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ON SN-SPACES

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Abstract
The aim of this paper is to introduce and study the concept of SN-spaces via the notation of simply-open sets as well as to investigate their relationship to other topological spaces and give some of its properties.

Keywords: regular open, regular closed, SN-space, semi-continuous , pre-semi open function ,so-continuous function, weakly closed function, pre-closed set, **G_δ set,** nd-preserving function

حول الفضاءات من النوع SN

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الخلاصة:
يهدف هذا البحث الى تقديم ودراسة مفهوم الفضاءات من النوع SN من خلال استخدام المجموعات المفتوحة ببساطة كما يقدم هذا البحث بعض العلاقات بيت هذا النوع من الفضاءات وبعض الانواع الاخرى من الفضاءات التبولوجية بالاضافة الى اعطاء بعض الخواص للفضاءات من النوع SN وتقديم بعض تطبيقاته على انواع معينة من الدوال المستمرة.

1. Introduction

Let S be a subset of a topological space (X,T). By the symbols ***clS***, ***intS*** and($\boldsymbol{X}$-$\boldsymbol{S}$ = $\boldsymbol{S}^c$) we mean the closure of S, the interior of S and the complement of S respectively. Recall that a subset S of a topological space is called semi-open [1] If S$\subseteq$cl(int(S)) .In [2] the concept of a simply-open set was defined, a subset *S* of a topological space *(X, T)* is called simply-open if S=O$\cup$N where O is open and N is nowhere dense subsets of X. There are other equivalents definitions of simply-open set for example In [3] Ganster, Reilly and Vamanmurthy showed that a subset *S* of a space *(X,T)* is simply-open if and only if it is intersection of semi-open and semi-closed subsets of a space *(X,T)***,**and In [4] and [5] simply-open sets called as semi-locally closed set and NDB-set, respectively. A subset *S* of a space (*X*,T) is called ***regular open*** (resp. ***regular closed***)[6] if *S* = int (cl *S*) (resp. *S* = cl (int *S*)). The main results in this work can be found in theorems 2.4 in this theorem we give some equivalent definitions of SN-space. Also in theorem 3.6 we showed that the property of being SN-space can be preserved by so-continuous function.

2. SN- spaces

We would like to give the first lemma which we will need in our work

LEMMA 2.1 [7]. Every semi-open subset of a topological space (*X*, T) is simply-open.

Now we will introduce the concept of SN-space.

Definition 2.2 : A topological space X is said to be SN-space if for each pair of closed sets F_1 and F_2 such that $F_1 \cap F_2 = \emptyset$, there exist disjoint simply-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

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Clearly, every normal space is SN-space as every semi-open set is simply-open, but not conversely as we will show in the next example.

Example 2.3 : let (X,T) be a topological space where X={a_1,a_2,a_3,a_4 } and T= { ∅, {a_2,a_4}, {a_1,a_2,a_4}, {a_2,a_3,a_4} , X} . Then X is SN-space, but normality is not satisfied since {a_1} and {a_3} cannot be contained in open and disjoint sets.

Now we will discuss some of SN-space properties, which is the first result in this paper.

Theorem 2.4 : In any topological space (X,T) the following statements are equivalent:

(a) X is SN-space.

(b) If G and H are open subsets of X such that X= G ∪ H, then there exist simply-open sets A and B s.t. A⊂G and B⊂H and A∪B=X.

(c) If D open set contained in the closed set G, then there exists a simply-open set A such that D ⊂ A⊂G.

Proof : (a)⇒(b) : Let G and H be open sets in a SN- space X such that X =U∪V. Then X-G,X-H are disjoint. Since X is SN-space there exist disjoint simply-open sets say G_1 and H_1 such that X-U ⊂ G_1 and X-V ⊂ H_1. Let A= X-G_1, B = X – H_1 .Then A and B are required simply-open sets.

(b) ⇒(c) :Let D be a closed set and G be an open set such that D ⊂G. Then X-D∪G=X. Then by hypothesis, there exist simply-open sets S_1 and S_2 such that S_1 ⊂ X-D and S_2 ⊂ G and S_1 ∪S_2 = X. Then D ⊂ X-S_1 , X-G ⊂ X-S_2 and (X-S_1)∩(X-S_2)= ∅. Let A= X-S_1 and B= X-S_2 . Then A satisfy that D ⊂ A ⊂G .

(c) ⇒ (a) :Let G_1 and G_2 closed subsets of X have empty intersection . Let S = X-G_2, then G_1∩S =∅ and F_1 ⊂ G where S is an open set .Then there is a simply-open set say A of X such that G_1 ⊂ A ⊂G . It follows that G_2 ⊂ X-A = B , then B is simply-open and A ∩ B =∅. Therefore X is SN-space■

Now we will consider one of the main properties of SN-space.

Theorem 2.5 : A regular closed subspace of SN-space is SN-space.

Proof : Let Y⊆X where Y is regular and closed subspace of X and X is SN- space .Let G and F closed subsets of Y and disjoint . G and F are closed sets of X. Then there is a disjoint simply-open sets S_1 and S_2 in X such that G ⊂S_1 and F⊂S_2. But S_1∩Y and S_2∩Y are simply-open in Y such that G⊂S_1∩Y and F⊂S_2∩Y. Then Y is SN- space■

The proof of following lemma is very easy so it omitted.

Lemma 2.5.1. For any disjoint simply-open sets A and B in a topological space (X,T) we have that cl(A)⊆ B^c.

The property of a topological space of being SN-space can be characterized also in the following result.

Since the homeomorphic image of open subset of a topological space is open and the homeomorphic image of nowhere dense set is nowhere dense then simply-open preserved under homeomorphism. So we have the following result is obvious and it will be given without proof.

Theorem 2.5.2 The property of a topological space of being SN-space is topological property.

Theorem 2.6 Any topological space (X,T) is SN-space if and only if there exists an simply-open set G such that F⊆G⊆cl(G)⊆H for any open superset H of a closed set F.

Proof. Let X be SN-space then there exists disjoint simply-open subsets G and G_1 such that F⊆G and H-X ⊆G_1 then G_1-X⊆H then by using Lemma 2.6 we have the inclusion F⊆G⊆cl(G) ⊆G_1-X ⊆H. Conversely let F_1 and F_2 be any closed subsets of X such that F_1∩ F_2 =∅.Then by necessarily condition we have simply-open set G such that F_1⊆G⊆cl(G)⊆F_2-X but cl(G)⊆F_2-X→F_2⊆cl(G)-X and cl(G)-X is simply open and G∩ cl(G)-X=∅, so X is SN-space■

In the next definition we will consider weak form of SN-space

Definition 2.7 : A topological space X is called WSN-space if for each disjoint closed ,regular closed subsets F_1 and F_2 respectively , there exist simply-open sets G_1 and G_2 such that F_1⊂G_1 and F_2⊂G_2 such that G_1 and G_2 are disjoint sets.

Clearly, every SN-space is WSN-space. The next example showed that the converse is not necessarily true.

Example 2.8:Let (X,T) be topological space where X={a_1,a_2,a_3} and T= {X ∅,{a_1},{a_1,a_2},{a_1,a_3}}. Then X is WSN- space , but it is not SN- since there are closed sets {a_2} and {a_3} have no disjoint simply-open sets containing them.

The main result in this section will be given in the next theory.

Theorem 2.9: If X is WSN-space then if $G_1 \cup G_2 = X$ where G_1 is open and G_2 is regular open, then there exist simply-open sets A and B such that $A \subset G_1$, $B \subset G_2$ and $A \cup B = X$.

Proof: Let G_1 and G_2 be open and regular open subsets of a WSN- space X respectively such that $G_1 \cup G_2 = X$. Then $(X-G_1)$ and ,$(X-G_2)$ is disjoint closed and regular closed respectively . So there exist disjoint simply-open sets A and B such that $X-G_1 \subset A$ and $X-G_2 \subset B$. Let $G= X- U_1$ and $H= X-V_1$. Then A-X and B-X are simply-open sets such that $A-X \subset G_1$ $B-X \subset G_2$ and $(A-X) \cup (B-X) = X$.

Finally in this section we noted that in example.1. [8] Below this example is still true if we replace the topological space by SN-space.

Examp2.10.[8]. The space (X,T) where X{a,b,c} and T={X,∅,{a},{b,c}} is SN-space, but {a, b} and {c} are disjoint sets which cannot be separated by disjoint simply-open sets.

3. The Main Results

In this section the property of a topological space for being SN-space discussed if it's preserved by so-continuous function.

Definition 3.1[4] The function f: (X, T) $\longrightarrow$ (Y, F) is said to be **so-continuous** if $f^{-1}(S)$ is simply-open in (X, T) whenever S is open in (Y, F)

By Proposition 2.1.the next lemma is very easy to prove

Lemma3.2. Every continuous function is so-continuous.

From upper Lemma we have directly the following Lemma:

Lemma 3.3. If f: $(X,T) \longrightarrow (Y,F)$ is one-to-one, onto, pre-semi-open, and so-continuous function between two SN- spaces X and Y then f is semi –homeomorphism.

Remark 3.4. The inverse image of SN- space under so-continuous bijective function is not necessarily SN- space.

Proof. Let X be an uncountable set such that (X, T_d) is a discrete space and (X, T_i) indiscrete space, then (X, T_d) is not SN- space and (X, T_i) is SN- space. Now, let $f : (X, T_d) \longrightarrow (X, T_i)$ be a function defined by $f(x) = x$, $\forall x \in X$. Then $f^{-1}(x) = x$, therefore f is so-continuous, bijection and the inverse image of SN- space (X, T_i) is not SN- space (X, T_d) ■

If we define $f : (X, T_i) \longrightarrow (X, T_d)$ by $f(x) =x$, $\forall x \in X$ .We then prove that; the bijection image of SN- space should not be simply-normal space■

THEOREM 3.5. let (X,T) and (Y,F) be two SN- spaces., the function $f : (X,T) \longrightarrow (Y,F)$ is so-continuous function if and only if , for every closed subset B of Y , $f^{-1}(B)$ is simply-closed in X.

Proof. Necessity. If $f : (X,T) \longrightarrow (Y,F)$ is so-continuous function , then for every open subset O of Y, $f^{-1}(O)$ is simply-open in X. If B is any closed subset of Y, then B^c is open. Thus $f^{-1}(B^c)$ is simply-open, but $f^{-1}(B^c)= (f^{-1}(B))^c$ so that $f^{-1}(B)$ is simply-closed.

Sufficiency. If, for all closed subset B of Y , $f^{-1}(B)$ is simply-closed in X, and if O is any open subset of Y, then O^c is closed . Also , $f^{-1}(O^c)=(f^{-1}(O))^c$ is simply-open. Thus $f^{-1}(O)$ is simply-open■

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