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Orthogonal Generalized Higher k-Derivation on Semi Prime Γ-Rings

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Abstract
The definition of orthogonal generalized higher k-derivation is examined in this paper and we introduced some of its related results.

Key words: Semi prime Γ-Ring, k-derivation, higher k-derivations, orthogonal, generalized higher k-derivation.

تعامد تعميمات المشتقات العليا من النمط k على الحلقات شبه الاولية من النمط Γ

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الخلاصة
في هذا البحث سوف ندرس مفهوم تعامد تعميمات المشتقات العليا من النمط k على الحلقات شبه الاولية من النوع Γ ودراسة بعض الخصائص المتعلقة بها.

Introduction

The definition of Γ-ring was introduced for the first time in [4] and it was circulated in [2]. The definition of prime Γ-ring and semi-prime Γ-ring was introduced in [6].The definition of 2-torsion free ring was introduced in [2]. In 1966 Kandamar introduced k-derivation and Jordan k-derivation on Γ-ring in [3]. The definition of higher k-derivations and Jordan higher k-derivations on Γ-rings presented in [5]. In [1], Ashraf and Jamal defined orthogonal derivations in Γ-ring . Orthogonal higher K-derivation on semiprime Γ-rings introduced in [7]. One of must important result in our study is the following : Let M be 2-torsion free semiprime Γ-ring ,$D = (D_n)_{i\in N}$ and $G = (G_n)_{i\in N}$ generalized higher K-derivation, where $K = (K_i)_{i\in N}$ family of additive mappings on Γ with associated higher K-derivation $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$, respectively, if $D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(y) = G_n(y)K_n(\Gamma)MK_n(\Gamma)G_n(x)$ then $(D_n - G_n)$ *and* $(D_n + G_n)$ *are orthogonal*

1. Orthogonal Generalized K-Derivations on Semi-prime Γ-Rings

In this paper we need the following lemma

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Lemma 1.1 : *[8]*
Let M be a 2-torsion free semi-prime Γ-ring and a, $b \in$ M , then the following conditions are equivalent.
(1) $a\Gamma x\Gamma b$ = 0, for all $x \in$ M.
(2) $b\Gamma x\Gamma a$ = 0, for all $x \in$ M.
(3) $a\Gamma x\Gamma b$ + $b\Gamma x\Gamma a$ = 0, for all $x \in$ M.
If one of the above conditions is fulfilled , then $a\Gamma b$ = $b\Gamma a$ = 0.

Definition 1.2
Two generalized higher K-derivation $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ defined on Γ-ring M, where $K = (K_i)_{i\in N}$ family of additive mappings on Γ , are called orthogonal if for every n$\in N$ and x,y ∈M $D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)MK_n(\Gamma)D_n(x)$, where $D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = \sum_{i=1}^{n} D_i(x)K_i(\Gamma)MK_i(\Gamma)G_i(y)$.

Example 1.3
Let D=$(D_i)_{i\in N}$ and G=$(G_i)_{i\in N}$ be two generalized higher K-derivations on Γ- ring M associated with K-derivations d=$(d_i)_{i\in N}$ and g=$(g_i)_{i\in N}$ on M . Let S=M×M , we define $D_n^{`} = (D_i^{`})_{i\in N}$, $G_n^{`} = (G_i^{`})_{i\in N}$ are generalized higher K-derivations on S as $D_n^{`}(x,y) = (D_n(x), 0)$
$G_n^{`}(x,y) = (0, G_n(y))$
$D_n^{`}(x,y)K_n(\Gamma)(z,w)K_n(\Gamma)G_n^{`}(m,v) = (D_n(x),0)K_n(\Gamma)(z,w)K_n(\Gamma)\big(0,G_n(v)\big) = (0,0)$
$G_n^{`}(m,v)K_n(\Gamma)(z,w)K_n(\Gamma)D_n^{`}(x,y) = \big(0,G_n(v)\big)K_n(\Gamma)(z,w)K_n(\Gamma)(D_n(x),0) = (0,0)$.
Therefore $D_n^{`}$ and $G_n^{`}$ are orthogonal.

Theorem 1.4
Let D=$(D_i)_{i\in N}$ *and* $G = (G_i)_{i\in N}$ be two generalized higher k-derivations with associated higher K-derivations d=$(d_i)_{i\in N}$ and g=$(g_i)_{i\in N}$, respectively where $D_n and G_n$ are commutative , if D_n *and* G_n are orthogonal then the following hold:
1) $D_n(x)k_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)D_n(x)$ *hence* $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$,
2) d_n *and* G_n *are orthogonal higher* $K-$ *derivation and* $d_n(x)K_n(\Gamma)G_n(y) = G_n(y)K_n(\Gamma)d_n(x) = 0$و
3) D_n *and* g_n *are orthogonal higher* $K-$ *derivations* and $g_n(x)K_n(\Gamma)D_n(y) =$,$D_n(y)K_n(\Gamma)g_n(x) = 0$
4) d_n *and* g_n *are orthogonal higher* $K-$ *derivations* ,
5) $d_nG_n = G_nd_n = 0$ *and* $g_nD_n = D_ng_n = 0$,
(6) D_nG_n= G_nD_n*=0.*
1)D_n *and* G_n *are orthogonal then* $D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)MK_n(\Gamma)D_n(x)$.
By lemma 1.1
$D_n(x)k_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)D_n(x)$.
Hence $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$
Proof 2
By (1) $D_n(x)K_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)D_n(x)$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$
Replace x by mβx
$\sum_{i=1}^{n} D_i(m\beta x)K_i(\alpha)G_i(y) = 0$
$\sum_{i=1}^{n} D_i(m)K_i(\beta)d_i(x)K_i(\alpha)G_i(y) = 0$
Replace $D_i(m)$ by $d_i(x)K_i(\alpha)G_i(y)$

$\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y)K_i(\beta)d_i(x)K_i(\alpha)G_i(y)=0$
Replace $K_i(\beta)$ by $K_i(\beta)mK_i(\beta)$
$\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y)\, K_i(\beta)mK_i(\beta)d_i(x)K_i(\alpha)G_i(y)=0$
Since M is semiprime $\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y)=0$
$d_n(x)K_n(\alpha)G_n(y)=0$
G_n is commuting $G_n(\quad)K_n(\Gamma)d_n(x)=0.$
Proof (3)
By (1) $D_n(x)k_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)D_n(x)$
$G_n(y)K_n(\Gamma)D_n(x)=0$
$\sum_{i=1}^{n} G_i(x)K_i(\alpha)D_i(y)=0$
Replace x by mβx
$\sum_{i=1}^{n} G_i(m\beta x)K_i(\alpha)D_i(y)=0$
$\sum_{i=1}^{n} G_i(m)K_i(\beta)g_i(x)K_i(\alpha)D_i(y)=0$
Replace $G_i(m)$ by $g_i(x)K_i(\alpha)D_i(y)$
$\sum_{i=1}^{n} g_i(x)K_i(\alpha)D_i(y)K_i(\beta)g_i(x)K_i(\alpha)D_i(y)=0$
Replace $K_i(\beta)$ by $K_i(\beta)mK_i(\beta)$
$\sum_{i=1}^{n} g_i(x)K_i(\alpha)D_i(y)\, K_i(\beta)mK_i(\beta)g_i(x)K_i(\alpha)D_i(y)=0$
Since M is semiprime $\sum_{i=1}^{n} g_i(x)K_i(\alpha)D_i(y)=0$
$g_n(x)K_n(\alpha)D_n(y)=0$
D_n is commuting $D_n(y)K_n(\Gamma)g_n(x)=0.$
Proof (4)
By (1) $D_n(x)K_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)D_n(x)$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y)=0$
Replace x by mβx and y by wβy
$\sum_{i=1}^{n} D_i(m\beta x)K_i(\alpha)G_i(w\beta y)=0$
$\sum_{i=1}^{n} D_i(m)K_i(\beta)d_i(x)K_i(\alpha)G_i(w)K_i(\beta)g_i(y)=0$
$\sum_{i=1}^{n} d_i(x)K_i(\beta)D_i(m)K_i(\alpha)G_i(w)K_i(\beta)g_i(y)=0$
Replace $D_i(m)$ by $g_i(y)$ and $G_i(w)$ by $d_i(x)$
$\sum_{i=1}^{n} d_i(x)K_i(\beta)g_i(y)K_i(\alpha)d_i(x)K_i(\beta)g_i(y)=0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} d_i(x)K_i(\beta)g_i(y)K_i(\alpha)mK_i(\alpha)d_i(x)K_i(\beta)g_i(y)=0$
Since M semiprime
$\sum_{i=1}^{n} d_i(x)K_i(\beta)g_i(y)=0$
$d_n(x)K_n(\Gamma)g_n(y)=0$
By lemma 1.1

$$d_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(y)=0 \quad \textit{and} \quad g_n(y)K_n(\Gamma)MK_n(\Gamma)d_n(x)=0$$

Hence d_n *and* g_n are orthogonal
Proof (5)
By (2) $d_n(x)K_n(\Gamma)G_n(y)=0$
$\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y)=0$
$\sum_{i=1}^{n} G_i(d_i(x)K_i(\alpha)G_i(y))=0$
$\sum_{i=1}^{n} G_i\big(d_i(x)\big)K_i(\alpha)g_i\big(G_i(y)\big)=0$
$\sum_{i=1}^{n} G_i\big(d_i(x)\big)K_i(\alpha)G_i\big(g_i(y)\big)=0$
Replace $g_i(y)$ by $d_i(x)$
$\sum_{i=1}^{n} G_i\big(d_i(x)\big)K_i(\alpha)G_i\big(d_i(x)\big)=0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} G_i\big(d_i(x)\big)K_i(\alpha)mK_i(\alpha)G_i\big(d_i(x)\big)=0$
Since M is semiprime $\sum_{i=1}^{n} G_i\big(d_i(x)\big)=0$

$G_n d_n = 0$
By (2) $G_n(y)K_n(\Gamma)d_n(x) = 0$
$\sum_{i=1}^{n} G_i(x)K_i(\alpha)d_i(y) = 0$
$\sum_{i=1}^{n} d_i(G_i(x)K_i(\alpha)d_i(y)) = 0$
$\sum_{i=1}^{n} d_i\big(G_i(x)\big)K_i(\alpha)d_i\big(d_i(y)\big) = 0$
Replace $d_i(y)$ by $G_i(x)$
$\sum_{i=1}^{n} d_i\big(G_i(x)\big)K_i(\alpha)d_i\big(G_i(x)\big) = 0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} d_i\big(G_i(x)\big)K_i(\alpha)mK_i(\alpha)d_i\big(G_i(x)\big) = 0$
Since M is semiprime
$\sum_{i=1}^{n} d_i\big(G_i(x)\big) = 0$
$d_n G_n = 0$
By (3) $g_n(x)K_n(\Gamma)D_n(y) = 0$
$\sum_{i=1}^{n} g_i(x)K_i(\alpha)D_i(y) = 0$
$\sum_{i=1}^{n} D_i(g_i(x)K_i(\alpha)D_i(y)) = 0$
$\sum_{i=1}^{n} D_i\big(g_i(x)\big)K_i(\alpha)d_i\big(D_i(y)\big) = 0$
$\sum_{i=1}^{n} D_i\big(g_i(x)\big)K_i(\alpha)D_i\big(d_i(y)\big) = 0.$
Replace $d_i(y)$ by $g_i(x)$
$\sum_{i=1}^{n} D_i\big(g_i(x)\big)K_i(\alpha)D_i\big(g_i(x)\big) = 0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} D_i\big(g_i(x)\big)K_i(\alpha)mK_i(\alpha)D_i\big(g_i(x)\big) = 0$
Since M is semiprime
$\sum_{i=1}^{n} D_i\big(g_i(x)\big) = 0$
$D_n g_n = 0$
By (3) $D_n(x)K_n(\Gamma)g_n(y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)g_i(y) = 0$
$\sum_{i=1}^{n} g_i(D_i(x)K_i(\alpha)g_i(y)) = 0$
$\sum_{i=1}^{n} g_i\big(D_i(x)\big)K_i(\alpha)g_i\big(g_i(y)\big) = 0$
Replace $g_i(y)$ by $D_i(x)$
$\sum_{i=1}^{n} g_i\big(D_i(x)\big)K_i(\alpha)g_i\big(D_i(x)\big) = 0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} g_i\big(D_i(x)\big)K_i(\alpha)mK_i(\alpha)g_i\big(D_i(x)\big) = 0$
Since M is semiprime
$\sum_{i=1}^{n} g_i\big(D_i(x)\big) = 0$
$g_n D_n = 0$
Proof (6)
$D_n(x)K_n(\Gamma)G_n(y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$
$\sum_{i=1}^{n} G_i(D_i(x)K_i(\alpha)G_i(y)) = 0$
$\sum_{i=1}^{n} G_i\big(D_i(x)\big)K_i(\alpha)g_i\big(G_i(y)\big) = 0$
$\sum_{i=1}^{n} G_i\big(D_i(x)\big)K_i(\alpha)G_i\big(g_i(y)\big) = 0$
Replace $g_i(y)$ by $D_i(x)$
$\sum_{i=1}^{n} G_i\big(D_i(x)\big)K_i(\alpha)G_i\big(D_i(x)\big) = 0$
Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$
$\sum_{i=1}^{n} G_i\big(D_i(x)\big)K_i(\alpha)mK_i(\alpha)G_i\big(D_i(x)\big) = 0$
Since M is semiprime

$\sum_{i=1}^{n} G_i(D_i(x)) = 0$
By the same way we get
$G_n D_n = 0,\ and\ D_n G_n = 0$.

Theorem 1.5

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively then* D_n *and* G_n *are orthogonal if and only if for all x,y ∈M*

(1) $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0.$

(2) $d_n(x)K_n(\Gamma)G_n(y) + g_n(y)K_n(\Gamma)D_n(x) = 0.$

Where D_n *and* G_n *are commuting mappings*

Proof:
Suppose $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$
Replace x by xαy
$\sum_{i=1}^{n} D_i(x\alpha y)K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)D_i(x\alpha y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)d_i(y)K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)D_i(x)K_i(\alpha)d_i(y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)d_i(y)K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)d_i(y)K_i(\alpha)D_i(x) = 0$
By lemma 1.1
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)d_i(y)K_i(\alpha)G_i(y) = 0$
$\sum_{i=1}^{n} G_i(y)K_i(\alpha)d_i(y)K_i(\alpha)D_i(x) = 0$
Hence D_n *and* G_n *are orthogonal*
Conversely
Let D_n *and* G_n *are orthogonal*
$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)MK_i(\alpha)G_i(y) = 0$
By lemma 1.1
$D_n(x)K_n(\Gamma)G_n(y) = 0\ \ and\ G_n(y)K_n(\Gamma)D_n(x) = 0$
Hence, $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$
Also $D_n(x)K_n(\Gamma)G_n(y) = 0$
$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$
$\sum_{i=1}^{n} d_i(D_i(x)K_i(\alpha)G_i(y)) = 0$
$\sum_{i=1}^{n} d_i(D_i(x))K_i(\alpha)d_i(G_i(y)) = 0$
$\sum_{i=1}^{n} d_i(D_i(x))K_i(\alpha)G_i(d_i(y)) = 0$
Replace $D_i(x)$ by x and $d_i(y)$ by y
$\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y) = 0$
$d_n(x)K_n(\Gamma)G_n(y) = 0$
And
$G_n(x)K_n(\Gamma)D_n(y) = 0$
$\sum_{i=1}^{n} G_i(x)K_i(\alpha)D_i(y) = 0$
$\sum_{i=1}^{n} g_i(G_i(x)K_i(\alpha)D_i(y)) = 0$
$\sum_{i=1}^{n} g_i(G_i(x))K_i(\alpha)g_i(D_i(y)) = 0$
$\sum_{i=1}^{n} g_i(G_i(x))K_i(\alpha)D_i(g_i(y)) = 0$
Replace $G_i(x)$ by y and $g_i(y)$ by x
$\sum_{i=1}^{n} g_i(y)K_i(\alpha)D_i(x) = 0$
$g_n(y)K_n(\Gamma)D_n(x) = 0$
$d_n(x)K_n(\Gamma)G_n(y) + g_n(y)K_n(\Gamma)D_n(x) = 0.$

Theorem 1.6

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively then* D_n *and* G_n *are orthogonal if and only if for all x,y* ∈*M*

$D_n(x)K_n(\Gamma)G_n(y) = d_n(x)K_n(\Gamma)G_n(y) = 0$

Where D_n *and* G_n *are commutative*

Proof

Suppose $D_n(x)K_n(\Gamma)G_n(y) = 0$

Replace x by xαy

$\sum_{i=1}^{n} D_i(x\alpha y)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)d_i(y)K_i(\alpha)G_i(y) = 0$

Since D_n *and* G_n *are commutative* $\sum_{i=1}^{n} G_i(y)K_i(\alpha)d_i(y)K_i(\alpha)D_i(x) = 0$

Hence D_n *and* G_n *are orthogonal*

Conversely

D_n *and* G_n *are orthogonal*

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$

$D_n(x)K_n(\Gamma)G_n(y) = 0.$ By lemma 1.1

And by using the same way we have

$d_n(x)K_n(\Gamma)G_n(y) = 0$

Theorem 1.7

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively then* D_n *and* G_n *are orthogonal if and only if for all x,y* ∈*M*

$D_n(x)K_n(\Gamma)G_n(y) = 0$ *and* $d_nG_n = d_ng_n = 0$

Proof

Suppose D_n *and* G_n *are orthogonal.*

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$

$D_n(x)K_n(\Gamma)G_n(y) = 0$ By lemma 1.1

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} d_i(D_i(x)K_i(\alpha)G_i(y)) = 0$

$\sum_{i=1}^{n} d_i\big(D_i(x)\big)K_i(\alpha)d_i\big(G_i(y)\big) = 0$

Replace $D_i(x)$ by $G_i(y)$

$\sum_{i=1}^{n} d_i\big(G_i(y)\big)K_i(\alpha)d_i\big(G_i(y)\big) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} d_i\big(G_i(y)\big)K_i(\alpha)mK_i(\alpha)d_i\big(G_i(y)\big) = 0$

Since M is semiprime

$\sum_{i=1}^{n} d_i\big(G_i(y)\big) = 0$

$d_nG_n = 0$

And by theorem 3.1 [7]

$d_ng_n = 0$

Conversely

$D_n(x)K_n(\Gamma)G_n(y) = 0$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)mK_i(\alpha)G_i(y) = 0$

By lemma 1.1

$\sum_{i=1}^{n} G_i(x)K_i(\alpha)mK_i(\alpha)D_i(y) = 0$

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)MK_n(\Gamma)D_n(x) = 0.$
Hence D_n *and* G_n *are orthogonal*

Theorem 1.8

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively if* $D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(y) = G_n(y)K_n(\Gamma)MK_n(\Gamma)G_n(x)$ *then* $(D_n - G_n)$ *and* $(D_n + G_n)$ *are orthogonal*

Proof

$(D_n + G_n)K_n(\Gamma)MK_n(\Gamma)(D_n - G_n)(x) + (D_n - G_n)K_n(\Gamma)MK_n(\Gamma)(D_n + G_n)(x)$
$(D_n(x) + G_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x) - G_n(x)) + (D_n(x) - G_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x) + G_n(x)$
$= (D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) - (D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x)) + (G_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$
$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x)) + (D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) + (D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$
$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) - (G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$
=0

By lemma 1.1
$(D_n + G_n)K_n(\Gamma)MK_n(\Gamma)(D_n - G_n)(x) = 0$
And $(D_n - G_n)K_n(\Gamma)MK_n(\Gamma)(D_n + G_n)(x) = 0.$
Hence $(D_n - G_n)$ *and* $(D_n + G_n)$ *are orthogonal.*

Theorem 1.9

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively if* $D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(y) = g_n(y)K_n(\Gamma)MK_n(\Gamma)g_n(x)$ *then* $(D_n - g_n)$ *and* $(D_n + g_n)$ *are orthogonal.*

Proof

$(D_n + g_n)K_n(\Gamma)MK_n(\Gamma)(D_n - g_n)(x) + (D_n - g_n)K_n(\Gamma)MK_n(\Gamma)(D_n + g_n)(x)$
$=(D_n(x) + g_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x) - g_n(x))$
$+(D_n(x) - g_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x) + g_n(x)$
$= (D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) - (D_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x)) + (g_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$
$-(g_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x)) + (D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) + (D_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))$
$-(g_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x)) - (g_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))$=0

By lemma 1.1
$(D_n + g_n)K_n(\Gamma)MK_n(\Gamma)(D_n - g_n)(x) = 0$
And $(D_n - g_n)K_n(\Gamma)MK_n(\Gamma)(D_n + g_n)(x) = 0.$
Hence $(D_n - g_n)$ *and* $(D_n + g_n)$ *are orthogonal.*

Theorem 1.10

Let M be a 2-torsion free semiprime Γ-ring , $D = (D_n)_{i\in N}$ *and* $G = (G_n)_{i\in N}$ *generalized higher K-derivations with associated higher K-derivation* $d = (d_i)_{i\in N}$ *and* $g = (g_i)_{i\in N}$ *respectively if* $d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(y) = G_n(y)K_n(\Gamma)MK_n(\Gamma)G_n(x)$ *then* $(d_n - G_n)$ *and* $(d_n + G_n)$ *are orthogonal.*

Proof

$(d_n + G_n)K_n(\Gamma)MK_n(\Gamma)(d_n - G_n)(x) + (d_n - G_n)K_n(\Gamma)MK_n(\Gamma)(d_n + G_n)(x)$

$(d_n(x)+G_n(x))K_n(\Gamma)MK_n(\Gamma)(d_n(x)-G_n(x))\ +(d_n(x)-G_n(x))K_n(\Gamma)MK_n(\Gamma)(d_n(x)+G_n(x)=$

$=(d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x))-(d_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))+(G_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x))$

$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))+(d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x))+(d_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$

$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x))-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))=0$

By lemma 1.1

$(d_n+G_n)K_n(\Gamma)MK_n(\Gamma)(d_n-G_n)(x)=0$

And $(d_n-G_n)K_n(\Gamma)MK_n(\Gamma)(d_n+G_n)(x)=0$

Hence $(d_n-G_n)\ and\ (d_n+G_n)\ are\ orthogonal,$

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