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Single Stage Shrinkage Estimation Methods for Reliability Function of Generalized Exponential Distribution Using Simulation

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Abstract

The reliability in stress–strength model (s–s) is estimated in this article when the system has parallel and series components subject to one of the stresses and it follows generalized exponential distribution by taking via different estimation methods, namely the maximum likelihood estimation, unbiased estimation, moment estimation and shrinkage estimation. Also, the Mont Carlo method is used to compare these methods under mean squared error.

Keywords: Generalized exponential distribution (GED), Maximum likelihood, Unbiased estimation, Moment estimation, Shrinkage estimation.

طرق تقدير التقلص بمرحلة واحدة لدالة الموثوقية للتوزيع الأسي العام باستخدام المحاكاة

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الخلاصة

يتم تقدير الموثوقية في نموذج القوةالإجهاد في هذا البحث عندما يكون للنظام مكونات متوازية ومتوالية تخضع لإحدى الإجهادات ويتبع التوزيع الأسي العام عن طريق أخذ طرق تقدير مختلفة ، طريقة الامكان الاعظم للتقدير ، تقدير غير متحيز. تقدير العزوم وتقدير التقلص .أيضًا يتم استخدام طريقة مونت كارلو لمقارنة هذه الطرق تحت متوسط الخطأ التربيعي.

1- Introduction

The term stress–strength (s–s) refers to a component that has a random strength X subject to a random stress Y to evaluate reliability, A failure occurs in the simplest form of the stress–strength model when the unit's strengths drop below the stress. Then, we will study two models as follows;

1- The series system reliability Rs for (s–s) model[1],[2] and [3];

$Rs = P\,[\mathrm{Max}\,(Y_{r+1}, Y_{r+2}) < \min(Y_1, Y_2, \ldots, Y_r)],$

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where Y_i ; $(i = 1, 2, \ldots, r)$ is strength subject to common stress Y_i ; $(i = k + 1, k + 2)$.
2- The parallel system reliability Rp for (s – s) model;
$\mathrm{Rp} = \mathrm{p}\,[\mathrm{Max}\,(\mathrm{Y}_{r+1}, \mathrm{Y}_{r+2}) < \mathrm{Max}(\mathrm{Y}_1, \mathrm{Y}_2, \ldots, \mathrm{Y}_r)]$
Where Y_i ; $(i = 1, 2, \ldots, r)$ is strength and Y_i ; $(i = r + 1, r + 2)$ are stress.

Many researchers studied the parallel and series of system reliability in the stress–strength model. For example, in 1996, Hanagal studied the reliability consist many parallel components and two strength that belongs to bivariate Petro[6]. In 2011, Munoli and Rolest derived the reliability function for a parallel system of two components. In 2013, Srinivasa estimated an (s–s) reliability system of the multi–component model when the random stress and strength followed the generalized exponential distribution[15]. In 2013, Alaa estimated stress–strength reliability for a parallel system using Lomax distribution. In 2018, Cheng demon started the reliability of the system in the (s–s) model of parallel components based on the Pareto exponential distribution[1]. On the other hand, the generalized exponential distribution denoted by (GED) is also known as two parameters was important distribution with extensive uses in all tests of life as it enters in the tests of population growth, and engineering. In 1967, Ahuja and Nash reached the generalized exponential distribution (GED) as a special case of Gompertz distribution. In 1999, Gupta and Kundu found the generalized exponential distribution as special case of the generalized Weibull distribution[5]. In this paper, the estimation of $Rs = p\,[Max\,(Y_{r+1}, Y_{r+2}) < \mathrm{Min}(Y_1, Y_2, \ldots, Y_r)]$ and $Rp = p\,[Max\,(Y_{r+1}, Y_{r+2}) < \mathrm{Max}\,(Y_1, Y_2, \ldots, Y_r)]$ are considered.

When $(Y_1, Y_2, \ldots, Y_r)$ are strengths subject to one of the stresses (Y_{r+1}, Y_{r+2}). Assuming that $Y_1, Y_2, \ldots, Y_r, Y_{r+1}, Y_{r+2}$ follow the generalized exponential distribution, we also derived the maximum likelihood estimation, unbiased estimation, moment estimation and shrinkage estimation of unknown parameters $(\, , \propto_1, \ldots, \propto_r, \propto_{r+1}, \propto_{r+2})$ and for the system reliability of parallel and series components in (s – s) model.

2- System Reliability

We take a multi–component system with r identical components. Let $(Y_1, Y_2, \ldots, Y_r)$ be the strengths of components subject to one of the stresses subject to one of the stresses (Y_{r+1}, Y_{r+2}). Also, let $(Y_1, Y_2, \ldots, Y_r, Y_{r+1}, Y_{r+2})$ be independent and it follows the generalized exponential distribution with shape parameters α_i $(i = 1, 2, \ldots, r, r + 1, r + 2)$ and the common scale parameter . The p.d.f. and c.d.f. of Y_i are respectively given below [9] and [10]

$$f_i(y) = \alpha_i\, \tau\, e^{-\tau y}(1 - e^{-\tau y})^{\alpha_i - 1}$$

$$F_i(y) = (1 - e^{-\tau y})^{\alpha_i}, \quad y > 0, \alpha_i > 0, \tau > 0, \quad i = 1, 2, \ldots, r, r + 1, r + 2$$

So, the distribution function of $w = \max(Y_{r+1}, Y_{r+2})$ given as below

$$H(w) = p(W < w) = \prod_{i=r+1}^{r+2} p(Y_i < w)$$
$$= (1 - e^{-\tau w})^{\propto_{r+1}} . (1 - e^{-\tau w})^{\propto_{r+2}}$$
$$= (1 - e^{-\tau w})^{\propto_{r+1} + \propto_{r+2}}$$

Also, distribution function of $v = \max\,(Y_1, Y_2 \ldots, Y_r)$) is given below

$$G_v(v) = p(V < v) = \prod_{i=1}^{r}(Y_i < v)$$
$$= p(Y_1 < v)\, p\,(Y_2 < v) \ldots p\,(Y_r < v)$$
$$= (1 - e^{-\tau v})^{\propto_1} (1 - e^{-\tau v})^{\propto_2} \ldots (1 - e^{-\tau v})^{\propto_r}$$
$$= (1 - e^{-\tau v})^{\propto_1 + \propto_2 + \cdots + \propto_r}$$

In a parallel system, the system reliability will be

$$Rp = p[max(Y_{r+1}, Y_{r+2}) < \max(Y_1, Y_{2, \ldots,} Y_r)]$$

$$= p[W < V] = \int_0^{\infty} p[w < v] \quad dGv\,(v)$$

$= \int_0^{\infty} (1 - e^{-\tau v})^{\sum_{i=r+1}^{r+2} \propto_i} . \sum_{i=1}^{r} \alpha_i \ \tau \ e^{-\tau v} \ (1 - e^{-\tau v})^{\sum_{i=1}^{r} \propto_i - 1} \ dv$

$$= \sum_{i=1}^{r} \alpha_i \int_0^{\infty} (1 - e^{-\tau v})^{\sum_{i=1}^{r+1} \propto_i - 1} . \tau . \ e^{-\tau v} \ dv$$

$= \frac{\sum_{i=1}^{r} \propto_i}{\sum_{i=1}^{r+2} \propto_i}$

So, we have

$$\text{Rp} = \frac{\sum_{i=1}^{r} \propto_i}{\sum_{i=1}^{r+2} \propto_i}$$

In the series system, the system reliability is given as below

$Rs = p[\max(Y_{r+1}, Y_{r+2}) < \min(Y_1, Y_2, \dots, Y_r)]$

$= \int_0^{\infty} \prod_{i=1}^{r} \left(1 - (1 - e^{-\tau w})^{\propto i}\right) . \sum_{i=r+1}^{r+2} \propto_i \ (1 - e^{-\tau w})^{\sum_{i=r+1}^{r+2} \propto_i - 1} \ \tau . e^{-\tau w} \ dw$

$$= \sum_{i=r+1}^{r+2} \propto_i \ . \ \tau \int_0^{\infty} \prod_{i=1}^{r} \left(1 - (1 - e^{-\tau w})^{\propto i}\right) . \ e^{-\tau w} . (1 - e^{-\tau w})^{\sum_{i=r+1}^{r+2} \propto_i - 1} * dw$$

Let $Z = (1 - e^{-\tau w})$ then $w = \frac{-\ln(1-z)}{\tau}$ and $dw = \frac{dz}{\tau(1-z)}$

Thus, $\prod_{i=1}^{r} (1 - (1 - e^{-\tau w})^{\propto i}) = \prod_{i=1}^{r} \left(1 - z^{\propto i}\right)$

$$= 1 - \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_{1\neq}, i_{2\neq}, \dots, \neq i_{r=1}}^{r} z^{\sum_{j=1}^{m} \propto ij}$$

Therefore,

$$Rs = \sum_{i=r+1}^{r+2} \propto i . \tau \int_0^1 (1 - \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_{1\neq}, i_{2\neq}, \dots, \neq i_{r=1}}^{r} z^{\sum_{j=1}^{m} \propto ij}) . (1 - z) \ z^{\sum_{i=r+1}^{r+2} \propto_i - 1} \ \frac{dz}{\tau(1 - z)}$$

$$= \sum_{i=r+1}^{r+2} \propto i . \int_0^1 (1 - \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_{1\neq}, i_{2\neq}, \dots, \neq i_{r=1}}^{r} z^{\sum_{j=1}^{m} \propto ij}) \ . \ z^{\sum_{i=r+1}^{r+2} \propto_i - 1} \ dz$$

$$= \sum_{i=r+1}^{r+2} \propto i . \int_0^1 z^{\sum_{i=r+1}^{r+2} \propto_i - 1} \ dz$$

$$- \sum_{i=r+1}^{r+2} \propto_i \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_{1\neq}, i_{2\neq}, \dots, \neq i_{r=1}}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \propto_{ij} + Z^{\sum_{i=r+1}^{r+2} \propto_i - 1}} \ dZ$$

So, we have

$$Rs = 1 - \sum_{i=r+1}^{r+2} \propto_i \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_{1\neq}, i_{2\neq}, \dots, \neq i_{r=1}}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \propto_i j + \sum_{i=r+1}^{r+2} \propto_i - 1} \ dZ$$

3- The Maximum Likelihood Estimators (MLE)

Let $Y_{i1}, Y_{i2}, \dots, Y_{ir}$ $(i = 1,2, \dots, n)$ be a random sample on strengths of (n) systems following the generalized exponential distribution with shape parameters $\propto_i$ $(i = 1,2, \dots, r)$ and scale parameters τ. Let Y_{ir+1}, Y_{ir+2} $(i = 1,2, \dots, n)$ be a random sample on stresses of (n) systems which are following the generalized exponential distribution with shape parameter $\propto_i$ $(i = r + 1, r + 2)$ and common scale parameter τ. The likelihood function of y_{ij}; $i = 1,2, \dots, n$ and $j = 1,2, \dots, r, r + 1, r + 2$

$$L\left(\propto_i, {}^{\tau}/_{y_{ij}}\right) = \prod_{i=1}^{r+2} \alpha_i^n \, \tau^{n(r+2)} . e^{-\tau \sum_{j=1}^{n} \sum_{i=1}^{r+2} y_{ij}} \prod_{j=1}^{n} \prod_{i=1}^{r+2} (1 - e^{-\tau y_{ij}})^{(\propto_i - 1)}$$

The log – likelihood function will be

$\text{Ln } L(\propto_i, \tau/y_{ij}) = n \sum_{i=1}^{r=2} \ln \propto_i + n(r+2) \ln \tau - \tau \sum_{j=1}^{n} \sum_{i=1}^{r+2} y_{ij} + (\propto_i - 1) \sum_{j=1}^{n} \sum_{i=1}^{r+2} \ln(1 - e^{-\tau y_{ij}})$

Differentiating log – likelihood function with respect to $\propto_i$ $(i = 1,2, \dots, r, r+1, r+2)$ and equating it to zero, we get

$$\frac{n}{\propto_i} + \sum_{j=1}^{n} \ln(1 - e^{-\tau y_{ij}}) = 0$$

$$\hat{\propto}_{imle} = \frac{-n}{\sum_{j=1}^{n} \ln(1 - e^{-\tau y_{ij}})}$$

Hence, the estimates of Rp and Rs are respectively given by

$$\hat{R}_{pmle} = \frac{\sum_{i=1}^{r} \hat{\propto}_{imle}}{\sum_{i=1}^{r+2} \hat{\propto}_{imle}}$$

And

$$\hat{R}_{smle} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{imle} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{ijmle} + \sum_{i=r+1}^{r+2} \hat{\propto}_{imle} - 1} \, dz$$

4- Unbiased Estimation (UBE)

The bias of an estimator is the difference between the expected value of the estimator and the true value of the parameter being estimated.

IF $Y_{ij} \sim G\,E\,D\,(\tau, \propto_i)$ for $j = 1,2, \dots, n : i = 1,2, \dots, r, r+1, r+2$

When τ is known, then $ln(1 - e^{-\tau y_{ij}})^{-1} \sim exp\,(\frac{1}{\propto_i})$

But $\sum_{j=1}^{n} ln(1 - e^{-\tau y_{ij}})^{-1} \sim Gamma\,(n, \frac{1}{\propto_i})$

While $\frac{n}{\sum_{j=1}^{n} \ln(1 - e^{-\tau y_{ij}})} \sim$ inverted $Gamma\,(n, \propto_i n)$ for $i = 1,2, \dots, r, r+1, r+2$

That means $\hat{\propto}_{imle} \sim Gamma\,(n, \propto_i n)\ since\ E(\hat{\propto}_{imle}) = \frac{n}{n-1} \propto_i$

Thus, $\hat{\propto}_{iub} = \frac{n-1}{n}\ \hat{\propto}_{imle} = \frac{n-1}{n} \frac{-n}{\sum_{j=1}^{n} \ln(1 - e^{-\tau y_{ij}})} = \frac{n-1}{-\sum_{j=1}^{n} \ln(1 - e^{-\tau y_{ij}})}$

It becomes unbiased estimator of $\propto_i$ $\ for\ i = 1,2, \dots, r, r+1, r+2$ with

$var\,(\hat{\propto}_{iub}) = \frac{(\propto_i)^2}{(n-2)}$

Hence, the estimates of Rp and Rs are respectively given by

$$\hat{R}_{pub} = \frac{\sum_{i=1}^{r} \hat{\propto}_{iub}}{\sum_{i=1}^{r+2} \hat{\propto}_{iub}}$$

And

$$\hat{R}_{sub} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{iub} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{iub} + \sum_{i=r+1}^{r+2} \hat{\propto}_{iub} - 1} \, dz$$

5- Moment Estimation (MOM)

Let $Y_{i1}, Y_{i2}, \dots, Y_{ir}$ $; i = 1,2, \dots, n$ be a random sample of strength of (n) system following the generalized exponential destitution with shape parameters $\propto_i$; $(i = 1,2, \dots, k)$, And scale parameter . Let Y_{ik+1}, Y_{ik+2} $(i = 1,2,\dots,n)$ be a random sample on stresses having (n) system which are following generalized exponential distribution with shape parameters $\propto_i$ $(i = r+1, r+2)$ and common scale parameter τ. The population mean of random variables y_i which follows the generalized exponential distribution is given below

$$\mathrm{E}\,(y_i) = \frac{1}{\tau}\left(\Psi(\propto_i + 1) - \Psi(1)\right)$$

equaling the sample mean with the corresponding population mean, we obtain

$$\bar{y}_i = \frac{\sum_{j=1}^{n} y_{ij}}{n} = E(y_i); \quad i = 1,2,\dots,k, k+1, k+2$$

$$\bar{y}_i = \frac{1}{\tau}[\Psi(\propto_i + 1) - \Psi(1)]$$

$$= \frac{1}{\tau}\left[\frac{\acute{\Gamma}(\propto_i + 1)}{\Gamma(\propto_i + 1)} - \Psi(1)\right]$$

$$= \frac{1}{\tau}\left[\frac{\acute{\Gamma}(\propto_i + 1)}{\propto_i \Gamma(\propto_i)} - \Psi(1)\right]$$

The moment estimator for the unknown shape parameters $\propto_i$ $(i = 1,2,\dots,r,r+1,r+2)$ will be

$$\hat{\propto}_{imom} = \frac{\acute{\Gamma}(\propto_{i0} + 1)}{\Gamma(\propto_{i0})[\tau\bar{y}_i + \Psi(1)]}$$

Now, $\propto_{i0}$ can be found below.

The population median of Random variable y which follows generalized exponential distribution will be

$$F(y, \propto_i, \tau) = \frac{1}{2}$$

$$(1 - e^{-\tau y})^{\propto_{i0}} = \frac{1}{2}$$

$y_{i\,med} = -\frac{1}{\tau}\ln(1 - (\frac{1}{2})^{\frac{1}{\propto_{i0}}})$

Equating ($Y_{i\,med}$) with sample median ($y_{i\,med}$), we get

($y_{i\,med}$) = ($Y_{i\,med}$)

$$y_{i\,med} = -\frac{1}{\tau}\ln\left(1 - \left[\frac{1}{2}\right]^{\frac{1}{\propto_{i0}}}\right)$$

$$1 - e^{-\tau y_{i\,med}} = (\frac{1}{2})^{\frac{1}{\propto_{i0}}}$$

$\propto_{i0} = \frac{\ln\frac{1}{2}}{\ln(1 - e^{-\tau y_{i\,med}})}$ for $i = (= 1,2,\dots,r,r+1,r+2)$ Where $y_{i\,med} = -\frac{1}{\tau}\ln\left[1 - (0.5)^{\frac{1}{\propto_{i0}}}\right]$

Hence, the estimates of Rp and Rs are respectively given by

$$\hat{R}_{pmom} = \frac{\sum_{i=1}^{r} \hat{\propto}_{imom}}{\sum_{i=1}^{r+2} \hat{\propto}_{imom}}$$

And

$$\hat{R}_{smom} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{imom} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{imom} + \sum_{i=r+1}^{r+2} \hat{\propto}_{imom} - 1}\, dz$$

6- Shrinkage Estimation

Thompson in 1968 suggested the problem of shrink a usual estimator $\hat{\propto}$ of the parameter $\propto$ to prior in formation $\propto_o$ using shrinkage weigh factor $\varphi(\hat{\propto})$, such that $0 \leq \varphi(\hat{\propto}) \leq 1$. The shrinkage estimator of $\propto_i$ denoted by $\hat{\propto}_{ish}$ which is defined as below

$$\hat{\propto}_{ish} = \varphi(\hat{\propto}_i)\,\hat{\propto}_i + \left(1 - \varphi\,(\hat{\propto}_i)\right)\hat{\propto}_{io}$$

We apply the unbiased estimator $\hat{\propto}_{iub}$ as a usual estimator and $\hat{\propto}_{imom}$ as a prior estimator of $\propto_i$, the form of shrinkage estimator of the shape parameter of $\propto_i$ of the generalized exponential distribution is given by

$\hat{\propto}_{ish} = \varphi 1(\hat{\propto}_i)\,\hat{\propto}_{iub} + (1 - \varphi 1(\hat{\propto}_i)\,\hat{\propto}_{imom}) \quad for\ \ i = 1,2,\dots,r,r+1,r+2$

6.1 The shrinkage weight function (SH1)

The shrinkage weight factor as a function of size (n) will be considered and taking the form $\varphi(\hat{\propto}) = (sin/n)$. Therefore, the shrinkage estimator using the above weight function of $\propto_i$ is given by

$\hat{\propto}_{i\,sh1} = (sin/n)\,\hat{\propto}_{i\,ub} + (1 - \sin n/n)\,\hat{\propto}_{i\,mom} \quad for\ i = 1,2,\dots,r,r+1,r+2$

Hence, the estimates of Rp and Rs are respectively given by

$$\hat{R}_{psh1} = \frac{\sum_{i=1}^{r} \hat{\propto}_{ish1}}{\sum_{i=1}^{r+2} \hat{\propto}_{ish1}}$$

And

$$\hat{R}_{ssh1} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{ish1} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{ish1} + \sum_{i=r+1}^{r+2} \hat{\propto}_{ish1} - 1}\, dz$$

6.2 constant shrinkage weight function (SH2)

We suggest constant shrinkage weight factor $\varphi_1(\hat{\propto}_i) = 0.1$. Therefore, the shrinkage estimator using a specific constant weight factor will be as follows

$\hat{\propto}_{ish2} = (0.1)\,\hat{\propto}_{i\,ub} + (1 - 0.1)\,\hat{\propto}_{i\,mom} \quad for\ \ i = 1,2,\dots,r,r+1,r+2.$

Hence, the shrinkage estimation of Rp and Rs using the above constant shrinkage weight factor are respectively given by

$$\hat{R}_{psh2} = \frac{\sum_{i=1}^{r} \hat{\propto}_{ish2}}{\sum_{i=1}^{r+2} \hat{\propto}_{ish2}}$$

And

$$\hat{R}_{ssh2} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{ish2} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{ish2} + \sum_{i=r+1}^{r+2} \hat{\propto}_{ish2} - 1}\, dz$$

6.3 Modified Thompson type shrinkage weight function (TH)

The modification of the shrinkage weight factor is given by

$$£(\hat{\propto}_i) = \frac{(\hat{\propto}_{iub} - \hat{\propto}_{imom})}{(\hat{\propto}_{iub} - \hat{\propto}_{imom}) + vor\,(\hat{\propto}_{iub})} * 0.001$$

Where $\text{var}(\hat{\propto}_{iub}) = \frac{(\propto_i)^2}{(n-2)}$. Therefore, the shrinkage estimation of $\propto_i$ $for\ i = 1,2,\dots,r,r+1,r+2$ using the modified shrinkage weight factor which is given by $\hat{\propto}_{iTh} = £(\hat{\propto}_i)\,\hat{\propto}_{i\,ub} + (1 - £(\hat{\propto}_i))\,\hat{\propto}_{i\,mom} \quad for\ \ i = 1,2,\dots,r,r+1,r+2$

Then the shrinkage estimation of Rp and Rs based on the modified Thompson type shrinkage wright function are respectively given by

$$\hat{R}_{pTh} = \frac{\sum_{i=1}^{r} \hat{\propto}_{iTh}}{\sum_{i=1}^{r+2} \hat{\propto}_{iTh}}$$

And

$$\hat{R}_{sTh} = 1 - \sum_{i=r+1}^{r+2} \hat{\propto}_{iTh} \sum_{m=1}^{r} (-1)^{m-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_r = 1}^{r} \int_0^1 Z^{\sum_{j=1}^{m} \hat{\propto}_{iTh} + \sum_{i=r+1}^{r+2} \hat{\propto}_{iTh} - 1} \, dz$$

7- Simulation Study

The Monte Carlo simulation technique is used to make a comparison among the proposed estimation methods of reliability system of parallel (Rp) and series (Rs) in stress – strength models for the generalized exponential distribution with unknown shape parameters $(\propto_1, \propto_2, \ldots, \propto_r, \propto_{r+1}, \propto_{r+2})$ when the scale parameter (τ) is known. The Monte Carlo simulation involves generating different sample sizers (r) .

The experiment simulation was repeated 1000 times, (M. S. E.) is employed to compare the estimation methods; the maximum likelihood estimation, unbiased estimation, moment estimation and shrinkage estimation. The results we obtained in Tables (1-12) using programs that are written in Matlab version 2013b

Table 1: Estimation methods of reliability system of R_p
$k = 4, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1, a_3 = 3.2, a_4 = 3.3$

α_5	α_6	$\hat{R}_p$	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$
3.5	4	0.6268	0.6413	0.6289	0.6350	0.6356	0.6335	0.6350
4.5	5	0.5701	0.5783	0.5696	0.5834	0.5839	0.5776	0.5834
5.5	6	0.5228	0.5333	0.5244	0.5367	0.5371	0.5312	0.5366
6.5	7	0.4827	0.4919	0.4830	0.4966	0.4971	0.4902	0.4965

Table 2: MSEs for estimates of R_p
$k = 4, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1, a_3 = 3.2, a_4 = 3.3$

α_5	α_6	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$	Best
3.5	4	0.0028	0.0026	0.0022	0.0023	0.0004	0.0021	$\hat{R}_{psh2}$
4.5	5	0.0039	0.0038	0.0026	0.0028	0.0006	0.0026	$\hat{R}_{psh2}$
5.5	6	0.00411	0.00402	0.00273	0.00286	0.00062	0.00266	$\hat{R}_{psh2}$
6.5	7	0.00388	0.00379	0.00259	0.00276	0.00059	0.00252	$\hat{R}_{psh2}$

Table 3: MLEs for estimates of R_s
$k = 4, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1, a_3 = 3.2, a_4 = 3.3$

α_5	α_6	$\hat{R}_s$	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$
3.5	4	0.0470	0.0507	0.0455	0.0482	0.0475	0.0494	0.0483
4.5	5	0.0281	0.0300	0.0277	0.0307	0.0302	0.0299	0.0307
5.5	6	0.0179	0.0200	0.0183	0.0199	0.0196	0.0193	0.0199
6.5	7	0.0119	0.0132	0.0120	0.0133	0.0131	0.0128	0.0133

Table 4: MSEs for estimates of R_s
$k = 4, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1, a_3 = 3.2, a_4 = 3.3$

α_5	α_6	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$	Best
3.5	4	0.0004	0.0003	0.0003	0.0003	0.00008	0.00034	$\hat{R}_{ssh2}$
4.5	5	0.0002	0.0002	0.00019	0.00020	0.00004	0.00019	$\hat{R}_{ssh2}$
5.5	6	0.00015	0.00013	0.00009	0.00009	0.00002	0.00008	$\hat{R}_{ssh2}$
6.5	7	0.00006	0.00005	0.00004	0.00004	0.000009	0.00004	$\hat{R}_{ssh2}$

Table 5: MLEs for estimates of R_p
$k=3, m=10, n=15, \mu=3, a_1=3, a_2=3.1, a_3=3.2$

α_4	α_5	$\hat{R}_p$	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$
3.5	4	0.5535	0.5605	0.5516	0.5620	0.5626	0.5582	0.5620
4.5	5	0.4946	0.5021	0.4932	0.5038	0.5039	0.4997	0.5037
5.5	6	0.4471	0.4577	0.4488	0.4590	0.4594	0.4547	0.4589
6.5	7	0.4078	0.4183	0.4097	0.4219	0.4222	0.4162	0.4218

Table 6: MSEs for estimates of R_s
$k=3, m=10, n=15, \mu=3, a_1=3, a_2=3.1, a_3=3.2$

α_4	α_5	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$	Best
3.5	4	0.00421	0.00420	0.00353	0.00368	0.00066	0.00343	$\hat{R}_{psh2}$
4.5	5	0.00446	0.00440	0.00311	0.00332	0.00069	0.00302	$\hat{R}_{psh2}$
5.5	6	0.00435	0.00422	0.00309	0.00333	0.00066	0.00300	$\hat{R}_{psh2}$
6.5	7	0.00430	0.00414	0.00298	0.00321	0.00080	0.00289	$\hat{R}_{psh2}$

Table 7: MLEs for estimates of R_s
$k=3, m=10, n=15, \mu=3, a_1=3, a_2=3.1, a_3=3.2$

10	α_5	$\hat{R}_s$	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$
3.5	4	0.0731	0.0759	0.0715	0.0762	0.0756	0.0753	0.0762
4.5	5	0.0480	0.0505	0.0474	0.0506	0.0501	0.0496	0.0506
5.5	6	0.0332	0.0363	0.0339	0.0358	0.0356	0.0352	0.0358
6.5	7	0.0239	0.0262	0.0244	0.0264	0.0261	0.0257	0.0264

Table 8: MSEs for estimates of R_s
$k=3, m=10, n=15, \mu=3, a_1=3, a_2=3.1, a_3=3.2$

α_4	α_5	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$	Best
3.5	4	0.00112	0.00103	0.00083	0.00086	0.00017	0.00081	$\hat{R}_{ssh2}$
4.5	5	0.00060	0.00053	0.00043	0.00046	0.00008	0.00042	$\hat{R}_{ssh2}$
5.5	6	0.00034	0.00029	0.00022	0.00024	0.00005	0.00022	$\hat{R}_{ssh2}$
6.5	7	0.00020	0.00017	0.00013	0.00014	0.00003	0.00013	$\hat{R}_{ssh2}$

Table 9: MLEs for estimates of R_p
$k=2, m=10, n=15, \mu=3, a_1=3, a_2=3.1$

α_3	α_4	$\hat{R}_p$	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$
3.5	4	0.4485	0.4540	0.4452	0.4578	0.4581	0.4531	0.4577
4.5	5	0.3910	0.3954	0.3869	0.4049	0.4052	0.3973	0.4048
5.5	6	0.3465	0.3558	0.3477	0.3571	0.3574	0.3528	0.3570
6.5	7	0.3112	0.3185	0.3108	0.3231	0.3232	0.3176	0.3230

Table 10: MSEs for estimates of R_p
$k = 2, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1$

α_3	α_4	$\hat{R}_{pmle}$	$\hat{R}_{pub}$	$\hat{R}_{pmom}$	$\hat{R}_{psh1}$	$\hat{R}_{psh2}$	$\hat{R}_{pTh}$	Best
3.5	4	0.00537	0.00531	0.00469	0.00501	0.00104	0.00457	$\hat{R}_{psh2}$
4.5	5	0.00500	0.00492	0.00413	0.00448	0.00086	0.00402	$\hat{R}_{psh2}$
5.5	6	0.00450	0.00433	0.00350	0.00383	0.00069	0.00341	$\hat{R}_{psh2}$
6.5	7	0.00401	0.00386	0.00321	0.00357	0.00069	0.00312	$\hat{R}_{psh2}$

Table 11: MLEs for estimates of R_s
$k = 2, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1$

α_3	α_4	$\hat{R}_s$	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$
3.5	4	0.1296	0.1326	0.1268	0.1347	0.1342	0.1324	0.1347
4.5	5	0.0950	0.0972	0.0927	0.1018	0.1014	0.0983	0.1017
5.5	6	0.0726	0.0769	0.0731	0.0766	0.0764	0.0753	0.0766
6.5	7	0.0573	0.0606	0.0575	0.0620	0.0618	0.0599	0.0619

Table 12: MSEs for estimates of R_s
$k = 2, m = 10, n = 15, \mu = 3, a_1 = 3, a_2 = 3.1$

α_3	α_4	$\hat{R}_{smle}$	$\hat{R}_{sub}$	$\hat{R}_{smom}$	$\hat{R}_{ssh1}$	$\hat{R}_{ssh2}$	$\hat{R}_{sTh}$	Best
3.5	4	0.00231	0.00217	0.00200	0.00213	0.00043	0.00194	$\hat{R}_{ssh2}$
4.5	5	0.00147	0.00137	0.00129	0.00139	0.00026	0.00125	$\hat{R}_{ssh2}$
5.5	6	0.00103	0.00094	0.00075	0.00082	0.00015	0.00073	$\hat{R}_{ssh2}$
6.5	7	0.00068	0.00061	0.00054	0.00060	0.00011	0.00053	$\hat{R}_{ssh2}$

8- Conclusion

According to simulation results which have been presented in Tables (1-12), we conclude that the constant shrinkage weight functions $(\hat{R}_{sh2})$ is the best method than the other estimators for Rp and Rs and the second best is the modified Thompson estimators $(\hat{R}_{Th})$ for Rp ad Rs and for all (r) and Parameters $(\tau, \alpha_1, \alpha_2 \dots, \alpha_r, \alpha_{r+1}, \alpha_{r+2})$.

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