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New Subclasses of Meromorphically Uniformly of Multivalent Functions with Positive and Fixed Second Coefficients

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Abstract
In this paper, we consider new subclasses $\int^{*}_{p,i}\ (\alpha,\beta,\lambda,n,z_0)(i=0,1)$ of meromorphic uniformly of multivalent functions in $\mathbb{U}^* = \{z \in \mathbb{C}, 0 < |z| < 1\} = U/\{0\}$ with fixed second coefficient, we obtain the estimation of coefficients, distortion theorems, closure theorems and some other results.

Keywords and phrases: p-valent, Meromorphic, Distortion, Closure Theorems, Fixed Point.

فئات فرعية جديدة من الدوال الميمورفية المنتظمة المتعددة التكافؤ مع معاملات موجبة وثانية ثابتة

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الخلاصة
في هذا البحث ناقشنا فئات فرعية جديدة $\int^{*}_{p,i}\ (\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\lambda},\boldsymbol{n},\boldsymbol{z_0})(\boldsymbol{i}=\boldsymbol{0},\boldsymbol{1})$ من الدوال الميمورفية المنتظمة المتعددة التكافؤ داخل $\mathbb{U}^*$ مع تثبيت المعامل الثاني وقد حصلناعلى بعض النتائج كتخمين المعاملات وبعض النظريات لتلك الفئات.

1. Introduction

Let $\sum_P$ be the class of functions of the form:

$$f(z) = a_0 z^{-p} + \sum_{m=p}^{\infty} a_m z^m\ (a_0 > 0,\ a_m \geq 0, p \in N = \{1,2,\dots\}), \tag{1}$$

which is meromorphic and multivalent in $\mathbb{U}^*$. For $f \in \sum_P$, let

$$\mathfrak{Y}_\lambda^0\ f(z) = f(z)\ ,$$
$$\mathfrak{Y}_\lambda^1\ f(z) = \mathfrak{Y}_\lambda f(z),$$
$$\mathfrak{Y}_\lambda^1\ f(z) = [1 - p\lambda] f(z) + \lambda z f'(z) \qquad (\lambda \geq 0),$$
$$\mathfrak{Y}_\lambda^1\ f(z) = a_0 z^{-p} + \sum_{m=p}^{\infty} [1 + \lambda(p+m)] a_m z^m,$$
$$\mathfrak{Y}_\lambda^2\ f(z) = \mathfrak{Y}_\lambda\ (\mathfrak{Y}_\lambda f(z))\ ,$$
$$\vdots$$
$$\mathfrak{Y}_\lambda^n\ f(z) = \mathfrak{Y}_\lambda\left(\mathfrak{Y}_\lambda^{n-1}\ f(z)\right),$$

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$$= a_0 z^{-p} + \sum_{m=p}^{\infty} [1 + \lambda(p+m)]^n a_m z^m \quad (n \in N_0 = N \cup \{0\}). \tag{2}$$

The linear operator $\mathfrak{Y}_\lambda$ was introduced by Saif and Kilieman [1].

Definition 1. The function f $in \sum_P$ is said to be meromorphic multivalent starlike (or convex) function of order α if

$$-Re\left\{\frac{z f'(z)}{f(z)}\right\} > \alpha, \qquad z \in \mathbb{U}^*, \quad 0 \le \alpha < p, \tag{3}$$

and

$$-Re\left\{1 + \frac{z f''(z)}{f'(z)}\right\} > \alpha, \qquad z \in \mathbb{U}^*; \quad 0 \le \alpha < p, \tag{4}$$

For some $\alpha \ge 0$, a subclass of $\sum_P$ denoted by $\int_p^* (\alpha, \beta, \lambda, n)$ which satisfies the following:

$$\left|\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} + \alpha + \alpha\beta + 1\right| \le Re\left\{-\left(\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} - \alpha + \alpha\beta + 1\right)\right\}. \tag{5}$$

Where $0 < \beta < 1,\ \lambda \ge 0\ \ and\ n \in N_0$.

For a given real number z_0 such that $0 < z_0 < 1$, let $\int_{p,i}$, $i = 1,0$ be the subclass of $\int_p^* (\alpha, \beta, \lambda, n)$ which satisfies the condition:

$$z_0^p f(z_0) = 1 \quad and \quad -\frac{z_0^{p+1}}{p} f'(z_0) = 1 \quad .$$

Let

$$\int_{p,i}^* (\alpha, \beta, \lambda, n, z_0) = \int_p^* (\alpha, \beta, \lambda, n, z_0) \cap \int_{p,i} (i = 1,0) \tag{6}$$

Meromorphic multivalent functions have been extensively studied by Aouf [2, 3,4], Adnan Aziz Hussein and Kassim A. Jassim [5,6], Joshi and Srivastava [7], Mogra [8,9], Owa et al. [10], Joshi and Aouf [11], Aouf and Darwish [12, 13], Raina and Srivastava [14], Uralegaddi and Ganigi [15], Raid Habib Buti and Kassim A. Jassim [16] and Yang [17].

2. Main Results

Theorem 1 (Coefficient Estimates): Let $\in \sum_P$, then $f \in \int_p^* (\alpha, \beta, \lambda, n)$ if and only if

$$\sum_{m=p}^{\infty} m[(1+p)(\alpha\beta + 1) + (m-1)][\lambda(p+m) + 1]^n a_m \le a_0 p(p+1)\alpha\beta \tag{7}$$

Proof: Assume that $f \in \int_p^* (\alpha, \beta, \lambda, n)$. Then,

$$\left|\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} + \alpha + \alpha\beta + 1\right| \le Re\left\{-\left(\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} - \alpha + \alpha\beta + 1\right)\right\}.$$

But,

$$Re\left\{\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} - \alpha + \alpha\beta + 1\right\} \le \left|\frac{z\left(\mathfrak{Y}_\lambda^n f(z)\right)''}{(1+p)\left(\mathfrak{Y}_\lambda^n f(z)\right)'} + \alpha + \alpha\beta + 1\right|$$

$$\leq Re\left\{-\left(\frac{z\left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)''}{(1+p)\left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)'}-\alpha+\alpha\beta+1\right)\right\},$$

that is

$$Re\left\{\frac{z\left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)''}{(1+p)\ \left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)'}+2(\alpha\beta+1)\right\}\leq 0,$$

by (2), we get

$$\sum_{m=p}^{\infty} m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^{n}a_m\leq a_0p(p+1)\alpha\beta$$

$$Re\left\{\frac{-2p(1+p)\alpha\beta a_0z^{-p-1}+\sum_{m=p}^{\infty}2m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^{n}a_mz^{m-1}}{-pa_0(1+p)\ z^{-p-1}+\sum_{m=p}^{\infty}m(1+p)[\lambda(p+m)+1]^{n}a_mz^{m-1}}\right\}$$
$$\leq 0$$

If we let z to take a real value such as $z\to 1^{-}$, then we get

$$\frac{-2p(1+p)\alpha\beta a_0+\sum_{m=p}^{\infty}2m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^{n}a_mz^{m+p}}{-pa_0(1+p)\ +\sum_{m=p}^{\infty}m(1+p)[\lambda(p+m)+1]^{n}a_mz^{m+p}}$$
$$\leq 0,$$

that is

$$-2p(1+p)\alpha\beta a_0+\sum_{m=p}^{\infty}2m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^{n}a_mz^{m+p}$$
$$\leq 0$$

which is equivalent to (7).
Conversely, suppose that the inequality (2.1) holds. Then, we have

$$\left|\frac{z\left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)''}{(1+p)\ \left(\mathfrak{Y}_{\lambda}^{n}\ f(z)\right)'}+\alpha+\alpha\beta+1\right|=$$

$$\left|\frac{-2p(1+p)(\alpha+\alpha\beta+1)a_0\ z^{-p-1}+\sum_{m=p}^{\infty}m[2(m-1)+(\alpha+\alpha\beta+1)(1+p)[\lambda(p+m)+1]^{n}]a_mz^{m-1}}{-pa_0(1+p)z^{-p-1}+\sum_{m=p}^{\infty}m(1+p)[\lambda(p+m)+1]^{n}a_mz^{m-1}}\right|$$

$$\leq\frac{-2p(1+p)(\alpha+\alpha\beta+1)a_0-\sum_{m=p}^{\infty}m[2(m-1)-(\alpha-\alpha\beta-1)(1+p)[\lambda(p+m)+1]^{n}]a_mz^{m+p}}{-pa_0(1+p)+\sum_{m=p}^{\infty}m(1+p)[\lambda(p+m)+1]^{n}a_mz^{m+p}}$$

Letting $z\to 1^{-}$ along the real axis, we obtain the desired inequality (7). Hence, the proof of Theorem 1 is completed

Corollary 1: If $f\in \int_p^{*}(\alpha,\beta,\lambda,n)$, then

$$a_m\leq\frac{p(1+p)\alpha\beta a_0}{m[(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^{n}}\ (m=p\in N=\{1,2,\dots\}).\qquad(8)$$

Theorem 2: Let $f(z) \in \int_p^* (\alpha, \beta, \lambda, n)$. Then, $f(z) \in \int_{p,i}^* (\alpha, \beta, \lambda, n, z_0)$ if and only if

$$\sum_{m=p}^{\infty}\left[\frac{m(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n}{p(p+1)\beta\alpha}+z_0^{m+p}\right]a_m \leq 1 \tag{9}$$

Proof: Since $f(z) \in \int_{p,0}^* (\alpha, \beta, \lambda, n, z_0)$, then

$$z_0^p f(z_0) = z_0^p\left(a_0 z^{-p} + \sum_{m=p}^{\infty} a_m z_0^m\right)$$
$$= a_0 + \sum_{m=p}^{\infty} a_m z_0^{m+p} = 1$$

$$a_0 = 1 - \sum_{m=p}^{\infty} a_m z_0^{m+p} \tag{10}$$

Substituting this value of a_0 in Theorem 1, we get the desire assertion.
Corollary 2: Let $f(z) \in \int_{p,0}^* (\alpha, \beta, \lambda, n, z_0)$. Then

$$a_m \leq \frac{p(1+p)\alpha\beta}{m[(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n+p(1+p)\alpha\beta z_0^{m+p}} \tag{11}$$

Theorem 3: Let $f(z) \in \int_p^* (\alpha, \beta, \lambda, n)$. Then $f(z) \in \int_{p,1}^* (\alpha, \beta, \lambda, n, z_0)$ if and only if

$$\sum_{m=p}^{\infty}\left[\frac{m(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n}{p(p+1)\beta\alpha}-\left[\frac{m}{p}\right]z_0^{m+p}\right]a_m \leq 1 \tag{12}$$

Proof : Since $f(z) \in \int_{p,1}^* (\alpha, \beta, \lambda, n, z_0)$, then we have

$$-\frac{z_0^p}{p}f'(z_0) = a_0 - \sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p} = 1$$

$$a_0 = 1 + \sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p} \quad , a_0 > 0, a_m \geq 0, p \in N \tag{13}$$

Replace a_0 from (13) by (7), we get

$$\sum_{m=p}^{\infty} m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n a_m z^{m+p}$$
$$\leq \left(1+\sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p}\right)p(p+1)\alpha\beta$$

that is

$$\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n - p(p+1)\left[\frac{m}{p}\right]\alpha\beta z_0^{m+p}\right]a_m$$
$$\leq p(p+1)\alpha\beta,$$

Which is equivalent to (12).
Conversely, let us suppose that the inequality (12) holds true. Then

$$\sum_{m=p}^{\infty}\frac{m(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n}{p(p+1)\beta\alpha}a_m-\sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p}$$
$$\leq a_0-\sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p}\ \leq 1$$

So that $-\frac{z_0^p}{p}f'(z_0)=1$, then $f(z)\in \int_{p,1}^{*}(\alpha,\beta,\lambda,n,z_0)$

Corollary 3: Let $f(z)\in \int_{p,1}^{*}(\alpha,\beta,\lambda,n,z_0)$. Then

$$a_m\leq\frac{p(1+p)\alpha\beta}{m[(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n-p(1+p)\alpha\beta\left[\frac{m}{p}\right]z_0^{m+p}}\qquad(14)$$

.

3.Distortion Theorem

Theorem 4: Let $f(z)\in \int_{p,0}^{*}(\alpha,\beta,\lambda,n,z_0)$. Then for
$0<|z|<1$, we have

$$|f(z)|\geq\frac{m[(m-1)+(1+p)(1+\beta\alpha)[\lambda(p+m)+1]^n-p(1+p)\beta\alpha r^{2p}}{r^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n+p(1+p)\beta\alpha z_0^{m+p}\right]}.\qquad(15)$$

Proof. Since $f(z)\in \int_{p,0}^{*}(\alpha,\beta,\lambda,n,z_0)$ and by (9) we get

$$\sum_{m=p}^{\infty}a_m\leq\frac{p(p+1)\beta\alpha}{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n+p(p+1)\beta\alpha z_0^{m+p}}\qquad(16)$$

From (10) we have

$$a_0=1-\sum_{m=p}^{\infty}a_m\, z_0^{m+p}$$

$$a_0\geq\frac{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n}{m(m-1)+(1+p)(\alpha\beta+1)[\lambda(p+m)+1]^n+p(p+1)\beta\alpha z_0^{m+p}}.\qquad(17)$$

Hence, we have

$$|f(z)|\geq a_0 r^{-p}-r^p\sum_{m=p}^{\infty}a_m$$

$$|f(z)|\geq\frac{m[(m-1)+(1+p)(1+\beta\alpha)[\lambda(p+m)+1]^n-p(1+p)\beta\alpha r^{2p}}{r^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n+p(1+p)\beta\alpha z_0^{m+p}\right]}\qquad(18)$$

By using (16) and (17). Further, the result is sharp for the function $f(z)$ which is given by

$$f(z)=\frac{m[(m-1)+(1+p)(1+\beta\alpha)[\lambda(p+m)+1]^n-p(1+p)\beta\alpha z^{2p}}{z^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n+p(1+p)\beta\alpha z_0^{m+p}\right]}$$

Theorem 5: Let $f(z) \in \int_{p,1}^{*}(\alpha,\beta,\lambda,n,z_0)$. Then

$$|f(z)| \leq \frac{m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n+p(1+p)\beta\alpha r^{2p}}{r^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n-p(1+p)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}\right]}. \quad (19)$$

For $0<|z|<1$, we have

Proof: Since $f(z) \in \int_{p,1}^{*}(\alpha,\beta,\lambda,n,z_0)$ and by (11) we get

$$\sum_{m=p}^{\infty} a_m \leq \frac{p(p+1)\beta\alpha}{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n-p(p+1)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}} \quad (20)$$

From (12) and (20) we have

$$a_0 = 1+\sum_{m=p}^{\infty}\left[\frac{m}{p}\right]a_m z_0^{m+p}$$

$$\leq 1+\frac{p(p+1)\beta\alpha}{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n-p(p+1)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}}\cdot\left[\frac{m}{p}\right]z_0^{m+p}$$

Or

$$a_0 \leq \frac{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n}{m[(m-1)+(1+p)(\alpha\beta+1)][\lambda(p+m)+1]^n-p(p+1)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}}. \quad (21)$$

Hence, we have

$$|f(z)| \geq a_0 r^{-p}+r^p\sum_{m=p}^{\infty}a_m$$

$$|f(z)| \geq \frac{m[(m-1)+(1+p)(1+\beta\alpha)[\lambda(p+m)+1]^n+p(1+p)\beta\alpha r^{2p}}{r^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n-p(1+p)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}\right]} \quad (m\geq p)$$

from (20) and (21). Further, the result is sharp for the function $f(z)$ which is given by

$$f(z) = \frac{m[(m-1)+(1+p)(1+\beta\alpha)[\lambda(p+m)+1]^n+p(1+p)\beta\alpha z^{2p}}{z^p\left[m[(m-1)+(1+p)(1+\beta\alpha)][\lambda(p+m)+1]^n-p(1+p)\beta\alpha\left[\frac{m}{p}\right]z_0^{m+p}\right]} \quad (22)$$

4. Closure Theorems:

In this section, we obtain closure theorems associate with the classes that are introduced in section 1.

Let $f_j(z)$, $j=1,2,\dots,q$, be defined as follows:

$$f_j(z) = a_{0,j}z^{-p}+\sum_{m=p}^{\infty}a_{m,j}z^m \qquad \left(a_{0,j}>0, a_{m,j}\geq 0, p\in N=\{1,2,\dots\}\right). \quad (23)$$

Theorem 6: Let $f_j(z)$ defined by (23) that belong to the class $\int_{p,0}^{*}(\alpha,\beta,\lambda,n,z_0)$ for $j=1,2,\dots,q$. Then, the function $h(z)$ which is defined by

$$h(z) = a_{0,j}z^{-p}+\sum_{m=p}^{\infty}c_m z^m \quad (24)$$

is also in the class $\int_{p,0}^{*}(\alpha,\beta,\lambda,n,z_0)$, where

$$c_m = \frac{1}{q}\sum_{j=1}^{q}a_{m,j} \quad (25)$$

Proof:_ Since $f_j(z) \in \int_{p,0}^{*} (\alpha,\beta,\lambda,n,z_0)(j=1,2,\dots,q)$ and by Theorem 2, we obtain

$$\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n+z_0^{m+p}p(p+1)\alpha\beta\right]a_{m,j} \le p(p+1)\alpha\beta \tag{26}$$

Hence,

$$\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n+z_0^{m+p}p(p+1)\alpha\beta\right]c_m$$
$$=\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n + z_0^{m+p}p(p+1)\alpha\beta\right]\left(\frac{1}{q}\sum_{j=1}^{q}a_{m,j}\right)$$
$$=\frac{1}{q}\sum_{j=1}^{q}\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n+z_0^{m+p}p(p+1)\alpha\beta\right]a_{m,j} \le p(p+1)\alpha\beta\,.$$

Therefore, $h(z) \in \int_{p,0}^{*} (\alpha,\beta,\lambda,n,z_0)$.

Theorem 7: Let $f_j(z)$ defined by (23) that belong to the class $\int_{p,1}^{*} (\alpha,\beta,\lambda,n,z_0)$ for $j=1,2,\dots,q$. Then the function $h(z)$ defined by (24) is also in the class $\int_{p,1}^{*} (\alpha,\beta,\lambda,n,z_0)$ for $j=1,2,\dots,q$, where c_m defined by (25).

Proof: Since $f_j(z) \in \int_{p,1}^{*} (\alpha,\beta,\lambda,n,z_0)(j=1,2,\dots,q)$ and by Theorem 3, we obtain

$$\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n-p(p+1)\alpha\beta\left[\frac{m}{p}\right]z_0^{m+p}\right]a_{m,j} \le p(p+1)\alpha\beta. \tag{27}$$

Hence,

$$\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n-p(p+1)\alpha\beta\left[\frac{m}{p}\right]z_0^{m+p}\right]c_m$$
$$=\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n - p(p+1)\alpha\beta\left[\frac{m}{p}\right]z_0^{m+p}\right]\left(\frac{1}{q}\sum_{j=1}^{q}a_{m,j}\right)$$
$$=\frac{1}{q}\sum_{j=1}^{q}\sum_{m=p}^{\infty}\left[m[(1+p)(\alpha\beta+1)+(m-1)][\lambda(p+m)+1]^n - p(p+1)\alpha\beta\left[\frac{m}{p}\right]z_0^{m+p}\right]a_{m,j} \le p(p+1)\alpha\beta. \tag{28}$$

Therefore, $h(z) \in \int_{p,0}^{*} (\alpha,\beta,\lambda,n,z_0)$.

5. Conclusions

New subclasses $\int_{p,i}^{*}(\alpha,\beta,\lambda,n,z_0)(i=0,1)$ of meromorphic uniformly of multivalent functions in $\mathbb{U}^* = \{z \in \mathbb{C}, 0 < |z| < 1\} = U/\{0\}$ with fixed second coefficient are considered. Many results are obtained, namely the estimation of coefficients, distortion theorems, closure theorems and others.

References

[1] Saif and A. Kilicman, "On certain subclasses of meromorphically p valent functions associated by the linear operator D_{ϑ}^{n} ", *J. Int. Appl*., vol. 2011, pp. 1-16, 2011.

[2] M. K. Aouf, "A generalization of meromorphic multivalent functions with positive coefficients", *Math., Japon*., vol. 35, pp. 609-614, 1990.

[3] M. K. Aouf, "Certain subclasses of meromorphically p-valent functions with positive or negative coefficients", *Math. Comput. Modelling*, vol. 47, no. 9-10, pp. 997-1008, 2008.

[4] M. K. Aouf, "On a class of meromorphic multivalent functions with positive coefficients", *Math. Japan*, vol. 35, pp. 603-608, 1990.

[5] Adnan Aziz Hussein and Kassim A. Jassim. "On Certain Class of Meromorphic Multivalent Functions Defined by Fractional Calculus Operator". *Iraqi Journal of Science*, vol. 60, no. 12, pp. 2685-2696, 2019.

[6] Adnan Aziz Hussein and Kassim A. Jassim. "Some Geometric Properties of Generalized Class of Meromorphic Multivalent Functions associated with Higher Ruschewyh Derivatives". *Iraqi Journal of Science*, vol. 60, no. 9, pp. 2036-2042, 2019.

[7] S. B. Joshi and H. M. Srivastava, "A certain family of meromorphically multivalent functions", *Comput. Math. Appl*., vol. 38, no. 3-4, pp. 201-211, 1999.

[8] M. L. Mogra, "Meromorphic multivalent functions with positive coefficients", *Math. Japonica*, vol. 35, no. 1, pp. 1-11, 1990.

[9] M. L. Mogra, "Meromorphic multivalent functions with positive coefficients", *Math. Japonica*, vol. 35, no. 6, pp. 1089-1098, 1990.

[10] S. Owa, H. E. "Darwish and M. K. Aouf, Meromorphic multivalent functions with positive and fixed second coefficients", *Math. Japonica*, vol. 46, no. 2, pp. 231-236, 1997.

[11] S. B. Joshi and M. K. Aouf, "A certain class of meromorphic multivalent functions with positive coefficients", *Ann Stiiut. Univ. Al. I. Cuza Iasi s. I. a., Mat*. (N. S.) vol. 41, pp. 211-228, 1995.

[12] M. K. Aouf and H. E. Darwish, "Certain meromorphically starlike functions with positive and fixed second coefficients", *Turkish J. Math*., vol. 21, pp. 311-316, 1997.

[13] M. K. Aouf and H. E. Darwish, "A certain subclass of p-valent meromorphically starlike functions with alternating coefficients", *Indian J. Pure Appl. Math*., vol. 39, no. 2, pp. 157-166, 2008.

[14] R. K. Raina and H. M. Srivastava, "A new class of meromorphically multivalent functions with applications to generalized hypergeometric functions", *Math. Comput. Modelling*, vol. 43, no. 3-4, pp. 350-356, 2006.

[15] A. Uralegaddi and M. D. Ganigi, "Meromorphic multivalent functions with positive coefficients", *Nepali Math. Sciences Report*, vol. 11, no. 2, pp. 95-102, 1986.

[16] Raid Habib Buti and Kassim A. Jassim. "On A Class of W-Valent Functions With Two Fixed Points Involving Hypergeomatric Function With Generalization Integral Operator". *Iraqi Journal of Science*, vol. 60, no. 80, pp, 1753-1759, 2019.

[17] G. Yang, "On new subclasses of meromorphic p-valent functions", *J. Math. Research and Exposition*, vol. 15, no. 1, pp. 7-13, 1995.