DOI: 10.24996/ijs.2023.64.5.41

ISSN: 0067-2904

Construction of Atmospheric Earth Modeling Using C++ Language

Malak. F. Hassan, Waleed I. Yaseen*

Department of Astronomy and Space -College of Science, University of Baghdad, Baghdad, Iraq

Received: 7/3/2022 Accepted: 26/8/2022 Published: 30/5/2023

Abstract

In this model, we use the C++ programming language to develop a program that calculates the atmospheric earth model from the surface to 250 kilometers. The balance forces theory is used to derive the pressure equation. The hydrostatic equation is utilized to calculate these parameters analytically. Variations of the parameters with altitude (density, pressure, temperature, and molecular weight) are investigated intensively. The equations for gravitational acceleration, sound speed, and scale height are also obtained. This model is used to investigate the effects of the earth's atmosphere on the space shuttle and the moving bodies inside it.

Keywords: Atmospheric Earth Modeling, density, pressure, temperature, molecular weight, C++ language,

بناء نموذج غلاف غازي ارضي باستخدام لغة السي بلص بلص

ملاك فلاح حسن, وليد ابراهيم ياسين*

قسم الفلك والفضاء, كلية العلوم, جامعة بغداد, بغداد, العراق

الخلاصة

في هذا النموذج ، استخدمنا لغة البرمجة سي بلص بلص لتطوير برنامج لحساب عناصر الغلاف الجوي الأرضي من السطح الارض إلى 250 كيلومترًا. تستخدم نظرية قوى التوازن لاشتقاق معادلة الضغط و يتم استخدام المعادلة الهيدروستاتيكية لحساب هذه المعلمات بشكل تحليلي. تمت دراسة التغيرات للعناصر مع الارتفاع (الكثافة ، الضغط ، درجة الحرارة ، والوزن الجزيئي) بشكل دقيق. تم أيضًا الحصول على معادلات التعجيل الارضي وسرعة الصوت وارتفاع القياسي. يستخدم هذا النموذج لدراسة تأثيرات الغلاف الجوي للأرض على مكوك الفضاء والأجسام المتحركة بداخله.

1. Introduction

About fifty years ago, some scientists studied the high atmosphere. At the time balloons were used for the first time to determine conditions of atmosphere at altitudes up to 3000 m, it wasn't until the development of rocket-sounding techniques that information about conditions of atmosphere at altitudes very high became available. As it was sent to greater altitudes to explore atmospheric conditions, rockets ushered in a new age in scientific and technological tools. Atmospheric characteristics were previously studied at heights greater than 100 kilometers, and when rocket-sounding began, rockets rose to altitudes greater than 200

*Email: waleed.yaseen@sc.uobaghdad.edu.iq

kilometers, and atmosphere qualities were solely estimated by extrapolation from data collected at lower altitudes [1]. Density, temperature, and pressure are the qualities of the atmosphere, with molecular weight having a considerable impact. The gases N_2 (78%) and O_2 (21%) dominate the Earth's atmosphere, with the remaining gases accounting for less than 1% of the total volume of the atmosphere. While nitrogen is an essential component for all life on Earth [2] this abundant N_2 generates the majority of the Earth's surface pressure, which is vital for liquid water stability.

The atmosphere is unstable due to changes in solar radiation, and its temperature, pressure, chemical constituent particle presence, and electrical characteristics are all continually changing. Altitude variations were converted from differences in air properties at a particular altitude [3].

Knowledge of the vertical distribution of such quantities as pressure, temperature, density, and sound speed is necessary for pressure altimeter calibrations, aircraft and rocket performance and their purposes design, and so on. Because the real environment never stays the same at any given time or location, a hypothetical model must be used to approximate what can be expected. The standard atmosphere is the name given to this model. The model assumes that the air is free of dust, humidity, and water vapor and that it is at rest with regard to the Earth (i.e., no winds or turbulence) [4].

An atmosphere model is built in this work, which calculates variations in “density, pressure, molecular weight, and temperature” as altitude changes from 0- 250 km.

.

2. Method of Model

2.1 Law of Ideal Gas

The gases that make up an earth's atmosphere can be considered ideal gases, which mean they follow the ideal gas law. The equation for the state of a hypothetical ideal gas is known as the ideal gas law. It's a good approximation of various gases' behavior under a variety of situations. The ideal gas law is commonly expressed as [5]:

$$pV = nR^{*}T \tag{1}$$

where P is the gas's absolute pressure (Pa), V is the gas's volume (m^3), n is the gas's quantity (kmol), T is the gas's absolute temperature (K), and R* is the universal gas constant.

$$p = \frac{\rho R^{*}T}{M} \tag{2}$$

or:

$$p = \frac{NR^{*}T}{N_A} \tag{3}$$

$$p = NKT \tag{4}$$

from Eq. (2) and (3):

$$\rho = \frac{NM}{N_A} \tag{5}$$

Where density of the gas: ρ (kg/m^3), means molecular weight: M (kg/kmol), total number density: N (m^{-3}), Avogadro constant: N_A ($kmol^{-1}$), and Boltzmann constant: k (J/K).

When M is constant, the equations containing R* and M can be substituted for the specific gas constant R, where R = R*/M [5].

2.2 Hydrostatic Equation

To calculate the standard pressure p at a particular height, the temperature is assumed to be constant and the air is assumed to be a perfect gas. Pressure altitude refers to the altitude obtained by measuring the pressure (PA). Pressure fluctuations can be calculated using the hydrostatic equation, perfect gas law, and temperature lapse rate equation. Consider a vertical column of air with a cross-sectional area of A=1, as illustrated in Figure 1. The mass (dm) of air in the column between (z) and (z+dz) is:

$$\rho(z) = \frac{m}{v} = \frac{dm}{Adz}$$

$$dm = \rho(z)Adz \quad (6)$$

Where ρ (z) is the density of air at height (z) as illustrated in Figure 1.
The force acting on this column as a result of the weight of the air area is:

$$fd_G = dm\, g(z) = g(z)\rho(z)Adz \quad (7)$$

Where g (z) is the acceleration due to gravity at height (z). The changes in pressures (p→Δp) leads to higher pressure:

$$\Delta f_p = A\, p\, \Delta p \rightarrow df_p = A[p - (p - dp)]$$

$$df_p = -A\, dp \quad (8)$$

(-dp) is a positive quantity and the balance of forces:

$$df_G = df_p$$

$$\frac{dp}{dz} = -g(z)\,\rho(z) \quad (9)$$

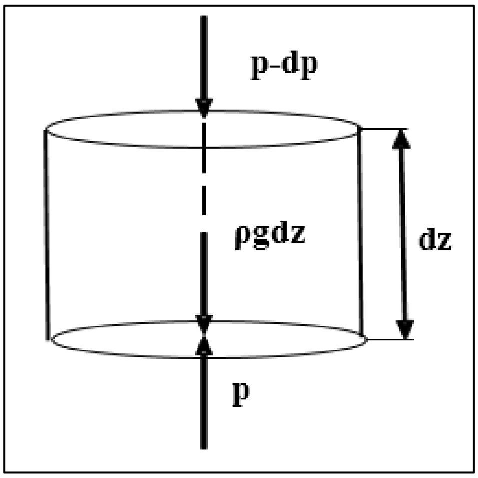

Figure 1: Atmospheric model element

Where g is the acceleration of gravity-dependent on height. The law of gravitation provides an expression for gas as a function of altitude [6]:

$$g = g_0\left(\frac{RE}{RE + z}\right)^2 \quad (10)$$

Where can symbolize the acceleration of gravity at sea level by g_o (m/s2) and also the effective radius of Earth at a specific latitude is denoted by RE.
Equations (2) and (9) are used and the hydrostatic equation can be written as [6]:

$$dp = -\left(\frac{pM}{R^{*}T}\right) gdz \quad (11)$$

Integrating equation (11) from z_1 to z_2 (1, 2) represents integration conditions:

$$p_2 = p_1 e^{\left[\frac{-gM}{R^{*}T}(z_2 - z_1)\right]} \quad (12)$$

Scale height H is defined as:

$$H = \frac{R^{*}T}{Mg} \quad (13)$$

Eq. (12) becomes:

$$p_2 = p_1 e^{\left[-\frac{(z_2 - z_1)}{H}\right]} \quad (14)$$

2.3 Geopotential Height

The transformation of geometric height z to a height of geopotential h has long been used in standard atmosphere calculations to successfully ignore the changing portion of gravity's acceleration, thereby simplifying both the equations of hydrostatic integration and the resulting expression for pressure computing. The idea of gravity governs the relationship between geometric and geopotential altitudes. The geopotential altitude h is calculated as follows:

$$h = \frac{1}{g_o} \int_0^z gdz \quad (15)$$

g_o is set equal to 9.80665 $m^2/(s^2 m')$ and the differential is expressed as:

$$g_o dh = gdz \quad (16)$$

$$h = \frac{R_e z}{R_E + z} \quad (17)$$

The link between geopotential height h and geometric height z is described as:

$$z = \frac{R_E h}{R_E - h} \quad (18)$$

2.4 Molecular-Scale Temperature

The molecular-scale temperature TM is the product of the kinetic temperature T and the ratio M/Mo at a position, where at that point the molecular weight of air is symbolized as M and the sea-level value of molecular weight of air is symbolized as Mo, therefore:

$$T_M = \frac{TM_o}{M} \quad (19)$$

$$H = \frac{RT_M}{M_o g_o} \quad (20)$$

$$p_2 = p_1 e^{\left[-\frac{(h_2 - h_1)}{H}\right]} \quad (21)$$

3. Development of Analytic Atmosphere Model

Our aim is to create an atmosphere model that examines density, pressure, temperature, and molecular weight fluctuations as altitude starts from sea level to 250 kilometers. The building of an analytic model of the atmosphere begins with the derivation of the hydrostatic Eq. (4), which functions in pressure (p) and geometric altitude (h). For the integration of this equation [6], the density and gravitational constant (g) are necessary.

3.1 Molecular Weight

In the region of the atmosphere below the Tropopause where the effect of diffusion and photochemical processes on mean molecular weight is insignificant. The fractional composition of each species is considered to be constant in this region, M= Mo.

3.2 Temperature

The temperature has been defined as a function of height in standard atmospheres. The function TM against h, and is written as a set of linear equations, with the usual form being [6]:

$$T_M = T_{M,b} + L_{M,b}(h - h_b) \qquad (22)$$

The molecular-scale temperature gradient, often known as the lapse rate, is denoted by L_M. Each consecutive layer is represented by the value of subscript b.

3.3 Pressure

Pressure is computed using two different equations within the homogeneity region, where pressure is a function of geopotential altitude. Where there are two cases: when $L_{M,b}$ for a specific layer is not equal to zero, and the other is for when $L_{M,b}$ is equal to zero as in the equations [6]:

$$p = p_b \left[\frac{T_{M,b}}{T_{M,b} L_{M,b}(h - h_b)} \right]^{\frac{g_o M_o}{R^* L_{M,b}}} \qquad (23)$$

and,

$$p = p_b e^{\left[\frac{-g_o M_o (h - h_b)}{R^* T_{M,b}}\right]} \qquad (24)$$

3.4 Mass Density

The ideal gas law can be used to calculate mass density after temperature and pressure have been calculated [7]:

$$\rho = \frac{p M_o}{R T_M} \qquad (25)$$

3.5 Speed of Sound

The speed of sound is symbolized as C and it's calculated using the equation [8]:

$$C = \left(\frac{\gamma R T_M}{M_o} \right)^{1/2} \qquad (26)$$

4. Result and Discussion:

The Atmosphere model is a perfect and stable simulation of average annual conditions of atmospheric Earth from the surface to 250 km at latitude 45^o North during a period of moderate solar activity for example the solar cycle 23. The temperature and pressure at sea-level, as well as a temperature-height profile to 250 km, are the defining meteorological components. The air is considered to be dry, and the atmosphere is assumed to be homogeneously mixed with a relative-volume composition that results in a constant mean molecular weight at heights below 86 km.

For the purposes of computation, the following constants are used, as indicated in Table 1. The atmosphere model uses the equations for altitudes below 86 km. Table 2 shows the values for h_b and $L_{M,b}$ (2). Gases that don't appear to contribute more than about 0.5 percent of the total composition at any point within this height range, or that don't behave consistently for various reasons, were left out on design. The gases N_2, O, O_2, Ar, and He are among those that fit these requirements. At altitudes of 250 kilometers and higher, atomic hydrogen is taken into account, but not at 86 kilometers because atomic hydrogen is dominant and more present than other gases in these altitudes. Figure 2 illustrates the flowchart of atmospheric model.

Table 1: International standard atmosphere, mean sea level conditions [9].

R*	8314.32 J/kg-K
k	1.380622×10-23 J/K
NA	6.022169×10-26 kmol-1
go	9.80665 m/s2
go'	9.80665 (m/s)2/m'
ro	6356.766 km
Mo	28.9644 kg/kmol
Po	101325 Pa
To	288.15 K
T∞	1000.0 K
γ	1.400

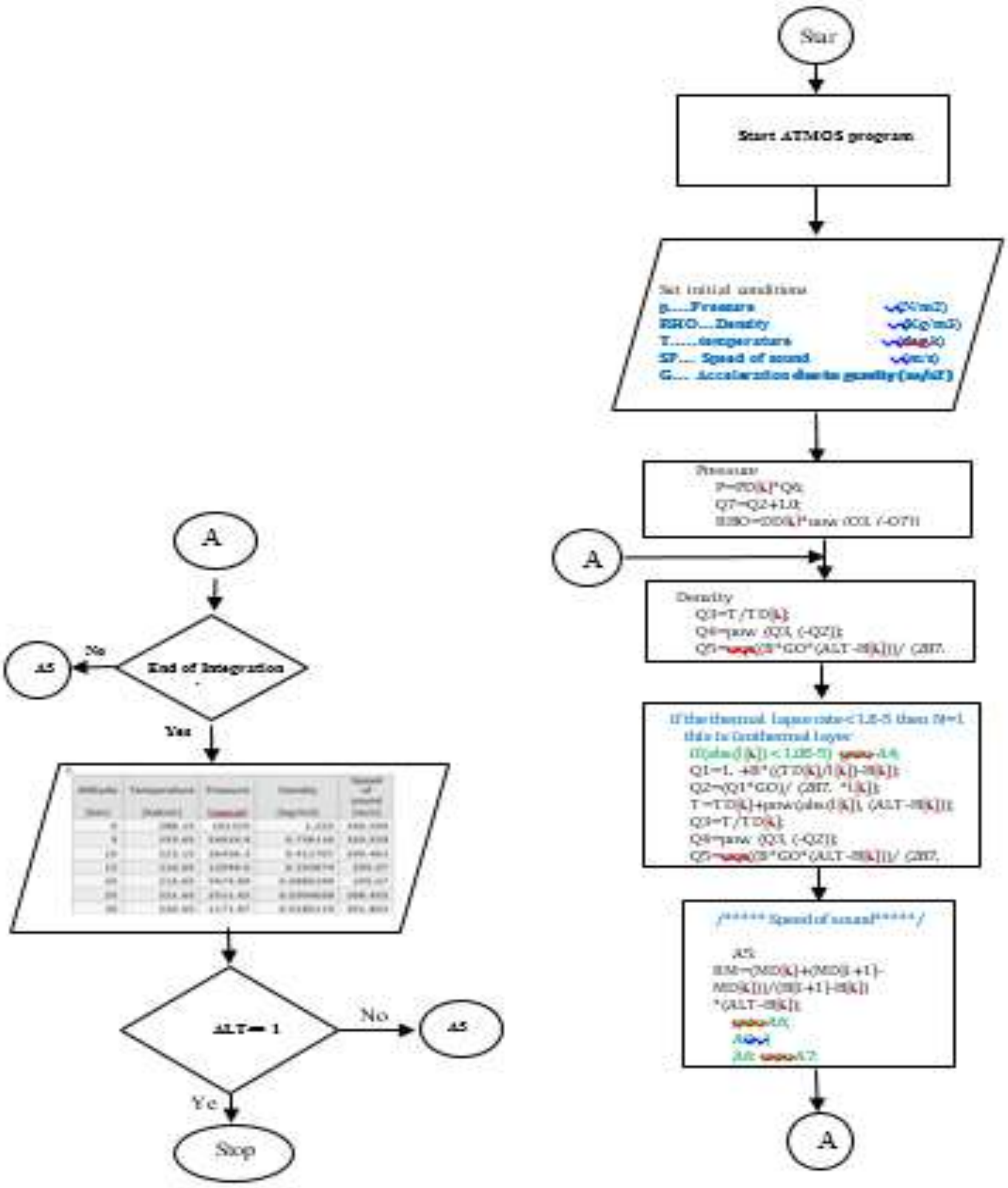

Figure 2: Flowchart of atmospheric model

Table 2: The reference levels and gradients of the linearly [9] .

Subscript b	Geopotential height, h_b (km')	Molecular-scale temperature gradient, $L_{M,b}$ (K/km')
0	0	−6.5
1	11	0.0
2	20	+1.0
3	32	+2.8
4	47	0.0
5	51	−2.8
6	71	−2.0
7	86	
8	91	
9	120	12
10	500	

The equations for calculating atmospheric parameters from 0 to 250 km are calculated by C++ language in this paper. The density and pressure depend on the altitude exponentially [10]. Figure 3 illustrates code of density and Figure 4 illustrates the density exponential change with height, where density decreases as altitude increases due to lower air molecular density. The atmospheric influences are considered below 30 km altitude, although the air molecular is relatively low above 30 km height [9].

```
p...............................Pressure                    (N/m2)
RHO.............................Density              (Kg/m3) T..............................Temperature
(deg k)
SP..............................Speed of sound          (m/s) G..............................Accelaration
due to gravity (m/s2)
/*********************************************************/
Density
Q3=T/TD[i];
Q4=pow (Q3, (-Q2));
Q5=exp((B*GO*(ALT-H[i]))/ (287. *l[i]));
Q6=Q4*Q5;
Pressure
P=PD[i]*Q6;
Q7=Q2+1.0;
RHO=DD[i]*pow (Q3, (-Q7)) *Q5;
goto A5;
```

Figure 3: Code of density variable with altitude.

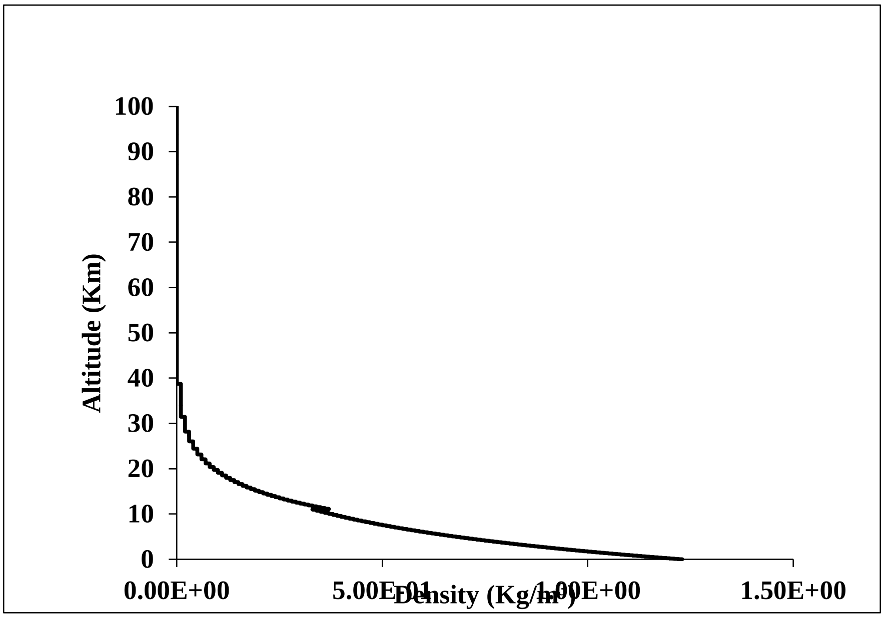

Figure 4: Density variable with altitude

Code of pressure is shown in Figure 5. The relationship between static pressure and altitude is depicted in Figure 6. Equation 4 illustrates pressure as a function of density, where static pressure decreases as altitude increases due to the reduction in air molecules at high altitude. The static pressure value at 10 km altitude is around half that at sea level.

```
/****Calculation of absolute pressure values****/
 PD[i]=PDC[i]*PO;
   DD[i]=PD[i]/(TD[i]*R); }
 Calculation of thermal lapse rate L (deg k/m)
 for (i=1; i<16; i++) {
 l[i]=(TD[i+1]-TD[i])/(H[i+1]-H[i]); }
 goto A1;
 A7: goto A2;
 A1: for (i=0; i<17; i++) {
 Maximum Altitude
 if(ALT>H[i+1]) goto A3;
if the thermal lapse rate<1.E-5 then N=1
this is isothermal layer
 if(abs(l[i]) <1.0E-5) goto A4;
 Q1=1. +B*((TD[i]/l[i])-H[i]);
 Q2=(Q1*GO)/ (287. *l[i]);
 T=TD[i]+pow(abs(l[i]), (ALT-H[i]));
 Q3=T/TD[i];
 Q4=pow (Q3, (-Q2));
 Q5=exp((B*GO*(ALT-H[i]))/ (287. *l[i]));
 Q6=Q4*Q5;
 Pressure
 P=PD[i]*Q6;
 Q7=Q2+1.0;
```

Figure 5: Code of pressure variable with altitude

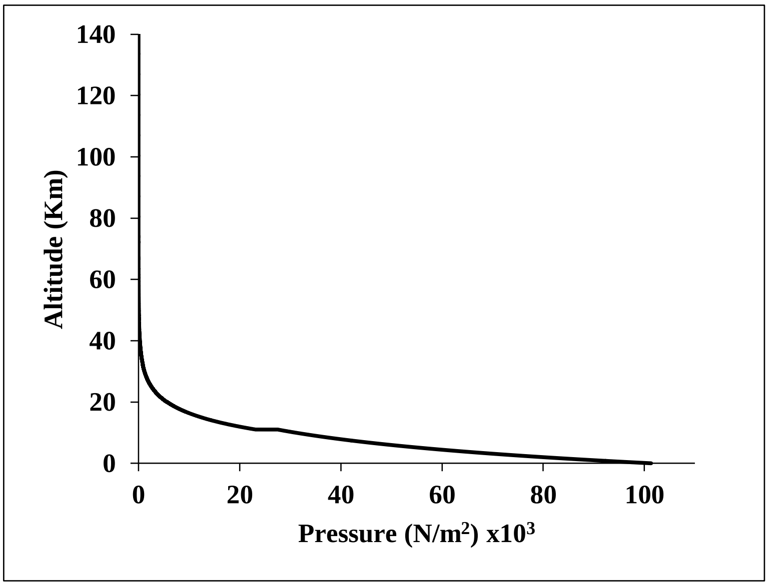

Figure 6: Pressure variable with altitude

On a typical day at sea level, the temperature ratio is 288.15k (15C). The temperature remains constant with altitude until it reaches the tropopause at 10 km, at which point it drops to about 200 K. Because the Earth's surface receives energy from the Sun on a daily basis, the temperature in the free troposphere drops as altitude increases. At the same time, the top of the troposphere radiates energy upward continually, cooling the upper troposphere. Because the troposphere's capacity to absorb solar energy is limited, it must rely on radiative, turbulent, and conductive energy transfers from the ground to keep its temperature stable [11]. At 150 km, the temperature rises by around 1375 degrees Celsius. The code of temperature is illustrated in Figure 7. The temperature varied with altitude is depicted in Figure 8.

```
/*****Temperature*****/

if the thermal lapse rate<1.E-5 then N=1
this is isothermal layer
 if(abs(l[i]) <1.0E-5) goto A4;
 Q1=1. +B*((TD[i]/l[i])-H[i]);
 Q2=(Q1*GO)/ (287. *l[i]);
 T=TD[i]+pow(abs(l[i]), (ALT-H[i]));
 Q3=T/TD[i];
 Q4=pow (Q3, (-Q2));
 Q5=exp((B*GO*(ALT-H[i]))/ (287. *l[i]));
 Q6=Q4*Q5;
```

Figure 7: Code of temperature variable with altitude

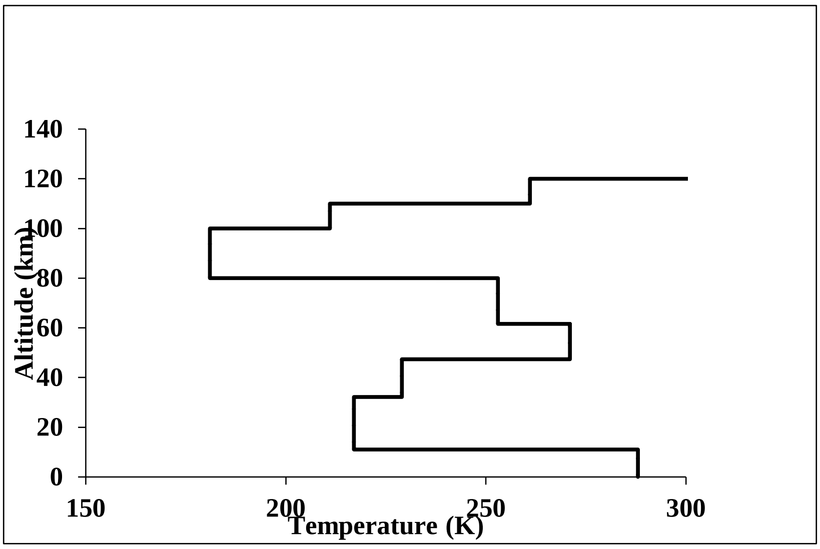

Figure 8: Temperature variable with altitude

The reason for the inversion in the stratosphere is due to the ozone for absorption of ultraviolet solar energy. Although maximum occurs at 25km, the lower air density at 50km allows solar energy to heat up temperature there to a much greater degree, agrees with [12]. Figure 9 illustrates the code of speed of sound and Figure 10 shows the variation speed of sound with altitude. From this figure it can be noted that the speed of sound is a function of temperature. Therefore, the speed of sound increases slowly with increasing altitude, but at a high altitude, the increase becomes very high. This result agrees with [13][14].

```
/***** Speed of sound*****/

   A5: RM=(MD[i]+(MD[i+1]-MD[i]))/(H[i+1]-H[i])
*(ALT-H[i]);
   goto A6;
   A3:;}
   A6: goto A7;
   A2: G=GO/ ((1. +ALT/RO) *(1. +ALT/RO));
   SP1=sqrt (1.4*8134. /(RM)*T);
```

Figure 9: Code of temperature variable with altitude

Figure 11 illustrates the gravitational acceleration code. Figure 12 illustrates the variation of gravitational acceleration with altitude and shows that it decreases linearly with increasing altitude. The parameters of atmospheric earth model are illustrated in Table 3.

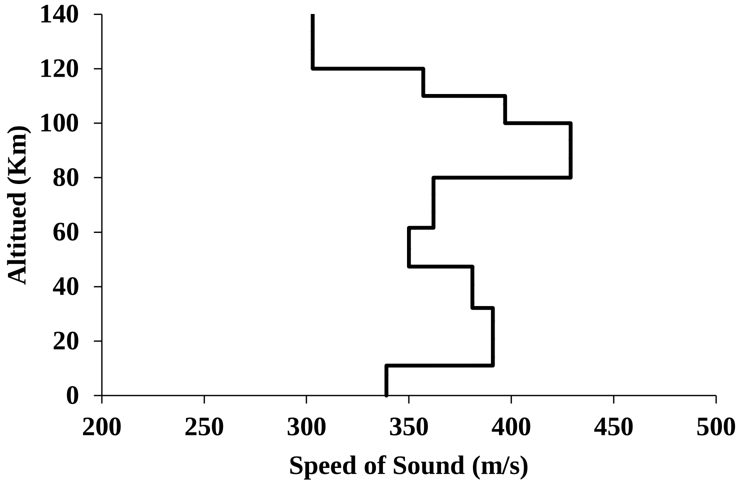

Figure 10: Speed of sound variable with altitude

```
/***** Acceleration due to gravity *****/

goto A1;
A7: goto A2;
A1: for (i=0; i<17; i++) {
Maximum Altitude
if(ALT>H[i+1]) goto A3;
Acceleration due to gravity
A2: G=GO/ ((1. +ALT/RO) *(1. +ALT/RO));
```

Figure 11: Code of temperature variable with altitude

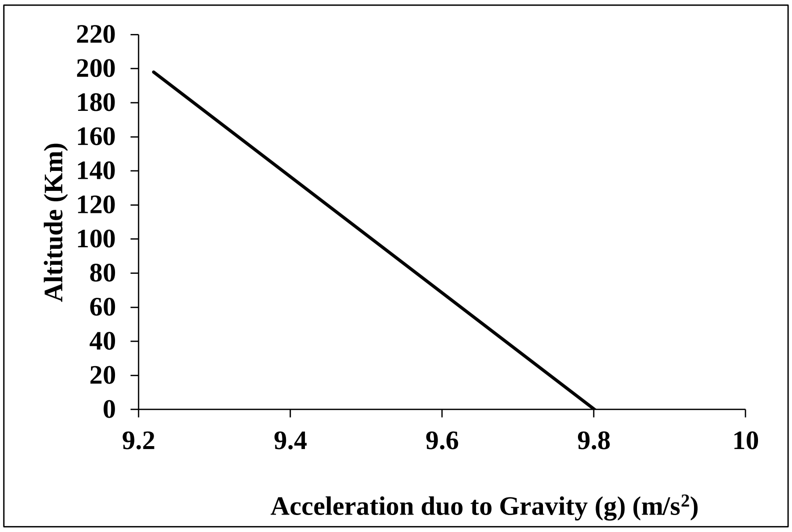

Figure 12: Acceleration due to gravity variable with altitude

Table 3: Parameters of atmospheric earth model

Altitude	Temperature	Pressure	Density	Speed of sound
[km]	[Kelvin]	[pascal]	[kg/m3]	[m/s]
0	288.15	101325	1.225	340.294
5	255.65	54019.9	0.736116	320.529
10	223.15	26436.3	0.412707	299.463
15	216.65	12044.6	0.193674	295.07
20	216.65	5474.89	0.0880349	295.07
25	221.65	2511.02	0.0394658	298.455
30	226.65	1171.87	0.0180119	301.803
35	237.05	558.924	0.00821392	308.649
40	251.05	277.522	0.00385101	317.633
45	265.05	143.135	0.00188129	326.369
50	270.65	75.9448	0.000977525	329.799
55	259.45	39.97	0.000536684	322.903
60	245.45	20.3143	0.000288321	314.07
65	231.45	9.92203	0.000149342	304.982
70	217.45	4.63422	0.000074243	295.614
75	206.65	2.06792	3.48607E-05	288.179
80	196.65	0.88628	1.57005E-05	281.12
85	186.946	0.36342	6.77222E-06	274.096

These predictions are at variance with the experimental data, the reason for which is usually attributed to several extraneous effects, e.g. non-uniform composition of air, presence of the sun, conduction of heat away from the earth's surface, the occurrence of vapor, dust, ozone, wind, etc.

5. Conclusion

The program in C++ language was successfully used to build a model for calculating the earth's atmosphere parameters. When heights were below 86 km, the atmosphere model applied specific conditions dependent on values of b or the type of atmosphere layers. The layer type affected the temperature and speed of sound. At altitudes below 40 kilometers, the pressure and density of molecules increased substantially. The presumed standard atmosphere did not differ from the actual atmosphere, and the former is useful for aircraft and missile designs, ballistic trajectories, and other uses by pressure altimeter calibrations.

References

[1] E. Charles, *Flight Theory and Aerodynamics, New York: John Wiley and Sons*, 1960, pp. 1518.
[2] S. L. Olson, E. W. Schwieterman, C. T. Reinhard and a. T. W. Lyons, *Earth: Atmospheric Evolution of a Habitable Planet, California: University of California*, 2018, pp. 1-2.
[3] F. J.R., Re-entry Vehicle Dynamics, American Institute of Aeronautics, 1984, pp. 2-23.
[4] T. T.A., *Introduction to the Aerodynamics of Flight, Washington: National Aeronautics and Space Administration*, 1975, pp. 6-9.
[5] M. R. A., *The 1976 Standard Atmosphere Above 86-km Altitude, Goddard Space Flight Center, NASA*, 1976.
[6] Thomson, L. Milne and a. C.B., *Theoretical Aerodynamics, New York: Dover publications*, 2006, pp. 11-15.

[7] Haag and a. I. M. U. de, *The Atmosphere, Arizona: Arizona State University*, 2017.
[8] Sohrab, K. A. Krout and a. S. H., "On the Speed of Sound," *International Journal of Thermodynamics* (IJoT), vol. 19, no. 1, pp. 29-34, 2016.
[9] W. I. Y., Y. C.B. and a. M. N., "Calculation of The Parameters for Atmospheric Model for The Earth," *Journal of Al-Nahrain University*, vol. 12, no. 3, pp. 64-69, 2009.
[10] Agrawal, V. J. Menon and a. D. C, "Density, pressure and temperature in the earth's atmosphere," *Indian Journal of Pure & Applied Physics*, vol. 28, pp: 49-50, 1990
[11] Jacobson and a. M. Z., *Fundamentals of Atmospheric Modeling, Cambridge University Press*, 2005, pp. 24-25.
[12] Y. Y. Jin, *Composition and Structure of the Atmospherethe Atmosphere, Kalinga University*, 2016.
[13] G. K. Vladimir, "Speed of Sound in Atmosphere of the Earth," *Journal of Acoustics*, vol. 2, pp. 80-85, 2012.
[14] Airbus, *Getting to Grips with Aircraft performance, Airbus Industrie, Customer Services, Blagnac*, 2000, pp. 11-16.