed density is given by [12, 13]

$$\rho(\vec{r},\vec{r}') = \int_{0}^{\infty} |f(x)|^2 \rho_x(\vec{r},\vec{r}') dx,$$
(10)

since

$$\rho_{x}(\vec{r},\vec{r}') = 3\rho_{0}(x)\frac{j_{1}(k_{F}(x)|\vec{r}-\vec{r}'|)}{k_{F}(x)|\vec{r}-\vec{r}'|} \times \theta\left(\vec{x} - \frac{|\vec{r}+\vec{r}'|}{2}\right),$$
(11)

is the density matrix for Z protons uniformly distributed in the sphere with radius x and density $\rho_0(x) = 3Z/4\pi x^3$. The Fermi momentum is defined as [12, 13]

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{V}{x}; \qquad V = \left(\frac{9\pi Z}{8}\right)^{1/3}$$
(12)

and the step function θ , in Eq. (10), is defined by

$$\theta(y) = \begin{cases} 1, & y \ge 0\\ 0, & y < 0 \end{cases}$$
(13)

According to the density matrix definition of Eq.(10), one-particle density $\rho(r)$ is given by its diagonal element as [12, 13]

$$\rho_{c}(r) = \rho_{c}(r, r')|_{r=r'} = \int_{0}^{\infty} \left| f(x) \right|^{2} \rho_{x}(r) dx,$$
(14)

In Eq. (14), $\rho_x(r)$ and $|f(x)|^2$ have the following forms [12, 13] $\rho_x(r) = \rho_0(x)\theta(x-|\vec{r}|)$ (15)

$$\left|f(x)\right|^{2} = \frac{-1}{\rho_{0}(x)} \frac{d\rho_{c}(r)}{dr}\Big|_{r=x}.$$
(16)

The weight function $|f(x)|^2$ of Eq. (16), determined in terms of the ground state $\rho_c(r)$, satisfies the following normalization condition [12, 13]

$$\int_{0}^{\infty} |f(x)|^2 dx = 1,$$
(17)

and holds only for monotonically decreasing $\rho_c(r)$, i.e. $\frac{d\rho_c(r)}{dr} < 0$.

On the basis of Eq. (14), the PMD, n(k), is given by [12, 13]

$$n(k) = \int_{0}^{\infty} |f(x)|^{2} n_{x}(k) dx,$$
(18)
where $n_{x}(k) = \frac{4}{3} \pi x^{3} \theta \left(k_{F}(x) - \left| \vec{k} \right| \right),$
(19)

is the Fermi-momentum distribution of the system with density $\rho_0(x)$. By means of Eqs. (16), (18) and (19), an explicit form for the PMD is expressed in terms of $\rho_c(r)$ [12,13] as

$$n^{CDFM}\left(k:\left[\rho_{c}\right]\right) = \left(\frac{4\pi}{3}\right)^{2} \frac{4}{Z} \times \left[6\int_{0}^{V/k} \rho_{c}(x)x^{5}dx - \left(\frac{V}{k}\right)^{6} \rho_{c}\left(\frac{V}{k}\right)\right],$$
(20)

with normalization condition

$$Z = \int n^{CDFM} (k) \frac{d^3 k}{(2\pi)^3}.$$
 (21)

The elastic monopole charge form factors $F_{C0}(q)$ of the target nucleus are also expressed in the CDFM as [12, 13]

$$F_{C0}(q) = \frac{1}{Z} \int_{0}^{\infty} \left| f(x) \right|^{2} F(q, x) dx,$$
(22)

where the form factor of uniform charge density distribution is given by [12,13]

$$F(q,x) = \frac{3Z}{(qx)^2} \left[\frac{\sin(qx)}{(qx)} - \cos(qx) \right],$$
(23)

Inclusion of the correction due to the finite nucleon size $f_{fs}(q)$ and the center of mass correction $f_{cm}(q)$ in the calculations requires multiplying the form factor of Eq. (22) by these corrections. Here, $f_{fs}(q)$ is considered as free nucleon form factor which is assumed to be the same for protons and neutrons [17]

$$f_{fs}(q) = \exp[-\frac{0.43q^2}{4}],$$
(24)

The correction $f_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and is given by [17]

$$f_{cm}(q) = \exp[\frac{q^2 b^2}{4A}],$$
 (25)

Multiplying the right hand side of Eq. (22) by these corrections yields:

$$F_{C0}(q) = \frac{1}{Z} \int_{0}^{\infty} |f(x)|^{2} F(q, x) dx f_{fs}(q) f_{cm}(q).$$
⁽²⁶⁾

It is important to point out that all physical quantities studied above in the framework of the CDFM such as n(k) and $F_{C0}(q)$, are expressed in terms of the weight function $|f(x)|^2$. In the previous work [12, 13], the weight function was obtained from the NDD of the 2PF, extracted by analyzing elastic electron-nuclei scattering experiments. In the present work, the theoretical weight function $|f(x)|^2$ is expressed, by introducing the derived CDD of Eq. (3) into Eq. (16), as

$$\left|f(x)\right|^{2} = \frac{8\pi x^{4}}{3Zb^{2}}\rho_{c}(x) - \frac{16x^{4}e^{-x^{2}/b^{2}}}{3Zb^{5}\pi^{1/2}} \begin{cases} \frac{11}{6}\gamma_{1} + \frac{5}{6}(Z - 20 - \gamma_{2}) \\ + \left(4 - 2\gamma_{1} - \frac{4}{3}(Z - 20 - \gamma_{2})\right)\left(\frac{x}{b}\right)^{2} \\ + \left(\frac{4}{35}\gamma_{2} + \frac{2}{5}(Z - 20 - \gamma_{2}) + \frac{2}{5}\gamma_{1}\right)\left(\frac{x}{b}\right)^{4} \end{cases}$$
(27)

Here, the quadrupole charge from factors are described by the undeformed fp – shell model, where the ground state charge density distributions of these deformed nuclei are described by [18]

$$\rho_{ch}(\vec{r}) = \rho_{0ch}(r) + \rho_{2ch}(r)Y_{20}(\cos\theta) + \dots$$
(28)

The normalization of the spherically symmetric part $\rho_{0ch}(r)$ gives $4\pi \int \rho_{0ch}(r) r^2 dr = Ze$. here, the $\rho_{0ch}(r)$ is calculated by Eq. (3), i.e., $\rho_{0ch}(r) \equiv \rho_c(r)$. The quadrupole part of the charge density $\rho_{2ch}(r)$ is related to the electric quadrupole moment Q by [18]

$$Q = 2\left(\frac{4\pi}{5}\right)^{1/2} \int \rho_{2ch}(r) r^4 dr$$
(29)

The quadrupole charge form factor, which contains the non-spherical part of the charge density distribution, is then given by [19]

$$F_{C2}(q) = \frac{\langle r^2 \rangle}{Q} \left(\frac{4}{5P_J}\right)^{1/2} \int j_2(qr) \rho_{2ch}(r) r^2 dr,$$
(30)

where $j_2(qr)$ is the spherical Bessel function of order two, P_J is a quadrupole projection factor given by $P_J = J(2J-1)/(J+1)(2J+3)$, and J is the ground state angular momentum (J = 7/2)for ⁵⁹Co nucleus and J = 3/2 for both (⁶³Cu, ⁶⁵Cu) nuclei). According to the undeformed fp – shell model [20], where the qudrupole moment arises from protons moving in the fp – shell of undeformed potential, the radial dependence of the quarupole charge density distributions $\rho_{2ch}(r)$ is assumed to be the same as that of the fp – shell part $\rho_{0ch}(r)$. In this study, the quadrupole moment Q is considered as a free parameter so as to fit the theoretical form factors with those of experimental data.

Results and Discussion

The proton momentum distributions n(k) and elastic form factors, F(q), for ${}^{59}Co$, ${}^{63}Cu$ and ${}^{65}Cu$ nuclei are studied by means of the CFM. The PMD of Eq. (20) is calculated in term of the CDD and obtained firstly from theoretical consideration, as in Eq. (3) and secondly from the fit to the electron-nuclei scattering experiments, such as 2PF [14]. The harmonic oscillator size parameters b are chosen in such a way so as to imitate the experimental root mean square (rms) charge radii of nuclei. The values of γ_1 are determined by Eq. (8). In Table-1, we display the values of the parameters b, γ_1 and γ_2 together with the experimental values of the parameters c, z of 2PF distribution as well as the corresponding value of the central densities ρ_{exp}^{2PF} (0), the root mean square charge radii $< r^2 > \frac{1}{exp}^2$ and Q for ${}^{59}Co$, ${}^{63}Cu$ and ${}^{65}Cu$ nuclei.

Nucleus	Model	с [14]	<i>z</i> [14]	$ \rho_{exp}(0) $ [14] (fm ⁻³)	$< r^{2} >_{\exp}^{1/2}$ [14] (fm)	b (fm)	${\cal Y}_1$	γ_2	Q
⁵⁹ Co	2PF	4.158	0.575	0.1646	3.864	2.0903	0.777623	6.40	150
⁶³ Cu	2PF	4.218	0.596	0.1672	3.947	2.1130	0.636387	8.336	290
⁶⁵ Cu	2PF	4.252	0.589	0.1695	3.954	2.1166	0.669720	8.220	300

Table 1- The Values of various parameters employed in the present calculations together With the value of $\rho_{\exp}(0)$ and $\langle r^2 \rangle_{\exp}^{1/2}$.

In Figure-1, we explore the dependence of the CDD (in fm^{-3}) on r (in fm) for ⁵⁹Co Figure-1(a), ⁶³Cu Figure-1(b) and ⁶⁵Cu Figure-1(c) nuclei. The dashed and solid curves correspond to the calculated CDD, using Eq. (3) with (γ_1 , $\gamma_2 = 0$) and (γ_1 , $\gamma_2 \neq 0$), respectively whereas the dotted symbols correspond to the experimental data [14]. It is obvious that the dashed curves are in poor

agreement with the experimental data, mainly for small r. Introducing the parameter γ_1 and γ_2 (i.e., taking into account the higher orbitals) into our calculations leads to a good agreement with the experimental data as displayed by the solid curves.



Figure 1- Dependence of the CDD on *r* for (a) ${}^{59}Co$ (b) ${}^{63}Cu$ and (c) ${}^{65}Cu$ nuclei. The dashed and solid curves are the calculated CDD of Eq. (3) when $\gamma_1 = \gamma_2 = 0$ and $\gamma_1 \neq \gamma_2 \neq 0$, Table-1, respectively. The dotted symbols are the experimental of 2PF data taken from ref. [14].

In Figure-2, we demonstrate the dependence of the n(k) (in fm^3) on k (in fm^{-1}) for ⁵⁹Co Figure-2(a), ⁶³Cu Figure-2(b) and ⁶⁵Cu Figure-2(c) nuclei. The long-dashed curves correspond to the PMD's of Eq. (9) evaluated by the shell model utilizing the single particle harmonic oscillator wave functions in the momentum space. The dotted symbols and solid curves correspond to the PMD's obtained by the CDFM of Eq. (20) utilizing the experimental and theoretical CDD, respectively. It is clear that the manner of the long-dashed distributions obtained by the shell model is in dissimilarity with the distributions imitated by the CDFM. The majer property of the long-dashed distributions is the steep slope mode, when k increases. This behavior is in disagreement with the studies [12, 13, 21- 23] and it is recognized to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into consideration the significant effects of the short range dynamical correlation functions. Therefore, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the PMD [21, 22].

It is noted that the general structure of the dotted and solid distributions at the region of high momentum components is almost the same for ${}^{59}Co$, ${}^{63}Cu$ and ${}^{65}Cu$ nuclei, where these distributions have the property of long-tail manner at momentum region $k \ge 2 \text{ fm}^{-1}$. The property of long-tail manner obtained by the CDFM, which is in agreement with the studies [12, 13, 21- 23], is connected to the presence of high densities $\rho_x(r)$ in the decomposition of Eq. (14), though their fluctuation functions $|f(x)|^2$ are small.



Figure 2- Dependence of PMD on k for (a) ${}^{59}Co$ (b) ${}^{63}Cu$ and (c) ${}^{65}Cu$ nuclei. The solid curves and dotted symbols are the calculated PMD obtained in terms of the CDFM of Eq. (20) using the theoretical CDD of Eq. (3) and the experimental data of ref. [14], respectively. The long-dashed curves are the calculated PMD of Eq. (9) obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum representation.

The elastic electron scattering charge form factors from the considered nuclei are calculated in the framework of the CDFM through introducing the theoretical weight functions $|f_c(x)|^2$ of Eq. (27) into Eq. (26). In Figure-3, we present the dependence of the form factors F(q) on the momentum transfer q (in fm^{-1}) for ⁵⁹Co Figure-3(a), ⁶³Cu Figure-3(b) and ⁶⁵Cu Figure-3(c) nuclei. Here, the effect of the quadrupole form factor $F_{C2}(q)$ is considered by the undeformed 1f - 2p shell

model as given in Eq. (30). The dashed and long-dashed curves correspond to the contributions of the monopole form factors $|F_{C0}(q)|^2$ and quadrupole form factors $|F_{C2}(q)|^2$, respectively, whereas the solid curves correspond to the total contribution, which is obtained as the sum of the monopole and quadrupole form factors. Figures-3(a) - 3(b), show that the contribution of the monopole form factors for the considered nuclei underestimated the experimental data [24] (filled circles symbols) at $1.03\langle q \rangle (2.08 \text{ (for } {}^{59}Co), 1.02\langle q \rangle (1.71 \text{ (for } {}^{63}Cu \text{) and } 1.02\langle q \rangle (1.71 \text{ (for } {}^{65}Cu \text{) respectively.}$

Inclusion the effect of quadrupole form factors in the calculations leads to improve the calculated form factors, especially at the regions where the experimental data are not explained by the dashed curve. The locations of the diffraction minima in Figures-3(a)-3(c) are approximately located in the correct places when the contribution of the quadrupole form factors is considered in the calculations. This figure gives the conclusion that the contribution of the quadrupole form factors gives a strong modification to the monopole form factors and brings the calculated values very close to the experimental data.



Figure 3- Dependence of the charge from factors on q for (a) ${}^{59}Co$, (b) ${}^{63}Cu$ and (c) ${}^{65}Cu$ nuclei. The dashed and long-dashed curves represent the contributions of the monopole form factors $|F_{C0}(q)|^2$ and the quadrupole form factors $|F_{C2}(q)|^2$, respectively. The solid curves represent the total form factors for both contributions. The experimental data (filled circles) for ${}^{59}Co$ ${}^{63}Cu$ and ${}^{65}Cu$ are taken from ref. [24].

Conclusions

The PMD and elastic charge form factors F(q), which are evaluated by the *CDFM*, are formulated via the weight function $|f(x)|^2$. The weight function, which is related with the local density $\rho_c(r)$, is obtained from experiment and from theory. The property of the long-tail behavior of the PMD, which is in agreement with the other studies [12, 13, 22, 23], is achieved by both theoretical and experimental weight functions and is connected to the presence of high densities $\rho_x(r)$ in the decomposition of Eq. (14), though their weight functions are small. It is found that the contribution of the quadrupole form factors in ${}^{59}Co$, ${}^{63}Cu$ and ${}^{65}Cu$ nuclei, which are described by the undeformed 1f - 2p shell model, is essential in obtaining a good agreement between the theoretical and experimental form factors. It is found that the theoretical CDD of Eq. (3) employed in the determination of theoretical weight function of Eq. (27) is capable to reproduce information about the PMD and elastic charge form factors as do those of the experimental data. **References**

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