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Modified Iterative Method for Solving Sine - Gordon Equations

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Abstract

The basic goal of this research is to utilize an analytical method which is called the Modified Iterative Method in order to gain an approximate analytic solution to the Sine-Gordon equation. The suggested method is the amalgamation of the iterative method and a well-known technique, namely the Adomian decomposition method. A method minimizes the computational size, averts round-off errors, transformation and linearization, or takes some restrictive assumptions. Several examples are chosen to show the importance and effectiveness of the proposed method. In addition, a modified iterative method gives faster and easier solutions than other methods. These solutions are accurate and in agreement with the series of solutions that are provided by analytical results. To evaluate the outcomes in the modified iterative process, we have used the Matlab symbolic manipulator.

Keywords: Iterative Method, Adomian Decomposition Method, Modified Iterative Method, Sine-Gordon equation.

الطريقة التكرارية المطورة لحل معادلات ساين – جوردون

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الخلاصة

الهدف الأساسي لهذا البحث ، هو أسلوب تحليلي يسمى الطريقة التكرارية المطورة يتم استخدامه للحصول على الحل التحليلي التقريبي لمعادلة ساين – جوردون. الطريقة المقترحة هي دمج الطريقة التكرارية والتقنية المعروفة جيدا طريقة ادومين التحليلية. تقلل الطريقة من الحجم الحسابي ، تتفادى أخطاء التقريب ، التحويل ، الخطية أو اتخاذ بعض الافتراضات المقيدة. لقد اخترنا العديد من الأمثلة لإظهار أهمية وفعالية الطريقة المقترحة. بالإضافة إلى ذلك ، فإن الطريقة التكرارية المطورة تعطي حلولاً أسرع وأسهل ، وتأتي هذه الحلول دقيقة ومتوافقة مع الحل المتسلسل الذي توفره النتائج التحليلية. لحساب النتائج في العمليات التكرارية المطورة استخدم البرنامج الحاسوبي الماتلاب.

1. Introduction

The Sine-Gordon equation is one of the most critical nonlinear evolution equations that plays a vital part in engineering and physical science such as stability of fluid motions,

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propagation of magnetic flux and in applied sciences. In the nineteenth century, the Sine-Gordon equation was first introduced to study the different problems in differential geometry, relativistic field theory, and mechanical transmission lines. [1-6].

In this article, we apply the modified iterative method(MIM) to find the solution to the Sine-Gordon nonlinear equation with initial conditions(ICs):

$$z_{tt}(x,t) - \alpha^2 z_{xx}(x,t) - \beta \sin(z(x,t)) = 0, \quad (1.1)$$
$$z(x,0) = p(x), z_t(x,0) = v(x), \quad (1.2)$$

where α and β are constants.

The Sine-Gordon equation equations play significant a role in specifying such problems. These equations are very efficient tools to characterize real-life phenomena and appear in many applications in different fields of the sciences. In most cases, theoretical or numerical solutions are intractable to find. Thus, their exact solution is more complicated to find a comparison to the other solutions of linear equations. In recent years, searching for new methods to solve nonlinear partial differential equations has received great interest, for example, in [7-9] Adomian decomposition method (ADM), homotopy analysis method (HAM) [10], variational iteration method(VIM) [11,12], modified variational iteration method (MVIM)[13, 14], the natural decomposition method (NDM)[15], the Daftardar-Gejji and Jafari method (DJM) [16]. These methods have been applied to gain an approximate solution to the Sine-Gordon equation.

The iterative method (IM) was presented by Temimi and Ansari [17], it was successfully used for solving nonlinear and linear functional equations, ordinary differential equations (ODEs), partial differential equations (PDEs), higher-order integro differential equations (HOIDEs), nonlinear delay differential equations(NDDEs), Korteweg-de Vries equations (KdVs) and Volterra - Fredholm integro differential equations (VFIDEs), see [18-23].

In this work, the approximate solution to the Sine-Gordon equation (1.1) is found by using a new modified iterative method. To the best of our knowledge, the modified method is not yet implemented to resolve the Sine-Gordon equation. The nonlinear idioms are cunningly computed utilizing Adomian polynomials. A reliable method gives a series solution which rapidly converges to an exact or an approximate solution with swiftly computational idioms. The obtained results are symmetrical with the variational iteration method, the natural decomposition method and the reduced differential transform method(RDTM) [3,14,15] and exact solution results.

2. Essential concept of the iterative method[17]

Basic steps of the purposed method, each partial differential equation can be created as follows**:**

$$L\big(z(x,t)\big) + N\big(z(x,t)\big) + q(x,t) = 0, \quad (2.1)$$

With the conditions $C(z, \frac{\partial z}{\partial t}) = 0.$ (2.2)

Unknown function $z(x,t)$ is independent variables point to x and t, while L and N indicate to linear and nonlinear operators, respectively. A known function $q(x,t)$ performs in-homogeneous expression, and C is the provision operator for a problem. The fundamental goal of the iterative method to resolve Eq. (2.1) with the initial conditions Eq.(2.2), which is an initial guess, via presuming that the premier guess $z_0(x,t)$ is the solution of a problem $z(x,t)$ and solution of an equation

$$L(z_0(x,t)) + q(x,t) = 0,\ C(z_0, \frac{\partial z_0}{\partial t}) = 0, \quad (2.3)$$

To create the next iteration of a solution, we set Eq.(2.1) to follow

$$L(z_1(x,t)) + q(x,t) + N(z_0(x,t)) = 0,\ C(z_1, \frac{\partial z_1}{\partial t}) = 0, \quad (2.4)$$

After several iterations strides of the solution, the general form of an equation can be get

$$L(z_{n+1}(x,t)) + q(x,t) + N(z_n(x,t)) = 0,\ C(z_{n+1}, \frac{\partial z_{n+1}}{\partial t}) = 0, \quad (2.5)$$

Obviously, every iterate of the function $z_n(x,t)$ performs efficacious the alone solution of Eq.(2.1)

3. Analysis of the Modified Iterative Method

The simplest idea of the reliable analysis method, the Eq.(1.1) can be expressed as follows :

$$L(z(x,t)) = \alpha^2 z_{xx}(x,t) + \beta \sin(z(x,t)), \quad (3.1)$$

The differential operator $L(z(x,t))$ is the highest order derivative in the Eq.(3.1), via utilizing the specified the initial conditions in Eq.(1.2) and a known function $q(x,t)$in the Sine-Gordon equation is equal to zero, then we integrate both sides of the Eq.(3.1) from 0 to t twice we get

$$z(x,t) = \Psi(x,t) + \int_0^t \int_0^t (\alpha^2 z_{xx}(x,t) + \beta\, sin(z(x,t)))\, dt, \quad (3.2)$$

A function $\Psi(x,t)$ is arising via integrating the source idiom of applying the initial conditions in Eq.(1.2) that are specified.

The following algorithm explains how this technique works:

Step1: To get $z_0(x,t)$ we solve $L(z_0(x,t)) = q(x,t)$ with the initial conditions in Eq.(1.2) and $q(x,t) = 0$, (3.3)

Integrating both sides of Eq.(3.3) from 0 to t twice, we obtain

$z_0(x,t) = \Psi(x,t)$,

Step2: The next iteration is $L(z_1(x,t)) = q(x,t)$, with the initial conditions in Eq.(1.2) and $q(x,t) = 0$, (3.4)

Integrating both sides of Eq.(3.4) from 0 to t twice, then use Eq.(3.1) to obtain

$z_1(x,t) = \Psi(x,t) + \int_0^t \int_0^t (\alpha^2 z_{0xx}(x,t) + \beta\, sin(z_0(x,t)))\, dt$,

Step3: After iteration strides of the solution, the general form of the equation is given as follows:

$$L(z_{n+1}(x,t)) = q(x,t), \text{ with ICs in Eq.(1.2) and } q(x,t) = 0, \quad (3.5)$$

Solving and integrating both sides of Eq.(3.5) from 0 to t twice, then use Eq.(3.1) to obtain

$$z_{n+1}(x,t) = \Psi(x,t) + \int_0^t \int_0^t (\alpha^2 z_{nxx}(x,t) + \beta\, sin(z_n(x,t)))\, dt, \quad (3.6)$$

It is clear that every iteration of the function $z_{n+1}(x,t)$ performs efficacious the only solution of Eq. (3.1). The basic idea of the modified iterative method is to use the Adomian's polynomials with the iterative method for solving nonlinear Sine-Gordon equation, the nonlinear terms are elegantly computed using Adomian polynomials. A modified version of the iterative method is gained by coupling the correction functional (3.6) of the iterative method with Adomian's polynomials [5, 24] and it is given via

$$z_{n+1}(x,t) = \Psi(x,t) + \int_0^t \int_0^t (\alpha^2 z_{nxx}(x,t) + \beta\, A_n)\, dt\,, \tag{3.7}$$

Nonlinear idiom $\sin(z(x,t))$ is decomposed as in [25]:

$$\sin\big(z(x,t)\big) = A_n, \tag{3.8}$$

where A_n are so-called Adomian's polynomials and the above composition (3.7) is the suggested method which is called the modified iterative method and it can be easily calculated with the next formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_0^n \lambda^i z_i(x,t)\right)\right]_{\lambda=0}, \tag{3.9}$$

where n = 0, 1, 2, ...

Some few components of A_n are calculated below

$A_0 = \sin(z_0(x,t))$,

$A_1 = z_1(x,t)\cos\big(z_0(x,t)\big)$,

$A_2 = z_2(x,t)\cos\big(z_0(x,t)\big) - \frac{1}{2!} z_1^2(x,t)\sin(z_0(x,t))$,

$A_3 = z_3(x,t)\cos\big(z_0(x,t)\big) - z_2(x,t) z_1(x,t)\sin(z_0(x,t)) - \frac{1}{3!} z_1^3(x,t)\cos(z_0(x,t))$,

$\vdots$

In the previous algorithm, the modified iterative method has the worthiness that the solution can be found with easier strides manually.

4. Applications [3,14,15]

In this part, the enforcement of the modified iterative method to the nonlinear Sine-Gordon equation is clearly illustrated to show its high accuracy and simplicity.

Example 1

Consider the nonlinear Sine-Gordon equation of the form:

$$z_{tt}(x,t) - z_{xx}(x,t) = \sin(z(x,t)), \tag{4.1}$$

subject to the initial conditions :

$$z(x,0) = \frac{\pi}{2}, z_t(x,0) = 0, \tag{4.2}$$

By applying the same steps in the previous part, the modified iterative method algorithm will be applied at the following equations (4.1), (4.2), $\alpha = \beta = 1$ and a known function $q(x,t)$in the Sine-Gordon equation is equal to zero. We firstly start by resolving the initial conditions so as to find the premier guess $z_0(x,t)$ as:

$$L\big(z(x,t)\big) = z_{tt}(x,t) + q(x,t)\ ,\ N\big(z(x,t)\big) = z_{xx}(x,t) + \sin(z(x,t)) \text{ and } q(x,t) = 0, \tag{4.3}$$

So, the primary step is $L\,(z_0(x,t)) = 0$,with $z_0(x,0) = \frac{\pi}{2}$,$(z_0)_t(x,0) = 0$, (4.4)

Then, the general relation as follows

$$L\big(z_{n+1}(x,t)\big) - N\,\big(z_n\,(x,t)\big) = 0,\ z_{n+1}(x,0) = \frac{\pi}{2}\ , (\,z_{n+1})_t(x,0) = 0 \text{ and } q(x,t) = 0, \tag{4.5}$$

resolving the problem which is determined in Eq. (4.4), we get: $z_0(x,t) = \frac{\pi}{2}$,

The first iteration can be get is as follows:

$$(z_1(x,t))_{tt} = (z_{0xx}(x,t) + A_0\,), \text{ with } z_1(x,0) = \frac{\pi}{2},\quad (z_1)_t(x,0) = 0, \tag{4.6}$$

And the non-linear idiom $\sin(z(x,t))$ is decomposed as: $A_0 = \sin(z_0(x,t))$.

Thus, we gain the following approximations of Eq. (4.6) as: $z_1(x,t) = \frac{\pi}{2} + \frac{1}{2}t^2$,
The second iteration is

$$(z_2(x,t))_{tt} = (z_{1xx}(x,t) + A_1), \text{ with } z_2(x,0) = \frac{\pi}{2}, (z_2)_t(x,0) = 0, \quad (4.7)$$

And the non-linear idiom $\sin(z(x,t))$ is decomposed as: $A_1 = z_1(x,t)\cos(z_0(x,t))$,
Thus, the solution of Eq. (4.7) as: $z_2(x,t) = \frac{\pi}{2}$,
The third iteration is:

$$(z_3(x,t))_{tt} = (z_{2xx}(x,t) + A_2), \text{ with } z_3(x,0) = \frac{\pi}{2}, \quad (z_3)_t(x,0) = 0, \quad (4.8)$$

And the non-linear idiom $\sin(z(x,t))$ is decomposed as:
$A_2 = z_2(x,t)\cos(z_0(x,t)) - \frac{1}{2!}z_1^2(x,t)\sin(z_0(x,t))$,
Then, the solution of Eq.(4.8) as: $z_3(x,t) = \frac{\pi}{2} - \frac{\pi^2}{16}t^2 - \frac{\pi}{48}t^4 - \frac{1}{240}t^6$,

As well, by the same strides, other resolutions can be generated from computing such problems via using Matlab symbolic manipulator, we get: $z_4(x,t) = \frac{\pi}{2} - \frac{\pi^2}{8}t^2 - \frac{\pi}{48}t^4$. Hence, in iteration steps, we have the series solution as follows

$$z_n(x,t) = \frac{\pi}{2} + \frac{1}{2}t^2 - \frac{1}{240}t^6 + \frac{1}{34560}t^{10} + \cdots$$

Example 2

Consider the nonlinear Sine-Gordon equation in Eq.(4.1) with the following initial conditions

$$z(x,0) = \frac{\pi}{2}, \; z_t(x,0) = 1, \quad (4.9)$$

By implementing the same algorithm in the previous example, we firstly start by resolving the initial conditions in Eq. (4.9) so as to find the premier guess $z_0(x,t)$, the modified iterative method start by the same step in Eq. (4.1) and a known function $q(x,t)$in the Sine-Gordon equation is equal to zero. So, the primary step is $L(z_0(x,t)) = z_{tt}(x,t) + q(x,t)$,with $z_0(x,0) = \frac{\pi}{2}$, $(z_0)_t(x,0) = 1$ and $q(x,t) = 0$ (4.10)

Then, the general relation is as follows:

$$L(z_{n+1}(x,t)) - N(z_n(x,t)) = 0, \; z_{n+1}(x,0) = \frac{\pi}{2}, (z_{n+1})_t(x,0) = 1, \text{ and } q(x,t) = 0, \quad (4.11)$$

Solving the elementary problem specified in Eq. (4.10) to get: $z_0(x,t) = \frac{\pi}{2} + t$,
The first iteration can be get as:

$$(z_1(x,t))_{tt} = (z_{0xx}(x,t) + A_0), \text{ with } z_1(x,0) = \frac{\pi}{2}, (z_1)_t(x,0) = 1, \quad (4.12)$$

And the nonlinear idiom $\sin(z(x,t))$ is decomposed as: $A_0 = \sin(z_0(x,t))$,

Thus, we gain the following approximations of Eq. (4.12) as: $z_1(x,t) = \frac{\pi}{2} + t + 1 - \cos t$,
The second iteration is

$$(z_2(x,t))_{tt} = (z_{1xx}(x,t) + A_1), \text{ with } z_2(x,0) = \frac{\pi}{2}, \quad (z_2)_t(x,0) = 1, \quad (4.13)$$

And the nonlinear idiom $\sin(z(x,t))$ is decomposed as: $A_1 = z_1(x,t)\cos(z_0(x,t))$,
Thus, the solution of Eq. (4.13) as:
$z_2(x,t) = \frac{\pi}{2} + t + \sin t - \frac{3}{4}t - \frac{1}{8}\sin 2t - 4\sin(\frac{t}{2})^2 - \frac{\pi}{2}t + \frac{\pi}{2}\sin t + t\sin t$,

$$\vdots$$

Hence, in iteration steps, we have the series solution is given by

$$z_n(x,t) = \frac{\pi}{2} + t + \frac{1}{2!}t^2 - \frac{1}{4!}t^4 + \cdots$$

Example 3

Consider the nonlinear Sine-Gordon equation in Eq.(4.1) with the following initial conditions

$$z(x,0) = \pi,\ z_t(x,0) = 1, \tag{4.14}$$

Applying the suggested method to Eq. (4.1), Eq.(4.14) and a known function $q(x,t)$in the Sine-Gordon equation is equal to zero. By the same steps, in order to find the premier guess $z_0(x,t)$, we obtain:

$z_0(x,t) = \pi + t.$

Hence, in iteration steps, we have: $z_1(x,t) = \pi + \text{sint}$,

$z_2(x,t) = \pi\text{cost} + \frac{3}{4}t + \frac{1}{4}\text{cost sint}$,

$z_3(x,t) = \pi + 2t + \frac{\pi^2}{2}t - \frac{29}{16}\text{sint} - \frac{\pi^2}{2}\sin t - \frac{\pi}{2}\sin t^2 - \frac{1}{48}\text{sint}^3 + \frac{1}{16}\sin t\cos t^2 + \frac{3}{4}t\cos t$,

$$\vdots$$

The series solution is gained via : $z_n(x,t) = \pi + t - \frac{1}{3}t^3 + \cdots$

Example 4

Consider the nonlinear Sine-Gordon equation in Eq.(4.1) with the following initial conditions

$$z(x,0) = \frac{3\pi}{2}, z_t(x,0) = 1\ . \tag{4.15}$$

Applying the suggested method to Eq. (4.1), Eq.(4.15) and a known function $q(x,t)$in the Sine-Gordon equation is equal to zero. By the same steps, in order to find the initial guess $z_0(x,t)$, we obtain:

$$z_0(x,t) = \frac{3\pi}{2} + t$$

Hence, in iteration steps, we have: $z_1(x,t) = \frac{3\pi}{2} + \ t + \text{cost} - 1$

$$z_2(x,t) = \frac{3\pi}{2} + \frac{3\pi}{2}t + \text{sint} + \frac{1}{4}t - \text{tsint} - \frac{3\pi}{2}\sin t - \frac{1}{4}\cos t\sin t + 4\sin(\tfrac{t}{2})^2$$

$$\vdots$$

We have the series solution obtained via

$$z_n(x,t) = \frac{3\pi}{2} + t - \frac{1}{4}t^2 + \cdots$$

Our examples are solved by the series solution of the modified iterative method which is in close agreement with the outcome gained by (RDTM), (MVIM) and (NDM), respectively [3-14-15]. The core of the method in comparison with other analytical methods,does not require big calculations like the Lagrange multiplier in the VIM or extended transformation and deductive homotopy polynomials in HPM. Moreover, the technique showed that it is effective in overcoming the risque in computation and resolving the nonlinear Sin-Gordon Equation with easier steps.

5. Error Analysis[17]

Error exemplifies an axial part of approximate solutions, when a lower error is shown, a more precise solution and nearer to an accurate solution is obtained, which points to the reliability and promptness of the proposed technique. An approximate solution is usually applied to puzzle out different problems which cannot be solved within analytic mathematical methods. That means there is an error value we have to compute. If one can determine an accurate error, so the perfect solution will be got. Thus, getting an accurate error is impossible. Then, we attempt to obtain an estimate of the error (i.e. a value which is not overridden by error). For the error analysis of results, we offer the consecutive errors as follows:

$$\mathbb{E}_n = \|z_{n+1}(x,t) - z_n(x,t)\|$$

for n= 0, 1, 2, … that are the differences between two consecutive duplicate solutions. We are transaction for an analytic continued solution, so as to calculate that variances we use the L_2-norm [17].

$$\|z_{n+1}(x,t) - z_n(x,t)\| = \sqrt{\int (z_{n+1}(x,t) - z_n(x,t))^2}$$

Error term $\mathbb{E}_n$ amidst two successive solutions for problems

n	$\mathbb{E}_4$ of Ex1	$\mathbb{E}_2$ of Ex2	$\mathbb{E}_3$ of Ex3	$\mathbb{E}_2$ of Ex4
0	2.23607 E^{-001}	2.10671 E^{-001}	6.05939E^{-002}	**2.10671E^{-001}**
1	0.05	1.13495E^{-001}	4.29733E^{-001}	**2.63761E^{-001}**
2	2.97882E^{-001}	-	4.18682E^{-001}	**-**
3	2.74829E^{-001}	-	-	**-**

6. Conclusions

In this paper, the modified iterative method is successfully applied to gain the solution of the nonlinear Sine-Gordon equation with smooth steps. The modified iterative method gives a series of solution which swiftly converges to an approximate or exact solution. Moreover, the method minimizes the computational size and averts round-off errors. Thus, the proposed method can be easily used to obtain the series solutions, so the outcomes of this study are discussed to be seen how fruitful this method is in terms of being a perfect, accurate and rapid tool with a little effort compared to other iterative methods. The modified iterative method is a very promising technique for solving the nonlinear Sine-Gordon equation due to its efficiency and high accuracy.

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