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On Complete Intuitionistic Fuzzy Roh-Ideals in Roh-Algebras

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Abstract
In this paper, we shall investigate and study some kinds of $\rho -$ideals in an intuitionistic fuzzy setting, they are called complete intuitionistic fuzzy $\rho -$subalgebra, complete intuitionistic fuzzy $\rho -$ideal, and complete intuitionistic fuzzy $\bar{\rho} -$ ideal. In this study, we have also proposed some hypotheses to explain some of the relationships between these kinds of intuitionistic fuzzy ideals.

Keywords: Intuitionistic fuzzy ρ–subalgebra, Intuitionistic fuzzy ρ–ideal, intuitionistic fuzzy $\bar{\rho}$-ideal.
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حول مثاليات رو الضبابية الحدسية الكاملة في جبور رو

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الخلاصة
في هذا البحث ، سوف يتم التطرق لدراسة أنواعًا مختلفة من مثاليات رو في بيئة ضبابية حدسية ،هذه المثاليات تسمى جبر رو الجزئي الضبابي الحدسي الكامل ، مثالية رو الضبابية الحدسية الكاملة ، مثالية رو بار الضبابية الحدسية الكاملة. كذلك في هذه الدراسة, قدمنا بعض النظريات لشرح بعض العلاقات بين هذه الفئات من المثاليات الضبابية الحدسية.

1. Introduction

The invention of the fuzzy subset of a set is investigated by Zadeh [1] in 1965. Scholars have been interested in extending the notions and outcomes of every concept in mathematics to the boarder framework of fuzzy setting. BCK-algebras were proposed by Imai and Iseki [2] as a generalization of the concept of set theoretic difference and propositional calculus. In the same year, Iseki [3] introduced BCI-algebra, which is a generalization of BCK-algebra. In BCK-algebra, Xi [4] introduced fuzzy ideals as well as the concept of fuzzy subalgebra. In 1996, the d-algebras class is given by Neggers and Kim [5] which is a generalization of BCK-algebras, and examined the relationship between them. Akram and Dar [6] considered the connotations of the fuzzy d-(algebra/subalgebra/ideal). In 2009, Kim [7] investigated the connotation of

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fuzzy dot subalgebra in d-algebra. After that, the connotation of fuzzy dot d-ideals of a d-algebra is introduced by Al-Shehrie [8]. Yun et al. [9] employed the connotation of intuitionistic fuzzy set in d-algebra and considered intuitionistic fuzzy topological d-algebra with intuitionistic fuzzy d-algebra. Mahmood & Abud Alradha [10] introduced the ρ −algebra. After that, many of structures and applications in $\rho -$ algebras using fuzzy sets [11] and soft sets [12,13] are considered. Here, in this paper, we introduce the notion of complete intuitionistic fuzzy ρ −subalgebra, complete intuitionistic fuzzy ρ −ideal, and complete intuitionistic fuzzy $\bar{\rho}$ −ideal of ρ −algebra. The goal of this work is to study and investigate new ideals in intuitionistic fuzzy ρ −ideals. In addition, several ideas are presented to explain some of the links between these intuitionistic fuzzy ρ −ideals.

2. Preliminaries and Some Results.

In the present section, we will recall some basic concepts and results that are necessary for this article.

Definition 2.1.[10] A ρ-algebra is a non-empty set $\mho$ with a constant 0 and a binary operation "¤" which satisfies the following axioms:
(1) $\alpha¤\alpha = 0$,
(2) $0¤\alpha = 0$,
(3) $\alpha¤\beta = 0 = \beta¤\alpha$ *imply that* $\alpha = \beta$,
(4) For all $\alpha \neq \beta \in \mho - \{0\}$ *imply that* $\alpha¤\beta = ¤\alpha \neq 0$.

Definition 2.2. [10] Let $\emptyset \neq \Upsilon \subseteq \mho$ and $(\mho, ¤, 0)$ be a ρ-algebra. We say Υ is a $\rho -$ subalgebra of $\mho$ if $\alpha¤\beta \in \Upsilon$ for any $\alpha, \beta \in \Upsilon$.

Definition 2.3.[10] A non-empty subset Υ of a ρ −algebra $\mho$ is called an $\rho -$ ideal of $\mho$ if the following conditions hold::
(1) $\alpha, \beta \in \Upsilon \Longrightarrow \alpha¤\beta \in \Upsilon$,
(2) $\alpha¤\beta \in \Upsilon$ & $\beta \in \Upsilon \Longrightarrow \alpha \in \Upsilon$.

Remark 2.4[10]. If Υ is any a $\rho -$ ideal, then Υ is $\rho -$ subalgebra. However, the converse may be not holding.
Definition2.5.[10] A non- empty subset Υ of a ρ-algebra $\mho$ is called an $\bar{\rho} -$ ideal of $\mho$ if it satisfies the following:
(1) $0 \in \Upsilon$,
(2) $\alpha \in \Upsilon$ & $\beta \in \mho \Longrightarrow \alpha¤\beta \in \Upsilon$.

Proposition 2.6. [10] Assume that $\emptyset \neq \Upsilon \subseteq \mho$, where $\mho$ is ρ-algebra. Then Υ is a ρ- subalgebra of $\mho$ if it is $\bar{\rho}$- Ideal.

Definition 2.7. [14] An Intuitionistic fuzzy set (briefly,IFS) over the universal $\mho$ is defined by $I = \{\prec \alpha, I_T(\alpha), I_F(\alpha) \succ \mid \alpha \in \mho\}$,where $I_T(\alpha)$ *and* $I_F(\alpha): \mho \longrightarrow [0,1]$ are maps,
with $I_T(\alpha)$ and $I_F(\alpha)$ are real numbers and their values represent the degree of membership and non- membership of α to I, respectively.

Definition 2.8[15] A complement an intuitionistic fuzzy set I^c over the universal $\mho$ is defined by $I^c = 1 - I = 1 - \{\prec \alpha, I_T(\alpha), I_F(\alpha) \succ \mid \alpha \in \mho\}$
$= \{\prec \alpha, 1 - I_T(\alpha), 1 - I_F(\alpha) \succ \mid \alpha \in \mho\}$
$= \{\prec \alpha, I_{T^c}(\alpha), I_{F^c}(\alpha) \succ \mid \alpha \in \mho\}$.
Definition 2.9.[14] Let I be an IFS over the universal $\mho$ and $t \in [0,1]$ then the set $\{\prec \alpha, I_T(\alpha)$

$\geq t, I_F(\alpha) \leq t \succ \mid \alpha \in \mho\}$ is called an intuitionistic fuzzy (t-cut), (briefly, IF -t-cut) and denoted by I_t.

Definition 2.10. [13] An IFS I in $\mho$ is called an intuitionistic fuzzy ρ -subalgebra (briefly, IF $-\rho - SA$) of $\mho$ if it satisfies the following conditions:
(i) $I_T(\alpha ¤ \beta) \geq min\{I_T(\alpha), I_T(\beta)\}$,
(ii) $I_F(\alpha ¤ \beta) \leq max\{I_F(\alpha), I_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Definition 2.11.[13] Let ($\mho$,¤, 0) be a ρ-algebra and I be an IFS of $\mho$. We say that I is an intuitionistic fuzzy ρ -ideal of $\mho$ (briefly, IF $-\rho - I$)) if the following conditions hold:
(i) $I_T(\alpha ¤ \beta) \geq min\{I_T(\alpha), I_T(\beta)\}$,
(ii) $I_F(\alpha ¤ \beta) \leq max\ \{I_F(\alpha), I_F(\beta)\}$,
(iii) $I_T(\alpha) \geq min\{I_T(\alpha ¤ \beta), I_T(\beta)\}$,
(iv) $I_F(\alpha) \leq max\{I_F(\alpha ¤ \beta), I_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Definition 2.12.[13] Let ($\mho$,¤, 0) be a ρ-algebra and I an IFS of $\mho$. We say that I is an intuitionistic fuzzy $\bar{\rho}$ −ideal of $\mho$ (briefly, IF $-\bar{\rho} - I$) if the following conditions hold:
(i)$I_T(0) \geq I_T(\alpha)$,
(ii) $I_F(0) \leq I_F(\alpha)$,
(iii) $I_T(\alpha ¤ \beta) \geq min\{I_T(\alpha), I_T(\beta)\}$,
(iv) $I_F(\alpha ¤ \beta) \leq max\{I_F(\alpha), I_F(\beta)\}$, for any $\alpha, \beta \in \mho$.

Remarks 2.13.[13]:
1- Every (IF $-\rho - I$)is (IF $-\rho - SA$).
2-Let I be (IF $-\bar{\rho} - I$) then I is (IF $-\rho - SA$).

Lemma 2.14.[13] Let I be an IF $-\rho - SA$ of $\mho$ then:
(i) $I_T(0) \geq I_T(\alpha)$,
(ii) $I_F(0) \leq I_F(\alpha)$, for any $\alpha \in \mho$.

3. The Complete Intuitionistic Fuzzy ρ – Subalgebra.

Definition 3.1. Let I be an IFS of $\rho -$ algebra ($\mho$,¤,0), then $K(I) = \{\alpha \in \mho | I_T(\alpha) = I_T(0), I_F(\alpha) = I_F(0)\}$ is a subset of $\mho$ and called an intuitionistic fuzzy ρ −kernel of an intuitionistic fuzzy set over $\mho$.

Example 3.2. Let $\mho$ = {0,1,2,3}, define ¤on the set $\mho$ as in Table 1. Then ($\mho$, ¤ ,0) is a ρ-algebra, we define an (IFS) I in $\mho$ as follows:

$$I_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0.4 & 0 & 0.4 \end{pmatrix}, I_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0.3 & 0 & 0.3 \end{pmatrix}, \ K(I) = \{0,2\}.$$

Table 1: $K(I) = \{0,2\}$.

¤	0	1	2	3
0	0	0	0	0
1	1	0	1	2
2	2	1	0	2
3	3	2	2	0

Proposition 3.3. If I is $(IF -\rho - SA)$ of (Ʊ, ¤ ,0), then K(I) is a $(\rho - SA)$.
Proof: Let $\alpha,\beta \in K(\mathrm{I})$, then $\mathrm{I}_T(\alpha) = \mathrm{I}_T(\beta) = \mathrm{I}_T(0), \mathrm{I}_F(\alpha) = \mathrm{I}_T(\beta) = \mathrm{I}_T(0)$. Also, since I is $(\mathrm{IF} - \rho - SA)$, we obtain $\mathrm{I}_T(\alpha¤\beta) \geq min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\} = min\{\mathrm{I}_T(0), \mathrm{I}_T(0)\}= \mathrm{I}_T(0)$. From Lemma (2.14), we get $\mathrm{I}_T(\alpha¤\beta) = \mathrm{I}_T(0)\}$. Also $\mathrm{I}_F(\alpha¤\beta) \leq max\{\mathrm{I}_F(\alpha),\mathrm{I}_F(\beta)\}= max\{\mathrm{I}_F(0),\mathrm{I}_F(0)\} = \mathrm{I}_F(0)$, and from Lemma (2.14), we get $\mathrm{I}_F(\alpha¤\beta) = \mathrm{I}_F(0)\}$. This implies $\alpha¤\beta \in K(I)$. Hence, $K(I)$ is$(\rho - SA)$.

Definition 3.4.
Let (Ʊ,¤ ,0) be a ρ-algebra and I be an IFS of Ʊ. We say that I is an intuitionistic ρ -constant of Ʊ if all maps I_T , I_F : Ʊ → [0,1] are constant maps.
Example 3.5. Let Ʊ = {α, β, γ, δ}, define ¤ on the set Ʊ as in Table 2. Then (Ʊ, ¤ , α) is a ρ-algebra, we define an (IFS) I in Ʊ as follows:
$\mathrm{I}_T = \mathrm{I}_F = \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$,
Hence, I is intuitionistic ρ -constant.

Table 2: I is an intuitionistic ρ -constant.

¤	α	β	γ	δ
Α	Α	α	α	α
Β	Β	α	β	γ
Γ	Γ	β	α	γ
Δ	Δ	γ	γ	α

Lemma 3.6. Let I be $(\mathrm{IF} -\rho - SA)$ of Ʊ then:
(i)$\mathrm{I}_{T^c}(\alpha) \geq \mathrm{I}_{T^c}(0)$,
(ii)$\mathrm{I}_{F^c}(\alpha) \leq \mathrm{I}_{F^c}(0)$ for any $\alpha \in$ Ʊ.

Proof: Let I be $(\mathrm{IF} -\rho - SA)$, then from Lemma (2.14)
We obtain: $\mathrm{I}_T(0) \geq \mathrm{I}_T(\alpha)$, $\mathrm{I}_F(0) \leq \mathrm{I}_F(\alpha)$, for any $\alpha \in$ Ʊ.
Since, $\mathrm{I}^c = 1 - \mathrm{I}$,thus
$\mathrm{I}_{T^c}(\alpha) = 1 - \mathrm{I}_T(\alpha) \geq 1 - \mathrm{I}_T(0) = \mathrm{I}_{T^c}(0)$,
$\mathrm{I}_{F^c}(\alpha) = 1 - \mathrm{I}_F(\alpha) \leq 1 - \mathrm{I}_F(0) = \mathrm{I}_{F^c}(0)$, This completes the proof.

Proposition 3.7. Let I be an IFS of $\rho -$ algebra (Ʊ, ¤ ,0), then I is $(\mathrm{IF} - \rho - SA)$ if it is I ={$\prec \alpha, \mathrm{I}_T(\alpha) = \mathrm{I}_T(0), \mathrm{I}_F(\alpha) = \mathrm{I}_F(0) \succ | \ \alpha \in$ Ʊ}.
Proof: Let I be an IFS and $\mathrm{I}_T(\alpha) = \mathrm{I}_T(0), \mathrm{I}_F(\alpha) = \mathrm{I}_F(0)$, for any $\alpha \in$I. Now, Let $\alpha,\beta \in$ Ʊ, then $\mathrm{I}_T(\alpha¤\beta) = \mathrm{I}_T(0) = min\{\mathrm{I}_T(0), \mathrm{I}_T(0)\}= min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\}$, thus $\mathrm{I}_T(\alpha¤\beta) \geq min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\}$, $\mathrm{I}_F(\alpha¤\beta) = \mathrm{I}_F(0)= max\{\mathrm{I}_F(0), \mathrm{I}_F(0)\}= max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\}$, thus $\mathrm{I}_F(\alpha¤\beta) \leq max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\}$, Hence, I is $(\mathrm{IF} - \rho - SA)$.

Proposition 3.8. Let I be an IFS of $\rho -$ algebra (Ʊ, ¤ ,0), then I^c is $(\mathrm{IF} - \rho - SA)$. If $\mathrm{I}^c = \{\prec \alpha, \mathrm{I}_{T^c}(\alpha) = \mathrm{I}_{T^c}(0), \mathrm{I}_{F^c}(\alpha) = \mathrm{I}_{F^c}(0) \succ | \alpha \in$ Ʊ}

Proof: Let $I^c = \{\prec \alpha, I_{T^c}(\alpha) = I_{T^c}(0), I_{F^c}(\alpha) = I_{F^c}(0) \succ \mid \alpha \in \mho\}$ and let $\alpha, \beta \in \mho$, then $I_{T^c}(\alpha¤\beta)= I_{T^c}(0)= min\{I_{T^c}(0), I_{T^c}(0)\}= min\{I_{T^c}(\alpha), I_{T^c}(\beta)\}$, thus $I_{T^c}(\alpha¤\beta) \geq min\{I_{T^c}(\alpha), I_{T^c}(\beta)\}$,$I_{F^c}(\alpha¤\beta)=I_{F^c}(0)= max\{I_{F^c}(0),I_{F^c}(0)\}=max\{I_{F^c}(\alpha),I_{F^c}(\beta)\}$.
Thus, $I_{F^c}(\alpha¤\beta) \leq max\{I_{F^c}(\alpha), I_{F^c}(\beta)\}$. Hence, I^c is $(IF - \rho - SA)$.

Proposition 3.9. If I is $(IF - \rho - SA)$ of ($\mho$, ¤ ,0), then $K(I^c)$ is a $(\rho - SA)$.
Proof: Let $\alpha, \beta \in K(I^c)$, then $I_{T^c}(\alpha) = I_{T^c}(\beta) = I_{T^c}(0), I_{F^c}(\alpha) = I_{F^c}(\beta) = I_{F^c}(0)$. Also,
$I_{T^c}(\alpha¤\beta) = 1 - I_T(\alpha¤\beta) \leq 1 - min\{I_T(\alpha), I_T(\beta)\}$ [since $I_T(\alpha¤\beta) \geq min\{I_T(\alpha), I_T(\beta)\}$].
$= max\{1 - I_T(\alpha), 1 - I_T(\beta)\}$
$= max\{I_{T^c}(\alpha), I_{T^c}(\beta)\}$
$= max\{I_{T^c}(0), I_{T^c}(0)\}= I_{T^c}(0)$,
$I_{F^c}(\alpha¤\beta) = 1 - I_F(\alpha¤\beta) \geq 1 - max\{I_F(\alpha), I_F(\beta)\}$[since $I_F(\alpha¤\beta) \leq max\{I_F(\alpha), I_F(\beta)\}$].
$= min\{1 - I_F(\alpha), 1 - I_F(\beta)\}$
$= min\{I_{F^c}(\alpha), I_{F^c}(\beta)\}$
$= min\{I_{F^c}(0), I_{F^c}(0)\}= I_{F^c}(0)$.
Now, from Lemma (3.6), we obtain $I_{T^c}(\alpha¤\beta) \geq I_{T^c}(0)$, $I_{F^c}(\alpha¤\beta) \leq I_{F^c}(0)$, thus $I_{T^c}(\alpha¤\beta)$ $=I_{T^c}(0), I_{F^c}(\alpha¤\beta) =I_{F^c}(0)$, this implies $\alpha¤\beta \in K(I^c)$, hence $K(I^c)$ is $(\rho - SA)$.

Proposition 3.10. Let I be $(IF - \rho - SA)$ then I_t is $(\rho - SA)$.
Proof: Assume that I is $(IF - \rho - SA)$and $\alpha, \beta \in I_t$, then $(I_T(\alpha) \geq t, I_F(\alpha) \leq t)$ and $(I_T(\beta) \geq t, I_F(\beta) \leq t)$. Also,
$I_T(\alpha¤\beta) \geq min\{I_T(\alpha), I_T(\beta)\} \geq t, I_F(\alpha¤\beta) \leq max\{I_F(\alpha), I_F(\beta)\} \leq t$, this implies $\alpha¤\beta \in I_t$, hence I_t is $(\rho - SA)$.

Proposition 3.11. Let ($\mho$,¤,0) be a $\rho-$algebra and I be an IFS of $\mho$. Then I is $(IF - \rho - SA)$if it is an intuitionistic fuzzy ρ −constant.

Proof: Assume that I is constant. Then for all $\alpha \in \mho, I_T(\alpha) = I_T(0)$, $I_F(\alpha) = I_F(0)$, and so $I_T(0)\geq I_T(\alpha)$,$I_F(0)\leq I_F(\alpha)$. Next, for all $\alpha, \beta \in \mho$, $I_T(\alpha¤\beta)= I_T(0) = min\{I_T(0), I_T(0)\} \geq min\{I_T(\alpha), I_T(\beta)\}$, $I_F(\alpha¤\beta)= I_F(0)=max\{I_F(0),I_F(0)\} \leq max\{I_F(\alpha), I_F(\beta)\}$,hence I is $(IF - \rho - SA)$.

Proposition 3.12. Let I be $(IF - \rho - SA)$. Then $0 \in I_t$, if $I_t \neq \emptyset$.
Proof: Assume that I is $(IF - \rho - SA)$ and $I_t \neq \emptyset$ then there is at least $\alpha \in I_t$. From Lemma (2.14) and Definition (2.9), we obtain, $I_T(0) \geq I_T(\alpha) \geq t$, $I_F(0) \leq I_F(\alpha) \leq t$, this means $0 \in I_t$.
Corollary 3.13. If I an intuitionistic fuzzy ρ -constant then I_t is $(\rho - SA)$.
Proof: It is directly obtained the proof from Proposition (3.11) and Proposition (3.10).
Definition 3.14. Let I be an IFS in $\mho$. We say that I is a complete an intuitionistic fuzzy ρ -subalgebra (briefly, $CIF -\rho - SA$) of $\mho$ if it satisfies the following:
(i) $I_T(\alpha¤\beta) \leq max\{I_T(\alpha), I_T(\beta)\}$,
(ii) $I_F(\alpha¤\beta) \geq min\{I_F(\alpha), I_F(\beta)\}$ for any $\alpha, \beta \in \mho$.

Example 3.15. Let $\mho = \{o, \mu, \nu, \xi\}$ and define ¤on the set $\mho$ as in Table 3. Then ($\mho$, ¤ , o) is a ρ-algebra, we define an (IFS) I in $\mho$ as follows:
$I_T=\begin{pmatrix} o & \mu & \nu & \xi \\ 0.4 & 0.6 & 0.6 & 0.6 \end{pmatrix}$, $I_F=\begin{pmatrix} o & \mu & \nu & \xi \\ 0.8 & 0.7 & 0.7 & 0.7 \end{pmatrix}$. Hence, I is $(CIF -\rho - SA)$.

Table 3: I is $(CIF -\rho - SA)$.

¤	o	μ	ν	ξ
o	o	o	o	o
μ	μ	o	ξ	ν
ν	ν	ξ	o	ν
ξ	ξ	ν	ν	o

Lemma 3.16. Let I be $(\mathrm{CIF}-\rho-SA)$ of Ʊ then:
(i) $\mathrm{I}_T(0) \leq \mathrm{I}_T(\alpha)$,
(ii) $\mathrm{I}_F(0) \geq \mathrm{I}_F(\alpha)$, for any $\alpha \in$ Ʊ.

Proof: Let I be $(\mathrm{CIF}-\rho-SA)$ then,
(i) $\mathrm{I}_T(0) = \mathrm{I}_T(\alpha¤\alpha) \leq max\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\alpha)\} = \mathrm{I}_T(\alpha)$.
(ii) $\mathrm{I}_F(0) = \mathrm{I}_F(\alpha¤\alpha) \geq min\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\alpha)\} = \mathrm{I}_F(\alpha)$.This completes the proof.

Proposition 3.17. If I is a $(\mathrm{CIF}-\rho-SA)$, then $K(\mathrm{I})$ is $(\rho-SA)$.
Proof: Let $\alpha, \beta \in K(\mathrm{I})$, then $\mathrm{I}_T(\alpha) = \mathrm{I}_T(\beta) = \mathrm{I}_T(0), \mathrm{I}_F(\alpha) = \mathrm{I}_F(\beta) = \mathrm{I}_F(0)$. Also, $\mathrm{I}_T(\alpha¤\beta) \leq max\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\} = max\{\mathrm{I}_T(0), \mathrm{I}_T(0)\} = \mathrm{I}_T(0)$, $\mathrm{I}_F(\alpha¤\beta) \geq min\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\} = min\{\mathrm{I}_F(0), \mathrm{I}_F(0)\} = \mathrm{I}_F(0)$, and from Lemma (3.16) $\mathrm{I}_T(0) \leq \mathrm{I}_T(\alpha¤\beta)$, $\mathrm{I}_F(0) \geq \mathrm{I}_F(\alpha¤\beta)$. Thus, $\mathrm{I}_T(\alpha¤\beta) = \mathrm{I}_T(0)$, $\mathrm{I}_F(\alpha¤\beta) = \mathrm{I}_F(0)$, and $\alpha¤\beta \in K(\mathrm{I})$ hence $K(\mathrm{I})$ is$(\rho-SA)$.

Proposition 3.18. Let I be an IFS then I is $(\mathrm{IF}-\rho-SA)$ if and only if I^c is $(\mathrm{CIF}-\rho-SA)$.
Proof: Let I be $(\mathrm{IF}-\rho-SA)$ then $\mathrm{I}_T(\alpha¤\beta) \geq min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\}$, $\mathrm{I}_F(\alpha¤\beta) \leq max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\}$, for any $\alpha, \beta \in$ Ʊ, Now, $\mathrm{I}_{T^c}(\alpha¤\beta) = 1-\mathrm{I}_T(\alpha¤\beta) \leq 1-min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\} = max\{1-\mathrm{I}_T(\alpha), 1-\mathrm{I}_T(\beta)\} = max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\}$, $\mathrm{I}_{F^c}(\alpha¤\beta) = 1-\mathrm{I}_F(\alpha¤\beta) \geq 1-max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\} = min\{1-\mathrm{I}_F(\alpha), 1-\mathrm{I}_F(\beta)\} = min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\}$. Hence I^c is $(\mathrm{CIF}-\rho-SA)$.
Conversely, let I^c be $(\mathrm{CIF}-\rho-SA)$, then $\mathrm{I}_{T^c}(\alpha¤\beta) \leq max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\}$, $\mathrm{I}_{F^c}(\alpha¤\beta) \geq min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\}$, for any $\alpha, \beta \in$ Ʊ. Now, $\mathrm{I}_T(\alpha¤\beta) = 1-\mathrm{I}_{T^c}(\alpha¤\beta) \geq 1-max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\} = min\{1-\mathrm{I}_{T^c}(\alpha), 1-\mathrm{I}_{T^c}(\beta)\} = min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\}$, $\mathrm{I}_F(\alpha¤\beta) = 1-\mathrm{I}_{F^c}(\alpha¤\beta) \leq 1-min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\} = max\{1-\mathrm{I}_{F^c}(\alpha), 1-\mathrm{I}_{F^c}(\beta)\} = max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\}$. Hence, I is $(\mathrm{IF}-\rho-SA)$.

Corollary 3.19.
1- Let I^c be a $(\mathrm{CIF}-\rho-SA)$, then I_t is $(\rho-SA)$.
2- Let I be an intuitionistic fuzzy ρ -constant then I_t is $(\rho-SA)$.

Proof 1: From Proposition (3.18) and Proposition (3.10), the proof is obtained. Or we can also get the proof from Proposition (3.11) and Proposition (3.10).

Corollary 3.20.
1- Let I be $(\mathrm{IF}-\rho-I)$ then I_t is $(\rho-SA)$.
2-Let I be $(\mathrm{IF}-\rho-I)$ then $\mathrm{I}^c$is $(\mathrm{CIF}-\rho-SA)$.
Proof 1: From Remarks (2.13**)**-1 and Proposition (3.10)**,** the proof is obtained. Or we can also get the proof from Remarks (2.13**)**-1 and Proposition (3.18)**.**

Lemma 3.21. Let I be $(\mathrm{IF}-\rho-I)$ of Ʊ then:
(i) $\mathrm{I}_T(0) \geq \mathrm{I}_T(\alpha)$,
(ii) $\mathrm{I}_F(0) \leq \mathrm{I}_F(\alpha)$, for any $\alpha \in$ Ʊ.
Proposition 3.22. Let I be $(\mathrm{IF}-\rho-I)$ then $K(\mathrm{I})$ is $-$ideal .

Proof: Let I be $(\text{IF} - \rho - I)$ and let $\alpha, \beta \in K(\text{I})$, then $\text{I}_T(\alpha) = \text{I}_T(\beta) = \text{I}_T(0), \text{I}_F(\alpha) = \text{I}_F(\beta) = \text{I}_F(0)$. Also, $\text{I}_T(\alpha ¤ \beta) \geq min\{\text{I}_T(\alpha), \text{I}_T(\beta)\} = min\{\text{I}_T(0), \text{I}_T(0)\} = \text{I}_T(0)$, $\text{I}_F(\alpha ¤ \beta) \leq max\{\text{I}_F(\alpha), \text{I}_F(\beta)\} = max\{\text{I}_F(0), \text{I}_F(0)\} = \text{I}_F(0)$, and from Lemma (3.21), we obtain $\text{I}_T(0) \geq \text{I}_T(\alpha ¤ \beta)$, $\text{I}_F(0) \leq \text{I}_F(\alpha ¤ \beta)$. Hence, $\alpha ¤ \beta \in K(\text{I})$. Now, assume that $\alpha ¤ \beta \in K(\text{I})$ & $\beta \in K(\text{I})$, then $\text{I}_T(\alpha ¤ \beta) = \text{I}_T(0), \text{I}_F(\alpha ¤ \beta) = \text{I}_F(0)$, and $\text{I}_T(\beta) = \text{I}_T(0), \text{I}_F(\beta) = \text{I}_F(0)$. Thus $\text{I}_T(\alpha) \geq min\{\text{I}_T(\alpha ¤ \beta), \text{I}_T(\beta)\} = \text{I}_T(0)$, $\text{I}_F(\alpha) \leq max\{\text{I}_F(\alpha ¤ \beta), \text{I}_F(\beta)\} = \text{I}_F(0)$, and from Lemma (3.21), we obtain $\text{I}_T(0) = \text{I}_T(\alpha)$, $\text{I}_F(0) = \text{I}_F(\alpha)$, thus $\alpha \in K(\text{I})$, hence $K(\text{I})$ is $\rho -$ ideal.

Proposition 3.23. Let I be $(\text{IF} - \rho - I)$ then $K(\text{I}^c)$ is $-$ideal .

Proof: Let I be $(\text{IF} - \rho - I)$ and let $\alpha, \beta \in K(\text{I}^c)$, then $\text{I}_{T^c}(\alpha) = \text{I}_{T^c}(\beta) = \text{I}_{T^c}(0), \text{I}_{F^c}(\alpha) = \text{I}_{F^c}(\beta) = \text{I}_{F^c}(0)$. Also,

$$\begin{aligned}\text{I}_{T^c}(\alpha ¤ \beta) &= 1 - \text{I}_T(\alpha ¤ \beta) \\ &\leq 1 - min\{\text{I}_T(\alpha), \text{I}_T(\beta)\} \\ &= max\{1 - \text{I}_T(\alpha), 1 - \text{I}_T(\beta)\} \\ &= max\{\text{I}_{T^c}(\alpha), \text{I}_{T^c}(\beta)\} \\ &= max\{\text{I}_{T^c}(0), \text{I}_{T^c}(0)\} = \text{I}_{T^c}(0),\end{aligned}$$

$$\begin{aligned}\text{I}_{F^c}(\alpha ¤ \beta) = 1 - \text{I}_F(\alpha ¤ \beta) &\geq 1 - max\{\text{I}_F(\alpha), \text{I}_F(\beta)\} \\ &= min\{1 - \text{I}_F(\alpha), 1 - \text{I}_F(\beta)\} \\ &= min\{\text{I}_{F^c}(\alpha), \text{I}_{F^c}(\beta)\} \\ &= min\{\text{I}_{F^c}(0), \text{I}_{F^c}(0)\} = \text{I}_{F^c}(0),\end{aligned}$$

and from Lemma (3.6), we obtain $\text{I}_{T^c}(\alpha ¤ \beta) \geq \text{I}_{T^c}(0)$, $\text{I}_{F^c}(\alpha ¤ \beta) \leq \text{I}_{F^c}(0)$, thus $\text{I}_{T^c}(\alpha ¤ \beta) = \text{I}_{T^c}(0), \text{I}_{F^c}(\alpha ¤ \beta) = \text{I}_{F^c}(0)$, this implies $\alpha ¤ \beta \in K(\text{I}^c)$. Now, let $\alpha ¤ \beta, \beta \in K(\text{I}^c)$, then $\text{I}_{T^c}(\alpha ¤ \beta) = \text{I}_{T^c}(0)$, $\text{I}_{F^c}(\alpha ¤ \beta) = \mathbb{I}_{F^c}(0)$, and $\text{I}_{T^c}(\beta) = \text{I}_{T^c}(0), \text{I}_{F^c}(\beta) = \text{I}_{F^c}(0)$. Moreover, since I is $(\text{IF} - \rho - I)$, then $\text{I}_{T^c}(\alpha) = 1 - \text{I}_T(\alpha) \leq 1 - min\{\text{I}_T(\alpha ¤ \beta), \text{I}_T(\beta)\} = max\{1 - \text{I}_T(\alpha ¤ \beta), 1 - \text{I}_T(\beta)\} = max\{\text{I}_{T^c}(\alpha ¤ \beta), \text{I}_{T^c}(\beta)\} = max\{\text{I}_{T^c}(0), \text{I}_{T^c}(0)\} = \text{I}_{T^c}(0)$, $\text{I}_{F^c}(\alpha) = 1 - \text{I}_F(\alpha) \geq 1 - max\{\text{I}_F(\alpha ¤ \beta), \text{I}_F(\beta)\} = min\{1 - \text{I}_F(\alpha ¤ \beta), 1 - \text{I}_F(\beta)\} = min\{\text{I}_{F^c}(\alpha ¤ \beta), \text{I}_{F^c}(\beta)\} = min\{\text{I}_{F^c}(0), \text{I}_{F^c}(0)\} = \text{I}_{F^c}(0)$, and from Lemma (3.6), we obtain $\text{I}_{T^c}(\alpha) \geq \text{I}_{T^c}(0)$, $\text{I}_{F^c}(\alpha) \leq \text{I}_{F^c}(0)$, thus $\text{I}_{T^c}(\alpha) = \text{I}_{T^c}(0), \text{I}_{F^c}(\alpha) = \text{I}_{F^c}(0)$, this implies $\alpha \in K(\text{I}^c)$, hence $K(\text{I}^c)$ is $\rho -$ ideal.

Proposition 3.24. Let I be $(\text{IF} - \rho - I)$ then I_t is $\rho -$ideal.

Proof: Assume that I is $(\text{IF} - \rho - I)$ and $\alpha, \beta \in \text{I}_t$,
then $\text{I}_T(\alpha ¤ \beta) \geq min\{\text{I}_T(\alpha), \text{I}_T(\beta)\} \geq t, \text{I}_F(\alpha ¤ \beta) \leq max\{\text{I}_F(\alpha), \text{I}_F(\beta)\} \leq t$,
this implies $\alpha ¤ \beta \in \text{I}_t$,
Now, assume that $\alpha ¤ \beta \in \text{I}_t$ and $\beta \in \text{I}_t$, since I is $(\text{IF} - \rho - I)$,
we obtain $\text{I}_T(\alpha) \geq min\{\text{I}_T(\alpha ¤ \beta), \text{I}_T(\beta)\} \geq t$, $\text{I}_F(\alpha) \leq max\{\text{I}_F(\alpha ¤ \beta), \text{I}_F(\beta)\} \leq t$,
thus $\alpha \in \text{I}_t$, hence I_t is $\rho -$ideal.

Definition 3.25. Assume that $(\mho, ¤, 0)$ is a ρ-algebra and let I be an IFS of $\mho$. We say I is a complete an intuitionistic fuzzy ρ -ideal of $\mho$ (briefly, CIF $-\rho - I$)if the following conditions hold:
(i) $\text{I}_T(\alpha ¤ \beta) \leq max\{\text{I}_T(\alpha), \text{I}_T(\beta)\}$,
(ii) $\text{I}_F(\alpha ¤ \beta) \geq min\{\text{I}_F(\alpha), \text{I}_F(\beta)\}$,
(iii) $\text{I}_T(\alpha) \leq max\{\text{I}_T(\alpha ¤ \beta), \text{I}_T(\beta)\}$,
(iv) $\text{I}_F(\alpha) \geq min\{\text{I}_F(\alpha ¤ \beta), \text{I}_F(\beta)\}$,for any $\alpha, \beta \in \mho$.

Example 3.26. Let $\mho = \{p, q, r, s\}$ and define ¤on the set $\mho$ as in Table 4. Then $(\mho, ¤, p)$ is a ρ-algebra, we define an (IFS) I in $\mho$ as follows:
$\text{I}_T = \begin{pmatrix} p & q & r & s \\ 0.1 & 0.3 & 0.2 & 0.2 \end{pmatrix}$, $\text{I}_F = \begin{pmatrix} p & q & r & s \\ 0.6 & 0.5 & 0.4 & 0.4 \end{pmatrix}$. Hence, I is $(\text{CIF} - \rho - I)$.

Table 4: I is $(\text{CIF} - \rho - I)$.

¤	p	q	r	s
p	p	p	p	p
q	q	p	q	r
r	r	q	p	r
s	s	r	r	p

Proposition 3.27. Let I be an IFS. Then I is $(\text{IF}-\rho-I)$ if and only if I^c is $(\text{CIF}-\rho-I)$.
Proof: Let I be $(\text{IF}-\rho-I)$. From proof of Proposition (3.18), we obtain $\text{I}_{T^c}(\alpha¤\beta)\leq max\{\text{I}_{T^c}(\alpha), \text{I}_{T^c}(\beta)\}$, $\text{I}_{F^c}(\alpha¤\beta)\geq min\{\text{I}_{F^c}(\alpha),\text{I}_{F^c}(\beta)\}$. Now, $\text{I}_{T^c}(\alpha)=1-\text{I}_T(\alpha)\leq 1-min\{\text{I}_T(\alpha¤\beta), \text{I}_T(\beta)\}= max\{1-\text{I}_T(\alpha¤\beta), 1-\text{I}_T(\beta)\}=max\{\text{I}_{T^c}(\alpha¤\beta), \text{I}_{T^c}(\beta)\}$, $\text{I}_{F^c}(\alpha)=1-\text{I}_F(\alpha)\geq 1-max\{\text{I}_F(\alpha¤\beta), \text{I}_F(\beta)\}=min\{1-\text{I}_F(\alpha), 1-\text{I}_F(\beta)\}=min\{\text{I}_{F^c}(\alpha¤\beta), \text{I}_{F^c}(\beta)\}$. Hence I^c is $(\text{CIF}-\rho-I)$.
Conversely: Let I^c be $(\text{CIF}-\rho-I)$ then from proof of Proposition (3.18), we obtain $\text{I}_T(\alpha¤\beta)\geq min\{\text{I}_T(\alpha), \text{I}_T(\beta)\}$, $\text{I}_F(\alpha¤\beta)\leq max\{\text{I}_F(\alpha),\text{I}_F(\beta)\}$. Now, $\text{I}_T(\alpha)= 1-\text{I}_{T^c}(\alpha)\geq 1-max\{\text{I}_{T^c}(\alpha¤\beta), \text{I}_{T^c}(\beta)\}=min\{1-\text{I}_{T^c}(\alpha¤\beta), 1-\text{I}_{T^c}(\beta)\}=min\{\text{I}_T(\alpha¤\beta), \text{I}_T(\beta)\}$, $\text{I}_F(\alpha)=1-\text{I}_{F^c}(\alpha)\leq 1-min\{\text{I}_{F^c}(\alpha¤\beta), \text{I}_{F^c}(\beta)\}= max\{1-\text{I}_{F^c}(\alpha¤\beta), 1-\text{I}_{F^c}(\beta)\}=max\{\text{I}_F(\alpha¤\beta), \text{I}_F(\beta)\}$. Hence, I is $(\text{IF}-\rho-I)$

Corollary 3.28. Let I^c be is $(\text{CIF}-\rho-I)$. Then,
1- I_t is $(\rho-SA)$.
2- I^c is $(\text{CIF}-\rho-SA)$.
3- I_t is ρ −ideal.

Proof 1: From Proposition (3.27) and Corollary (3.20)-1, we get the proof. We can also get the proof from Proposition (3.27) and Corollary (3.20)-2. Further, we can obtain the proof from Proposition (3.27) and Proposition (3.24).

Corollary 3.29. Let I be $(\text{IF}-\bar{\rho}-I)$ then :
1- I_t is $(\rho-SA)$.
2- I^c is $(\text{CIF}-\rho-SA)$.
Proof 1: From Remarks (2.13)-2 and Proposition (3.10), the proof is got, we can also get the proof from Remarks (2.13)-2 and Proposition (3.18).

Proposition 3.30. Let I be $(\text{IF}-\bar{\rho}-I)$ then I_t is $\bar{\rho}$ −ideal.
Proof: Assume that I is $(\text{IF}-\bar{\rho}-I)$ and $\alpha,\beta\in \text{I}_t$, then $\text{I}_T(\alpha¤\beta)\geq min\{\text{I}_T(\alpha), \text{I}_T(\beta)\}\geq t$, $\text{I}_F(\alpha¤\beta)\leq max\{\text{I}_F(\alpha), \text{I}_F(\beta)\}\leq t$, this implies $\alpha¤\beta\in \text{I}_t$. Now, since I is $(\text{IFS}-\bar{\rho}-I)$, and $\text{I}=\{\prec \alpha, \text{I}_T(\alpha)\geq \text{t}, \text{I}_F(\alpha)\leq \text{t} \succ\mid \alpha\in \mho\}$, we obtain that $\text{I}_T(0)\geq \text{I}_T(\alpha)\geq \text{t}$, $\text{I}_F(0)\leq \text{I}_F(\alpha)\leq t$, thus $0\in \text{I}_t$, hence I_t is $\bar{\rho}$ −ideal.

Definition 3.31. Let $(\mho, ¤, 0)$ be a ρ-algebra and I be an IFS of $\mho$. We say that I is a complete an intuitionistic fuzzy $\bar{\rho}$-ideal of $\mho$ (briefly, $\text{CIF}-\bar{\rho}-I$) If the following conditions hold:
(i) $\text{I}_T(0)\leq \text{I}_T(\alpha)$,
(ii) $\text{I}_F(0)\geq \text{I}_F(\alpha)$,
(iii) $\text{I}_T(\alpha¤\beta)\leq max\{\text{I}_T(\alpha), \text{I}_T(\beta)\}$,
(iv) $\text{I}_F(\alpha¤\beta)\geq min\{\text{I}_F(\alpha),\text{I}_F(\beta)\}$, for any $\alpha,\beta\in\mho$.

Example 3.32. Let $\mho=\{x, y, z, w\}$ and define ¤ on the set $\mho$ as in Table 5. Hence, $(\mho, ¤, x)$ is a ρ-algebra. We define a (IFS) I in $\mho$ as follows:

$I_T = \begin{pmatrix} x & y & z & w \\ 0.1 & 0.2 & 0.2 & 0.2 \end{pmatrix}$, $I_F = \begin{pmatrix} x & y & z & w \\ 0.6 & 0.4 & 0.4 & 0.4 \end{pmatrix}$. Then, I is (CIF $-\bar{\rho} - I$).

Table 5: I is (CIF $-\bar{\rho} - I$).

¤	*x*	*y*	*z*	*w*
x	*x*	*x*	*x*	*x*
y	*w*	*x*	*w*	*w*
z	*z*	*w*	*x*	*z*
w	*y*	*w*	*z*	*x*

Lemma 3.33. If I is (CIF $-\bar{\rho} - I$), then I is(CIF $-\rho - SA$) .
Proposition 3.34. Let I be (CIF $-\bar{\rho} - I$), then $K(\mathrm{I})$ is ($\rho - SA$) .
Proof: Let $\alpha, \beta \in K(\mathrm{I})$, then $\mathrm{I}_T(\alpha) = \mathrm{I}_T(\beta) = \mathrm{I}_T(0)$, $\mathrm{I}_F(\alpha) = \mathrm{I}_F(\beta) = \mathrm{I}_F(0)$. Also, $\mathrm{I}_T(\alpha¤\beta) \leq max\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\} = max\{\mathrm{I}_T(0), \mathrm{I}_T(0)\} = \mathrm{I}_T(0)$, $\mathrm{I}_F(\alpha¤\beta) \geq min\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\} = min\{\mathrm{I}_F(0), \mathrm{I}_F(0)\} = \mathrm{I}_F(0)$, and $\mathrm{I}_T(0) \leq \mathrm{I}_T(\alpha¤\beta)$, $\mathrm{I}_F(0) \geq \mathrm{I}_F(\alpha¤\beta)$ by Lemma (3.16). Thus $\mathrm{I}_T(\alpha¤\beta) = \mathrm{I}_T(0)$,$\mathrm{I}_F(\alpha¤\beta) = \mathrm{I}_F(0)$,and $\alpha¤\beta \in K(\mathrm{I})$, hence $K(\mathrm{I})$ is($\rho - SA$) .

Proposition 3.35. Let I be an IFS then I is (IF $-\bar{\rho} - I$) if and only if I^c is (CIF $-\bar{\rho} - I$).
Proof: Let I be (IF $-\bar{\rho} - I$), we obtain $\mathrm{I}_T(0) \geq \mathrm{I}_T(\alpha)$, $\mathrm{I}_F(0) \leq \mathrm{I}_F(\alpha)$, Thus

$\mathrm{I}_{T^c}(\alpha) = 1 - \mathrm{I}_T(\alpha) \geq 1 - \mathrm{I}_T(0) = \mathrm{I}_{T^c}(0)$

$\mathrm{I}_{T^c}(\alpha) = 1 - \mathrm{I}_F(\alpha) \leq 1 - \mathrm{I}_F(0) = \mathrm{I}_{F^c}(\alpha)$. Now,

$$\begin{aligned}\mathrm{I}_{T^c}(\alpha¤\beta) &= 1 - \mathrm{I}_T(\alpha¤\beta) \leq 1 - min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\} \\ &= max\{1 - \mathrm{I}_T(\alpha), 1 - \mathrm{I}_T(\beta)\} \\ &= max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\},\end{aligned}$$

$$\begin{aligned}\mathrm{I}_{F^c}(\alpha¤\beta) &= 1 - \mathrm{I}_F(\alpha¤\beta) \geq 1 - max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\} \\ &= min\{1 - \mathrm{I}_F(\alpha), 1 - \mathrm{I}_F(\beta)\} \\ &= min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\}.\end{aligned}$$

Hence I^c is (CIF $-\bar{\rho} - I$).
Conversely, let I^c be (CIF $-\bar{\rho} - I$), then $\mathrm{I}_{T^c}(0) \leq \mathrm{I}_{T^c}(\alpha)$, $\mathrm{I}_{F^c}(0) \geq \mathrm{I}_{F^c}(\alpha)$, $\mathrm{I}_T(0) = 1 - \mathrm{I}_{T^c}(0) \geq 1 - \mathrm{I}_{T^c}(\alpha) = \mathrm{I}_T(\alpha)$,
$\mathrm{I}_F(0) = 1 - \mathrm{I}_{F^c}(0) \leq 1 - \mathrm{I}_{F^c}(\alpha) = \mathrm{I}_F(\alpha)$, and from the following $\mathrm{I}_{T^c}(\alpha¤\beta) \leq max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\}$, $\mathrm{I}_{F^c}(\alpha¤\beta) \geq min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\}$, for any $\alpha, \beta \in \mho$. We obtain

$$\begin{aligned}\mathrm{I}_T(\alpha¤\beta) &= 1 - \mathrm{I}_{T^c}(\alpha¤\beta) \geq 1 - max\{\mathrm{I}_{T^c}(\alpha), \mathrm{I}_{T^c}(\beta)\} \\ &= min\{1 - \mathrm{I}_{T^c}(\alpha), 1 - \mathrm{I}_{T^c}(\beta)\} \\ &= min\{\mathrm{I}_T(\alpha), \mathrm{I}_T(\beta)\},\end{aligned}$$

$$\begin{aligned}\mathrm{I}_F(\alpha¤\beta) &= 1 - \mathrm{I}_{F^c}(\alpha¤\beta) \leq 1 - min\{\mathrm{I}_{F^c}(\alpha), \mathrm{I}_{F^c}(\beta)\}, \\ &= max\{1 - \mathrm{I}_{F^c}(\alpha), 1 - \mathrm{I}_{F^c}(\beta)\}, \\ &= max\{\mathrm{I}_F(\alpha), \mathrm{I}_F(\beta)\}.\end{aligned}$$

Hence, I is (IF $-\bar{\rho} - I$).
Corollary 3.36. Let I^c be is (CIF $-\bar{\rho} - I$). Then,
1- I_t is ρ −subalgebra.
2- I^c is (CIF $-\rho - S$)
3- I_t is $\bar{\rho}$ −ideal.

Proof 1: From Proposition (3.35) and Corollary (3.29)-1, we get the proof. We can also get the proof from Proposition (3.35) and Corollary (3.29)-2. Further, we get the proof from Proposition (3.35) and Proposition (3.30).

4. CONCLUSION

This paper investigates and discusses several new ideals in intuitionistic fuzzy ρ −algebra. This study will be helpful in the future if we apply the neutrosophic fuzzy sets theory to consider new conceptions in the neutrosophic fuzzy ρ −algebra.

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